

PROGRAM STATEMENT: Solve the system of equations, by Gauss-elimination method,

$$\begin{array}{rcl} 2x_1 + 3x_2 + x_3 & = & 9 \\ x_1 + 2x_2 + 3x_3 & = & 6 \\ 3x_1 + x_2 + 2x_3 & = & 8 \end{array}$$

Correct upto 3-significant figures.

THEORY: It is a direct method for finding the solution or the values of unknown of a system of linear equations based on the principle of elimination of unknowns in successive steps.

Consider the following system of linear equations

$$\begin{array}{rcl} 3x_1 + 6x_2 - 2x_3 + 2x_4 & = & -22 \\ -3x_1 - 4x_2 + 6x_3 - 4x_4 & = & 34 \\ 6x_1 + 16x_2 + x_3 + x_4 & = & -33 \\ -6x_1 - 18x_2 - 2x_3 - 2x_4 & = & 36 \end{array}$$

Writing a_{ij} for the coefficient of x_j in the i th equation and b_i for the right-hand side of the i th equation, this system can be written as

$$(1) \quad \sum_{j=1}^n a_{ij}x_j = b_i \quad (1 \leq i \leq n)$$

with $n = 4$. To solve this system of equation, we write $m_{i1} = a_{i1}/a_{11}$, and subtract m_{i1} times the first equation from the i th equation for $i = 2, 3$, and 4 . We have $m_{21} = -1$, $m_{31} = 2$, and $m_{41} = -2$. The following equations will result (the first equation is written down unchanged):

$$\begin{array}{rcl} 3x_1 + 6x_2 - 2x_3 + 2x_4 & = & -22 \\ 2x_2 + 4x_3 - 2x_4 & = & 12 \\ 4x_2 + 5x_3 - 3x_4 & = & 11 \\ -6x_2 - 6x_3 + 2x_4 & = & -8 \end{array}$$

Write $a_{ij}^{(2)}$ for the coefficient of x_j in the i th equation. We will continue the above process: writing $m_{i2} = a_{i2}^{(2)}/a_{22}^{(2)}$ and subtract m_{i2} times the second equation from the i th equation for $i = 3$ and $i = 4$. We have $m_{32} = 2$ and $m_{42} = -3$. We obtain the equations

$$\begin{aligned} 3x_1 + 6x_2 - 2x_3 + 2x_4 &= -22 \\ 2x_2 + 4x_3 - 2x_4 &= 12 \\ -3x_3 + x_4 &= -13 \\ 6x_3 - 4x_4 &= 28 \end{aligned}$$

Write $a_{ij}^{(3)}$ for the coefficient of x_j in the i th equation. Write $m_{43} = a_{43}^{(3)}/a_{33}^{(3)}$, and subtract m_{43} times the third equation from the fourth equation. We have $m_{43} = -2$, and the following equations result:

$$\begin{aligned} 3x_1 + 6x_2 - 2x_3 + 2x_4 &= -22 \\ 2x_2 + 4x_3 - 2x_4 &= 12 \\ -3x_3 + x_4 &= -13 \\ -2x_4 &= 2 \end{aligned}$$

This is the Gauss elimination and now we shall solve it to obtain the result.

PROGRAM CODE :

```
//C Program to Solve a System of equations by Gauss Elimination Method
#include <stdio.h>
#include <math.h>
int main()
{
    int n,i,j,k;
    printf("Enter the number of Equations and the number of variables::");
    scanf("%d",&n);
    printf("Enter the coefficients of the equations\n");
    double a[n][n+3],s;
    for(i=0;i<n;i++)
    {
        printf("Equation %d\n",i+1);
        for(j=0;j<=n;j++)
        {
            scanf("%lf",&a[i][j]);
        }
    }
    for(i=1;i<=n;i++)
        printf("x%d\t",i);
    printf("b\tm\n");
    for(i=0;i<n-1;i++)
    {
        printf("Step %d\n",i+2);
        for(j=i+1;j<n;j++)
        {
```

```

        a[j][n+1]=-a[j][i]/a[i][i];
        for(k=0;k<=i;k++)
            printf("\t");
        for(k=i+1;k<=n;k++)
        {
            a[j][k]=a[j][k]+(a[j][n+1]*a[i][k]);
            printf("%.3lf\t",a[j][k]);
        }
        printf("%.3lf\n",a[j][n+1]);
    }
}
printf("The Solutions are::\n");
for(i=n-1;i>=0;i--)
{
    s=0;
    for(j=i;j<=n;j++)
    {
        s=s+a[i][j]*a[j][n+2];
    }
    a[i][n+2]=(a[i][n]-s)/a[i][i];
    printf("x%d=%.3lf\n",i+1,a[i][n+2]);
}
return 0;
}

```

OUTPUT:

Enter the number of Equations and the number of variables::3

Enter the coefficients of the equations

Equation 1

2 3 1 9

Equation 2

1 2 3 6

Equation 3

3 1 2 8

x1 x2 x3 b m

Step 2

0.500 2.500 1.500 -0.500

-3.500 0.500 -5.500 -1.500

Step 3

18.000 5.000 7.000

The Solutions are::

x3=0.278

x2=1.611

x1=1.944