Duke Statistical Science

Hierarchical modeling of annual rainfall data with spatial covariates

STA 702 Fa24 Course Project

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November 26, 2024

Data Set

- Y_{ij}; $i = 1901, \ldots, 2022$ and $j = 1, \ldots, 357$. Annual rainfall at j-th location in i-th year.

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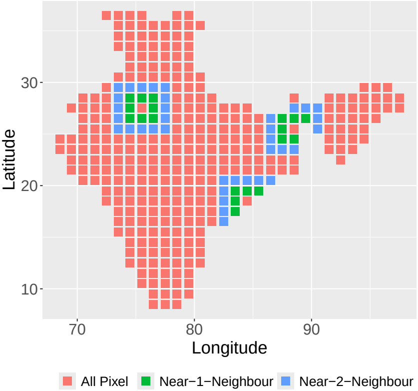

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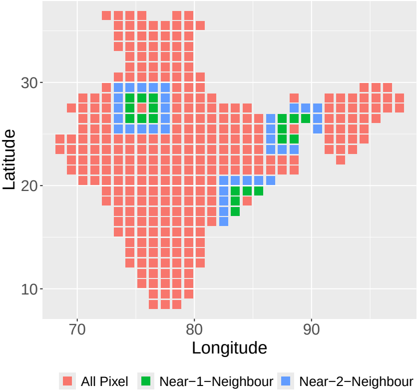
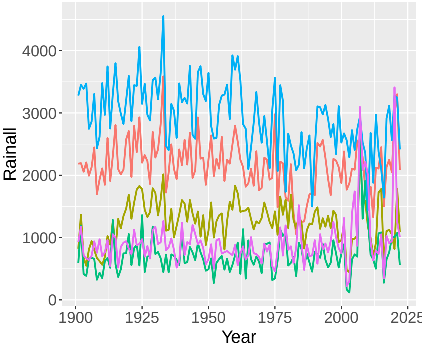
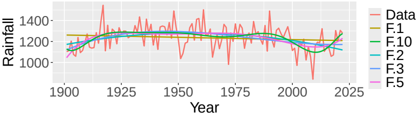




- Hard to model as time series.

Initial Models

- Simple Linear Regression



F.i is the fitted regression line with time variable taken up to i−th order

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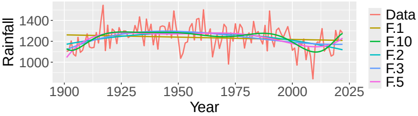

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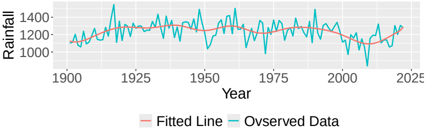
Basis Spline Regression



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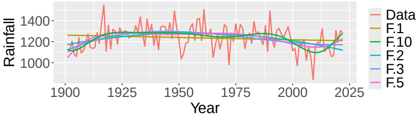

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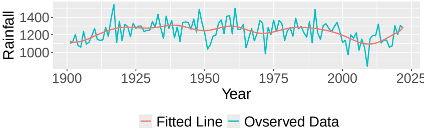
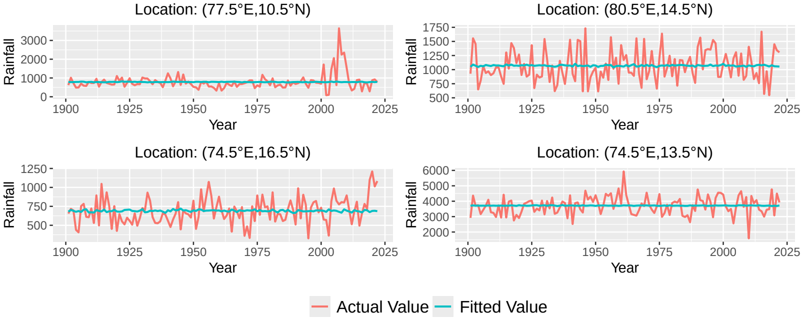
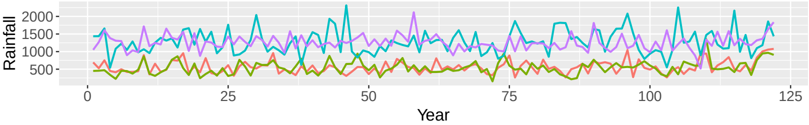


Gaussian Process



EDA



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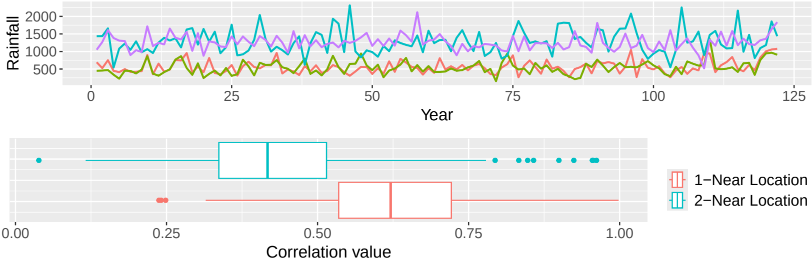
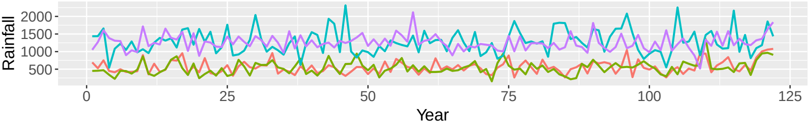

