Mechanism Calculator

AMB Calculator

This calculator is used to find the properties of a mechanism given its gear ratio, or to find the proper gear ratio needed to give the mechanism certain properties. A mechanism is defined as a system powered by one or more motors with a constant gear ratio connecting the motor to a constant load.

Motor Equations

In FRC we describe DC motors using four parameters. The free speed, ω_f, is the maximum speed the motor can spin when not mechanically connected to anything. The stall torque, T_s, is the maximum torque the motor can produce when not allowed to rotate. The free current, I_f, is the minimum current the motor will draw (which occurs when spinning at max speed), and the stall current, I_s, is the maximum current the motor will draw (which occurs when it is not allowed to rotate). These four values are measured when the motor is given a certain voltage, called the specification voltage V_{spec}. In FRC this is almost always 12V.

To find the motor speed ω and current draw I at any applied torque T, we use the following equations:

$$\omega(T) = \omega_f \left(1 - \frac{T}{T_s}\right) \tag{1}$$

$$I(T) = \left[(I_s - I_f) \cdot \frac{T}{T_s} + I_f\right] \tag{2}$$

The mechanical power generated by the motor P is equal to the output speed multiplied by the output torque, $P(T) = T \cdot \omega(T)$. Plugging in the function $\omega(T)$, we see that the maximum power P_{max} occurs at $T = \frac{1}{2}T_s$, or:

$$P_{max} = \Big[T \cdot \omega(T)\Big]_{T=\frac{1}{2}T_s} = \frac{T_s\,\omega_f}{4} \tag{3}$$

The motor efficiency, η_{motor}, is the ratio of the output power to input power. The input power, in the form of electricity, is the voltage multiplied by the current drawn. So:

$$\eta_{motor}(T) = \frac{P(T)}{V \cdot I(T)} = \frac{T \cdot \omega_f \left(1 - \frac{T}{T_s}\right)}{V \cdot \left[(I_s - I_f) \cdot \frac{T}{T_s} + I_f\right]} = \frac{T \cdot \omega_f \,(T_s - T)}{V\left[T\,(I_s - I_f) + T_s I_f\right]} \tag{4}$$

Motor Systems

We can combine multiple identical motors that are mechanically tied together into a motor system, with parameters derived from those of the individual motors. We will use $\widetilde{\square}$ to denote the property of the motor system which contains n motors. When combining the motors in a gearbox, there is an imperfect power transmission; this efficiency percentage will be denoted as η. All of the parameters scale linearly with the voltage applied to the motor, so we can adjust the parameters for the applied voltage V as well.

The speed of the motor system is identical to the speed of each individual motor:

$$\widetilde{\omega}_f = \omega_f \left(\frac{V}{V_{spec}}\right) \tag{5}$$

The torque produced by the motor system is equal to the sum of the torques produced by each of the motors, multiplied by our gearbox efficiency fraction. Since all of the motors are identical:

$$\widetilde{T}_s = T_s \cdot n\eta \left(\frac{V}{V_{spec}}\right) \tag{6}$$

Similarly, the total current drawn by the system is equal to the sum of the individual motor currents:

$$\widetilde{I}_f = I_f \cdot n \left(\frac{V}{V_{spec}} \right) \quad ; \qquad \widetilde{I}_s = I_s \cdot n \left(\frac{V}{V_{spec}} \right) \tag{7}$$

And plugging these into the equation for the motor efficiency:

$$\widetilde{\eta}_{motor} = \frac{T \cdot \widetilde{\omega}_f \left(\widetilde{T}_s - T \right)}{V \left[T \left(\widetilde{I}_s - \widetilde{I}_f \right) + \widetilde{T}_s \widetilde{I}_f \right]} \tag{8}$$

Mechanism Definition

We define a mechanism as any system powered by a motor or system of identical motors with a constant gear ratio G, which can have either a rotational or linear output. All mechanisms are loaded with a constant force F at a constant radius r, which depend on the specific geometry and usage. For linear mechanisms, the load radius is the radius of the wheel, pulley, sprocket, etc. that translates between rotational and linear movement. For rotational mechanisms, the it is the perpendicular distance between the force and the axis of rotation.

We will express the output speed in two ways. The free speed is the mechanism's output speed when no load is connected. The loaded speed is the output speed when the load force is applied. These speeds can be expressed as either linear velocities, v_{free} and v_{load}, or rotational velocities, ω_{free} and ω_{load}. In all cases, the speed in question is a steady-state speed, meaning the mechanism has been powered on for enough time to reach it's final speed. It does not take into account the acceleration of the mechanism while in start-up or as the load changes.

Since one common limitation is the breaker on each motor, we will calculate the per-motor current I when the given load is applied. In addition, we define the stall load, F_s, which is the force needed to get the loaded speed to zero (i.e. the mechanism cannot move). We also define the stall voltage, V_s, as the voltage that, when applied to the motor(s), would cause the loaded speed to fall to zero under the defined force. This is useful for ensuring the voltage to hold a certain force for a prolonged time is low enough to be safe for the motor.

