

# Partial Orders for Efficient BMC of Concurrent Software

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**Abstract.** The vast number of interleavings that a concurrent program can have is typically identified as the root cause of the difficulty of automatic analysis of concurrent software. Weak memory is generally believed to make this problem even harder. We address both issues by modelling programs' executions with partial orders rather than the interleaving semantics (SC). We implemented a software analysis tool based on these ideas. It scales to programs of sufficient size to achieve first-time formal verification of non-trivial concurrent systems code over a wide range of models, including SC, Intel x86 and IBM Power.

## 1 Introduction

Automatic analysis of concurrent programs is a practical challenge. Hardly any of the very few existing tools for concurrency will verify a thousand lines of code [21]. Most papers name the number of *thread interleavings* that a concurrent program can have as a reason for the difficulty. This view presupposes an execution model, namely *Sequential Consistency* (SC) [47], where an execution is a *total order* (more precisely an interleaving) of the instructions from different threads. The choice of SC as the execution model poses at least two problems.

First, the large number of interleavings modelling the executions of a program makes their enumeration intractable. *Context bounded* methods [59,54,45,23] (which are unsound in general) and *partial order reduction* [56,31,26] can reduce the number of interleavings to consider, but still suffer from limited scalability. Second, modern multiprocessors (e.g., Intel x86 or IBM Power) serve as a reminder that SC is an inappropriate model. Indeed, the *weak memory models* implemented by these chips allow more behaviours than SC.

We address these two issues by using *partial orders* to model executions, following [58,64,10,57]. We also aim at practical verification of concurrent programs [17,19,23]. Rarely have these two communities met. Notable exceptions are [61,62], forming with [14] the closest related work. We show that the explicit use of partial orders generalises these works to concurrency at large, from SC to weak memory, without affecting efficiency.

Our method is as follows: we map a program to a formula consisting of two parts. The first conjunct describes the data and control flow for each thread of

the program; the second conjunct describes the concurrent executions of these threads as partial orders. We prove that for any satisfying assignment of this formula there is a valid execution w.r.t. our models; and conversely, any valid execution gives rise to a satisfying assignment of the formula.

Thus, given an analysis for sequential programs (the per-thread conjunct), we obtain an analysis for concurrent programs. For programs with bounded loops, we obtain a sound and complete model checking method. Otherwise, if the program has unbounded loops, we obtain an exhaustive analysis up to a given bound on loop unrollings, i.e., a bounded model checking method.

To experiment with our approach, we implement a *symbolic decision procedure* answering reachability queries over concurrent C programs w.r.t. a given memory model. We support a wide range of models, including SC, Intel x86 and IBM Power. To exercise our tool w.r.t. weak memory, we verify 4500 tests used to validate formal models against IBM Power chips [60,50]. Our tool is the first to handle the subtle *store atomicity relaxation* [4] specific to Power and ARM.

We show that mutual exclusion is not violated in a queue mechanism of the Apache HTTP server software. We confirm a bug in the worker synchronisation mechanism in PostgreSQL, and that adding two fences fixes the problem. We verify that the Read-Copy-Update mechanism of the Linux kernel preserves data consistency of the object it is protecting. For all examples we perform the analysis for a wide range of memory models, from SC to IBM Power via Intel x86.

We provide the sources of our tool, our experimental logs and our benchmarks at <http://www.cprover.org/wpo>.

## 2 Related Work

We start with models of concurrency, then review tools proving the absence of bugs in concurrent software, organised by techniques.

*Models of concurrency* Formal methods traditionally build on Lamport’s SC [47]. A year earlier, Lamport defined *happens-before models* [46]. The happens-before order is the smallest partial order containing the program order and the relation between a write, and a read from this write.

These models seem well suited for analyses relative to synchronisation, e.g., [22,25,40], because the relations they define are oblivious to the implementation of the idioms. Despite happens-before being a partial order, most of [46] explains how to linearise it. Hence, this line of work often relies on a notion of total orders. Partial orders, however, have been successfully applied in verification in the context of Petri nets [53], which have been linked to software verification in [41] for programs with a small state space.

We (and [15,29,61,62]) reuse the *clocks* of [46] to build our orders. Yet we do not aim at linearisation or a transitive closure, as this leads to a polynomial overhead of redundant constraints.

Our work goes beyond the definition and simulation of memory models [32,37,63,60,50]. Implementing an executable version of the memory models is an important step,

but we go further by studying the validity of systems code in C (as opposed to assembly or toy languages) w.r.t. both a given memory model and a property.

The style of the model influences the verification process. Memory models roughly fall into two classes: operational and axiomatic. The operational style models executions via interleavings, with transitions accessing buffers or queues, in addition to the memory (as on SC). Thus this approach inherits the limitations of interleaving-based verification. For example, [9] (restricted to Sun Total Store Order, TSO) bounds the number of context switches.

Other methods use operational specifications of TSO, Sun Partial Store Order (PSO) and Relaxed Memory Order (RMO) to place fences in a program [44,43,49]. Abdulla et al. [3] address this problem on an operational TSO, for finite state transition systems instead of programs. The methods of [44,43] have, in the words of [49], “severely limited scalability”. The dynamic technique presented in [49] scales to 771 lines but does not aim to be sound: the tool picks an invalid execution, repairs it, then iterates.

Axiomatic specifications categorise behaviours by constraining relations on memory accesses. Several hardware vendors adopt this style [1,2] of specification; we build on the axiomatic framework of [8] (cf. Sec. 3). CheckFence [14] also uses axiomatic specifications, but does not handle the store atomicity relaxation of Power and ARM.

*Running example* Below we use Prog. 1 (from the TACAS Software Verification Competition [11]) as an illustration. The shared variables  $x$  and  $y$  can reach the  $(2N)$ -th Fibonacci number, depending on the interleaving of  $\text{thr1}$  and  $\text{thr2}$ . Prog. 1 permits at least  $\mathcal{O}(2^{6N})$  interleavings of  $\text{thr1}$  and  $\text{thr2}$ . In each loop iteration,  $\text{thr1}$  reads  $x$  and then  $y$ , and then writes  $x$ ;  $\text{thr2}$  reads  $y$  and  $x$ , and then writes  $y$ . Each interleaving of these two writes yields a unique sequence of shared memory states. Swapping, e.g., the read of  $y$  in  $\text{thr2}$  with the write of  $x$  in  $\text{thr1}$  does not affect the memory states, but swapping the accesses to the same address does.

```
#define N 5
int x=1, y=1;

void thr1() {
    for(int k=0; k<N; ++k)
        x=x+y; }

void thr2() {
    for(int k=0; k<N; ++k)
        y=y+x; }

int main() {
    start_thread(thr1);
    start_thread(thr2);
    assert(x<=144 && y<=144);
    return 0; }

Prog. 1. Fibonacci from [11]
```

*Interleaving tools* Traditionally, tools are based on interleavings, and do not consider weak memory. By contrast, we handle weak memory by reasoning in terms of partial orders.

*Explicit-state model checking* performs a search over states of a transition system. SPIN [36], VeriSoft [30] and Java PathFinder [35,38] implement this approach; they adopt various forms of *partial order reduction* (POR) to cope with the number of interleavings.

POR reduces soundly the number of interleavings to study [56,31,26] by observing that a partial order gives rise to a class of interleavings [51], then

picking only one interleaving in each class. Prog. 1 is an instance where the effect of POR is limited. We noted in Sec. 2 that amongst the  $\mathcal{O}(2^{6N})$  interleavings permitted by Prog. 1, only the interleavings of the writes give rise to unique sequences of states. Hence distinct interleavings of the threads representing the same interleavings of the writes are candidates for reduction. POR reduces the number of interleavings by at least  $2^{2N}$ , but  $\mathcal{O}(2^{4N})$  interleavings remain.

Explicit-state methods may fail to cope with large state spaces, even in a sequential setting. Symbolic encodings [13] can help, but the state space often needs further reduction using, e.g., *bounded model checking* (BMC) [12] or *predicate abstraction* [33]. These techniques may again also be combined with POR. ESBMC [19] implements BMC. An instance of Prog. 1 has a fixed  $N$ , i.e., bounded loops. Thus BMC with  $N$  as bound is sound and complete for such an instance. ESBMC verifies Prog. 1 for  $N = 10$  within 30 mins (cf. Sec. 6, Fig. 8). SatAbs [17] uses predicate abstraction in a CEGAR loop; it completes no more than  $N = 3$  in 30 mins as it needs multiple predicates per interleaving, resulting in many refinement iterations. Our approach easily scales to, e.g.,  $N = 50$ , in less than 20 s, and more than  $N=300$  within 30 mins, as we build only a polynomial number of constraints, at worst cubic in the number of accesses to a given shared memory address.

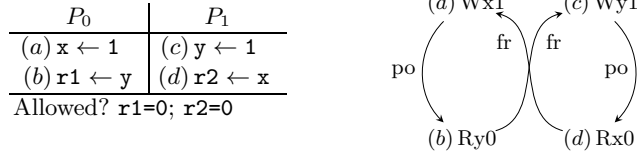
*Non-interleaving tools* Another line of tools is not based on interleavings. The existing approaches do not handle weak memory and are either incomplete (i.e., fail to prove the absence of a bug) or unsound (i.e., might miss a bug due to the assumptions they make).

*Thread-modular reasoning* [39,24,28,27,34] is sound, but usually incomplete. Each read presumes guarantees about the values provided by the environment. Empty guarantees amount to fully non-deterministic values, thus this is a trivially sound approach. Our translation of Sec. 4 corresponds to empty guarantees. The constraints of Sec. 5, however, make our encoding complete.

In Prog. 1, if we guarantee  $x \leq 144$  &&  $y \leq 144$ , the problem becomes trivial, but finding this guarantee automatically is challenging. Threader [34] fails for  $N=1$  (cf. Sec. 6, Fig. 8).

Context bounded methods fix an arbitrary bound on *context switches* [59,54,45,23]. This supposes that most bugs happen with few context switches. Our method does not make this restriction. Moreover, we believe that there is no obvious generalisation of these works to weak memory, other than instrumentation as [9] does for TSO, i.e., adding information to a program so that its SC executions simulate its weak ones. We used our tool in SC mode, and applied the instrumentation of [9] to it. On average, the instrumentation is 9 times more costly (cf. Sec. 6, Fig. 8).

In Prog. 1, we need at least  $N$  context switches to disprove the assertion `assert (x <= 143 && y <= 143)` (or any upper bound to  $x$  and  $y$  that is the  $(2N)$ -th Fibonacci number minus 1). The hypothesis of the approach (i.e., small context bounds suffice to find a bug) does not apply here; Poirot fails for  $N \geq 1$  (cf. Sec. 6, Fig. 8).



**Fig. 1.** Store Buffering (**sb**)

Our work relates the most to [14,29,61,62]; we discuss [29] below and detail [14,61,62] in Sec. 5.7. These works use axiomatic specifications of SC to compose the distinct threads. CheckFence [14] models SC with total orders and transitive closure constraints; [61,62] use partial orders like us. [61,62] note redundancies of their constraints, but do not explain them; our semantic foundations (Sec. 3) allow us both to explain their redundancies and avoid them (cf. Sec. 5.7).

The encodings of [14,61,62] are  $\mathcal{O}(N^3)$  for  $N$  shared memory accesses *to any address*; [29] is quadratic, but in the number of threads times the number of per-thread transitions, which may include arbitrary many local accesses. Our encoding is  $\mathcal{O}(M^3)$ , with  $M$  the maximal number of events for a *single address*. By contrast, the encodings of [29,61,62] quantify over all addresses. Prog. 1 has two addresses only, but the difference is already significant:  $(6N)^3$  for [14,29,61,62] vs.  $2 \times (3M)^3$  in our case, i.e. 1/4 of the constraints (cf. Sec. 6, Fig. 7 for other case studies).

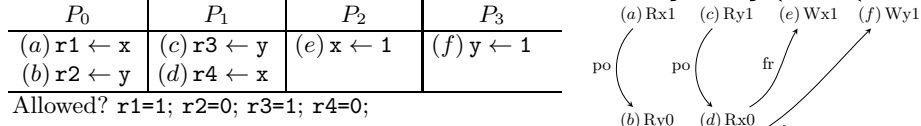
### 3 Context: Axiomatic Memory Model

We use the framework of [8], which provably embraces several *architectures*: SC [47], Sun TSO (i.e. the x86 model [55]), PSO and RMO, Alpha, and a fragment of Power. We present this framework via *litmus tests*, as shown in Fig. 1.

The keyword *allowed* asks if the architecture permits the outcome “ $r1=1; r2=0; r3=1; r4=0$ ”. This relates to the event graphs of this program, composed of relations over *read and write memory events*. A store instruction (e.g.  $x \leftarrow 1$  on  $P_0$ ) corresponds to a write event  $((a) \text{ Wx1})$ , and a load (e.g.  $r1 \leftarrow y$  on  $P_0$ ) to a read  $((b) \text{ Ry0})$ . The validity of an execution boils down to the absence of certain cycles in the event graph. Indeed, an architecture allows an execution when it represents a *consensus* amongst the processors. A cycle in an event graph is a potential violation of this consensus.

