AMB Design Spreadsheet Equations

Ari Meles-Braverman

Version 1.0

1 General Motor Equations

1.1 Motor System Equations

$\tilde{x}$ denotes the property of a system of equivalent motors connected 1:1 in a gearbox

$$\tilde{\omega_f} = \omega_f \cdot \left(\frac{V}{V_{spec}}\right) \tag{1}$$

$$\tilde{\tau_s} = \tau_s \cdot \eta n \left(\frac{V}{V_{spec}}\right) \tag{2}$$

$$\tilde{i_f} = i_f \cdot n \left(\frac{V}{V_{spec}}\right) \tag{3}$$

$$\tilde{i_s} = i_s \cdot n \left(\frac{V}{V_{spec}}\right) \tag{4}$$

$$P = \frac{2\pi\omega_f \cdot \tau_s}{4} \tag{5}$$

$$\tilde{P} = \frac{2\pi\tilde{\omega_f} \cdot \tilde{\tau_s}}{4} = P \cdot \eta n \left(\frac{V}{V_{spec}}\right)^2 \tag{6}$$

$$K_T = \frac{\tilde{\tau_s}}{\tilde{i_s} - \tilde{i_f}} \tag{7}$$

ω_f = Motor Free Speed
$\tilde{\omega_f}$ = Adjusted Free Speed
τ_s = Motor Stall Torque
$\tilde{\tau_s}$ = Adjusted Stall Torque
i_f = Motor Free Current
$\tilde{i_f}$ = Adjusted Free Current
i_s = Motor Stall Current
$\tilde{i_s}$ = Adjusted Stall Current
P = Motor Power
$\tilde{P}$ = Adjusted Motor Power
n = # of Motors
η = Gearbox Efficiency
V = Applied Voltage
V_{spec} = Specification Voltage (Almost Always 12)
K_T = Motor Torque Constant

1.2 Instantaneous Motor Equations

$$\omega = \tilde{\omega_f} \cdot (1 - \frac{\tau}{\tilde{\tau_s}}) \tag{8}$$

$$i = (\tilde{i_s} - \tilde{i_f})\frac{\tau}{\tilde{\tau_s}} + \tilde{i_f} \tag{9}$$

$$\eta_{motor} = \frac{W_{out}}{W_{in}} = \frac{\tau \cdot \omega(\tau)}{V \cdot i(\tau)} = \frac{\tau(\tilde{\tau_s} - \tau)\tilde{\omega_f}}{V(\tilde{i_s}\tau + \tilde{i_f}(\tilde{\tau_s} - \tau))} \tag{10}$$

ω = Instantaneous Motor Speed
τ = Instantaneous Motor Torque
i = Instantaneous Motor Current
η_{motor} = Instantaneous Motor Efficiency

2 Mechanism Gear Ratio Calculator

2.1 General Equations

$$\omega_{free} = \frac{\tilde{\omega_f}}{G} \tag{11}$$

$$F_s = \frac{\tilde{\tau_s} G}{r} \tag{12}$$

$$\omega_{load} = \omega_{free} \cdot (1 - \frac{F}{F_s}) \tag{13}$$

$$v_{free} = \omega_{free} \cdot 2\pi r \tag{14}$$

$$v_{load} = \omega_{load} \cdot 2\pi r \tag{15}$$

$$i = \frac{rF}{K_T G n} + \frac{\tilde{i_f}}{n} \tag{16}$$

$$V_s = V_{spec} \frac{rF}{\tilde{\tau_s} \eta n G} \tag{17}$$

G = Gear Ratio
η = Gearbox Efficiency
n = # of Motors
F = Load Applied
r = Load Radius
ω_{free} = Output Rotational Free Speed
ω_{load} = Output Rotational Loaded Speed
v_{free} = Output Linear Free Speed
v_{load} = Output Linear Loaded Speed
i = Current Per Motor
F_s = Stall Load
V_s = Stall Voltage

2.2 Gear Ratio Calculations

2.2.1 Maximum Power

$$F_s = 2F \tag{18}$$

Substituting into (12) and solving for G gives:

$$G = \frac{2rF}{\tilde{\tau_s}} \tag{19}$$

2.2.2 Maximum Efficiency

$$G = \frac{rF}{\tilde{\tau_s}} \left(1 + \sqrt{\frac{\tilde{i_s}}{\tilde{i_f}}}\right) \tag{20}$$