These quantities are related to each other through the following equations:

$$\omega_{free} = \frac{\widetilde{\omega}_f}{G} \quad ; \qquad v_{free} = 2\pi r \cdot \omega_{free} \tag{9}$$

$$\omega_{load} = \omega_{free} \left(1 - \frac{Fr}{\widetilde{T}_s G} \right) \quad ; \qquad v_{load} = 2\pi r \cdot \omega_{load} \tag{10}$$

$$I = \frac{1}{n} \left[\frac{Fr}{\widetilde{T}_s G} \left(\widetilde{I}_s - \widetilde{I}_f \right) + \widetilde{I}_f \right] \tag{11}$$

$$F_s = \frac{\widetilde{T}_s G}{r} \tag{12}$$

$$V_s = \frac{Fr}{T_s n \eta G} \cdot V_{spec} \tag{13}$$

Gear Ratio Calculation

In order to set certain characteristics of the mechanism, we will derive equations to find the gear ratio when given each characteristic.

For the free rotational and linear velocities, we solve for G to get:

$$G = \frac{\widetilde{\omega}_f}{\omega_{free}} \qquad\qquad G = \frac{2\pi r\widetilde{\omega}_f}{\omega_{free}} \tag{14}$$

For the loaded rotational velocity, we will substitute (9) into (10) and solve for G:

$$\omega_{load} = \frac{\widetilde{\omega}_f}{G}\left(1 - \frac{Fr}{\widetilde{T}_s G}\right) \implies G = \frac{\widetilde{\omega}_f}{2\omega_{load}}\left(1 + \sqrt{1 - 4\frac{Fr}{\widetilde{T}_s}\cdot\frac{\omega_{load}}{\widetilde{\omega}_f}}\right) \tag{15}$$

And for the loaded linear velocity we substitute in $v_{load} = 2\pi r \cdot \omega_{load}$:

$$G = \frac{\pi r\widetilde{\omega}_f}{v_{load}}\left(1 + \sqrt{1 - 4\frac{F}{\widetilde{T}_s}\cdot\frac{v_{load}}{2\pi\widetilde{\omega}_f}}\right) \tag{16}$$

For the per-motor current, we solve (11) for G to get:

$$G = \frac{\widetilde{T}_s}{Fr}\left(\frac{In - \widetilde{I}_f}{\widetilde{I}_s - \widetilde{I}_f}\right) \tag{17}$$

For stall load, we solve (12) for G:

$$G = \frac{F_s r}{\widetilde{T}_s} \tag{18}$$

And for stall voltage we solve (13) for G:

$$G = \frac{Fr}{T_s n\eta\left(\frac{V_s}{V_{spec}}\right)} \tag{19}$$

Finally we define three important characteristic points: Max Power, Max Efficiency, and Stall. At stall, the stall load is equal to the applied load (i.e. $F_s = F$). Substituting into (18):

$$G = \frac{Fr}{\widetilde{T}_s} \tag{20}$$

We know that maximum power for a single motor occurs at half of the maximum load. Expanding this to an entire mechanism, maximum power occurs when $F = \frac{1}{2}F_s$. Inserting this into (18) gives:

$$G = \frac{2Fr}{\widetilde{T}_s} \tag{21}$$

In order to find the maximum efficiency, we will take the derivative of the efficiency equation and set it equal to zero:

$$0 = \frac{d\eta_{motor}}{dT} = \frac{\widetilde{\omega}_f\left[\widetilde{I}_f(T - \widetilde{T}_s)^2 - \widetilde{I}_s T^2\right]}{V\left[\widetilde{I}_s T + \widetilde{I}_f(\widetilde{T}_s - T)\right]^2} \tag{22}$$

Solving for the value of T at which η_{motor} is maximized:

$$T_{max} = \widetilde{T}_s\frac{\sqrt{\widetilde{I}_f}}{\sqrt{\widetilde{I}_s} + \sqrt{\widetilde{I}_f}} \tag{23}$$

We know that $T_{manip} = G \cdot T_{motor}$, so we can substitute T_{max} for T_{motor}:

$$G = \frac{T_{manip}}{T_{motor}} = \frac{Fr}{T_{max}} = \frac{Fr}{\widetilde{T}_s\frac{\sqrt{\widetilde{I}_f}}{\sqrt{\widetilde{I}_s}+\sqrt{\widetilde{I}_f}}} = \frac{Fr}{\widetilde{T}_s}\left(1 + \sqrt{\frac{\widetilde{I}_s}{\widetilde{I}_f}}\right) \tag{24}$$

Gearbox Options Selector

AMB Calculator

This calculator allows the user to find sets of gears that produce the desired overall ratio, filtered by certain restrictions.

Gears lists are taken from the websites of Vex, AndyMark, and REV. Last updated on 7/12/2022. All gears are 20dp, with various bores.