If a graph has a cycle, we check if the architecture *relaxes* some relations. The consensus ignores relaxed relations, hence becomes acyclic, i.e. the architecture allows the final state. In Fig. 1, on SC where nothing is relaxed, the cycle forbids the execution. x86 relaxes the program order (**po** in Fig. 1) between writes and reads, thus a forbidding cycle no longer exists for  $(a, b)$  and  $(c, d)$  are relaxed.

*Executions* Formally, an *event* is a read or a write memory access, composed of a unique identifier, a direction R for read or W for write, a memory address,



**Fig. 2.** Independent Reads of Independent Writes (**iriw**)

and a value. We represent each instruction by the events it issues. In Fig. 2, we associate the store  $x \leftarrow 1$  on processor  $P_2$  to the event  $(e) Wx1$ .

We associate the program with an *event structure*  $E \triangleq (\mathbb{E}, \text{po})$ , composed of its events  $\mathbb{E}$  and the *program order*  $\text{po}$ , a per-processor total order. We write  $\text{dp}$  for the relation (included in  $\text{po}$ , the source being a read) modelling *dependencies* between instructions, *e.g.* an *address dependency* occurs when computing the address of a load or store from the value of a preceding load.

Then, we represent the *communication* between processors leading to the final state via an *execution witness*  $X \triangleq (\text{ws}, \text{rf})$ , which consists of two relations over the events. First, the *write serialisation*  $\text{ws}$  is a per-address total order on writes which models the *memory coherence* widely assumed by modern architectures. It links a write  $w$  to any write  $w'$  to the same address that hits the memory after  $w$ . Second, the *read-from* relation  $\text{rf}$  links a write  $w$  to a read  $r$  such that  $r$  reads the value written by  $w$ .

We include the writes in the consensus via the write serialisation. Unfortunately, the read-from map does not give us enough information to embed the reads as well. To that aim, we derive the *from-read* relation  $\text{fr}$  from  $\text{ws}$  and  $\text{rf}$ . A read  $r$  is in  $\text{fr}$  with a write  $w$  when the write  $w'$  from which  $r$  reads hit the memory before  $w$  did. Formally, we have:  $(r, w) \in \text{fr} \triangleq \exists w', (w', r) \in \text{rf} \wedge (w', w) \in \text{ws}$ .

In Fig. 2, the outcome corresponds to the execution on the right if each memory location and register initially holds 0. If  $r1=1$  in the end, the read  $(a)$  read its value from the write  $(e)$  on  $P_2$ , hence  $(e, a) \in \text{rf}$ . If  $r2=0$ , the read  $(b)$  read its value from the initial state, thus before the write  $(f)$  on  $P_3$ , hence  $(b, f) \in \text{fr}$ . Similarly, we have  $(f, c) \in \text{rf}$  from  $r3=1$ , and  $(d, e) \in \text{fr}$  from  $r4=0$ .

*Relaxed or safe* A processor can commit a write  $w$  first to a store buffer, then to a cache, and finally to memory. When a write hits the memory, all the processors agree on its value. But when the write  $w$  transits in store buffers and caches, a processor can read its value through a read  $r$  before the value is actually available to all processors from the memory. In this case, the read-from relation between the write  $w$  and the read  $r$  does not contribute to the consensus, since the reading occurs in advance.

We model this by some subrelation of the read-from  $\text{rf}$  being *relaxed*, i.e. not included in the consensus. When a processor can read from its own store buffer [4] (the typical TSO/x86 scenario), we relax the internal read-from  $\text{rfi}$ . When two processors  $P_0$  and  $P_1$  can communicate privately via a cache (a case of *write atomicity* relaxation [4]), we relax the external read-from  $\text{rfe}$ , and call

the corresponding write *non-atomic*. This is the main particularity of Power or ARM, and cannot happen on TSO/x86.

Some program-order pairs are relaxed (e.g. write-read pairs on x86), i.e. only a subset of **po** is guaranteed to occur in this order.

When a relation is not relaxed, we call it *safe*. Architectures provide special *fence* (or *barrier*) instructions, to prevent weak behaviours. Following [8], the relation  $\text{fence} \subseteq \text{po}$  induced by a fence is *non-cumulative* when it orders certain pairs of events surrounding the fence, i.e. **fence** is safe. The relation **fence** is *cumulative* when it makes writes atomic, e.g. by flushing caches. The relation **fence** is *A-cumulative* (resp. *B-cumulative*) if **rfe**; **fence** (resp. **fence**; **rfe**) is safe. When stores are atomic (i.e. **rfe** is safe), e.g. on TSO, we do not need cumulativity.

*Architectures* An *architecture*  $A$  determines the set  $\text{safe}_A$  of the relations safe on  $A$ , i.e. the relations embedded in the consensus. Following [8], we consider the write serialisation **ws** and the from-read relation **fr** to be always safe. SC relaxes nothing, i.e. **rf** and **po** are safe. TSO authorises the reordering of write-read pairs and store buffering (i.e. **po<sub>WR</sub>** and **rfi** are relaxed) but nothing else. We denote the safe subset of read-from, i.e. the read-from relation globally agreed on by all processors, by **grf**.

Finally, an execution  $(E, X)$  is *valid* on  $A$  when the three following conditions hold. 1. SC holds per address, i.e. the communication and the program order for accesses with same address **po-loc** are compatible:  $\text{uniproc}(E, X) \triangleq \text{acyclic}(\text{ws} \cup \text{rf} \cup \text{fr} \cup \text{po-loc})$ . 2. Values do not come out of thin air, i.e. there is no causal loop:  $\text{thin}(E, X) \triangleq \text{acyclic}(\text{rf} \cup \text{dp})$ . 3. There is a consensus, i.e. the safe relations do not form a cycle:  $\text{consensus}(E, X) \triangleq \text{acyclic}((\text{ws} \cup \text{rf} \cup \text{fr} \cup \text{po}) \cap \text{safe}_A)$ . Formally:  $\text{valid}_A(E, X) \triangleq \text{uniproc}(E, X) \wedge \text{thin}(E, X) \wedge \text{consensus}(E, X)$ .

From the validity of executions we deduce a comparison of architectures: We say that an architecture  $A_2$  is *stronger* than another one  $A_1$  when the executions valid on  $A_2$  are valid on  $A_1$ . Equivalently we would say that  $A_1$  is *weaker* than  $A_2$ . Thus, SC is stronger than any other architecture discussed above.

## 4 Symbolic event structures

For an architecture  $A$  and *one* execution witness  $X$ , the framework of Sec. 3 determines if  $X$  is valid on  $A$ . To prove reachability of a program state, we need to reason about all its executions. To do so efficiently, we use symbolic representations capturing all possible executions in a single constraint system. We then apply SAT or SMT solvers to decide if a valid execution exists for  $A$ , and, if so, get a satisfying assignment corresponding to an execution witness.

As said in Sec. 1, we build two conjuncts. The first one, **ssa**, represents the data and control flow per thread. The second, **pord**, captures the communications between threads (cf. Sec. 5). We include a reachability property in **ssa**; the program has a valid execution violating the property iff **ssa**  $\wedge$  **pord** is satisfiable.

We mostly use *static single assignment form* (SSA) of the input program to build **ssa** (cf. [42] for details). In this SSA variant, each equation is augmented

with a *guard*: the guard is the disjunction over all conjunctions of branching guards on paths to the assignment. To deal with concurrency, we use a fresh index for each occurrence of a given shared memory variable, resulting in a fresh symbol in the formula. CheckFence [14] and [61,62] use a similarly modified encoding.

Together with *ssa*, we build a *symbolic event structure* (*ses*). As detailed below, it captures basic program information needed to build the second conjunct *pord* in Sec. 5. Fig. 3 illustrates this section: the formula *ssa* on top corresponds to the *ses* beneath.

main	$P_0$	$P_1$	$P_2$	$P_3$
$x_0 = 0$				
$\wedge y_0 = 0$	$\wedge r1_0^1 = x_1$	$\wedge r3_0^2 = y_2$	$\wedge x_3 = 1$	$\wedge y_3 = 1$
	$\wedge r2_0^1 = y_1$	$\wedge r4_0^2 = x_2$		
$\wedge \text{prop}$				
<hr/>				
$(i_0) Wx x_0$				
$(i_1) Wy y_0$				
$(a) Rx x_1 \quad (c) Ry y_2 \quad (e) Wx x_3 \quad (f) Wy y_3$				
$(b) Ry y_1 \quad (d) Rx x_2$				

**Fig. 3.** The formula *ssa* for *iriw* (Fig. 2) with *prop* =  $(r1_0^1 = 1 \wedge r2_0^1 = 0 \wedge r3_0^2 = 1 \wedge r4_0^2 = 0)$ , and its *ses* (guards omitted since all true)

*Static single assignment form (SSA)* To encode *ssa* we use a variant of SSA [20] and loop unrolling. The details of this encoding are in [42], except for differences in the handling of shared memory variables, as explained below.

In SSA, each occurrence of a program variable is annotated with an index. We turn assignments in SSA form into equalities, with distinct indexes yielding distinct symbols in the resulting equation. For example, the assignment  $x := x + 1$  results in the equality  $x_1 = x_0 + 1$ . We use unique indexes for assignments in loops via loop unrolling: repeating  $x := x + 1$  twice yields  $x_1 = x_0 + 1$  and  $x_2 = x_1 + 1$ . Control flow join points yield additional equations involving the guards of branches merging at this point (see [42] for details).

In concurrent programs, we also need to consider join points due to communication between threads, i.e., *concurrent SSA form* (CSSA) [48]. To deal with weaker models, we use a fresh index for each occurrence of a given shared memory variable, resulting in a fresh symbol in the formula. Thus, each occurrence may take non-deterministic values, i.e. this approach over-approximates the behaviours of a program. If  $x$  is shared in the above example, the modified SSA encoding of the second loop unrolling becomes  $x_3 = x_2 + 1$ , breaking any causality between the first loop iteration (encoded as  $x_1 = x_0 + 1$ ) and the second one. Sinha and Wang [61,62] use the same approach, but since they consider SC only, their use of fresh indexes may produce more symbols than necessary.



By adding the negation of the reachability property to be checked to our (over-approximating) SSA equations, we obtain a formula `ssa` that is satisfiable if there exists a (concurrent) counterexample violating the property. As this is an over-approximation, the converse need not be true, i.e., a satisfying assignment of `ssa` may constitute a spurious counterexample. Sec. 5 restores precision using the `pord` constraints derived from the `ses`.

In `ssa`, memory addresses map to unique symbols via the (symbolic) pointer dereferencing of [42, Sec. 4]. In the weak memory case, we ensure this by using analyses sound for this setting [6].

The top of Fig. 3 gives `ssa` for Fig. 2. We print a column per thread, vertically following the control flow, but it forms a single conjunction. Each occurrence of a program variable carries its SSA index as a subscript. Each occurrence of the shared memory variables `x` and `y` has a unique SSA index. Here we omit the guards, as this program does not use branching or loops.

*From SSA to symbolic event structures* A symbolic event structure (`ses`)  $\gamma \triangleq (\mathbb{S}, \text{po})$  is a set  $\mathbb{S}$  of *symbolic events* and a *symbolic program order* `po`. A symbolic event holds a *symbolic value* instead of a concrete one as in Sec. 3. We define  $g(e)$  to be the Boolean guard of a symbolic event  $e$ , which corresponds to the guard of the SSA equation as introduced above. We use these guards to build the executions of Sec. 3: a guard evaluates to true if the branch is taken, false otherwise. The symbolic program order  $\text{po}(\gamma)$  gives a list of symbolic events per thread of the program. The order of two events in  $\text{po}(\gamma)$  gives the program order in a concrete execution if both guards are true.

Note that  $\text{po}(\gamma)$  is an implementation-dependent linearisation of the branching structure of a thread, induced by the path merging applied while constructing the SSA form. For instance, `if  $e_1$  then  $e_2$  else  $e_3$`  could be linearised as either  $(e_1, e_2, e_3)$  or  $(e_1, e_3, e_2)$  as any two events of a concrete execution ( $e_1$  and  $e_2$ , or  $e_1$  and  $e_3$ ) remain in program order. The original branching structure, i.e., the unlinearised symbolic program order induced by the control flow graph, is maintained in the relation  $\text{po-br}(\gamma)$ . For the above example,  $\text{po-br}(\gamma)$  contains  $(e_1, e_2)$  and  $(e_1, e_3)$ .

We build the `ses`  $\gamma$  alongside the SSA form, as follows. Each occurrence of a shared program variable on the right-hand side of an assignment becomes a *symbolic read*, with the SSA-indexed variable as symbolic value, and the guard is taken from the SSA equation. Similarly, each occurrence of a shared program variable on the left-hand side becomes a *symbolic write*. Fences do not affect memory states in a sequential setting, hence do not appear in SSA equations. We simply add a fence event to the `ses` when we see a fence. We take the order of assignments per thread as program order, and mark thread spawn points.

At the bottom of Fig. 3, we give the `ses` of `iriw`. Each column represents the symbolic program order, per thread. We use the same notation as for the events of Sec. 3, but values are SSA symbols. Guards are omitted again, as they all are trivially true. We depict the thread spawn events by starting the program order in the appropriate row. Note that we choose to put the two initialisation writes in program order on the main thread.