For derivation, see Appendix A

2.2.3 At Stall

$$F_s = F \tag{21}$$

Substituting into (12) and solving for G gives:

$$G = \frac{rF}{\tilde{\tau_s}} \tag{22}$$

2.2.4 By Rotational Speed (ω_{load})

We can substitute (8) and (9) into (10):

$$\omega_{load} = \frac{\tilde{\omega_f}}{G} \cdot (1 - \frac{F}{\frac{\tilde{\tau_s} G}{r}}) \tag{23}$$

Solving for G, we get:

$$G = \frac{\tilde{\omega_f}}{2\omega_{load}} \left(1 + \sqrt{1 - 4rF \frac{\omega_{load}}{\tilde{\tau_s}\tilde{\omega_f}}} \right) \tag{24}$$

2.2.5 By Linear Speed (v_{load})

We can solve (15) for ω_{load}:

$$\omega_{load} = \frac{v_{load}}{2\pi r} \tag{25}$$

Now knowing ω_{load} we can use (24) to calculate G

2.2.6 By Per-Motor Current (i)

Solving (16) for G, we get:

$$G = \frac{rF}{K_T(ni - \tilde{i_f})} \tag{26}$$

2.2.7 By Stall Load (F_s)

We can solve (12) for G:

$$G = \frac{rF_s}{\tilde{\tau_s}} \tag{27}$$

2.2.8 By Stall Voltage (V_s)

Solving (17) for G, we get:

$$G = 12 \frac{rF}{\tau_s \eta n V_s} \tag{28}$$

3 Lead Screw Calculator

The basic transformation between rotational and linear motion is given by:

$$v = \omega \cdot np \tag{29}$$

Force & torque equations are taken from Shigley's Mechanical Engineering §8-2

$$d_m = d - \frac{p}{2} \tag{30}$$

$$T_R = \frac{Fd_m}{2}\left(\frac{\pi\mu d_p + np\cos(\alpha)}{\pi d_p\cos(\alpha) - \mu np}\right) \tag{31}$$

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi\mu d_p - np\cos(\alpha)}{\pi d_p\cos(\alpha) + \mu np}\right) \tag{32}$$

Backdrive-able if $T_L < 0$.

$$\eta = \frac{T_R|_{\mu=0}}{T_R} = \frac{np}{\pi d_p} \cdot \frac{\pi d_p\cos(\alpha) - \mu np}{\pi\mu d_p + np\cos(\alpha)} \tag{33}$$

Equivalent radius/load formulas are designed to allow for lead screws as outputs in the Mechanism Gear Ratio Calculator. Derivations can be found in Appendix B.

$$r_{eq} = \frac{np}{2\pi} \tag{34}$$

$$L_{R,eq} = \frac{T_R}{r_{eq}} \tag{35}$$

$$L_{L,eq} = \frac{T_L}{r_{eq}} \tag{36}$$

d = Screw Diameter
p = Screw Pitch
n = # of Starts
α = Half Thread Angle (i.e. $\frac{\text{thread angle}}{2}$)
μ = Coefficient of Friction of Screw
F = Applied Force
v = Instantaneous Linear Speed
ω = Instantaneous Rotational Speed
d_m = Mean (Average) Diameter
T_R = Raise Torque
T_L = Lower Torque
η = Efficiency
r_{eq} = Equivalent Radius
$L_{R,eq}$ = Equivalent Raise Load
$L_{L,eq}$ = Equivalent Lower Load

4 Drivetrain Calculator

Equations taken from JVN's Mechanical Design Calculator.

$$v_{free} = \frac{\tilde{\omega_f} \cdot \pi d}{G} \tag{37}$$

$$v_{adj} = v_{free} \cdot k_{SL} \tag{38}$$

$$i_{slip} = \frac{\tilde{W} \mu d}{2K_T \cdot nG} + \frac{\tilde{i_f}}{n} \tag{39}$$

v_{free} = Free Speed
v_{adj} = Adjusted Speed
i_{slip} = Wheel Slip Current
K_T = Motor Torque Constant
ω_f = Motor Free Speed
i_f = Motor Free Current
d = Wheel Diameter
μ = Wheel Coefficient of Friction
$\tilde{W}$ = Adjusted Robot Weight (i.e. weight resting on driven wheels)
G = Total Gear Ratio
k_{SL} = Speed Loss Constant

5 Beam Bend Calculator

5.1 Geometrical Deformation Resistance

Formulas for deformation resistance constants of different cross-sectional geometries are taken from StructX.