To calculate the center-to-center distance between two gears x and y in a stage, we use the following formula:

$$d_{xy} = \frac{n_x + n_y}{2 \cdot dp} \text{ [in]} \tag{1}$$

In order to calculate a gear's outer diameter, the following formula is used:

$$OD_x = \frac{n_x + 2}{dp} \text{ [in]} \tag{2}$$

The clearance of an axle is the space between the outer diameter of one gear and the center-to-center distance of the opposite axle. Therefore, for gear x on the same axle as gear y, which mates with gear z, the formula is:

$$\text{clearance}_{xyz} = d_{yz} - \tfrac{1}{2} OD_x \tag{3}$$

When "Axle Bore" is chosen for the clearance requirement, the any gearboxes with clearance less than the radius of the corresponding axle are ignored.

The calculator works by running a brute-force search for all combinations of the possible gears. Only those within the allowable deviation of the desired ratio are output, in order of their deviation. If no allowable deviation is given, only exact matches are shown.

Chain Belt C-C Distance Calculator

AMB Calculator

This calculator determines the proper center-to-center spacing between two sprockets or pulleys, as based on the belt/chain length or an approximate center-to-center distance.

The calculator comes loaded with the standard types of belt and chain used in FRC, and their respective pitch lengths, p. If you want a non-standard pitch length, you can set that by choosing a "Custom" type. For the standard types, useful dimensions are also shown including the width, thickness, and weight. These dimensions do not affect the calculations. Suggested adders (i.e. the recommended amount to over/under tension the system) and load ratings are given for some types, though these values may vary based on the source of the belt/chain and team preference.

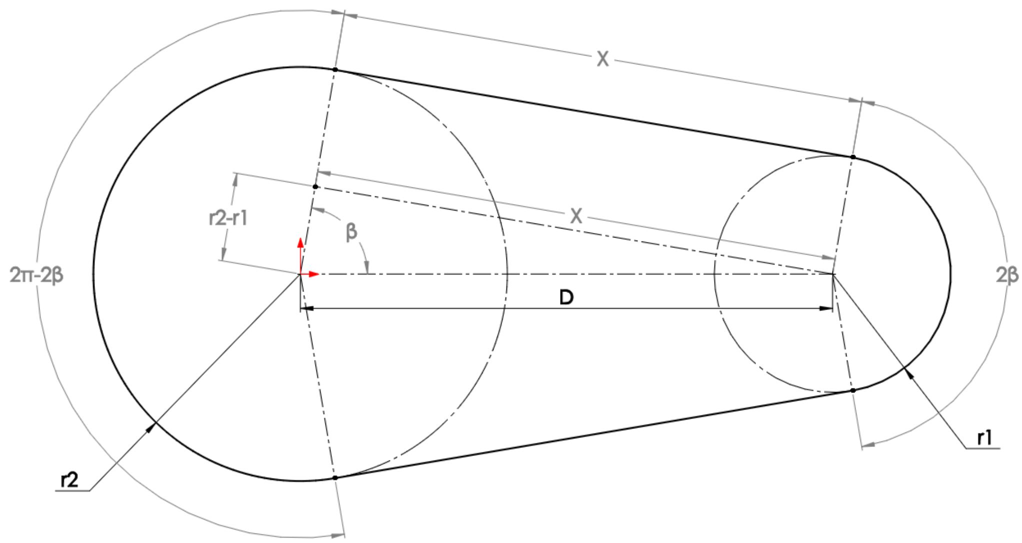
For this document we will use the nomenclature for chain, though the equations apply identically to belts.

To calculate the pitch diameter of each sprocket with n teeth, we use the formula:

$$d_p = \frac{n \cdot p}{\pi} \tag{1}$$

Halving this value gives us the pitch radius for each sprocket, $r_x = \frac{n_x p}{2\pi}$.

Defining the total chain length as L and the number of links as ℓ, we know $L = \ell \cdot p$. We can also find the total chain length geometrically using the sprocket radii and center-to-center distance D.



Adding up the various sections of the chain path, it is clear that:

$$L = 2X + r_1 \cdot 2\beta + r_2 \cdot (2\pi - 2\beta) \tag{2}$$

Using the Pythagorean theorem and trigonometry on the right triangle in the center of the diagram, we can see that:

$$D^2 = X^2 + (r_2 - r_1)^2 \implies X = \sqrt{D^2 - (r_1 - r_2)^2} \tag{3}$$

$$\beta = \cos^{-1}\left(\frac{r_2 - r_1}{D}\right) = \cos^{-1}\left(\frac{r_1 - r_2}{D}\right) \tag{4}$$

Substituting (3) and (4) into (2) gives:

$$\begin{aligned} L &= 2\sqrt{D^2-(r_1-r_2)^2} + r_1 \cdot 2\cos^{-1}\left(\frac{r_1-r_2}{D}\right) + r_2 \cdot \left(2\pi - 2\cos^{-1}\left(\frac{r_1-r_2}{D}\right)\right) \\ &= 2\sqrt{D^2-(r_1-r_2)^2} + 2\left(r_1-r_2\right)\cos^{-1}\left(\frac{r_1-r_2}{D}\right) + 2\pi r_2 \end{aligned} \tag{5}$$

Unfortunately, this equation cannot be solved analytically for D. So instead, the calculator solves it numerically using the Newton-Raphson Method. For the calculation based on number of links, our goal is $L = \ell \cdot p$ and our initial guess is $D = \frac{1}{2}L$, which would be the case if both sprockets had zero pitch diameter. For the approximate center-to-center distance calculation, our initial guess for D is the approximate distance. To get the target value, we plug in the approximate distance to (5) and find the non-integer number of links that would produce. That number is then rounded to an even multiple in the direction specified to get ℓ, and the target is $L = \ell \cdot p$. The numerical solver continues until a precision of at least 10^{-3} is reached.