*From symbolic to concrete event structures* To relate to the models of Sec. 3, we *concretise* symbolic events. A satisfying assignment to  $\text{ssa} \wedge \text{pord}$ , as computed by a SAT or SMT solver, induces, for each symbolic event, a concrete value (if it is a read or a write) and a valuation of its guard (for both accesses and fences). A valuation  $V$  of the symbols of  $\text{ssa}$  includes the values of each symbolic event. Since guards are formulas that are part of  $\text{ssa}$ ,  $V$  allows us to evaluate the guards as well. For a valuation  $V$ , we write  $\text{conc}(e_s, V)$  for the concrete event corresponding to  $e_s$ , if there is one, i.e., if  $g(e_s)$  evaluates to true under  $V$ .

The concretisation of a set  $S$  of symbolic events is a set  $E$  of concrete events, as in Sec. 3, s.t. for each  $e \in E$  there is a symbolic version  $e_s$  in  $S$ . We write  $\text{conc}(S, V)$  for this concrete set  $E$ . The concretisation  $\text{conc}(r_s, V)$  of a symbolic relation  $r_s$  is the relation  $\{(x, y) \mid \exists(x_s, y_s) \in r_s. x = \text{conc}(x_s, V) \wedge y = \text{conc}(y_s, V)\}$ .

Given an  $\text{ses } \gamma$ ,  $\text{conc}(\gamma, V)$  is the event structure (cf. Sec. 3), whose set of events is the concretisation of the events of  $\gamma$  w.r.t.  $V$ , and whose program order is the concretisation of  $\text{po}(\gamma)$  w.r.t.  $V$ . For example, the graph of Fig. 2 (erasing the rf and fr relations) is a concretisation of the  $\text{ses}$  of **iriw** (cf. Fig. 3).

## 5 Encoding the communication and weak memory relations symbolically

For an architecture  $A$  and an  $\text{ses } \gamma$ , we need to represent the communications (i.e., rf, ws and fr) and the weak memory relations (i.e.,  $\text{ppo}_A$ ,  $\text{grf}_A$  and  $\text{ab}_A$ ) of Sec. 3. We encode them as a formula  $\text{pord}$ , s.t.  $\text{ssa} \wedge \text{pord}$  is satisfiable iff there is an execution valid on  $A$  violating the property encoded in  $\text{ssa}$ . We avoid transitive closures to obtain a small number of constraints. We start with an informal overview of our approach, then describe how we encode partial orders, and finally detail the encoding for each relation of Sec. 3.

*Overview* We present our approach on **iriw** (Fig. 2) and its  $\text{ses } \gamma$  (Fig. 3). In Fig. 2, we represent only one possible execution, namely the one corresponding to the (non-SC) final state of the test at the top of the figure. In this section, we generate constraints representing all the executions of **iriw** on a given architecture. We give these constraints, for the address  $x$  in Fig. 4 in the SC case (for brevity we skip  $y$ , analogous to  $x$ ). Weakening the architecture removes some constraints: for Power, we omit the (rf-grf) and (ppo) constraints. For TSO, all constraints are the same as for SC.

In Fig. 4, each symbol  $c_{ab}$  is a *clock constraint*, representing an ordering between the events  $a$  and  $b$ . A variable  $s_{wr}$  represents a read-from between the write  $w$  and the read  $r$ .

The constraints of Fig. 4 represent the preserved program order (cf. Sec. 5.4), e.g., on SC or TSO the read-read pairs  $(a, b)$  on  $P_0$  (ppo  $P_0$ ) and  $(c, d)$  on  $P_1$  (ppo  $P_1$ ), but nothing on Power. We generate constraints for the read-from (cf. Sec. 5.1), for example (rf-some  $x$ ); the first conjunct  $s_{i_0a} \vee s_{ea}$  concerns the read  $a$  on  $P_0$ . This means that  $a$  can read either from the initial write  $i_0$  or from the write  $e$  on  $P_2$ . The selected read-from pair also implies equalities of the

(rf-val $x$ )	$(s_{i_0a} \Rightarrow x_1 = x_0) \wedge (s_{i_0d} \Rightarrow x_2 = x_0) \wedge$ $(s_{ea} \Rightarrow x_1 = x_3) \wedge (s_{ed} \Rightarrow x_2 = x_3)$
(rf-grf $x$ )	$(s_{i_0a} \Rightarrow c_{i_0a}) \wedge (s_{ea} \Rightarrow c_{ea}) \wedge$ $(s_{i_0d} \Rightarrow c_{i_0d}) \wedge (s_{ed} \Rightarrow c_{ed})$
(rf-some $x$ )	$(s_{i_0a} \vee s_{ea}) \wedge (s_{i_0d} \vee s_{ed})$
(ws $x$ )	$\neg c_{i_0e} \Rightarrow c_{ei_0}$
(fr $x$ )	$((s_{i_0a} \wedge c_{i_0e}) \Rightarrow c_{ae}) \wedge ((s_{i_0d} \wedge c_{i_0e}) \Rightarrow c_{de}) \wedge$ $((s_{ea} \wedge c_{ei_0}) \Rightarrow c_{ai_0}) \wedge ((s_{ed} \wedge c_{ei_0}) \Rightarrow c_{di_0})$
(ppo main)	$c_{i_0i_1} \quad (\text{ppo } P_0) \quad c_{ab} \quad (\text{ppo } P_1) \quad c_{cd}$

**Fig. 4.** Partial order constraints for address  $x$  in Fig. 2 on SC

values written and read (rf-val  $x$ ): for instance,  $s_{i_0a}$  implies that  $x_1$  equals the initialisation  $x_0$ . The architecture-independent constraints for write serialisation (cf. Sec. 5.2) and from-read (cf. Sec. 5.3) are specified as (ws  $x$ ) and (fr  $x$ ); (ws  $y$ ) and (fr  $y$ ) are analogous. As there are no fences in **iriw**, we do not generate any memory fence constraints (cf. Sec. 5.5).

We represent the execution of Fig. 2 as follows. For  $(e, a)$  and  $(i_0, d) \in \text{grf}$ , we have the constraint  $s_{ea} \Rightarrow c_{ea}$  and  $s_{i_0d} \Rightarrow c_{i_0d}$  in (rf-grf  $x$ ). This means that  $a$  reads from  $e$  (as witnessed by  $s_{ea}$ ), and that we record that  $e$  is ordered before  $a$  in **grf** (as witnessed by  $c_{ea}$ ); *idem* for  $d$  and  $i_0$ . To represent  $(d, e) \in \text{fr}$ , we pick the appropriate constraint in (fr  $x$ ), namely  $(s_{i_0d} \wedge c_{i_0e}) \Rightarrow c_{de}$ . This reads “if  $d$  reads from  $i_0$  and  $i_0$  is ordered before  $e$  (in **ws**, because  $i_0$  and  $e$  are two writes to  $x$ ), then  $d$  is ordered before  $e$  (in **fr**).”

Together with (ppo  $P_0$ ) and (ppo  $P_1$ ), these constraints represent the execution in Fig. 2. We cannot find a satisfying assignment of these constraints, as this leads to both  $a$  before  $b$  (by (ppo  $P_0$ )) and  $b$  before  $a$  (by (fr  $y$ ), (rf-grf  $y$ ), (ppo  $P_1$ ), (fr  $x$ ) and (grf  $x$ )). On Power, however, we neither have the ppo nor the grf constraints, hence we can find a satisfying assignment.

*Symbolic partial orders* We associate each symbolic event  $x$  of an **ses**  $\gamma$  with a unique *clock* variable  $\text{clock}_x$  (cf. [46,61]) ranging over the naturals. For two events  $x$  and  $y$ , we define the Boolean *clock constraint* as  $c_{xy} \triangleq (g(x) \wedge g(y)) \Rightarrow \text{clock}_x < \text{clock}_y$  (“ $<$ ” being less-than over the integers). We encode a relation  $r$  over the symbolic events of  $\gamma$  as the formula  $\phi(r)$  defined as the conjunction of the clock constraints  $c_{xy}$  for all  $(x, y) \in r$ , i.e.,  $\phi(r) \triangleq \bigwedge_{(x,y) \in r} c_{xy}$ .

Let  $C$  be a valuation of the clocks of the events of  $\gamma$ . Let  $V$  be a valuation of the symbols of the formula **ssa** associated to  $\gamma$ . As noted in Sec. 4,  $V$  gives us concrete values for the events of  $\gamma$ , and allows us to evaluate their guards. We show below that  $(C, V)$  satisfies  $\phi(r)$  iff the concretisation of  $r$  w.r.t.  $V$  is acyclic, provided that this relation has *finite prefixes*.

A prefix of  $x$  in a relation  $r$  is a (possibly infinite) list  $S = [x_0, x_1, x_2, \dots]$  s.t.  $x = x_0$  and for all  $i$ ,  $(x_{i+1}, x_i) \in r$  (observe that the prefix is reversed w.r.t. the order imposed by the relation). The relation  $r$  has finite prefixes if for each  $x$ , there is a bound  $l \in \mathbb{N}$  to the cardinality of the prefixes of  $x$  in  $r$ .

We write  $\text{card}(S)$  for the cardinality of a list  $S = [x_0, x_1, x_2, \dots]$ , i.e.,  $\text{card}(S) \triangleq \text{card}(\{x \mid \exists i. x = x_i\})$ . We write  $\text{pref}(r, x)$  for the set of prefixes of  $x$  in  $r$ . Formally,  $r$  has finite prefixes when  $\forall x. \exists l. \forall S \in \text{pref}(r, x). \text{card}(S) < l$ . In our proofs and in Alg. 4 we denote the concatenation of two lists  $S_1$  and  $S_2$  by  $S_1 ++ S_2$ .

In the following, we allow symbolic relations with infinite prefixes provided their concretisations have finite prefixes. Thus we do not consider executions with an infinite past, or running for more steps than the cardinality of  $\mathbb{N}$ . Our first lemma justifies why checking the acyclicity of a concrete relation amounts to checking the satisfiability of the formula encoding this relation symbolically:

**Lemma 1.**  *$(C, V)$  satisfies  $\phi(r)$  iff  $\text{conc}(r, V)$  is acyclic and has finite prefixes.*

*Proof.  $\Rightarrow$ :* We let  $r_c = \text{conc}(r, V)$ . One can show by induction that  $(*)$  if  $(C, V)$  satisfies  $\phi(r)$  then for all  $(x, y) \in r_c^+$ ,  $c_{xy}$  is true. Now, suppose  $\phi(r)$  satisfied, and as a contradiction,  $r_c$  cyclic, i.e.,  $\exists x. (x, x) \in r_c^+$ . Thus  $c_{xx}$  is true by  $(*)$ ; this contradicts the irreflexivity of  $<$  over the integers.

Now we show that  $r_c$  has finite prefixes, i.e., for each  $x$  we give a bound  $l$  over all  $S \in \text{pref}(r_c, x)$ . As a contradiction take  $S = [x_0, \dots, x_n] \in \text{pref}(r_c, x)$  s.t.  $x = x_0$  and  $\text{card}(S) > \text{clock}_x$ . Thus for all  $i$ , we have  $(x_{i+1}, x_i) \in r_c$  and  $\text{clock}_{x_{i+1}} < \text{clock}_{x_i}$  by  $(*)$ . Since  $n \geq \text{card}(S)$ ,  $\text{card}(S) > \text{clock}_x$  and  $\text{clock}_{x_0} = \text{clock}_x$ , we have  $\text{clock}_{x_n} < 0$ , which contradicts the fact that our clocks are naturals. Thus for each  $x$  we can take  $l = \text{clock}_x$ .

*$\Leftarrow$ :* Let  $r_c = \text{conc}(r, V)$ . For all  $e$  s.t.  $g(e) = \text{false}$ , take  $\text{clock}_e = 0$ . Thus  $c_{xy}$  is true if  $g(x)$  or  $g(y)$  is false. Now, have  $(x, y) \in r$  with both guards true, i.e.,  $(x, y) \in r_c$ . Take  $\text{clock}_x$  to be the maximal cardinality of the  $S$  in  $\text{pref}(r_c, x)$ , idem for  $y$ . We want to prove  $\text{clock}_x < \text{clock}_y$ . Take  $S$  s.t.  $\text{clock}_x = \text{card}(S)$ . From  $(x, y) \in r_c$ , we have  $[y] ++ S \in \text{pref}(r_c, y)$ . Now,  $\text{card}([y] ++ S) \leq \text{clock}_y$  by maximality of  $\text{clock}_y$ . It suffices to prove  $\text{card}(S) < \text{card}([y] ++ S)$ . Suppose  $\text{card}(S) \geq \text{card}([y] ++ S)$ . Then  $y$  appears in  $S$ . Thus  $(y, x) \in r_c^+$  since  $S$  is a prefix of  $x$ ; as  $(x, y) \in r_c$  by hypothesis, we have a cycle in  $r_c$ .

The formula  $\phi(r_1 \cup r_2)$  is equivalent to  $\phi(r_1) \wedge \phi(r_2)$ . Thus we encode unions of relations, e.g.,  $\text{ghb}_A \triangleq \text{ws} \cup \text{fr} \cup \text{grf}_A \cup \text{ppo}_A \cup \text{ab}_A$ , as the conjunction of their respective encodings. By Lem. 1, the acyclicity of  $\text{ghb}_A$  corresponds to the satisfiability of  $\phi(\text{ghb}_s)$ , where  $\text{ghb}_s$  is a symbolic encoding of  $\text{ghb}_A$ . To form  $\phi(\text{ghb}_s)$ , we form the conjunction of the formulas  $\phi(r)$ , for  $r$  being a symbolic encoding of  $\text{ws}$ ,  $\text{fr}$ ,  $\text{grf}_A$ ,  $\text{ppo}_A$  and  $\text{ab}_A$ .