I = Area (Second) Moment of Inertia
J = Torsion Constant

5.1.1 Hex

$$I = 0.0601a^4 \tag{40}$$

$$J = 0.1154a^4 \tag{41}$$

a = Distance Between Flats

5.1.2 Round

$$I = \frac{\pi}{64}d^4 \tag{42}$$

$$J = \frac{\pi}{32}d^4 \tag{43}$$

d = Diameter

5.1.3 Round Tube

$$I = \frac{\pi}{8} d^3 h \tag{44}$$

$$J = \frac{\pi}{4} d^3 h \tag{45}$$

d = Diameter
h = Wall Thickness

5.1.4 Square

$$I = \frac{a^4}{12} \tag{46}$$

$$J = \frac{9}{64} a^4 \tag{47}$$

a = Side Length

5.1.5 Rectangular Tube

$$I = \frac{1}{3} x^2 y h \tag{48}$$

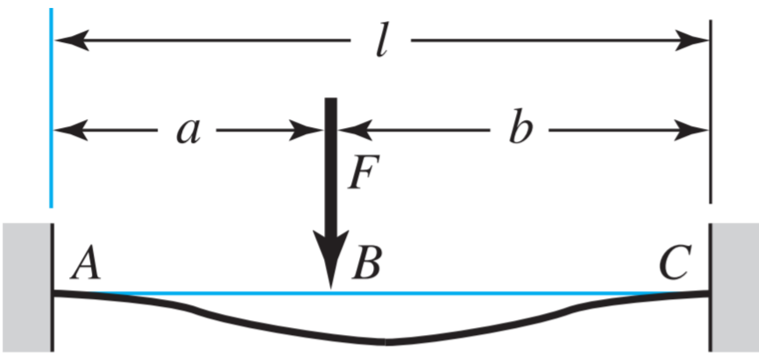
$$J = \frac{2h^2 (x-h)^2 (y-h)^2}{h(x+y-h)} \tag{49}$$

x = Length Parallel to Force
y = Length Perpendicular to Force
h = Wall Thickness

5.2 Displacement Equations

Displacement formulas are taken from Shigley's Mechanical Engineering Appendix A-9

5.2.1 Force Between Supports



$$y = \frac{Fx^2 b^2}{6EIl^3} [3al - x(3a+b)] \tag{50}$$

Substitute $b = l - a$ to get:

$$y = \frac{Fx^2 (a-l)^2}{6EIl^3} [3al - x(2a+l)] \tag{51}$$

Take the derivative, set equal to zero, and solve for x:

$$0 = \frac{\partial y}{\partial x} = \frac{Fx(l-a)^2}{2EIl^3}(2a(l-x) - lx) \tag{52}$$

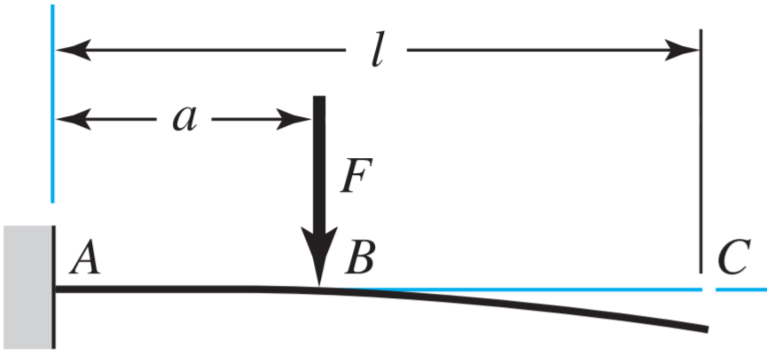
$$x = \frac{2al}{2a+l} \tag{53}$$

Substitute (53) into (51) to get:

$$y_{max} = \frac{2Fa^3(l-a)^2}{3EI(2a+l)^2} \tag{54}$$

y = Vertical Displacement
F = Load Force
a = Distance to Closer Support
l = Distance Between Supports
E = Modulus of Elasticity
I = Area Moment of Inertia