The derivation of this algorithm is based on work done by Clem McKown of FRC team 1640.

Reduction Swap Selector

AMB Calculator

This calculator is designed to help in the case where the robot is already manufactured and the team realizes they need to change the reduction for a mechanism. It allows you to input the reduction you currently have, the ratio you want, and options for how to achieve that reduction, and it outputs all of the possible ways to get the desired ratio without changing the spacing of the original reduction.

The original reduction can be input in the form of a gear pair, a belt or chain with two pulleys/sprockets, or a custom distance. To calculate the distance for a gear pair with gears n_1 and n_2 and diametrical pitch dp, the formula is:

$$d = \frac{n_1 + n_2}{2dp} \text{ [in]} \tag{1}$$

For a belt or chain, the algorithm is the same as the one shown in the Chain/Belt Calculator.

The user then chooses their desired ratio, the amount the solutions can deviate from that ratio, and the amount the solutions can deviate from the desired distance. Under "Ranges to Check", the user can select the maximum and minimum sizes of each of the types of reductions that they can use. Not all integer sizes in the range may be available for purchase, but at this time the algorithm cannot make that determination. The calculator runs a brute-force search through each of the reduction types to find gear, sprocket, and pulley pairs that produce an acceptable ratio with an acceptable center-to-center distance. It then outputs those options, as well as the resulting ratio, center distance, and belt/chain length if applicable, sorted by the minimum deviation from the desired distance.

Drivetrain Simulator

AMB Calculator

The drivetrain simulator is used to find the proper gear ratio for a robot's drivetrain. By choosing the desired distance to traverse, you can optimize the gear ratio to give the best combination of acceleration and top speed. This simulation takes into account a number of factors, including wheel slip, current limits, voltage ramps, and voltage sag. It also features a "Predictive Stop" mode, which attempts to decelerate the robot before it hits the target distance in order to arrive at the target at zero velocity.

Simulation Calculations

The simulation runs one timestep at a time, starting from time $t = 0$ at position $x = 0$ and velocity $v = 0$. We will start the simulation by applying the full battery voltage in the forward direction.

Between the battery and motor, there is voltage sag due to high current draw through a system resistance defined as R_{sys}. To model this, when the battery is connected, resistive losses are subtracted from the magnitude of the battery voltage to get the voltage at the motor. Since we do not yet know what current will be drawn, we use the current drawn in the previous timestep I_{prev} for the calculation.

$$V_{motor} = \pm V_{batt} - I_{prev} \cdot R_{sys} \cdot \text{sign}\left(V_{batt}\right) \tag{1}$$

Rather than use the FRC motor parameters ω_f, T_s, I_s, I_f, we will use the more standard DC motor parameters: torque constant k_m, back-EMF constant k_e, and armature resistance R. These can be calculated from the FRC motor parameters with the formulas:

$$k_m = \frac{T_s}{I_s - I_f} \ ; \qquad k_e = \frac{12}{\omega_f} \ ; \qquad R = \frac{12}{I_s - I_f} \tag{2}$$

We can then use the standard DC motor equations for applied voltage V, output speed ω, current draw I, and motor torque T_{motor}:

$$V = I \cdot R + k_e \cdot \omega \ ; \qquad T_{motor} = k_m \cdot (I - I_f) \tag{3}$$

The rotational output speed of the motor is equal to the robot's velocity divided by the wheel radius r, and multiplied by the gear ratio G. So we can calculate the total current draw across all n motors as:

$$I = \frac{n}{R}\left(V - k_e \cdot \frac{v \cdot G}{r}\right) \tag{4}$$

We can apply the current limit by limiting the magnitude of this calculated current to be less than the desired value. In reality, this is done through complex algorithms to change the applied voltage in order to give the proper current draw. For the purposes of this simulation, we will assume the current is properly limited so $I \leq I_{limit}$.

With this current, we can use the second DC motor equation to calculate the torque generated by the motor. From there, the torque at the wheels T is the motor torque multiplied by the gear ratio, minus parasitic losses that scale linearly with the robot speed (an approximation, but good enough for the purposes of this simulation):

$$T = T_{motor} \cdot G \cdot \eta - T_s\left(1 - \eta\right) \cdot \frac{v}{v_{max}} \tag{5}$$

In order to account for wheel slip, we calculate the maximum torque that can be transferred through static and kinetic friction. This is the friction force multiplied by the wheel radius:

$$T_{slip_s} = \mu_s \cdot mg \cdot (\%W) \cdot r \ ; \qquad T_{slip_k} = \mu_k \cdot mg \cdot (\%W) \cdot r \tag{6}$$

At $t = 0$ the wheels are not slipping. If $T > T_{slip_s}$, the wheels begin to slip. If $T < T_{slip_k}$, the wheels stop slipping. If the wheels are slipping, the magnitude of the torque at the wheels is limited to T_{slip_k}, and we work backwards to calculate the associated current draw I.