We now present these encodings, in that order. Sec. 3 also relies on the program order per location for the uniproc check, and the dependencies for the thin check, omitted for brevity. We compute them alongside the preserved program order; they use independent sets of clock variables, but the same clock constraints.

We define auxiliaries over symbolic events:  $\text{tid}(e)$  is the thread identifier of  $e$ ,  $\text{addr}(e)$  the memory address read from or written to (e.g.,  $x$  for  $(e) \text{R}xy$ ), and  $\text{val}(e)$  its (symbolic) value. Each algorithm outputs constraints, whose conjunction we add to  $\text{pord}$ .

**input:**  $\gamma, A$     **output:**  $C_{\text{wf}}, C_{\text{rf}}, C_{\text{grf}}$

```

1 reads :=  $\{(\alpha, \{r_1 \dots r_n\}) \mid r_i \text{ is read} \wedge \text{addr}(r_i) = \alpha\}$ 
2 writes :=  $\{(\alpha, \{w_1 \dots w_n\}) \mid w_i \text{ is write} \wedge \text{addr}(w_i) = \alpha\}$ 
3  $C_{\text{rf}} := \emptyset; C_{\text{grf}} := \emptyset; C_{\text{wf}} := \emptyset$ 
4 foreach  $\alpha$  s.t.  $\exists R, W. (\alpha, R) \in \text{reads} \wedge (\alpha, W) \in \text{writes}$  do
5   foreach  $r \in R$  do
6     rf_some :=  $\emptyset$ 
7     foreach  $w \in W$  do
8       if  $(r, w) \notin \text{po}(\gamma)$  then
9         rf_some := rf_some  $\cup \{s_{wr}\}$ 
10         $C_{\text{wf}} := C_{\text{wf}} \cup \{s_{wr} \Rightarrow (g(w) \wedge \text{val}(r) = \text{val}(w))\}$ 
11         $C_{\text{rf}} := C_{\text{rf}} \cup \{s_{wr} \Rightarrow c_{wr}\}$ 
12        if  $(w, r)$  not relaxed on A and  $\text{tid}(w) \neq \text{tid}(r)$  then
13           $C_{\text{grf}} := C_{\text{grf}} \cup \{s_{wr} \Rightarrow c_{wr}\}$ 
14         $C_{\text{wf}} := C_{\text{wf}} \cup \{g(r) \Rightarrow \bigvee_{s \in \text{rf\_some}} s\}$ 

```

**Algorithm 1:** Constraints for read-from

For each algorithm we state and prove a lemma about its correctness. These follow the scheme of Lem. 1, i.e. we show the encoding correct for any satisfying valuation of clocks and **ssa**. Thus we will introduce symbolic encodings of sets  $\mathbf{r}(\gamma)$ , where membership in the set is given by a formula and thus depends on the actual valuation under  $\mathbf{C}$  and  $\mathbf{V}$ .

### 5.1 Read-from

For an architecture  $A$  and an **ses**  $\gamma$ , Alg. 1 encodes the read-from (resp. safe read-from) as the set of constraints  $C_{\text{rf}}$  (resp.  $C_{\text{grf}}$ ). Following Sec. 3, we add constraints to  $C_{\text{grf}}$  depending on: first, the relation being within one thread or between distinct threads (derivable from  $\text{tid}(w)$  and  $\text{tid}(r)$ ); second, whether  $A$  exhibits store buffering, store atomicity relaxation, or both.

Alg. 1 groups the reads and writes by address, in the sets **reads** and **writes** (lines 1 and 2). For **iriw**, **reads** =  $\{(x, \{a, d\}), (y, \{b, c\})\}$  and **writes** =  $\{(x, \{i_0, e\}), (y, \{i_1, f\})\}$ .

The next step forms the potential read-from pairs. To that end, Alg. 1 introduces a free Boolean variable  $s_{wr}$  for each pair  $(w, r)$  of write and read to the same address (line 9), unless such a pair contradicts program order (line 8). Indeed, if  $(w, r)$  is in **rf** and  $(r, w)$  is in **po**, this violates the uniproc check of Sec. 3.

The variable **rf\_some**, initialised in line 6, collects the variables  $s_{wr}$  in line 9. For **iriw**, the memory address  $x$ , and the read  $a$ , we have **rf\_some** =  $\{s_{i_0a}, s_{ea}\}$ , i.e., the read  $a$  can read either from  $i_0$  (the initial write to  $x$ ), or from the write  $e$  on  $P_2$ .

Following Sec. 3, each read must read from some write. We ensure this at line 14, by gathering in  $C_{\text{wf}}$ , for a given  $r$ , the union of all the potential read-from  $s_{wr}$  collected in **rf\_some**.

Going back to **iriw**, recall from Sec. 4 that an event has a guard indicating the branch of the program it comes from. In **iriw**, the guard of  $a$  is true (as all the others), i.e., the read  $a$  is concretely executed. Hence there exists a write (either  $i_0$  or  $e$ ) from which  $a$  reads, as expressed by the constraint  $s_{i_0a} \vee s_{ea}$  formed at line 14.

If  $s_{wr}$  evaluates to true (i.e.,  $r$  reads from  $w$ ), we record the *value constraint*  $\text{val}(r) = \text{val}(w)$  in the set  $C_{\text{wf}}$  (line 10). For **iriw**, we obtain the following for  $x$ :  $(s_{i_0a} \Rightarrow x_1 = x_0) \wedge (s_{i_0d} \Rightarrow x_2 = x_0) \wedge (s_{ea} \Rightarrow x_1 = x_3) \wedge (s_{ed} \Rightarrow x_2 = x_3)$ . The constraint  $s_{i_0a} \Rightarrow x_1 = x_0$  reads “if  $s_{i_0a}$  is true (i.e.,  $a$  reads from  $i_0$ ) then the value  $x_1$  read by  $a$  equals the value  $x_0$  written by  $i_0$ .”

The constraint added to  $C_{\text{rf}}$  is such that only if  $s_{wr}$  evaluates to true, the clock constraint  $c_{wr}$  is enforced (line 11). For **iriw** we add the following to  $C_{\text{rf}}$ , for the address  $x$ :  $(s_{i_0a} \Rightarrow c_{i_0a}) \wedge (s_{ea} \Rightarrow c_{ea}) \wedge (s_{i_0d} \Rightarrow c_{i_0d}) \wedge (s_{ed} \Rightarrow c_{ed})$ .

If  $(w, r)$  is not relaxed on  $A$ , we also add its clock constraint  $c_{wr}$  to  $C_{\text{grf}}$  (line 13). In **iriw**, all reads read from an external thread. Thus on an architecture that does not relax store atomicity (i.e., stronger than Power), we add the constraints that we added to  $C_{\text{rf}}$  to  $C_{\text{grf}}$  as well. On Power,  $C_{\text{grf}}$  remains empty.

We now write  $\text{grf}_A$  for both the function over concrete relations given by the definition of  $A$  as in Sec. 3, and the corresponding function over symbolic relations. Given an architecture  $A$ , we have  $\text{grf}_A(r) = \{(w, r) \in r \mid (w, r) \text{ is not relaxed on } A\}$ . For example if  $A$  is TSO, all thread-local read-from pairs are relaxed:  $\text{grf}_A(r) = \{(w, r) \in r \mid \text{tid}(w) \neq \text{tid}(r)\}$ . We write  $(w, r) \in \text{WR}_\alpha$  when  $w$  writes to an address  $\alpha$  and  $r$  reads from the same  $\alpha$ , and  $\text{prf}(\gamma) \triangleq \{(w, r) \in \bigcup_\alpha \text{WR}_\alpha \mid (r, w) \notin \text{po}(\gamma)\}$ . We write  $\text{rf}(\gamma)$  for the set  $\{(w, r) \in \text{prf}(\gamma) \mid s_{wr}\}$  (with  $s_{wr}$  of Alg. 1), and  $\text{grf}(\gamma)$  for  $\text{grf}_A(\text{rf}(\gamma))$ . Note that we build the external safe read-from ( $\text{grfe}(\gamma)$ ) only, i.e., between two events from distinct threads. We compute the internal one as part of  $\text{ppo}_A$ , in Alg. 4.

Given an  $\text{ses } \gamma$ , Alg. 1 outputs  $C_{\text{rf}}, C_{\text{grf}}$  and  $C_{\text{wf}}$ . Let  $\text{WR}$  be a valuation of the  $s_{wr}$  variables of  $\gamma$ . We write  $\text{inst}(\text{rf}(\gamma), \text{WR})$  (resp.  $\text{inst}(\text{grf}(\gamma), \text{WR})$ ) for  $\text{rf}(\gamma)$  (resp.  $\text{grfe}(\gamma)$ ) where  $\text{WR}$  instantiates the  $s_{wr}$  variables (thus  $\text{rf}(\gamma)$  is a symbolic encoding of the set as noted before this sub-section; we use this notation similarly in the remainder of this section). We show that Alg. 1 gives the clock constraints encoding  $\text{grf}$  (we omit the corresponding lemma for  $\text{rf}$ ):

**Lemma 2.**  $(C, V, \text{WR})$  satisfies  $\bigwedge_{c \in C_{\text{wf}} \cup C_{\text{grf}}} c$  iff  $(C, V)$  satisfies

- i) for all  $r$  s.t.  $g(r)$  is true, there is  $w$  s.t.  $(w, r) \in \text{inst}(\text{rf}(\gamma), \text{WR})$  and
- ii) for all  $(w, r) \in \text{inst}(\text{rf}(\gamma), \text{WR})$ ,  $g(w)$  is true and  $\text{val}(w) = \text{val}(r)$  and
- iii)  $\bigwedge_{(w, r) \in \text{inst}(\text{grfe}(\gamma), \text{WR})} c_{wr}$ .

*Proof.* An induction on  $R, W$  s.t.  $(\alpha, R) \in \text{reads}$  and  $(\alpha, W) \in \text{writes}$  for some address  $\alpha$ , then union for all  $\alpha$  shows that  $C_{\text{grf}} = \{s_{wr} \Rightarrow c_{wr} \mid (w, r) \in \text{prf}(\gamma) \cap \text{grfe}_A\}$ , and  $C_{\text{wf}} = \bigcup_{r \text{ is read}} \{g(r) \Rightarrow \bigvee_{(w, r) \in \text{prf}(\gamma)} s_{wr}\} \cup \{s_{wr} \Rightarrow (g(w) \wedge \text{val}(r) = \text{val}(w)) \mid (w, r) \in \text{prf}(\gamma)\}$ . The first component of  $C_{\text{wf}}$  is equivalent to i); the second to ii).  $C_{\text{grf}}$  is equivalent to iii).

The model described in Sec. 3 suggests that  $\text{rf}$  must be encoded to be exclusive, i.e., to link a read to only one write. An explicit encoding thereof, however,

**input:**  $\gamma$     **output:**  $C_{ws}$

- 1  $writes := \{(\alpha, \{w_1 \dots w_n\}) \mid w_i \text{ is write} \wedge \text{addr}(w_i) = \alpha\}$
- 2  $C_{ws} := \emptyset$ ; **foreach**  $\alpha$  *s.t.*  $\exists W. (\alpha, W) \in writes$  **do**
- 3    **foreach**  $w \in W$  **do**
- 4     **foreach**  $w' \in W$  *s.t.*  $\text{tid}(w') \neq \text{tid}(w)$  **do**
- 5        $C_{ws} := C_{ws} \cup \{\neg c_{ww'} \Rightarrow c_{w'w}\}$

**Algorithm 2:** Constraints for write serialisation

would be redundant, as this is already enforced by **ws** and **fr**. Hence it suffices to consider *at least one* write per read, as Alg. 1 does:

**Lemma 3.**  $\text{uniproc}(E, X) \Rightarrow \forall r. \neg(\exists w \neq w'. (w, r) \in rf \wedge (w', r) \in rf)$

*Proof.* By contradiction, have  $w \neq w'$  *s.t.*  $(w, r) \in rf$  and  $(w', r) \in rf$ . By totality of **ws**,  $(w, w') \in ws$  or  $(w', w) \in ws$ . W.l.o.g. have  $(w, w') \in ws$ . Then  $(r, w') \in fr$ , i.e., a cycle in  $rf \cup fr$ :  $w', r, w'$ , forbidden by **uniproc**.

## 5.2 Write serialisation

Given an **ses**  $\gamma$ , Alg. 2 encodes the write serialisation **ws** as the set of constraints  $C_{ws}$ . By definition, **ws** is a total order over writes to a given address. Alg. 2 implements the totality by ensuring that for two writes  $w \neq w'$  to the same address either  $c_{ww'}$  or  $c_{w'w}$  holds. For implementation reasons we choose to express this as  $\neg c_{ww'} \Rightarrow c_{w'w}$  rather than  $c_{ww'} \vee c_{w'w}$ .

Alg. 2 groups the writes per address. For each address  $\alpha$  and write  $w$  to  $\alpha$  (lines 2 and 3) we choose another write  $w'$  to  $\alpha$  (line 4), and build the disjunction of clock constraints over  $w$  and  $w'$  (line 5). For **iriw** we have  $writes = \{(x, \{i_0, e\}), (y, \{i_1, f\})\}$ , and the constraints:  $(\neg c_{i_0e} \Rightarrow c_{ei_0}) \wedge (\neg c_{i_1f} \Rightarrow c_{fi_1})$ .

