PROGRAM STATEMENT: Solve the equation $x^3-9x+1=0$ for the root lying between 2 and 3 , correct upto 3 significant figures.

THEORY: It is an iterative method and is based on a well known theorem which states that if f(x) be a continuous function in a closed interval [a,b] and f(a)f(b)<0, then there exists b at least one real root of the equation f(x)=0, between a and b. If further f'(x) exists and f'(x) maintains same sign in same [a,b] i.e. f(x) is strictly monotonic, then there is only one real root of f(x)=0 in [a,b]. The method of bisection is nothing but a repeated application of the tabulation theorem.

Let the interval $[a_0,b_0]$ be divide into two equal parts by x_1 , i.e. $X_1 = (a_0+b_0)/2$ and $f(x_1)$ is calculated .If $f(x_1)=0$, then x_1 is exact root of $f(x_0)=0$. preceding in this manner ,we find $x_{n+1}=(a_n+b_n)/2$ which is the (n+1) th approximate of the root α of f(x)=0 an lies in interval $[a_n,b_n]$ where $b_{n-1}a_n=(b_0-a_0)/2^n$ and $a_0<=a_n< b_n<=b_0$ for all n.

PROGRAM CODE:

```
//C Program to find the Soln of an Equation using Bisection Method
#include <stdio.h>
#include <math.h>
double eq(double x)
{
     return (pow(x,3)-9*x+1);
double mod(double x)
     if(x<0)
           return -x;
     else
           return x;
}
double error(int a)
{
     return 5*pow(10, -a-1);
int main()
     int i=0;
     double a,b,x,e,f;
     printf("The equation we are solving is-> x^3-9*x+1=0\n");
     printf("Enter value of a::");
     scanf("%lf", &a);
     printf("Enter value of b::");
     scanf("%lf",&b);
     x = (a+b)/2;
     printf("You need the answer correct upto how many decimal
places? ::");
     scanf("%lf", &e);
     e=error(e);
     if ((eq(a)>0\&eq(b)<0) \mid |(eq(b)>0\&eq(a)<0)) / (Condition that proves
that the result lies between a and b
           printf("n ta(n) ttb(n) tttx(n+1) tttf(x(n+1)) n");
           while ((mod(a-b)>e) | | (mod(b-x)>e) | | (mod(x-a)>e))
                x = (a+b)/2;
```

```
f=eq(x);
               printf("%d\t%lf\t\t%lf\t\t%lf\n", i++, a, b, x, f);
               if(f>0)
                   b=x;
               else if(f<0)
                   a=x;
               else
                   break;
          }
          printf("The root is lf\n",x);
     else
          printf("The result doesn't lie between a and b. Program
Terminated.\n");
     }
     return 0;
}
OUTPUT:
The equation we are solving is->x^3-9*x+1=0
Enter value of a::2
Enter value of b::3
You need the answer correct upto how many decimal places? :: 3
                                  x(n+1)
     a(n)
                   b(n)
                                                      f(x(n+1))
    2.000000
0
                   3.000000
                                  2.500000
                                                 -5.875000
1
    2.500000
                   3.000000
                                  2.750000
                                                 -2.953125
2
    2.750000
                   3.000000
                                  2.875000
                                                 -1.111328
   2.875000
3
                   3.000000
                                  2.937500
                                                 -0.090088
4
   2.937500
                   3.000000
                                  2.968750
                                                 0.446259
5
    2.937500
                   2.968750
                                  2.953125
                                                 0.175922
6
   2.937500
                   2.953125
                                  2.945312
                                                 0.042378
7
    2.937500
                   2.945312
                                  2.941406
                                                 -0.023990
8
    2.941406
                   2.945312
                                  2.943359
                                                 0.009160
9
   2.941406
                   2.943359
                                  2.942383
                                                 -0.007423
10 2.942383
                   2.943359
                                  2.942871
                                                 0.000867
```

The root is 2.942871