With the limited torque at the wheel, we can calculate the net force on the robot, and therefore the robot's acceleration a:

$$F = \frac{T}{r} \qquad \implies \qquad a = \frac{F}{m} \tag{7}$$

And then we can apply that acceleration to change the robot's velocity and position:

$$v = v_{prev} + a \cdot dt \qquad \implies \qquad x = x_{prev} + v \cdot dt + \tfrac{1}{2} a \cdot dt^2 \tag{8}$$

Stopping Calculations

At the end of each timestep, we check if our conditions for stopping the simulation have been met, and if not how much voltage we should apply in the next timestep.

For stopping type "No Stop", the simulation is done when the robot reaches the sprint distance, $x \geq x_{sprint}$. For type "Stop After", the simulation is done when $x \geq x_{sprint}$ and the robot has come to rest, $v = 0$. For type "Predictive", the robot does not need to reach the sprint distance but it does need to have passed the stopping point (to be calculated shortly) and come to rest.

The stopping point x_{stop} is the distance at which the robot stops accelerating and begins decelerating, which is calculated based on the stopping type. If the stopping type is "No Stop" or "Stop After", the stopping point is the desired sprint distance, $x_{stop} = x_{sprint}$. If the stopping type is "Predictive", the stopping point is moved forward so that the robot should come to rest at the sprint distance. We will assume the deceleration a_{stop} is constant, which is not really the case but will give us a close enough result. Then from basic motion equations:

$$0 = v_f^2 = v_i^2 + 2a_{stop} \cdot \Delta x \implies x_{stop} = x_{sprint} - \frac{v^2}{2a_{stop}} \tag{9}$$

For "Brake" and "Reverse" stopping methods, we assume the wheels are slipping for the entirety of the deceleration, so $a_{stop} = -\frac{1}{m} F_{slip_k} = \mu_k g \cdot (\%W)$. For "Coast", the average deceleration is equal to approximately half of the maximum friction losses, $a_{stop} \approx -\frac{T_s(1-\eta)}{2mr}$.

If the simulation is not finished, we will calculate what voltage to apply to the system on the next timestep. If the robot has not yet reached its stopping point ($x < x_{stop}$), it is given full battery voltage. If it has reached the stopping point, the applied voltage is based on the selected stopping method. For "Coast", the battery is disconnected and no voltage is applied (i.e. the motor voltage is not constrained). For "Brake", the motor leads are "shorted", and the lead-to-lead voltage is forced to zero. For "Reverse", the full battery voltage is applied in reverse.

The calculations for this simulation are based in part on work done by Jesse Knight and FRC team 1885.

Projectile Trajectory Calculator

AMB Calculator

This calculator is used to help plan the trajectory of an object launched by a shooter. It can be used both to predict the object's final position and heading angle based on the initial values, or to calculate the needed initial values to arrive at the desired target values. All angles are measured in degrees above the horizontal.

Find Target Values

These modes iteratively simulates the object's trajectory from its launch until it hits the target, defined either by a horizontal distance or both a vertical height and direction of travel. In this mode, the simulation can incorporate drag and lift (Magnus) forces for spherical objects. In this configuration we will refer to the spherical object as a ball, but the purely parabolic equations hold for objects of any shape by setting the respective coefficients to zero.

We have a ball of radius r and mass m, moving at velocity $\bar{v}$ with rotational velocity $\bar{\omega}$. The ball has cross-sectional area $A_c = \pi r^2$, and drag and lift coefficients C_D and C_L respectively. At any time t there are three forces acting on the ball: the gravity force F_g, drag force F_D, and lift force F_L. These can be expressed in vector form as:

$$F_g = mg\left(-\hat{y}\right) \tag{1}$$

$$F_D = \frac{1}{2}\rho\left|\bar{v}\right|^2 A_c C_D\left(-\hat{v}\right) = \frac{\pi}{2}\rho r^2 v^2 C_D\left(-\hat{v}\right) \tag{2}$$

$$F_L = \frac{1}{2}\rho\left|\bar{v}\right|^2 A_c C_L\left(\hat{\omega}\times\hat{v}\right) = \frac{\pi}{2}\rho r^2 v^2 C_L\left(\hat{\omega}\times\hat{v}\right) \tag{3}$$

where ρ is the air density and g is the gravitational constant.

