Particle in 1-D Square-Well Potential: Normalization Constant

Wavefunction

$$\psi(x) = A\sin kx = A\sin\frac{n\pi}{L}x$$

Normalization

$$\int_0^L \psi^*(x)\cdot\psi(x)\cdot dx = A^2\int_0^L \sin^2\frac{n\pi}{L}x\cdot dx = 1$$

$$A = \sqrt{\frac{2}{L}} \qquad \psi(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x$$

Expectation values: Particle in 1-D Square-Well Potential

$$\langle x \rangle = \int \psi^* \cdot x \cdot \psi \cdot dx$$

$$= \int_0^L \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x \cdot x \cdot \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x \cdot dx$$

$$= \frac{2}{L} \int_0^L x \cdot \sin^2 \frac{n\pi}{L} x \cdot dx$$

$$= \frac{L}{2}$$

Expectation values: Particle in 1-D Square-Well Potential

$$\langle p_x \rangle = \int \psi^* \cdot \left(-i\hbar \frac{\partial}{\partial x} \right) \cdot \psi \cdot dx$$

$$= -i\hbar \int_0^L \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x \cdot \frac{\partial}{\partial x} \cdot \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x \cdot dx$$

$$= \frac{-2i\hbar n\pi}{L^2} \int_0^L \sin \frac{n\pi}{L} x \cdot \cos \frac{n\pi}{L} x \cdot dx$$

$$= 0$$

We need to find the most probable value of r, in the ground state of the hydrogen atom, the wavefunction of the ground state is given by:

$$\psi_{100} = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

the probability is therefore:

$$\begin{aligned} P &= |\psi|^2 4\pi r^2 dr \\ &= \frac{4}{a^3} e^{-2r/a} r^2 dr \\ &= p(r) dr \end{aligned}$$

where:

$$p(r) = \frac{4}{a^3} r^2 e^{-2r/a}$$

to find the most probable value of r, we take the derivative of this equation with respect to r, then we set it equal to zero, and finally solve for r to get the most probable value as:

$$\begin{aligned} \frac{dp}{dr} &= \frac{4}{a^3} \left(2re^{-2r/a} + r^2 \left[-\frac{2}{a} e^{-2r/a} \right] \right) \\ &= \frac{8r}{a^3} e^{-2r/a} \left(1 - \frac{r}{a} \right) \end{aligned}$$

$$\frac{dp}{dr} = 0$$

thus:

$$1 - \frac{r}{a} = 0$$

$$\boxed{r = a}$$