

PROGRAM STATEMENT: Solve the system of equations, by Gauss-Jacobi method,

$$\begin{array}{rcl} \mathbf{x}_1 + \mathbf{x}_2 + 4\mathbf{x}_3 & = & 9 \\ 8\mathbf{x}_1 - 3\mathbf{x}_2 + 2\mathbf{x}_3 & = & 20 \\ 4\mathbf{x}_1 + 11\mathbf{x}_2 - \mathbf{x}_3 & = & 33 \end{array}$$

THEORY:

In certain cases, one can solve the system of equations

$$\sum_{j=1}^n a_{ij}x_j = b_i \quad (1 \leq i \leq n)$$

by iteration. In Jacobi iteration, the k th approximation $x_j^{(k)}$ to the solution x_j of this equation is calculated as

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1, j \neq i}^n a_{ij}x_j^{(k)}}{a_{ii}} \quad (1 \leq i \leq n),$$

and in Gauss-Seidel iteration one takes

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij}x_j^{(k)}}{a_{ii}} \quad (1 \leq i \leq n);$$

that is, in Gauss-Seidel iteration, one makes use of $x_j^{(k+1)}$ as soon as it is available. One can use any starting values with these iterations; for example, $x_i^{(0)} = 0$ for each i is a reasonable choice.

Call an $n \times n$ matrix $A = (a_{ij})$ *row-diagonally dominant* if

$$|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^n |a_{ij}|$$

for each i with $1 \leq i \leq n$.

PROGRAM CODE:

```
//C Program to Solve a System of Equations by Gauss-Jacobi Method
#include <stdio.h>
#include <math.h>
double mod(double x)
{
    if(x<0)
        return -x;
    else
        return x;
}
double error(int a)
{
    return 5*pow(10,-a-1);
}
int main()
{
    int n,i,j,k=0,pos;
    printf("Enter the number of Equations and the number of variables::");
    scanf("%d",&n);
    printf("Enter the coefficients of the equations\n");
```

```

double a[n][n+1],x[n],y[n],e,c,f;
for(i=0;i<n;i++)
{
    printf("Equation %d\n",i+1);
    x[i]=0;
    for(j=0;j<=n;j++)
    {
        scanf("%lf",&a[i][j]);
    }
}
printf("You need the answer correct upto how many decimal places? ::");
scanf("%lf",&e);
e=error(e);
for(i=0;i<n;)
{
    c=0;
    for(j=0;j<n;j++)
    {
        if(i!=j)
            c=c+mod(a[i][j]);
    }
    if(mod(a[i][i])>c)
        i=i+1;
    else
    {
        f=mod(a[i][0]);
        for(j=0;j<n;j++)
            if(mod(a[i][j])>f)
            {
                f=mod(a[i][j]);
                pos=j;
            }
        for(j=0;j<=n;j++)
        {
            f=a[i][j];
            a[i][j]=a[pos][j];
            a[pos][j]=f;
        }
    }
}
printf("The rearranged eqns are::");
for(i=0;i<n;i++)
{
    printf("\nEquation %d\n",i+1);
    for(j=0;j<=n;j++)
    {
        printf("%lf\t",a[i][j]);
    }
}
printf("\nk\t");
for(i=0;i<n;i++)
    printf("x%d\t\t",i+1);
do
{
    c=0;

```

```

        for(i=0;i<n;i++)
            y[i]=x[i];
        for(i=0;i<n;i++)
        {
            x[i]=0;
            for(j=0;j<n;j++)
            {
                if(i!=j)
                    x[i]=x[i]-(y[j]*a[i][j]/a[i][i]);
                else
                    x[i]=x[i]+(a[i][n]/a[i][i]);
            }
        }
        printf("\n%d\t",k++);
        for(i=0;i<n;i++)
        {
            printf("%lf\t",y[i]);
            if(mod(x[i]-y[i])<e)
                c++;
        }
    }
    while(c!=n);
    printf("\nThe Solutions are::\n");
    for(i=0;i<n;i++)
        printf("x%d=%.2lf\n",i+1,x[i]);
    return 0;
}

```

OUTPUT:

```

Enter the number of Equations and the number of variables::3
Enter the coefficients of the equations
Equation 1
1 1 4 9
Equation 2
8 -3 2 20
Equation 3
4 11 -1 33
You need the answer correct upto how many decimal places? ::2
The rearranged eqns are::
Equation 1
8.000000  -3.000000  2.000000  20.000000
Equation 2
4.000000  11.000000  -1.000000  33.000000
Equation 3
1.000000  1.000000  4.000000  9.000000
k      x1      x2      x3
0      0.000000  0.000000  0.000000
1      2.500000  3.000000  2.250000
2      3.062500  2.295455  0.875000
3      3.142045  1.965909  0.910511
4      3.009588  1.940212  0.973011
5      2.984327  1.994060  1.012550
6      2.994635  2.006840  1.005403
7      3.001214  2.002442  0.999631
The Solutions are::

```

x1=3.00
x2=2.00
x3=1.00