We will define θ_v as the angle of $\hat{v}$ counter-clockwise relative to $+\hat{x}$. So we can combine the x and y components of all the forces:

$$\begin{aligned} ma_x = \sum F_x &= -F_L\sin\theta_v - F_D\cos\theta_v \\ &= -\frac{\pi}{2}\rho r^2 v^2\left(C_L\sin\theta_v + C_D\cos\theta_v\right) \\ &= -\frac{\pi}{2}\rho r^2 v\left(C_L\cdot v_y + C_D\cdot v_x\right) \end{aligned} \tag{4}$$

$$\begin{aligned} ma_y = \sum F_y &= F_L\cos\theta_v - F_D\sin\theta_v - F_g \\ &= \frac{\pi}{2}\rho r^2 v^2\left(C_L\cos\theta_v - C_D\sin\theta_v\right) - mg \\ &= \frac{\pi}{2}\rho r^2 v\left(C_L\cdot v_x - C_D\cdot v_y\right) - mg \end{aligned} \tag{5}$$

Starting with $\bar{x} = \bar{0}$ and $\bar{v} = v_i\angle\theta_i$, we can calculate the acceleration components a_x and a_y. We can then calculate the position and velocity of the ball at the next timestep as:

$$\bar{v}_{i+1} = \bar{v}_i + \bar{a}\cdot dt \qquad \Longrightarrow \qquad \bar{x}_{i+1} = \bar{x}_i + \bar{v}_i\cdot dt + \tfrac{1}{2}\bar{a}\cdot dt^2 \tag{6}$$

The simulation end condition is determined by the mode. For "Find Target Values by Target Distance", the simulation ends when $x_x \geq d$, the horizontal distance to the target. For "Find Target Values by Target Height", the simulation ends when $x_y \geq h_{target}$ and $v_y \leq 0$ if the target height direction arrow is up, or $x_y \leq h_{target}$ and $v_y \leq 0$ if the target direction arrow is down.

If drag and Magnus forces are ignored, we can use traditional parabolic trajectory equations to find the object's final position and angle:

$$\begin{aligned} h_{target} = h_0 + d \cdot \tan\theta_0 - \frac{g \cdot d^2}{2v_0^2 \cos^2\theta_0} \iff \\ d = \frac{v_0^2 \cos^2\theta_0}{g}\left(\tan\theta_0 \pm \sqrt{\tan^2\theta_0 - \frac{2g}{v_0^2\cos^2\theta_0}\left(h_{target} - h_0\right)}\right) \end{aligned} \tag{7}$$

$$\theta_{target} = \tan^{-1}\left(2\,\frac{h_{target} - h_0}{d} - \tan\theta_0\right) \tag{8}$$

Find Launch Values

This mode calculates the launch velocity and angle needed to arrive at the desired target values in the absence of drag or Magnus forces. //
Starting with basic projectile motion equations:

$$\Delta x = v_0 \cos\theta_0 \cdot t \tag{9}$$

$$\Delta y = v_0 \sin\theta_0 \cdot t - \tfrac{1}{2} g \cdot t^2 \tag{10}$$

$$\theta_{target} = \tan^{-1}\left(\frac{v_{y,target}}{v_{x,target}}\right) = \tan^{-1}\left(\frac{v_0 \sin\theta_0 - g \cdot t}{v_0\cos\theta_0}\right) \tag{11}$$

Solving (9) for t and substituting into (10) gives:

$$\Delta y = \Delta x \cdot \tan\theta_0 - \frac{g \cdot (\Delta x)^2}{2v_0^2\cos^2\theta_0} \tag{12}$$

And with $\Delta x = d$ and $\Delta y = h_{target} - h_0$:

$$h_{target} = h_0 + d \cdot \tan\theta_0 - \frac{g \cdot d^2}{2v_0^2\cos^2\theta_0} \tag{13}$$

Substituting t from (9) into (11):

$$\theta_{target} = \tan^{-1}\left(\frac{v_0\sin\theta_0 - g \cdot \frac{\Delta x}{v_0\cos\theta_0}}{v_0\cos\theta_0}\right) = \tan^{-1}\left(\tan\theta_0 - \frac{g \cdot d}{v_0^2\cos^2\theta_0}\right) \tag{14}$$

Further substituting in (13) and simplifying gives:

$$\theta_{target} = \tan^{-1}\left(2\frac{h_{target} - h_0}{d} - \tan\theta_0\right) \tag{15}$$

Which can easily be solved for θ_0:

$$\theta_0 = \tan^{-1}\left(2\frac{h_{target} - h_0}{d} - \tan\theta_{target}\right) \tag{16}$$

We can then substitute in (13):

$$\tan\theta_{target} - \tan\theta_0 = -\frac{g \cdot d}{v_0^2 \cos^2\theta_0} \tag{17}$$

Which can be solved for v_0:

$$v_0 = \sec\theta_0 \cdot \sqrt{\frac{g \cdot d}{|\tan\theta_{target} - \tan\theta_0|}} \tag{18}$$

Thus we have the initial velocity v_0 and angle θ_0. We can then run the simulator without drag or Magnus forces to track the projectile's trajectory.