5.2.2 Cantilevered Force



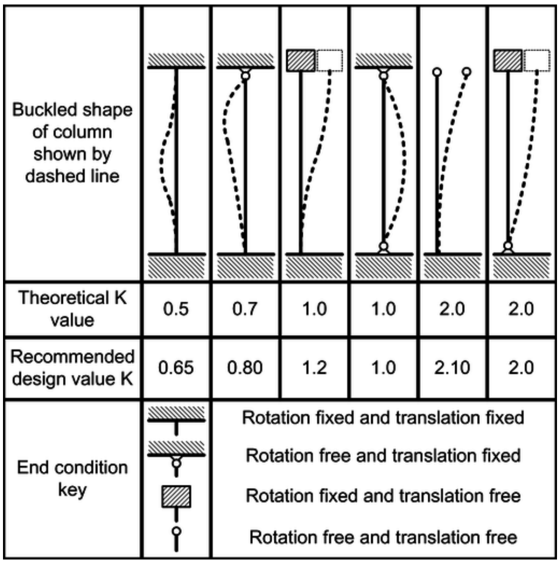
$$y_{max} = \frac{Fa^2}{6EI}(a - 3l) \tag{55}$$

y_{max} = Largest Displacement
F = Load Force
a = Distance to Support
l = Total Length
E = Modulus of Elasticity
I = Area Moment of Inertia

5.3 Buckling Force

This formula is taken from the Wikipedia page on Euler's Critical Load

$$F_{max} = \frac{\pi^2 EI}{(KL)^2} \tag{56}$$



F_{max} = Maximum Force
E = Modulus of Elasticity
I = Area Moment of Inertia
K = End Condition Constant
L = Column Length

5.4 Twisting Torque

This formula is taken from Shigley's Mechanical Engineering §3-12

$$\theta_{max} = \frac{TL}{GJ} \tag{57}$$

θ_{max} = Largest Angular Displacement
T = Applied Torque
L = Distance Between Torque and Support
G = Shear Modulus
J = Torsion Constant

6 Projectile Trajectory Calculator

6.1 Forward Calculation

$$h_f = h_i + d\tan\theta_i - \frac{g \cdot d^2}{2v_i^2\cos^2\theta_i} \tag{58}$$

$$\theta_f = \tan^{-1}\left(2\frac{h_f - h_i}{d} - \tan\theta_i\right) \tag{59}$$

6.2 Reverse Calculation

$$\theta_i = \tan^{-1}\left(2\frac{h_f - h_i}{d} - \tan\theta_f\right) \tag{60}$$

$$v_i = \sec\theta_i \cdot \sqrt{\frac{g \cdot d}{|\tan\theta_i - \tan\theta_f|}} \tag{61}$$

h_i = Release (Initial) Height
h_f = Target (Final) Height
θ_i = Release (Initial) Angle
θ_f = Target (Final) Angle
v_i = Release (Initial) Velocity
d = Horizontal Distance to Target
g = Acceleration Due to Gravity

For derivation see Appendix C.

7 Chain/Belt C-C Calculator

This method of solving for D is inspired by and based on the formulas used in Clem1640's Chain/Belt Calculator. The language in this section will only reference chain, sprockets, and chain links; but all equations also apply to belts, pulleys, and belt teeth, respectively.

First we start with basic equations for chain and sprockets:

$$2\pi r_i = n_i p \tag{62}$$

$$L = lp \tag{63}$$

$$C = D - (r_1 + r_2) \tag{64}$$

We can construct an equation to represent the path length of the chain relative to the center-to-center distance. A derivation of this equation can be found in Appendix D.

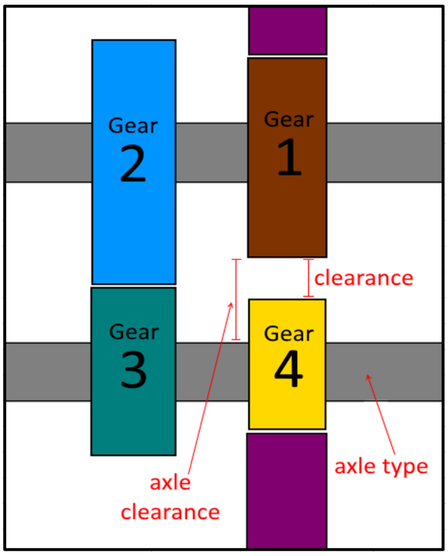
$$L = 2\sqrt{D^2 - (r_1 - r_2)^2} + 2(r_1 - r_2)\cos^{-1}\left(\frac{r_1 - r_2}{D}\right) + 2\pi r_2 \tag{65}$$