Note that we build the external **ws** only (**wse**). With  $WW_\alpha$  the pairs of writes to the address  $\alpha$ , and  $ws(\gamma)$  the set  $\{(w, w') \in \bigcup_\alpha WW_\alpha \mid c_{w'w} = \text{false}\}$ , we have  $wse(\gamma) \triangleq ws(\gamma) \cap \{(w, w') \mid \text{tid}(w) \neq \text{tid}(w')\}$ . We compute the thread-local **ws** as part of  $\text{ppo}_A$ , in Alg. 4. Given an input  $\gamma$  of Alg. 2, we now characterise the clock constraints given by  $C_{ws}$ . Basically we show that Alg. 2 gives the clock constraints encoding **ws**. The proof (omitted for brevity) is by induction as for Lem. 2:

**Lemma 4.**  $(C, V)$  satisfies  $\bigwedge_{c \in C_{ws}} c$  iff it satisfies  $\bigwedge_{(w, w') \in wse} c_{ww'}$ .

We quantify over all pairs of writes to the same address to build **ws**. Thus for  $w_0, w_1, w_2$  in **ws** in a concrete execution, we build  $(w_0, w_1), (w_1, w_2)$  and the redundant  $(w_0, w_2)$  in the symbolic world. This is inherent to the totality of **ws**.

## 5.3 From-read

Given an **ses**  $\gamma$ , Alg. 3 encodes from-read as the set of constraints  $C_{fr}$ . Recall that  $(r, w) \in fr$  means  $\exists w'. (w', r) \in rf \wedge (w', w) \in ws$ . The existential quantifier

**input:**  $\gamma$     **output:**  $C_{fr}$

- 1 **reads**  $:= \{(\alpha, \{r_1 \dots r_n\}) \mid r_i \text{ is read} \wedge \text{addr}(r_i) = \alpha\}$
- 2 **writes**  $:= \{(\alpha, \{w_1 \dots w_n\}) \mid w_i \text{ is write} \wedge \text{addr}(w_i) = \alpha\}$
- 3  $C_{fr} := \emptyset$
- 4 **foreach**  $\alpha$  *s.t.*  $\exists R, W. (\alpha, W) \in \text{writes}, (\alpha, R) \in \text{reads}$  **do**
- 5    **foreach**  $(w, w') \in W \times W$  *s.t.*  $w' \neq w$  **do**
- 6     **foreach**  $r \in R$  *with*  $\text{tid}(r) \neq \text{tid}(w)$  **do**
- 7        $C_{fr} := C_{fr} \cup \{(s_{w'r} \wedge c_{w'w} \wedge g(w)) \Rightarrow c_{rw}\}$

**Algorithm 3:** Constraints for from-read

corresponds to a disjunction:  $\bigvee_{w' \text{ is write}} (w', r) \in \text{rf} \wedge (w', w) \in \text{ws}$ . Since this disjunction can be large, which is undesirable in the expression simplification used in the implementation, we rewrite it as a conjunction of small implications, each of which are simplified in isolation:  $\bigwedge_{w' \text{ is write}} ((r, w) \in \text{fr} \Leftarrow (w', r) \in \text{rf} \wedge (w', w) \in \text{ws})$ . Thus Alg. 3 encodes from-read as a conjunction of the premise variables  $s_{wr}$  of  $C_{fr}$  and clock variables  $c_{ww'}$  of  $C_{ws}$  introduced in Alg. 1 and 2.

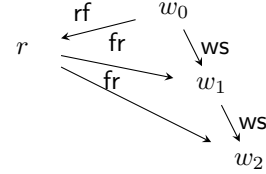
Again, we collect the sets of reads and writes per address. Alg. 3 considers triples  $(w', w, r)$  of events to the same address, where  $(w', w)$  is in the write serialisation, and  $(w', r)$  is in read-from. We enumerate the pairs of writes in line 5, and then pick a read in line 6. For each such triple we add in line 7 the clock constraint  $c_{rw}$  under the premise that i)  $(w', r) \in \text{rf}$ , witnessed by  $s_{w'r}$ , ii)  $(w', w) \in \text{ws}$ , witnessed by  $c_{w'w}$ , and that iii) the write  $w$  actually takes place in a concrete execution, i.e.,  $g(w)$  evaluates to true.

For **iriw** all guards are true. For  $x$ , we obtain:  $(s_{i_0a} \wedge c_{i_0e}) \Rightarrow c_{ae} \wedge ((s_{i_0d} \wedge c_{i_0e}) \Rightarrow c_{de}) \wedge ((s_{ea} \wedge c_{ei_0}) \Rightarrow c_{ai_0}) \wedge ((s_{ed} \wedge c_{ei_0}) \Rightarrow c_{di_0})$ . For example,  $(s_{i_0a} \wedge c_{i_0e}) \Rightarrow c_{ae}$ , reads “if  $s_{i_0a}$  is true (i.e., if  $a$  reads from  $i_0$ ), and if  $c_{i_0e}$  is true (i.e.,  $(i_0, e) \in \text{ws}$ ) then  $c_{ae}$  is true (i.e.,  $a$  is in **fr** before  $e$ ).”

Given an **ses**  $\gamma$ , Alg. 3 outputs  $C_{fr}$ . Note that we compute here the external from-read only (**fre**), and the internal one as part of **ppo**<sub>A</sub>, in Alg. 4. We show that Alg. 3 gives the clock constraints encoding **fr**. The (omitted) proof is as for Lem. 2:

**Lemma 5.**  $(C, V, WR)$  satisfies  $\bigwedge_{c \in C_{fr}} c$  iff  $(C, V)$  satisfies  $\bigwedge_{(r,w) \in \text{inst}(\text{fre}(\gamma), WR)} c_{rw}$ .

The **fr** defined above, together with **ws**, does introduce possible redundancies: given  $(w_0, r) \in \text{rf}$  with  $(w_0, w_1) \in \text{ws}$  and  $(w_1, w_2) \in \text{ws}$ , we have both  $(r, w_1) \in \text{fr}$  and  $(r, w_2) \in \text{fr}$  – but the latter is redundant as the same ordering is implied by  $(r, w_1) \in \text{fr}$  and  $(w_1, w_2) \in \text{ws}$ . We could thus, instead, build a fragment of **fr**, which we write  $\text{fr}_0$ . We define  $\text{fr}_0$  as  $\{(r, w_1) \mid \exists w_0. (w_0, r) \in \text{rf} \wedge (w_0, w_1) \in \text{ws} \wedge \nexists w'. ((w_0, w') \in \text{ws} \wedge (w', w_1) \in \text{ws})\}$ . In Fig. 5,  $(r, w_1)$  is in  $\text{fr}_0$  but not  $(r, w_2)$ , because there is a write  $(w_1)$  in **ws** between the write  $w_0$  from which  $r$  reads and  $w_2$ . One can show that  $(r, w) \in \text{fr}$  if  $(r, w) \in \text{fr}_0$  or



**Fig. 5.** **fr** derives from **rf** and **ws**



**input:**  $\gamma, A$     **output:**  $C_{ppo}$

```

1  $C_{ppo} := \emptyset$ ; foreach  $S \in po(\gamma) \wedge S \neq \emptyset$  do
2    $S = [e]++S' \cap \{e \mid e \text{ is not fence}\}$ 
3    $chains := [(e, \emptyset)]$ ;  $R := \text{true}$ 
4   foreach  $e' \in S'$  do
5      $T' := \emptyset$ 
6     foreach  $(e'', T'') \in chains$  s.t. there is no  $r$  s.t.
        $(e'', r) \in T'$  and  $((g(e') \wedge g(e'') \wedge R) \Rightarrow r)$  do
7        $r_{e''e'} := \text{not\_relax } A \gamma (e'', e')$ 
8       if  $r_{e''e'}$  is satisfiable then
9          $C_{ppo} := C_{ppo} \cup \{r_{e''e'} \Rightarrow c_{e''e'}\}$ 
10         $T' := T' \cup \{(e'', r_{e''e'})\}$ 
11        foreach  $(e, r) \in T''$  do
12          if  $\exists r'. (e, r') \in T'$  then
13             $R := R \wedge (\rho \Leftrightarrow r' \vee (r_{e''e'} \wedge r))$ 
14             $T' := \{(e, \rho)\} \cup T' \setminus (e, r')$ 
15          else  $T' := \{(e, r_{e''e'} \wedge r)\} \cup T'$ 
16    $chains := [(e', T')]++[chains]$ 

```

**Algorithm 4:** Constraints for preserved program order

there exists  $w'$  s.t.  $(r, w') \in fr_0$  and  $(w', w) \in ws$ , i.e., we can generate  $fr$  from  $fr_0$  and  $ws$ .

#### 5.4 Preserved program order

For an architecture  $A$  and an **ses**  $\gamma$ , Alg. 4 encodes the preserved program order as the set  $C_{ppo}$ . In Sec. 3, the function  $ppo_A$ , which is part of the definition of  $A$ , determines if  $A$  relaxes a pair  $(e, e')$  in program order in a concrete execution. For example, RMO and Power relax read-read pairs, but PSO and stronger do not.

We reuse the notation  $ppo_A$  for the function collecting non-relaxed pairs in symbolic program order. Unlike in Sec. 3, the non-relaxed pairs in symbolic program order also include the internal safe read-from, internal write serialisation, internal from-read, and the orderings due to Power's **isync** fence. We generate these constraints here, rather than in Alg. 1–3, to limit the redundancies. We write  $ppo_A(\gamma)$  for  $ppo_A(po-br(\gamma))$ , or only  $ppo_A$  if  $\gamma$  is clear from the context.

Alg. 4 avoids building redundant transitive closure constraints, taking into account the guards of events: for two events  $e_1, e_2$ , we build a constraint iff  $(e_1, e_2) \in ppo_A(\gamma)$ . If, e.g.,  $ppo_A(po-br) = po-br$  (on SC), Alg. 4 creates constraints only for neighbouring events in  $po-br(\gamma)$  in each control flow branch of the program.

As SSA and loop unrolling yield  $po(\gamma)$  (i.e., lists of symbolic events per thread) rather than  $po-br(\gamma)$  (the corresponding DAG), we cannot construct  $C_{ppo}$  by analysing control flow branches of the program. Building  $C_{ppo}$  from  $po(\gamma)$  requires some more work.

To build  $\text{ppo}_A$ , Alg. 4 uses the variable `chains`, a list of pairs  $(y, T)$ . For a given  $y$ , its companion set  $T$  contains the events  $x$  occurring before  $y$  in  $\text{ppo}_A^+$  together with a formula  $r$  that characterises all paths of  $\text{ppo}_A^+$  between  $x$  and  $y$ . We build  $r$  from formulas  $r_{e''e'}$  asserting that  $(e'', e') \in \text{ppo}_A$ , describing individual steps  $(e'', e')$  of a path between  $x$  and  $y$ .

We compute the formula  $r_{e''e'}$  at line 7, using the function `not_relax`. Given an ses  $\gamma$  and a pair  $(e'', e')$ , `not_relax`  $\gamma$   $(e'', e')$  returns a formula  $r_{e''e'}$  expressing the condition under which  $(e'', e')$  is not relaxed. For PSO or stronger models, `not_relax` only needs to take the direction of the events and their addresses into account. For instance, TSO relaxes write-read pairs, but nothing else. If a pair is necessarily relaxed, `not_relax` returns false, otherwise `not_relax`  $\gamma$   $(e'', e') = g(e'') \wedge g(e')$ . For models weaker than PSO, such as Alpha, RMO or Power, `not_relax` has to determine data- and control dependencies, and handle Power's `isync` fence. We resolve data dependencies via a definition-use data flow analysis [5] on the program part in program order between the two events. Control dependencies use the data dependency analysis to test whether there exists a branching instruction in program order between the events such that the branching decision is in data dependency with the first event. For `isync`, the approach is similar, except that in addition there must be an `isync` in program order between the branch and the second event. We then add the guard of the fence to the conjunction returned by `not_relax`.

For a given  $e'$ , we initialise its companion set  $T'$  at line 5, then increment it in lines 10–15. In line 14, we use fresh variables  $\rho$  constrained in the formula  $R$  (line 13) to avoid repeating sub-formulas, as is standard in, e.g., CNF encodings [16]. In line 7 we compute the condition  $r_{e''e'}$  for  $(e'', e')$  not being relaxed on  $A$  for each  $e''$  in `chains` (unless skipped for transitivity, see below). We generate the constraint  $r_{e''e'} \Rightarrow c_{e''e'}$  iff  $r_{e''e'}$  is satisfiable (line 9), i.e.,  $(e'', e')$  is not relaxed on  $A$ .

Now, suppose  $e_1, e_2, e_3$  on the same thread all in  $\text{ppo}_A$ ; the companion set of  $e_2$  is  $\{(e_1, r_{e_1e_2})\}$ , because  $(e_1, e_2) \in \text{ppo}_A$  and there is no other event before  $e_1$  on the thread. Suppose that Alg. 4 has already built the beginning of the chain formed by  $e_1, e_2$  and  $e_3$ , so that `chains` =  $[(e_2, \{(e_1, r_{e_1e_2})\}), (e_1, \emptyset)]$  (observe that the chains are in reverse order of `po`). At line 4, for each remaining  $e'$  on a given thread, i.e.,  $e_3$  in our example, Alg. 4 follows lines 5–9 and adds a constraint w.r.t. the immediate predecessor  $e_2$  of  $e_3$  in  $\text{ppo}_A$ . The subsequent elements of `chains` ( $e_1$  in our example) are also candidates for a clock constraint.