Experimentally Determining Drag and Lift Coeffieicnts

For the lift coefficient, we can use experiments done at the University of Illinois. They defined a dimensionless quantity called the spin factor, $s = \frac{r \cdot \omega}{v}$, based on the ball's rotational and linear velocities ω and v. They found that the lift coefficient is directly related to this parameter, following $C_L = 1.6s$ for $s < 0.1$ and $C_L = 0.6s + 0.1$ for $s > 0.1$. By entering the rotational velocity, assumed to be constant throughout the trajectory, the simulation automatically calculates the lift coefficient according to the spin factor for each timestep.

Determining the drag coefficient is more complicated. Earlier versions calculated the Reynolds number of the ball and used that to find the drag coefficient, but this method was not very accurate as typical values often fell in the transition range between laminar and turbulent flow. Instead, the recommended method now involves experimental testing by dropping the ball off a high surface of known height and measuring the time it takes to hit the ground.

We will define a constant $\alpha = \frac{\pi}{2m}\rho r^2 C_D$. In this case, the motion equation is:

$$ma = m\frac{dv}{dt} = \sum F = F_D - F_g = \alpha m v^2(t) - mg \tag{19}$$

This differential equation can be solved for $v(t)$ with initial condition $v(0) = 0$:

$$v(t) = -\sqrt{\frac{g}{\alpha}} \tanh\left(\sqrt{\alpha g} \cdot t\right) \tag{20}$$

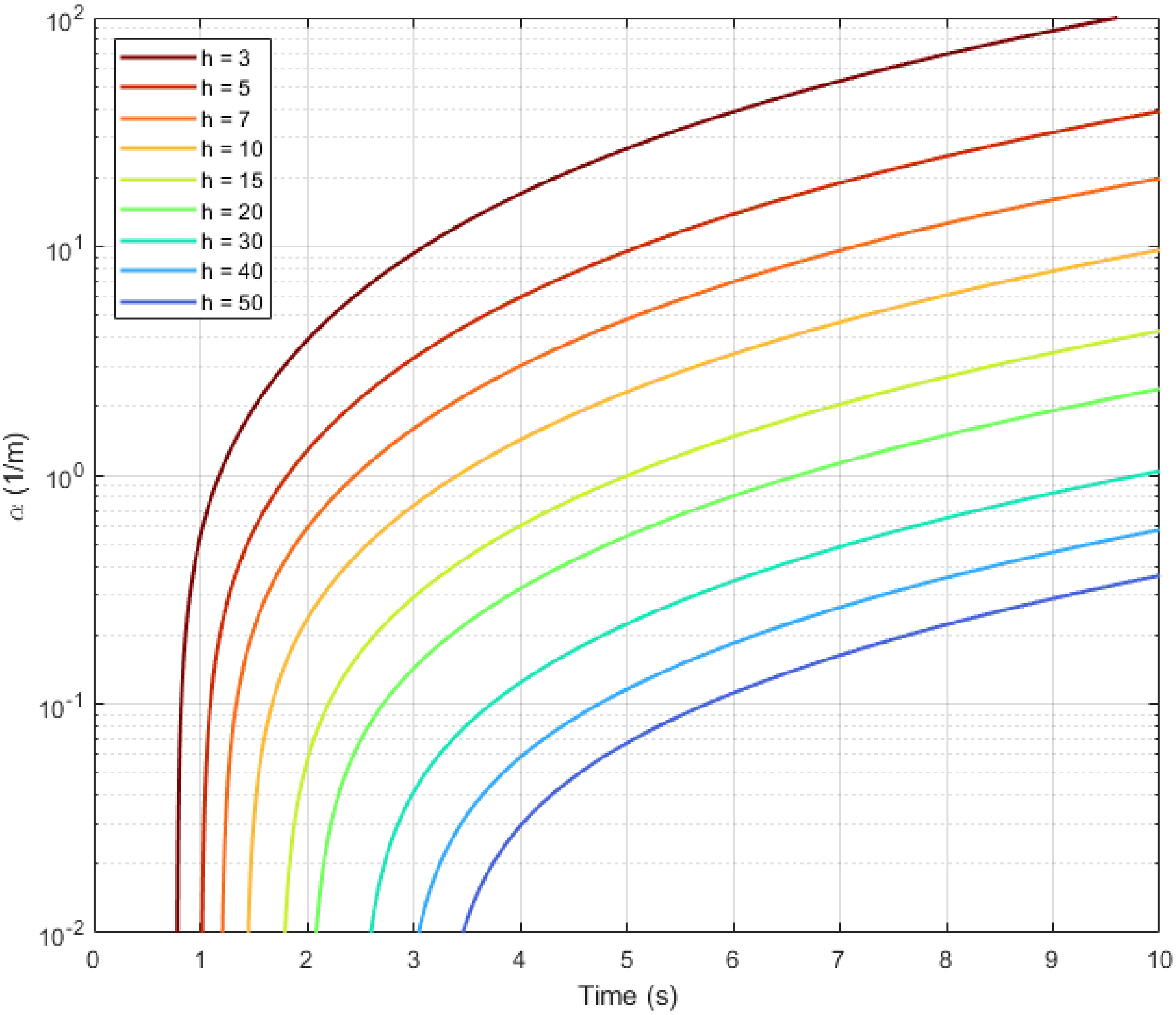
This can then be integrated with initial condition $x(0) = h$ to get the position equation:

$$x(t) = h - \frac{1}{\alpha} \ln\left(\cosh\left(\sqrt{g\alpha} \cdot t\right)\right) \tag{21}$$