Unfortunately, we cannot solve this equation empirically to get D, even with the help of a computer. So we solve the function numerically, using the Newton-Raphson Method. For the forward calculation our initial guess for D is $\frac{L}{2}$, which would be the case if both sprockets had diameters of 0. For the reverse calculation, we use the estimated value of

D as the initial guess. The Newton-Raphson Method then improves the accuracy of D to the correct solution until the error is less than 10^{-12}.

L = Chain Length
l = # of Links
D = Center to Center Distance
n_i = # of Teeth on Sprocket $i \in [1, 2]$
r_i = Radius of Sprocket $i \in [1, 2]$
p = Chain Pitch
C = Clearance

8 Gear Clearance Calculator



Starting with the basic equations:

$$r_p = \frac{n}{2p} \tag{66}$$

$$r_o = \frac{n+2}{2p} \tag{67}$$

$$C_a = r_{o2} + r_{o3} - r_{o1} - \frac{d_{axle}}{2} \tag{68}$$

$$C_g = r_{o2} + r_{o3} - r_{o1} - r_{o4} \tag{69}$$

Substituting (67) into (68) and (69), respectively, gives:

$$C_a = \frac{n_2 + n_3}{2p_{23}} - \frac{n_1}{2p_1} - \frac{d_{axle}}{2} \tag{70}$$

$$C_g = \frac{n_2 + n_3}{2p_{23}} - \frac{n_1}{2p_1} - \frac{n_4}{2p_4} \tag{71}$$

The axle or gear clears if $C_a > 0$ or $C_g > 0$, respectively.

C_a = Axle Clearance
C_g = Gear Clearance
r_p = Pitch Radius
r_o = Outside Radius
n = # of Teeth
p = Diametrical Pitch (a.k.a. dp)
d_{axle} = Axle Diameter

9 Ratio & Distance Calculator

We have the basic equations:

$$G = \frac{N}{n} \tag{72}$$

$$d = \frac{N + n}{2p} \tag{73}$$

Solving these equations for n and N gives:

$$n = \frac{2dp}{G+1} \tag{74}$$

$$N = \frac{2dp}{G+1}G \tag{75}$$

For each gear we can get the pitch diameter and outside diameter using (66) and (67), respectively.

N = # of Teeth on Large Gear
n = # of Teeth on Small Gear
G = Gear Ratio
d = Center-to-Center Distance
p = Diametrical Pitch (a.k.a. dp)

10 Pneumatics Calculator

The "simulation" is run at timesteps of $dt = 1$ second for a duration of 150 seconds.

The pressure at the current step of the simulation can be calculated by:

$$P_{n+1} = P_n + \frac{W_{comp} - \sum W_{cyl}}{V} \tag{76}$$

$$W_{comp} = \dot{V}_{comp}(P) \cdot dt \cdot 1\text{atm} \tag{77}$$

$$W_{cyl} = \left(\frac{\pi D^2}{4} P_{push} + \frac{\pi(D^2 - d^2)}{4} P_{pull} \right) L \cdot m \tag{78}$$

$\dot{V}_{comp}(P)$ is the compressor flow-rate as a function of the output pressure, taken from a fourth-degree polynomial interpolation of the data provided online. The volume of air is measured at atmospheric pressure, not at system pressure. m is the number of actuations per second (can be 0 when not firing, 1 when firing, or $m > 1$ if firing more than once per second).

Derivations of these equations can be found in Appendix E.