We do not add any constraint if  $(e_1, e_3)$  is guaranteed to be in  $\text{ppo}_A^+$ , as follows. Any remaining element of `chains` that belongs to the companion set  $T''$  of  $e''$  is added to  $T'$  at lines 11–15. As an instance, recall that  $e_1$  is in the companion set of  $e_2$ . Thus, after generating the constraint  $c_{e_2e_3}$  at line 9, we add  $e_1$  with its transitivity condition  $r_{e_2e_3} \wedge r_{e_1e_2}$  to  $T'$  at line 15. Then, line 6 iterates over the rest of `chains`, i.e.,  $(e_1, \emptyset)$ . With the updated set  $T'$  the test  $(e_1, r_{e_1}) \in T'$  yields  $r_{e_1} = r_{e_2e_3} \wedge r_{e_1e_2}$ , and thus amounts to checking the validity of  $(g(e_3) \wedge g(e_1)) \Rightarrow (r_{e_2e_3} \wedge r_{e_1e_2})$ . Remember that, unless there is an `isync`, the conditions  $r_{xy}$  amount to conjunctions over guards, hence in our example we

are checking the validity of  $(g(e_3) \wedge g(e_1)) \Rightarrow g(e_3) \wedge g(e_2) \wedge g(e_1)$ . If all three events  $e_1$ ,  $e_2$  and  $e_3$  are on the same control flow branch, the implication is valid because all guards are equal. This makes the test of line 6 fail and  $(e_1, e_3)$  will not be considered for adding another constraint  $c_{e_1 e_3}$ , which would have been redundant. When the implication is not valid, the test of line 6 succeeds; then we add another constraint  $c_{e_1 e_3}$ , as this is not redundant here.

This elaboration on guards is essential as witnessed by the following variant of our example: assume, in contrast to the above, that  $e_2$  is not a dominator of  $e_3$  on the control flow graph. This might occur in a program fragment (**if**  $e_1$  **then**  $e_2$ );  $e_3$ , where the guard of  $e_2$  would be different from that of  $e_1$  or  $e_3$ . If we were to skip  $(e_1, e_3)$  as above, the constraints would be insufficient to enforce the order of  $e_1$  before  $e_3$  when  $g(e_2)$  evaluates to false. In this case, the premises  $g(e_1) \wedge g(e_2)$  and  $g(e_2) \wedge g(e_3)$  of  $c_{e_1 e_2}$  and  $c_{e_2 e_3}$ , respectively, are false, hence the clock constraints  $\text{clock}_{e_1} < \text{clock}_{e_2}$  and  $\text{clock}_{e_2} < \text{clock}_{e_3}$  are not enforced, leaving the order of  $(e_1, e_3)$  unconstrained.

We illustrate Alg. 4 on the **ses**  $\gamma$  of **iriw** (cf. Fig. 3). Alg. 4 proceeds over  $\text{po}(\gamma)$ , equal to  $\{[i_0, i_1], [a, b], [c, d], [e], [f]\}$  for **iriw**. Given a non-empty list  $S$  of  $\text{po}(\gamma)$ , e.g.,  $S = [a, b]$  corresponding to  $P_0$ , the first non-fence event  $a$  of  $S$  initialises at line 3 the variable **chains** (explained below in detail). The loop at line 4 proceeds with the tail  $S'$  of the list  $S$ . Thus for  $P_0$  at this point we have **chains** =  $[(a, \emptyset)]$  and Alg. 4 proceeds with  $S' = [b]$ .

The contents of **chains** depend on the architecture  $A$ , as **iriw** shows. For  $P_0$ , recall that **chains** =  $[(a, \emptyset)]$  and only  $b$  remains in  $S'$ . If  $A$  relaxes read-read pairs, e.g., RMO or weaker, then  $(a, b)$  is relaxed. Thus we do not add any clock constraint to  $C_{\text{ppo}}$  at line 9 and eventually **chains** =  $[(b, \emptyset), (a, \emptyset)]$  in line 16. If  $A$  does not relax read-read pairs, e.g., PSO or stronger, we add  $c_{ab}$  to  $C_{\text{ppo}}$  at line 9 and add  $a$  with the guard conjunction true to  $T'$  at line 15. Thus **chains** =  $[(b, \{(a, \text{true})\}), (a, \emptyset)]$ . Let us now characterise the output of Alg. 4, given an input **ses**  $\gamma$ :

**Lemma 6.** *Alg. 4 outputs  $\{r_{xy} \Rightarrow c_{xy} \mid (x, y) \in \text{ppo}_A\}$ .*

*Proof.* We write  $L_1$  (resp.  $L_2$ ) for the loop from line 4 to 16 (resp. 6 to 15).  $L_1$  maintains the invariant that  $S = \text{rd}(\text{chains})++S'$ , where  $\text{rd}$  reverses its argument and deletes  $T$  for each element  $(e, T)$  of its argument. We write  $\text{path}_{x,y}^e(e_1, \dots, e_n)$  when there is a path from  $x$  to  $y$  in  $\text{ppo}_A(\gamma)$  passing by  $e$ , i.e.,  $e_1 = x$  and  $e_n = y$  and  $\forall i. (e_i, e_{i+1}) \in \text{ppo}_A(\gamma)$  and  $\exists i. e_i = e$ .  $L_2$  maintains the invariant that  $T' = \bigcup_{e \in [e'', e']} T_e^{e'}$ , where  $e \in [e'', e']$  means  $(e'', e) \in \text{po} \wedge (e, e') \in \text{po}$ , and  $T_e^{e'} = \{(x, r_x) \mid r_x = \bigvee_{\text{path}_{x,y}^e(e_1, \dots, e_n)} \bigwedge_{1 \leq i \leq n} r_{e_i e_{i+1}}\}$ . We conclude by double inclusion of  $C_{\text{ppo}}$  and  $\{r_{xy} \Rightarrow c_{xy} \mid (x, y) \in \text{ppo}_A\}$ , omitted for brevity.

Since the  $r_{xy}$  are guard conditions, we just need to evaluate the guards to evaluate them. We show that Alg. 4 gives the clock constraints encoding **ppo**; the proof is immediate by Lem. 6:

**Lemma 7.**  *$(C, V)$  satisfies  $\bigwedge_{c \in C_{\text{ppo}}} c$  iff it satisfies  $\bigwedge_{(x,y) \in \text{ppo}_A} c_{xy}$ .*

**input:**  $\gamma, A$     **output:**  $C_{ab'}$

```

1  $C_{ab'} := \emptyset$ ; foreach  $S \in \text{po}(\gamma) \wedge S \neq \emptyset$  do
2    $\text{fences} := \{s \mid s \in S \wedge s \text{ is fence}\}$ 
3   foreach  $e \in S \setminus \text{fences}$  do
4     foreach  $s \in \text{fences}$  do
5       if  $(e, s) \in \text{po}(\gamma)$  then
6          $C_{ab'} := C_{ab'} \cup \{g(s) \Rightarrow c_{es}\}$ 
7         if  $A$  is not store atomic then
8           foreach  $(w, e)$  being a  $w$ - $r$  pair s.t.  $\text{addr}(w) = \text{addr}(e)$  and
              $\text{tid}(w) \neq \text{tid}(e)$  do
9              $C_{ab'} := C_{ab'} \cup \{(g(s) \wedge s_{we}) \Rightarrow c_{ws}\}$ 
10        else  $C_{ab'} := C_{ab'} \cup \{g(s) \Rightarrow c_{se}\}$ 
11        if  $A$  is not store atomic then
12          foreach  $(e, r)$  being a  $w$ - $r$  pair s.t.  $\text{addr}(e) = \text{addr}(r)$  and
             $\text{tid}(e) \neq \text{tid}(r)$  do
13             $C_{ab'} := C_{ab'} \cup \{(g(s) \wedge s_{er}) \Rightarrow c_{sr}\}$ 

```

**Algorithm 5:** Constraints for memory fences

## 5.5 Memory fences and cumulativity

Given an architecture  $A$  and an ses  $\gamma$ , Alg. 5 encodes the fence orderings as the set  $C_{ab'}$ . A fence  $s$  potentially induces orderings over all  $(e, e')$  s.t.  $e$  is in **po** before  $s$  and  $e'$  after  $s$ , which is quadratic in the number of events in **po** for each fence. Cumulativity constraints depend on the read-from to appear in the concrete event structure, and again these are paired with all events before or after (in **po**) a fence. We alleviate this with the fence events (see below). The implementation supports x86's **mfence** and Power's **sync**, **lwsync** and **isync**. We handle **isync** as part of **ppo** in Alg. 4. We first present x86's **mfence** and Power's **sync**, then **lwsync**.

*Fences **mfence** and **sync*** Alg. 5 applies its procedure to **po**( $\gamma$ ) (line 1). For example, assume **sync** fences between the read-read pairs of  $P_0$  and  $P_1$  of **iriw**, associated with the fences events  $s_0$  and  $s_1$ . We then have  $\text{po}(\gamma) = \{[i_0, i_1], [a, s_0, b], [c, s_1, d], [e], [f]\}$ .

For each list  $S$  of **po**( $\gamma$ ) (i.e., per thread), we compute at line 2 the set **fences**, containing the fence events of  $S$ . For **iriw**, **fences** is empty for  $P_2$  and  $P_3$ . For  $P_0$ , we have **fences** =  $\{s_0\}$ , and  $\{s_1\}$  for  $P_1$ . We test at line 5 for each pair  $(e, s)$  s.t.  $e$  is a non-fence event and  $s$  is fence whether  $(e, s)$  is in program order, or rather  $(s, e)$ . We then build the according non-cumulative constraints, and constraints for A-cumulativity (for  $(e, s)$  in program order) or B-cumulativity (otherwise).

For non-cumulativity, if  $e$  is before (resp. after)  $s$  in program order, Alg. 5 produces at line 6 the clock constraint  $c_{es}$  (resp.  $c_{se}$  at line 10). In **iriw**, all guards are true, hence we generate  $c_{as_0}$  (resp.  $c_{cs_1}$ ) for the event  $a$  (resp.  $c$ ) in **po** before the fence  $s_0$  (resp.  $s_1$ ) on  $P_0$  (resp.  $P_1$ ). Line 10 generates  $c_{s_0b}$  (resp.  $c_{s_1d}$ ) for  $b$  (resp.  $d$ ), in **po** after the fence  $s_0$  (resp.  $s_1$ ) on  $P_0$  (resp.  $P_1$ ).

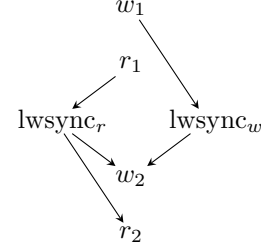
If  $A$  relaxes store atomicity, we build cumulativity constraints. For A-cumulativity, Alg. 5 adds at line 9 the constraint  $s_{we} \Rightarrow c_{ws}$ , for each  $(w, e)$  s.t.  $e$  is in **po** before

the fence  $s$ , and  $e$  reads from the write  $w$ . The constraint reads “if  $g(s)$  is true (i.e., the fence is concretely executed) and if  $s_{we}$  is true (i.e.,  $e$  reads from  $w$ ), then  $c_{ws}$  is true (i.e., there is a global ordering, due to the fence  $s$ , from  $w$  to  $s$ )”. All other constraints, i.e., the actual ordering of  $w$  before some event  $e'$  in **po** after  $s$ , follow by transitivity. We handle B-cumulativity in a similar way, given in lines 12 and 13.

As Power relaxes store atomicity, the **sync** fences between the read-read pairs of **iriw** create A-cumulativity constraints, namely for  $s_0$  (and analogous ones for  $s_1$ ):  $(s_{i_0a} \Rightarrow c_{i_0s_0}) \wedge (s_{ea} \Rightarrow c_{es_0})$ .

If we were not using fence events, we would create a clock constraint  $c_{we'}$  for every  $e'$  in program order after the fence  $s$  to implement Sec. 3, for each fence  $s$ . Thus the non-cumulative part would be cubic already, whereas fence events yield a quadratic number at most. For cumulativity, we would obtain a constraint for every pair  $(r, e')$  s.t.  $(w, r) \in \text{rfe}$  and  $r$  is in **po** before the fence  $s$ . The resulting number of constraints is the number of such pairs  $(r, e')$  times the number of pairs  $(w, r)$ , i.e., cubic in the number of events *per fence*  $s$ . Furthermore cases of both A- and B-cumulativity at the same fence  $s$  need to be taken into account, resulting in even higher complexity. Fence events, however, reduce all these cases, including the combined one, to cubic complexity (all triples of external writes, reads, and fence events).

*Fence **lwsync*** As **lwsync** does not order write-read pairs (cf. Sec. 3), we need to avoid creating a constraint  $c_{wr}$  between a write  $w$  and a read  $r$  separated by an **lwsync**. To do so, we use two distinct clock variables  $\text{clock}_s^r$  and  $\text{clock}_s^w$  for an **lwsync**  $s$ . This avoids the wrong transitive constraint  $c_{wr}$  implied by  $c_{ws}$  and  $c_{sr}$ . Fig. 6 illustrates this setup: the write-read pair  $(w_1, r_2)$  will not be ordered by any of the constraints, but all other pairs are ordered.



