## Chain Belt C-C Distance Calculator

## AMB Calculator

This calculator determines the proper center-to-center spacing between two sprockets or pulleys, as based on the belt/chain length or an approximate center-to-center distance.

The calculator comes loaded with the standard types of belt and chain used in FRC, and their respective pitch lengths, p. If you want a non-standard pitch length, you can set that by choosing a "Custom" type. For the standard types, useful dimensions are also shown including the width, thickness, and weight. These dimensions do not affect the calculations. Suggested adders (i.e. the recommended amount to over/under tension the system) and load ratings are given for some types, though these values may vary based on the source of the belt/chain and team preference.

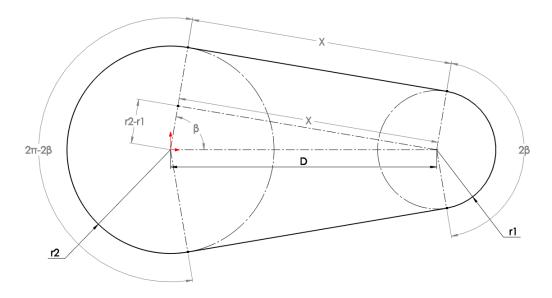
For this document we will use the nomenclature for chain, though the equations apply identically to belts.

To calculate the pitch diameter of each sprocket with n teeth, we use the formula:

$$d_p = \frac{n \cdot p}{\pi} \tag{1}$$

Halving this value gives us the pitch radius for each sprocket,  $r_x = \frac{n_x p}{2\pi}$ .

Defining the total chain length as L and the number of links as  $\ell$ , we know  $L = \ell \cdot p$ . We can also find the total chain length geometrically using the sprocket radii and center-to-center distance D.



Adding up the various sections of the chain path, it is clear that:

$$L = 2X + r_1 \cdot 2\beta + r_2 \cdot (2\pi - 2\beta) \tag{2}$$

Using the Pythagorean theorem and trigonometry on the right triangle in the center of the diagram, we can see that:

$$D^{2} = X^{2} + (r_{2} - r_{1})^{2} \implies X = \sqrt{D^{2} - (r_{1} - r_{2})^{2}}$$
(3)

$$\beta = \cos^{-1}\left(\frac{r_2 - r_1}{D}\right) = \cos^{-1}\left(\frac{r_1 - r_2}{D}\right)$$
 (4)

Substituting (3) and (4) into (2) gives:

$$L = 2\sqrt{D^2 - (r_1 - r_2)^2} + r_1 \cdot 2\cos^{-1}\left(\frac{r_1 - r_2}{D}\right) + r_2 \cdot \left(2\pi - 2\cos^{-1}\left(\frac{r_1 - r_2}{D}\right)\right)$$

$$= 2\sqrt{D^2 - (r_1 - r_2)^2} + 2(r_1 - r_2)\cos^{-1}\left(\frac{r_1 - r_2}{D}\right) + 2\pi r_2$$
(5)

Unfortunately, this equation cannot be solved analytically for D. So instead, the calculator solves it numerically using the Newton-Raphson Method. For the calculation based on number of links, our goal is  $L = \ell \cdot p$  and our initial guess is  $D = \frac{1}{2}L$ , which would be the case if both sprockets had zero pitch diameter. For the approximate center-to-center distance calculation, our initial guess for D is the approximate distance. To get the target value, we plug in the approximate distance to (5) and find the non-integer number of links that would produce. That number is then rounded to an even multiple in the direction specified to get  $\ell$ , and the target is  $L = \ell \cdot p$ . The numerical solver continues until a precision of at least  $10^{-3}$  is reached.