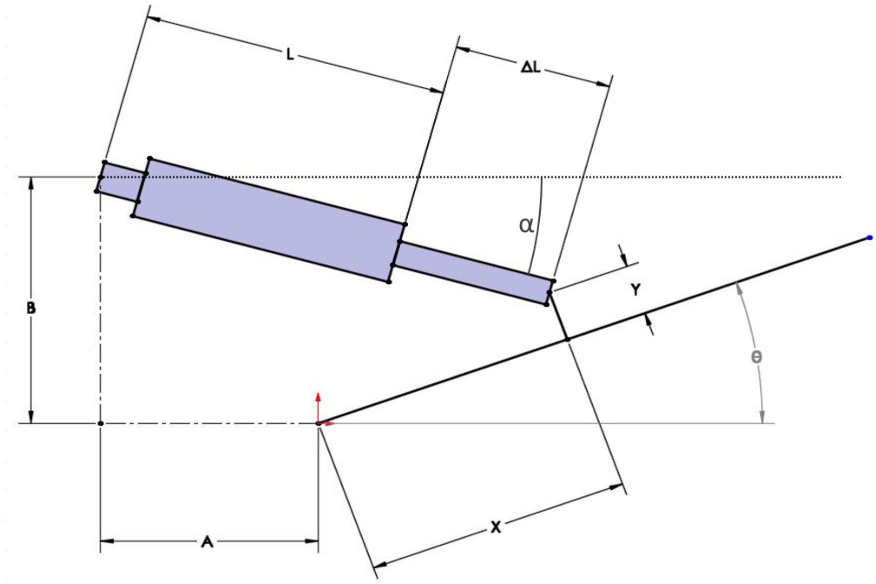
We can also calculate the pushing and pulling force of each cylinder using the formulas:

$$F_{push} = \frac{\pi D^2}{4} P_{push} \tag{79}$$

$$F_{pull} = \frac{\pi(D^2 - d^2)}{4} P_{pull} \tag{80}$$

P_n = System Pressure at Step n
V = System Volume
W_{comp} = Work Done by the Compressor
W_{cyl} = Work Done by Each Cylinder
D = Cylinder Diameter
d = Cylinder Rod Diameter
L = Cylinder Length
P_{push} = Pushing Pressure
P_{pull} = Pulling Pressure
F_{push} = Pushing Force
F_{pull} = Pulling Force

11 Pneumatic Linkage Calculator



Let θ_1 and α_1 represent the system when the cylinder is retracted, and θ_2 and α_2 represent it when extended.

In the horizontal direction we have:

$$A + X \cos\theta_1 - Y \sin\theta_1 = L \cos\alpha_1 \tag{81}$$

$$A + X \cos\theta_2 - Y \sin\theta_2 = (L + \Delta L) \cos\alpha_2 \tag{82}$$

And in the vertical direction we have:

$$X \sin\theta_1 + Y \cos\theta_1 = B - L \sin\alpha_1 \tag{83}$$

$$X \sin\theta_2 + Y \cos\theta_2 = B - (L + \Delta L) \sin\alpha_2 \tag{84}$$

We can combine (81) and (84) to remove α_1 and get:

$$L^2 = A^2 + B^2 + X^2 + Y^2 + 2AX \cos\theta_1 - 2BY \cos\theta_1 - 2BX \sin\theta_1 - 2AY \sin\theta_1 \tag{85}$$

And combine (82) and (84) to remove α_2 and get:

$$(L+\Delta L)^2 = A^2+B^2+X^2+Y^2+2AX \cos\theta_2-2BY \cos\theta_2-2BX \sin\theta_2-2AY \sin\theta_2 \tag{86}$$

To derive the forward formulas, you can solve (85) and (86) separately to get expressions for θ_1 and θ_2. To derive the reverse formulas, you can solve (85) and (86) as a system of equations to get expressions for A and B. I will not include these expressions in this document, because they are extremely long.

Derivations

A Maximum Efficiency Derivation

To find the maximum efficiency, we take the derivative of (10) and set it equal to 0:

$$0 = \frac{\partial \eta}{\partial \tau} = \frac{(\tilde{i_f}(\tau - \tilde{\tau_s})^2 - \tilde{i_s}\tau^2)\tilde{\omega_f}}{V(\tilde{i_s}\tau + \tilde{i_f}(\tilde{\tau_s} - \tau))^2} \tag{87}$$

Solving for the value of τ at which η is maximized:

$$\tau_{\eta_{max}} = \frac{\tilde{\tau_s}\sqrt{\tilde{i_f}}}{\sqrt{\tilde{i_f}} + \sqrt{\tilde{i_s}}} \tag{88}$$