**Fig. 6.** Constraints for **lwsync**

To create a clock constraint in lines 6, 9, 10, or 13, we then pick one or both of the clock variables, as follows. If  $e$  is a read, the clock constraint is  $\text{clock}_e < \text{clock}_s^r$  when  $e$  is before  $s$ , i.e., lines 6 or 9 (or  $\text{clock}_s^r < \text{clock}_e$  if  $e$  is after, i.e., lines 10 or 13). If  $e$  is a write preceding  $s$  (i.e., lines 6 or 9), the clock constraint is  $\text{clock}_e < \text{clock}_s^w$ . Finally, if  $e$  is a write after  $s$ , i.e., lines 10 or 13, the clock constraint is the conjunction  $(\text{clock}_s^w < \text{clock}_e) \wedge (\text{clock}_s^r < \text{clock}_e)$ . To make **lwsync** non-cumulative (cf. footnote in Sec. 3), we just need to disable the lines 8,9,12 and 13.

In **iriw**, if we use **lwsync** instead of **sync** as discussed above, we obtain the following constraints:  $(\text{clock}_a < \text{clock}_{s_0}^r) \wedge (\text{clock}_{s_0}^r < \text{clock}_b) \wedge (s_{i_0a} \Rightarrow \text{clock}_{i_0} < \text{clock}_{s_0}^w) \wedge (s_{ea} \Rightarrow \text{clock}_e < \text{clock}_{s_0}^w)$ . These constraints will *not* order the writes  $i_0$  or  $e$  with the read  $b$ , because  $i_0$  and  $e$  are ordered w.r.t. to  $\text{clock}_{s_0}^w$ , but  $b$  is only ordered w.r.t. the distinct  $\text{clock}_{s_0}^r$ . This corresponds to the fact that placing **lwsync** fences in **iriw** does not forbid the non-SC execution.

Given an **ses**  $\gamma$ , Alg. 5 outputs  $C_{ab'}$ . We let  $\text{rfe}(\gamma)$  be  $\{(w, r) \in \bigcup_{\alpha} \text{WR}_{\alpha} \mid \text{tid}(w) \neq \text{tid}(r) \wedge s_{wr}\}$ . We write  $\text{ab}'(\gamma)$  for  $\{(e_1, e_2) \mid \text{nc}'(e_1, e_2) \vee \text{ac}'(e_1, e_2) \vee$

$bc'(e_1, e_2)\}$ , where  $nc'(e_1, e_2)$  corresponds to non-cumulativity, i.e.,  $(e_1, e_2) \in po(\gamma) \wedge ((g(e_1) \wedge e_1 \text{ is fence}) \vee (g(e_2) \wedge e_2 \text{ is fence})) \wedge \text{not both } e_1 \text{ and } e_2 \text{ are fences}$ ,  $ac'(e_1, e_2)$  to A-cumulativity, i.e.,  $\exists r. (e_1, r) \in rfe(\gamma) \wedge (r, e_2) \in po(\gamma) \wedge g(e_2) \wedge e_2 \text{ is fence}$ , and  $bc'(e_1, e_2)$  corresponds to B-cumulativity, i.e.,  $\exists w. (w, e_2) \in rfe(\gamma) \wedge (e_1, w) \in po(\gamma) \wedge g(e_1) \wedge e_1 \text{ is fence}$ . We show that Alg. 5 gives the clock constraints encoding  $ab'$ . The proof is immediate like for Lem. 2:

**Lemma 8.**  $(C, V, WR)$  satisfies  $\bigwedge_{c \in C_{ab'}} c$  iff  $(C, V)$  satisfies  $\bigwedge_{(x,y) \in inst(ab'(\gamma), WR)} c_{xy}$ .

We let  $ab(\gamma)$  be the symbolic version of  $ab$  in Sec. 3, i.e., we let  $nc(e_1, s, e_2)$  be  $g(s) \wedge s \text{ is fence} \wedge (e_1, s) \in po(\gamma) \wedge (s, e_2) \in po(\gamma)$ ,  $ac(e_1, s, e_2)$  be  $\exists r. (e_1, r) \in rfe(\gamma) \wedge nc(r, s, e_2)$  and  $bc(e_1, s, e_2)$  be  $\exists w. nc(e_1, s, w) \wedge (w, e_2) \in rfe(\gamma)$ . We only prove this encoding sound w.r.t. Sec. 3, as  $ab'$  is more fine-grained than  $ab$  (to see why, note that one cannot express  $nc'(e_1, e_2)$  as a combination of  $nc, ac$  or  $bc$ ). Yet we prove our overall encoding complete in Thm. 1.

**Lemma 9.** If  $(C, V, WR)$  satisfies  $\bigwedge_{c \in C_{ab'}} c$  then  $(C, V)$  satisfies  $\bigwedge_{(e_1, e_2) \in inst(ab(\gamma), WR)} c_{e_1 e_2}$ .

*Proof.* We give only the case  $lwsync(\gamma)$ . Take  $(e_1, e_2) \in lwsync(\gamma)$ , i.e., there is an  $lwsync$   $s$  s.t.  $nc(e_1, s, e_2)$  or  $ac(e_1, s, e_2)$  or  $bc(e_1, s, e_2)$ . In the  $nc$  case, we know that  $s$  is a fence and  $g(s)$  is true, and  $(e_1, s) \in po(\gamma)$  and  $(s, e_2) \in po(\gamma)$ , i.e.,  $nc'(e_1, s)$  and  $nc'(s, e_2)$ . Thus  $c_{e_1 s}$  and  $c_{se_2}$  are in  $C_{ab'}$ . Now, by definition of  $lwsync(\gamma)$ ,  $(e_1, e_2) \notin WR$ . For  $(e_1, e_2) \in WW$ ,  $c_{e_1 s}$  is  $clock_{e_1} < clock_s^w$  and  $c_{se_2}$  is  $clock_s^w < clock_{e_2}$ . Thus  $clock_{e_1} < clock_{e_2}$ , i.e.,  $c_{e_1 e_2}$  holds. Writing  $RR$  for the read-read pairs, take  $(e_1, e_2) \in RR$ . Thus  $c_{e_1 s}$  is  $clock_{e_1} < clock_s^r$  and  $c_{se_2}$  is  $clock_s^r < clock_{e_2}$ . Hence  $clock_{e_1} < clock_{e_2}$ , i.e.,  $c_{e_1 e_2}$  holds. For  $(e_1, e_2) \in RW$ ,  $c_{e_1 s}$  is  $clock_{e_1} < clock_s^r$  and  $c_{se_2}$  is  $(clock_s^w < clock_{e_2}) \wedge (clock_s^r < clock_{e_2})$ . Thus  $clock_{e_1} < clock_{e_2}$ , i.e.,  $c_{e_1 e_2}$  holds. In the  $ac$  case,  $s$  is a fence and  $g(s)$  is true, and there is  $r$  s.t.  $(e_1, r) \in rfe(\gamma)$  and  $nc(r, s, e_2)$ . Thus  $e_1$  is a write (source of a  $rf$ ), and since  $(e_1, e_2) \notin WR$  (by definition of  $lwsync(\gamma)$ ),  $e_2$  is a write. So  $ac'(e_1, s)$ , and  $nc'(s, e_2)$ , i.e.,  $c_{e_1 s}$  and  $c_{se_2}$  hold. We are back to the  $WW$  case. In the  $bc$  case,  $s$  is a fence and  $g(s)$  is true, and there is  $w$  s.t.  $nc(e_1, s, w)$  and  $(w, e_2) \in rfe(\gamma)$ . Thus  $e_2$  is a read (target of a  $rf$ ), and since  $(e_1, e_2) \notin WR$ ,  $e_1$  is a read. So  $nc'(e_1, s)$  and  $bc'(s, e_2)$ , i.e.,  $c_{e_1 s}$  and  $c_{se_2}$  hold. We are back to the  $RR$  case.

## 5.6 Soundness and completeness of the encoding

Given an architecture  $A$  and a program, the procedure of Sec. 4 and Sec. 5 outputs a formula  $ssa \wedge pord$  and an  $ses \gamma$ . This formula provably encodes the executions of this program valid on  $A$  and violating the property encoded in  $ssa$  in a sound and complete way. Proving this requires proving that any assignment to the system corresponds to a valid execution of the program, and vice versa. This result requires three steps, one for uniproc, one for thin and one for the acyclicity of  $ghb$ . By lack of space, we show only the last one. Given an  $ses \gamma$ , we write  $\phi$  for  $\bigwedge_{c \in C_{ppo} \cup C_{grf} \cup C_{wrf} \cup C_{ws} \cup C_{ab'}} c$ :

**Theorem 1.** *The formula  $\text{ssa} \wedge \phi$  is satisfiable iff there are  $V$ , a valuation to the symbols of  $\text{ssa}$ , and a well formed  $X$  s.t.  $\text{ghb}_A(\text{conc}(\gamma, V), X)$  is acyclic and has finite prefixes.*

*Proof.* Let  $(C, V, WR)$  be a satisfying assignment of  $\text{ssa} \wedge \phi$ . By Lem. 7, 2, 4, 5 and 8, we know that  $(C, V, WR)$  satisfies  $\phi$  iff *i*) for all  $r$  s.t.  $\text{g}(r)$  is true, there is  $w$  s.t.  $(w, r) \in \text{inst}(\text{rf}(\gamma), WR)$  and *ii*) for all  $(w, r) \in \text{inst}(\text{rf}(\gamma), WR)$ ,  $\text{g}(w)$  is true and  $\text{val}(w) = \text{val}(r)$  and *iii*)  $(C, V, WR)$  satisfies  $\phi(\text{ppo}_A(\gamma)) \wedge \phi(\text{inst}(\text{grf}(\gamma), WR)) \wedge \phi(\text{wse}(\gamma)) \wedge \phi(\text{inst}(\text{fr}(\gamma), WR)) \wedge \phi(\text{inst}(\text{ab}'(\gamma), WR))$ .

$\Rightarrow$ : Take  $X = (\text{conc}(\text{inst}(\text{rf}(\gamma), WR), V), (\text{conc}(\text{ws}(\gamma), V)))$ . Note that *i*) and *ii*), together with Lem. 3, imply that  $\text{rf}(X)$  is well formed. For  $\text{ws}(X)$ , this comes from the totality of  $\text{ws}(\gamma)$  over writes to the same address, implied by the shape of  $C_{\text{ws}}$  (cf. Alg. 2) for the external  $\text{ws}$ , and by the totality of  $\text{po}(\gamma)$  for the internal  $\text{ws}$ .

By Lem. 8 and 9, *iii*) says that  $(C, V, WR)$  satisfies  $\phi(r)$ , with  $r = \text{ppo}_A(\gamma) \cup \text{inst}(\text{grf}_A(\gamma), WR) \cup \text{ws}(\gamma) \cup \text{inst}(\text{fr}(\gamma), WR) \cup \text{inst}(\text{ab}(\gamma), WR)$ . By Lem. 1, since  $\text{ghb}_A(\text{conc}(\gamma, V), X)$  is  $\text{conc}(r, V)$ , we have our result.

$\Leftarrow$ : We let  $E$  be  $\text{conc}(\gamma, V)$ . Take  $WR$  s.t.  $s_{wr}$  is true iff  $(w, r) \in \text{rf}(X)$ .  $\text{rf}(X)$  being well-formed implies *i*) and *ii*).

We let  $\text{ghb}'_A(E, X)$  be  $r_1 \cup \text{ab}'(E, X)$ , with  $r_1 = \text{ppo}_A(E) \cup \text{grf}_A(X) \cup \text{ws}(X) \cup \text{fr}(E, X)$ . Note that  $\text{ghb}_A(E, X)$  is  $r_1 \cup \text{ab}(E, X)$ . We show below that the acyclicity of  $\text{ghb}_A(E, X)$  implies the acyclicity of  $\text{ghb}'_A(E, X)$  (idem for finite prefixes). Then we take  $r = \text{ghb}'_A(E, X)$  in Lem. 1, and take  $C$  as in Lem. 1. Hence we have  $(C, V)$  satisfying  $\phi(\text{ppo}_A(\gamma)) \wedge \phi(\text{inst}(\text{grf}(\gamma), WR)) \wedge \phi(\text{wse}(\gamma)) \wedge \phi(\text{inst}(\text{fr}(\gamma), WR)) \wedge \phi(\text{inst}(\text{ab}'(\gamma), WR))$ , namely *iii*); our result follows.

We let  $r_2 = \{(e_1, e_2) \in \text{ab}' \mid \text{neither } e_1 \text{ nor } e_2 \text{ is a fence}\}$ . We write  $(x, y) \in r; r'$  for  $\exists z. (x, z) \in r \wedge (z, y) \in r'$ . One can show: (\*) if  $(e_1, e_2) \in \text{ghb}^+$ , we have  $(e_1, e_2)$  in  $\text{ab}'$ ;  $(r_1^+ \cup r_2^+)^+$ , or  $(r_1^+ \cup r_2^+)^+; \text{ab}'$ , or  $(r_1^+ \cup r_2^+)^+$ .