Rainfall
2000
1500
1000
500
0
25
50
75
100
125
Year
0.00
0.25
0.50
0.75
1.00
Correlation value
1−Near Location
2−Near Location

EDA



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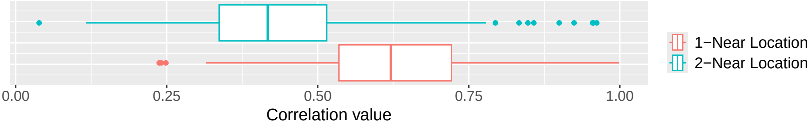

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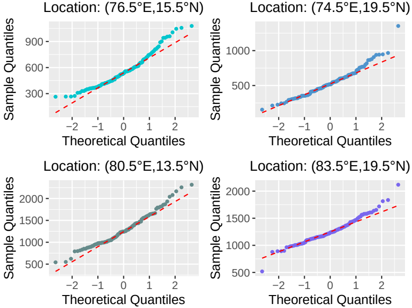
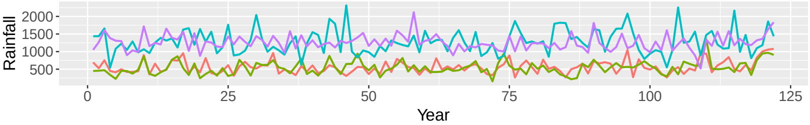


Location: (76.5°E,15.5°N)
Location: (74.5°E,19.5°N)
Location: (80.5°E,13.5°N)
Location: (83.5°E,19.5°N)
Sample Quantiles
Theoretical Quantiles

EDA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1500
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0
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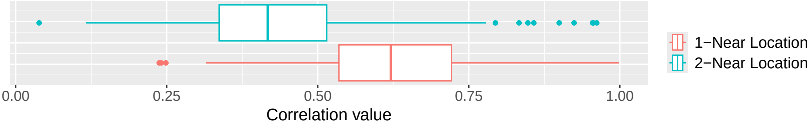
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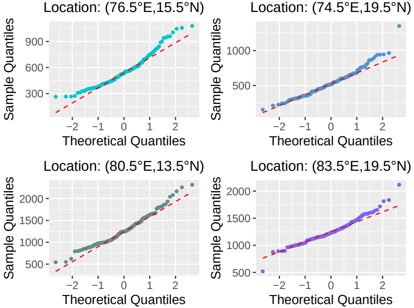
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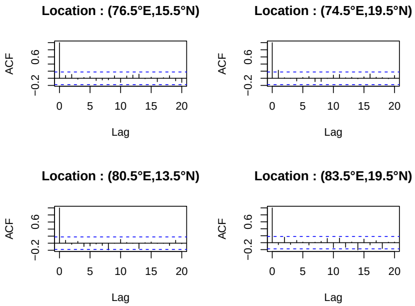
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ACF
Lag

Model Description

- **Model:**

For $i = 1, \ldots, T$, and $j = 1, \ldots, S$,

$$Y_{ij} = \beta_{j1}\,\bar{Y}_{ij,1} + \beta_{j2}\,\bar{Y}_{ij,2} + \beta_{j3}\,\bar{Y}_{i,1} + \beta_{j4}\,\bar{Y}_{i,2} + \epsilon_{ij}.$$

$$\Rightarrow Y_{ij} = \boldsymbol{x}_j^T \boldsymbol{\beta}_j + \epsilon_{ij}, \text{ where } \epsilon_{ij} \overset{\text{iid}}{\sim} N(0, \sigma^2).$$

Here $\boldsymbol{x}_j^T = [\bar{Y}_{ij,1}, \bar{Y}_{ij,2}, \bar{Y}_{i,1}, \bar{Y}_{i,2}]'$ and $\boldsymbol{\beta}_j = [\beta_{j1}, \ldots, \beta_{j4}]'$.

$$\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_S \overset{\text{iid}}{\sim} N_4(\boldsymbol{\theta}, \Sigma)$$

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- **Description:**

  $\bar{Y}_{ij,1}$: Mean of Near-1-Neighbor

  $\bar{Y}_{ij,2}$: Mean of Near-2-Neighbor

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- **Prior:**

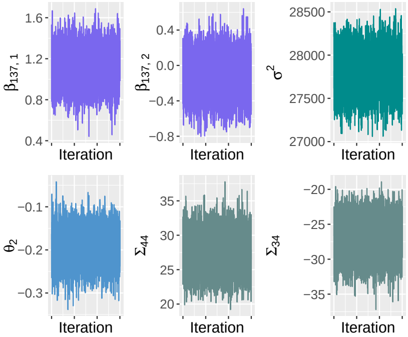
$$\boldsymbol{\theta} \sim N(\boldsymbol{\mu}_0, \boldsymbol{\Lambda}_0)$$

$$\Sigma \sim \text{Inv-Wish}(\eta_0, \boldsymbol{S}_0)$$

$$\sigma^2 \sim \text{Inv-Gam}\left(\frac{\nu_0}{2}, \frac{\nu_0 \sigma_0^2}{2}\right)$$