We know that $\tau_{manip} = G \cdot \tau_{motor}$, so we can substitute $\tau_{\eta_{max}}$ for τ_{motor}:

$$G = \frac{\tau_{manip}}{\tau_{motor}} = \frac{r \cdot F}{\tau_{\eta_{max}}} = \frac{r \cdot F}{\frac{\tilde{\tau_s}\sqrt{\tilde{i_f}}}{\sqrt{\tilde{i_f}}+\sqrt{\tilde{i_s}}}} \tag{89}$$

And simplifying gives:

$$G = \frac{rF}{\tilde{\tau_s}}\left(1 + \sqrt{\frac{\tilde{i_s}}{\tilde{i_f}}}\right) \tag{20}$$

B Equivalent Radius/Load Derivation

To find the equivalent radius, we equate the rotational $\rightarrow$ linear motion transformation equations for both a wheel (15) and lead screw (29)

$$2\pi r \cdot \omega = v = \omega \cdot np \tag{90}$$

We can then solve for $r = r_{eq}$ to find the radius that will provide the same rotational $\rightarrow$ linear motion transformation as the given lead screw

$$r_{eq} = \frac{np}{2\pi} \tag{34}$$

We want to conserve the work done by a lead screw and equivalent wheel across one rotation (where x can be L or R).

$$W = 2\pi r_{eq} \cdot L_{x,eq} = 2\pi T_x \tag{91}$$

Solving for $L_{x,eq}$, we get:

$$L_{x,eq} = \frac{T_x}{r_{eq}} \tag{92}$$

C Projectile Equation Derivations

Starting with basic projectile motion equations:

$$\Delta x = v_i \cos\theta_i \cdot t \tag{93}$$

$$\Delta y = v_i \sin\theta_i \cdot t - \tfrac{1}{2}gt^2 \tag{94}$$

$$\theta_f = \tan^{-1}\left(\frac{v_{f,y}}{v_{f,x}}\right) = \tan^{-1}\left(\frac{v_i \sin\theta_i - gt}{v_i \cos\theta_i}\right) \tag{95}$$

Solving (93) for t and substituting into (94) gives:

$$\Delta y = \Delta x \tan\theta_i - \frac{g \cdot (\Delta x)^2}{2v_i^2 \cos^2\theta_i} \tag{96}$$

We can then substitute in $\Delta x = d$ and $\Delta y = h_f - h_i$:

$$h_f = h_i + d\tan\theta_i - \frac{g \cdot d^2}{2v_i^2 \cos^2\theta_i} \tag{58}$$

Solving (93) for t and substituting into (95) gives:

$$\theta_f = \tan^{-1}\left(\frac{v_i \sin\theta_i - g \cdot \frac{\Delta x}{v_i \cos\theta_i}}{v_i \cos\theta_i}\right) = \tan^{-1}\left(\tan\theta_i - \frac{g \cdot d}{v_i^2 \cos^2\theta_i}\right) \tag{97}$$

Further substituting in (58) into (97) gives:

$$\theta_f = \tan^{-1}\left(\tan\theta_i - \frac{2}{d}(d\tan\theta_i - (h_f - h_i))\right) \tag{98}$$

And by simplifying we get:

$$\theta_f = \tan^{-1}\left(2\frac{h_f - h_i}{d} - \tan\theta_i\right) \tag{59}$$

To calculate the reverse equations, we can solve (98) for θ_i:

$$\theta_i = \tan^{-1}\left(2\frac{h_f - h_i}{d} - \tan\theta_f\right) \tag{60}$$