Acyclicity: by contradiction, take a cycle in  $\text{ghb}'_A(E, X)$ , i.e.,  $x$  s.t.  $(x, x) \in (\text{ghb}'_A(E, X))^+$ . In the first two cases (\*),  $\text{ab}'$  connects two non-fence events, a contradiction. Hence a cycle in  $\text{ghb}'$  implies one in  $r_1 \cup r_2$ , i.e., in  $\text{ghb}$  since  $r_2 \subseteq \text{ab}$ .

Prefixes: as a contradiction, take an infinite path in  $\text{ghb}'_A(E, X)^+$ . Only the cases  $(r_1^+ \cup r_2^+)^+$  and  $\text{ab}'$ ;  $(r_1^+ \cup r_2^+)^+$  of (\*) apply, and both imply an infinite path in  $(r_1^+ \cup r_2^+)^+$ . Hence, we have an infinite prefix in  $\text{ghb}^+$ , since  $r_2 \subseteq \text{ab}$ .

To decide the satisfiability of  $\phi$ , we can use any solver supporting a sufficiently rich fragment of first-order logic. The procedure reveals the concrete executions, as expressed by Thm. 1.

## 5.7 Comparison to [14] and [61,62]

Both [14] and [61,62] use an SSA encoding similar to our  $\text{ssa}$  of Sec. 4. The difference resides in the ordering constraints.

[14] encodes total orders over memory accesses. Thus, in contrast to our clock variables with less-than constraints, [14] uses a Boolean variable  $M_{xy}$  per

pair  $(x, y)$ , whose value places  $x$  and  $y$  in a total order: either  $x$  before  $y$ , or  $y$  before  $x$ . Prog. 1 has  $3 \cdot N$  memory accesses per thread, hence [14]’s encoding has  $6 \cdot N \cdot (6 \cdot N - 1)$  Boolean variables. [14] builds additional constraints for the transitive closure; their number is at least cubic in the number of variables  $M_{xy}$ , leading to  $\mathcal{O}(N^6)$  constraints.

We only consider relations per address, except for program order and fence orderings, and do not build transitive closures. The constraints for **fr** and **ab** are cubic in the worst case; all others are quadratic. In Prog. 1, the write serialisation is internal, hence **fr** is only quadratic. Hence our number of constraints is  $\mathcal{O}(N^2)$ .

[61,62] use partial orders like us; they note redundancies in their constraints in [62] but do not explain them, which we do below. Basically, [61,62] quantify over all events regardless of their address, whereas we mostly build constraints per address. Fig. 7 shows that the maximal number of events to a single address is experimentally much smaller than the total number of events.

Our notations correspond to the ones of [62] as follows (the original description [61] has different notations).  $\text{HB}(a, b)$  is our clock constraint  $c_{ab}$ . The functions  $\text{addr}$  and  $\text{val}$  map to ours;  $\text{en}(x)$  is our  $g(x)$ ;  $\text{link}(r, w)$  denotes that  $r$  reads from  $w$ , i.e., our  $s_{wr}$ . [62] expresses **po**, **rf**, **fr**, and **ws** as follows (since it is restricted to SC, it gives no encoding of **ppo**<sub>A</sub>, **grf**<sub>A</sub>, and **ab**<sub>A</sub>).

[62] encodes **po** as the conjunction of the  $c_{a_i a_j}$ , with  $a_i$  in **po** before  $a_j$ . If the implementation of [62] strictly follows this definition, it redundantly includes the transitive closure constraints, which we avoid by building the transitive reduction in Alg. 4.

[62] encodes **rf** in  $\Pi_1 := \forall r. \exists w. g(r) \Rightarrow (g(w) \wedge s_{wr})$  and  $\Pi_2 := \forall r. \forall w. s_{wr} \Rightarrow (c_{wr} \wedge \text{addr}(r) = \text{addr}(w) \wedge \text{val}(r) = \text{val}(w))$ . [62] forces **rf** to be exclusive. We explained in Sec. 5.1 why this is unnecessary in our case, which allows us to only build a disjunction over writes (cf. Alg. 1) linear in their number.

$\Pi_2$  combines our value and clock constraints, with one major difference:  $\Pi_2$  ranges over all reads and writes, regardless of their address. Our **rf** (Alg. 1) ranges over pairs to the same address, thus reaches this number only when all reads and writes have the same memory address, which is unlikely in non-trivial programs.

Ranging over the same address, as we do, and not all addresses, as in [62], becomes even more advantageous in  $\Pi_3$ , encoding **fr**:  $\Pi_3 := \forall r. \forall w. \forall w'. (s_{wr} \Rightarrow (g(w') \wedge \neg c_{w'w} \wedge \neg c_{rw'} \Rightarrow \text{addr}(r) \neq \text{addr}(w')))$ .  $\Pi_3$  ranges over all  $(r, w, w')$ , again independently of their addresses. For distinct addresses the conjunction holds trivially, but [62] builds it nevertheless. Our **fr** (cf. Alg. 3) quantifies only over the same address, thus spares these trivial constraints.

[62] does not encode **ws**. The totality of **ws** comes as a side effect: [62] initialises each write with a unique integer, hence writes are totally ordered by  $<$  over integers. This is again regardless of the addresses, whereas we order writes to the same address only.



## 6 Experimental Results

We detail here our experiments, which indicate that our technique is scalable enough to verify non-trivial, real-world concurrent systems code, including the worker-synchronisation logic of the relational database PostgreSQL, code for socket-handover in the Apache httpd, and the core API of the Read-Copy-Update mutual exclusion code from Linux 3.2.21.

We implement our technique within the bounded model checker CBMC [18], using a SAT solver as an underlying decision procedure. We see two primary comparison points to estimate the overhead introduced by the partial order constraints. First, we pass the benchmarks with a single, fixed interleaving to sequential CBMC. Our implementation performs comparably to sequential CBMC, as Fig. 7 shows (rows “sequential” and “concurrent”). Second, we compare to ESBMC [19], which also implements bounded model checking, but uses interleaving-based techniques.

In Fig. 7, we gather facts about all examples: the Fibonacci example from [11] with  $N=5$ , 4500 litmus tests (see below), the worker synchronisation in PostgreSQL, RCU, and fdqueue in Apache httpd. For each we give the number of lines of code (LOC), the number of distinct memory addresses “tot. addr” (including unused shared variables), the total number of shared accesses “tot. shared”, the maximal number of accesses to a single address “same addr”, the total number of constraints “all constr” and the relation with the most costly encoding, in terms of the number of constraints generated. We give the loop unrolling bounds “unroll”: we write “none” when there is no loop, and “bounded” when the loops in the program are natively bounded.

The total number of shared accesses is on average 13 times the maximal number of accesses to a single address. The most costly constraint is usually the read-from, or the barriers, which build on read-from. The time needed by our tool to analyse a program grows with the total number of constraints generated. ESBMC is 4 times slower than our tool on Fibonacci, 3050 times slower on the litmus tests, times out on PostgreSQL, and cannot parse RCU and Apache.

	Fibo.	Litmus	PgSQL	RCU	Apache
LOC	41	50.9	5412	5834	28864
unroll	5	none	2	bounded	5
tot. addr	2	11.8	6	3	8
tot. shared	45	58.7	233	107	88
same addr	11	3.7	72	4	5
all constr	308	874	3762	90	160
most costly	rf (178)	ab (342)	rf (1868)	rf (33)	rf (49)
sequential	0.3 s	0.1 s	4.1 s	0.8 s	1.7 s
concurrent	3.3 s	0.2 s	90.0 s	1.0 s	2.8 s
ESBMC	13.8 s	609.8 s	t/o	parse err	parse err

**Fig. 7.** Facts about all examples

	CBMC	CBMC	CBMC	CheckFence	CImpact	ESBMC	Poirot	SatAbs	Threader
	SC	TSO	Power	SC, TSO	SC	SC	SC	SC	SC
F	CE $N = 300$	CE $N = 220$	CE $N = 240$	conv err	t/o $N = 1$	CE $N = 10$	fails $N \geq 1$	V $N = 3$	t/o $N = 1$
L	100%	100%	100%	18%	20%	34%	47%	100%	8%
P	V	V	CE	conv err	aborts	t/o	parse err	t/o	n/a
Pf	V	V	V	conv err	aborts	t/o	parse err	t/o	n/a
R	V	V	V	conv err	aborts	parse err	parse err	ref err	n/a
A	V	V	V	conv err	aborts	parse err	parse err	aborts	n/a

**Fig. 8.** Comparison of all tools on all examples (time out 30 mins)

*Other tools* There are very few tools for verifying concurrent C programs, even on SC [21]. For weak memory, existing techniques are restricted to TSO, and its siblings PSO and RMO [14,44,43,9,3,49]. Not all of them have been implemented, and only few handle systems code given as C programs.

Thus, as a further comparison point, we implemented an instrumentation technique [7], similar to [9]. The technique of [9] is restricted to TSO, and consists in delaying writes, so that the SC executions of the instrumented code simulate the TSO executions of the original program. Our instrumentation handles all the models of Sec. 3.

We tried 5 ANSI-C model checkers: SatAbs, a verifier based on predicate abstraction [17]; ESBMC; CImpact, a variant of the Impact algorithm [52] extended to SC concurrency; Threader, a thread-modular verifier [34]; and Poirot, which implements a context-bounded translation to sequential programs [45]. These tools cover a broad range of techniques for verifying SC programs. We also tried CheckFence [14].

In Fig. 8, we compare all tools on all examples: F for Prog. 1, L for the litmus tests, P for PostgreSQL with its bug, Pf for our fix, R for RCU and A for Apache. For L, P, R and A, the bounds are as in Fig. 7; for Pf we take the one of P. For F we try the maximal  $N$  that the tool can handle within the time out of 30 mins. For each tool, we specify the model below. We write “t/o” when there is a timeout. We write “fail” when the tool gives a wrong answer. CheckFence provides a conversion module from C to its internal representation; we write “conv err” when it fails. We write “parse err” when the tool cannot parse the example. SatAbs uses a refinement procedure; we write “ref err” when it fails. When a tool verifies an example we write “V”; when it finds a counterexample we write “CE”.

*Fibonacci* All tools, except for ESBMC, SatAbs and ours, fail to analyse Fibonacci. Poirot claims the assertion is violated for any  $N$ , which is not the case for  $1 \leq N \leq 5$ . SatAbs does not reach beyond  $N = 4$ . Our tool handles more than  $N = 300$ , which is 30 times more loop unrolling than ESBMC, within the same amount of time.

*Litmus tests* We analyse 4500 tests exposing weak memory artefacts, e.g., instruction reordering, store buffering, store atomicity relaxation. These tests are generated by the diy tool [8], which generates assembly programs with a final state unreachable on SC, but reachable on a weaker model. For example, **iriw**

(Fig. 2) can only be reached on RMO (by reordering the reads) or on Power (*idem*, or because the writes are non-atomic).

We convert these tests into C code, of 50 lines on average, involving 2 to 4 threads. Despite the small size of the tests, they prove challenging to verify, as Fig. 8 shows: most tools, except Blender, SatAbs and ours, give wrong results or fail in other ways on a vast majority of tests, even for SC. For each tool we give the average percentage of correct results over all models. Our tool verifies all tests on all models in 0.22s on average.

*PostgreSQL* Developers observed that a regression test failed on a PowerPC machine<sup>4</sup>, and later identified the memory model as possible culprit: the processor could delay a write by a thread until after a token signalling the end of this thread’s work had been set. Our tool confirmed the bug, and proved a patch we proposed. A detailed description of the problem is in [7].

*RCU Read-Copy-Update* (RCU) is a synchronisation mechanism of the Linux kernel, introduced in version 2.5. Writers to a concurrent data structure prepare a fresh component (e.g., list element), then replace the existing component by adjusting the pointer variable linking to it. Clean-up of the old component is delayed until there is no process reading.

Thus readers can rely on very lightweight (and thus fast) lock-free synchronisation only. The protection of reads against concurrent writes is fence-free on x86, and uses only a light-weight fence (`lwsync`) on Power. We verify the original implementation of the 3.2.21 kernel for x86 (5824 lines) and Power (5834 lines) in less than 1s, using a harness that asserts that the reader will not obtain an inconsistent version of the component. On Power, removing the `lwsync` makes the assertion fail.

*Apache* The Apache httpd is the most widely used HTTP server software. It supports a broad range of concurrency APIs distributing incoming requests to a pool of workers.

The fdqueue module (28864 lines) is the central part of this mechanism, which implements the hand-over of a socket together with a memory pool to an idle worker. The implementation uses a central, shared queue for this purpose. Shared access is primarily synchronised by means of an integer keeping track of the number of idle workers, which is updated via architecture-dependent compare-and-swap and atomic decrement operations. Hand-over of the socket and the pool and wake-up of the idle thread is then coordinated by means of a conventional, heavy-weight mutex and a signal. We verify that hand-over guarantees consistency of the payload data passed to the worker in 2.45s on x86 and 2.8s on Power.

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<sup>4</sup> <http://archives.postgresql.org/pgsql-hackers/2011-08/msg00330.php>

## 7 Conclusion

Our experiments demonstrate that weakness is a virtue for programs with bounded loops. Our proofs suggest that this contention is not limited to bounded loops, but impracticable as is, since it involves infinite structures. Thus we believe that this work opens up new possibilities for over-approximation for programs with unbounded loops, which we hope to investigate in the future.

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