MCMC Diagnostistics

- Traceplot:



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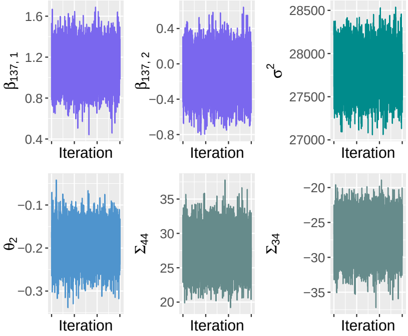
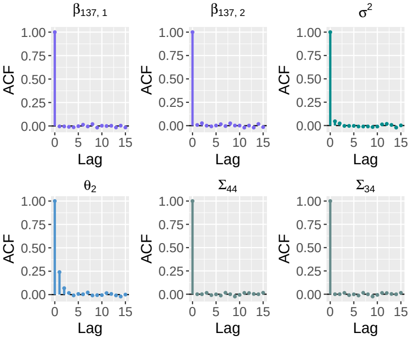


- ACF Plot:



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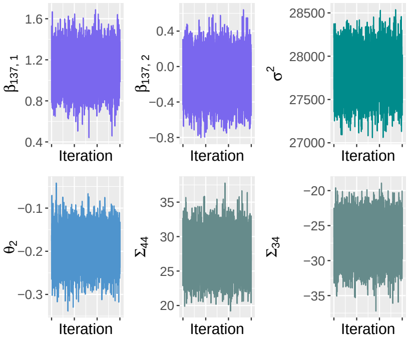

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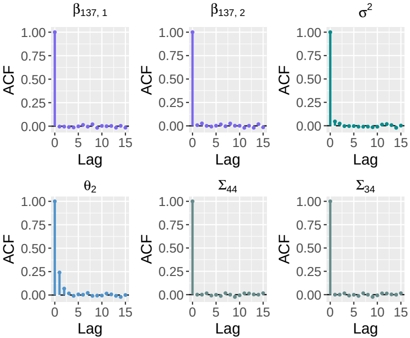


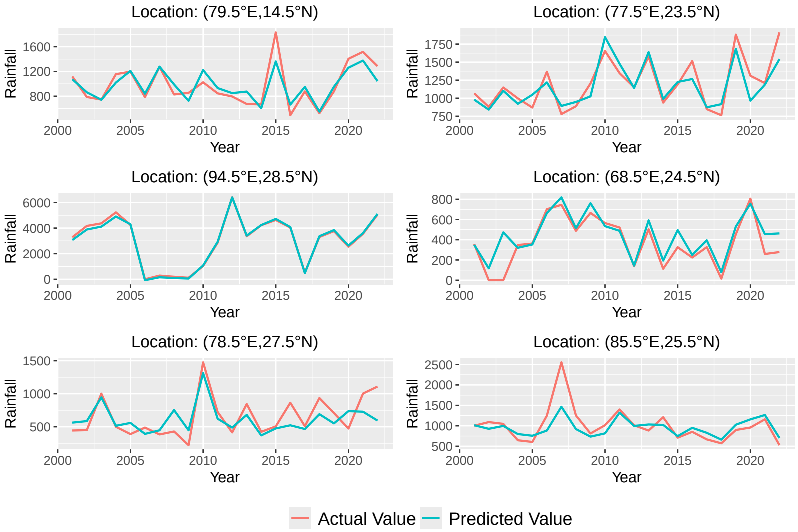
- ESS:

$\text{ESS}(\sigma^2) = 5249$

$\text{ESS}(\theta) = 3255$

	Min	Q1	Mean	Q3	Max
β	5010	5775	5938	6000	6000
Σ	5715	5719	5912	6000	6000

Prediction


Location: (79.5°E,14.5°N)
Rainfall
1600
1200
800
2000 2005 2010 2015 2020
Year
Location: (77.5°E,23.5°N)
Rainfall
1750
1500
1250
1000
750
2000 2005 2010 2015 2020
Year
Location: (94.5°E,28.5°N)
Rainfall
6000
4000
2000
0
2000 2005 2010 2015 2020
Year
Location: (68.5°E,24.5°N)
Rainfall
800
600
400
200
0
2000 2005 2010 2015 2020
Year
Location: (78.5°E,27.5°N)
Rainfall
1500
1000
500
2000 2005 2010 2015 2020
Year
Location: (85.5°E,25.5°N)
Rainfall
2500
2000
1500
1000
500
2000 2005 2010 2015 2020
Year
Actual Value
Predicted Value