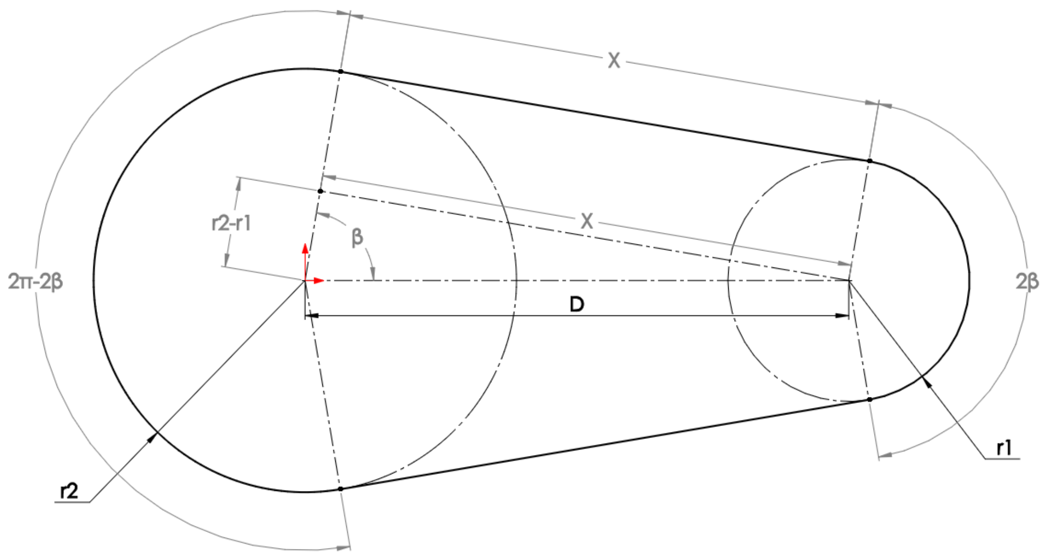
Then substitute (58) into (60):

$$\tan\theta_i + \tan\theta_f = \frac{2}{d}\left(d\tan\theta_i - \frac{g \cdot d^2}{2v_i^2 \cos^2\theta_i}\right) \tag{99}$$

And solve that for v_i:

$$v_i = \sec\theta_i \cdot \sqrt{\frac{g \cdot d}{|\tan\theta_i - \tan\theta_f|}} \tag{61}$$

D Chain Length Derivation



It is clear that:

$$L = 2X + r_1 \cdot 2\beta + r_2 \cdot (2\pi - 2\beta) \tag{100}$$

Looking at the right triangle in the center of the image, we can see that:

$$D^2 = X^2 + (r_2 - r_1)^2 \implies X = \sqrt{D^2 - (r_1 - r_2)^2} \tag{101}$$

$$\beta = \cos^{-1}\left(\frac{r_2 - r_1}{D}\right) = \cos^{-1}\left(\frac{r_1 - r_2}{D}\right) \tag{102}$$

Substituting (101) and (102) into (100) gives:

$$L = 2\sqrt{D^2 - (r_1 - r_2)^2} + r_1 \cdot 2\cos^{-1}\left(\frac{r_1 - r_2}{D}\right) + r_2 \cdot \left(2\pi - 2\cos^{-1}\left(\frac{r_1 - r_2}{D}\right)\right) \tag{103}$$

And rearranging gives:

$$L = 2\sqrt{D^2 - (r_1 - r_2)^2} + 2(r_1 - r_2)\cos^{-1}\left(\frac{r_1 - r_2}{D}\right) + 2\pi r_2 \tag{65}$$

E Pneumatics Simulation Derivation

We know that the energy of the system at any point can be represented by:

$$E = P \cdot V \tag{104}$$

We can find the energy of the system at any timestep by adding the total work done on the system in that time to the energy of the system at the previous step:

$$E_{n+1} = E_n + W_{tot} \tag{105}$$

Substituting (104) into (105) gives:

$$(PV)_{n+1} = (PV)_n + W_{comp} - \sum W_{cyl} \tag{106}$$

Since the volume of the system remains constant, we can divide by V to get:

$$P_{n+1} = P_n + \frac{W_{comp} - \sum W_{cyl}}{V} \tag{76}$$

Basic thermodynamics says that the work done by compression/expansion of gas at constant pressure can be represented by $W = P \cdot \Delta V$. In each actuation of a cylinder gas is compressed twice, once for extension and once for retraction. So for each timestep, the work done on the cylinder can be expressed as:

$$W_{cyl} = ((PAL)_{push} + (PAL)_{pull}) \cdot m \tag{107}$$

L is constant, so it can be factored out. We can also substitute A for the effective piston areas, giving:

$$W_{cyl} = \left(\frac{\pi D^2}{4} P_{push} + \frac{\pi (D^2 - d^2)}{4} P_{pull} \right) L \cdot m \tag{78}$$

Similarly, we can represent the work done by the compressor as:

$$W_{comp} = \dot{V}_{comp}(P) \cdot dt \cdot 1\text{atm} \tag{77}$$

Note: since $\dot{V}_{comp}$ is measured at atmospheric pressure, P must be 1 atm and not the system pressure