So solving for the time the ball hits the ground, $x(t) = 0$, gives:

$$t = \frac{1}{\sqrt{g\alpha}} \cosh^{-1}\left(\exp\left(\alpha \cdot h\right)\right) \tag{22}$$

We would like to solve for α in order to find C_D, but unfortunately this is not possible to do analytically. Instead, we will use the following graph to find α based on the drop height h (in meters) and the time the ball takes to hit the ground:



We can now find C_D using the definition of α:

$$C_D = \frac{2\alpha \cdot m}{\pi\rho \cdot r^2} \tag{23}$$

Where m is the ball's mass, r is its radius, and ρ is the density of air ($1.275\ \frac{\mathrm{kg}}{\mathrm{m}^3}$ at STP). Note that all of the units in the equation should match so that C_D is a dimensionless parameter.

The equations in this calculator are based in part by work done by the University of Illinois Department of Physics

Pneumatics System Simulator

AMB Calculator

This simulator is used to determine the air usage of your robot based on the number and size of its cylinders and how often they fire. It can be helpful for determining how many air tanks are needed to maintain pressure throughout the match and the effects of using a compressor during the match versus only between matches.

The simulation is run discretely in time with timesteps of one second. We know the energy of the system E at any time can be represented by the pressure of the system P times its volume V, $E = P \cdot V$. We can find the energy of the system at any timestep by adding the total work done on the system during that time to the energy at the previous step. That work comes in the form of positive work done by the compressor and negative work done by the cylinders:

$$E_{n+1} = E_n + W_{tot} \implies (PV)_{n+1} = (PV)_n + W_{comp} - \sum W_{cyl} \tag{1}$$

Since the volume of the system remains constant, we can divide by V to get:

$$P_{n+1} = P_n + \frac{W_{comp} - \sum W_{cyl}}{V} \tag{2}$$

According to the laws of thermodynamics, the work done by compression/expansion of a gas at constant pressure can be represented by $W = P \cdot \Delta V$. In each actuation of the cylinder gas is allowed to expand twice, once when it extends and once when it retracts. So for each actuation, the work lost by a cylinder with cross-sectional area A_c and length L is:

$$W_{cyl} = (P \cdot A_c L)_{push} + (P \cdot A_c L)_{pull} \tag{3}$$

L is constant between the extend and retract stroke, so it can be factored out. For the extend stroke, $A_c = \frac{\pi}{4} D^2$ for a piston of diameter D. For the retract stroke, $A_c = \frac{\pi}{4}\left(D^2 - d^2\right)$ with bore diameter d. So the work used in each actuation cycle is:

$$W_{cyl} = \frac{\pi}{4} L \left[D^2 P_{push} + \left(D^2 - d^2\right) P_{pull}\right] \tag{4}$$

We will define a variable m to represent the number of actuations of a given cylinder in a given timestep. m is necessarily zero if the timestep is before the cylinder's start time or after its end time. If the time per cycle (period) of the cylinder T is more than one (i.e. less than one actuation cycle per second), $m = 1$ if the remainder of the time elapsed since the cylinder's start divided by the cylinder period is less than the timestep size, otherwise zero. This way one cycle's work is lost for every cycle period. If the time per cycle is less than one (i.e. more than one actuation per cycle), $m = \frac{1}{T}$.

Similar to the cylinders, we can represent the work done by the compressor as:

$$W_{comp} = \Delta V \cdot P = \dot{V}(P) \cdot dt \cdot 1 \text{ atm} \tag{5}$$

Where $\dot{V}(P)$ is the compressor's volumetric flow rate as a function of the system pressure, measured at atmospheric pressure (hence multiplying by 1 atm). We can approximate this function using a cubic interpolation of the data provided by `andymark.com`. The work from the compressor is added to the system when the system pressure starts below the compressor trigger and until it hits the compressor threshold.

In order to ensure the robot will not run out of air during the match, we want to make sure that the pressure never falls below the highest regulated pressure used by any of the cylinders.

Beam Bend Calculator

AMB Calculator

This calculator is used to check the deflection or twist in a beam (extrusion of uniform cross-section). It can be helpful for determining whether a profile or axle will be strong enough to carry the desired load. Note that this is not a replacement for proper Finite Element Analysis simulation and does not return the material stress.

Material & Cross-Section

Materials are defined with three values: Young's Modulus E, Shear Modulus G, and density ρ. You can choose one of the pre-defined materials or enter these values manually.

Five types of cross-sections are defined: hex, round, round tube, rectangular, and rectangular tube. Each cross-section geometry has its own equations to find the corresponding Area A, Area Moment of Inertia I, and Torsional Constant J. These can also be entered manually.

For hex beams with distance a between the flat sides:

$$A = \frac{3\sqrt{3}}{8}a^2 \qquad I = 0.0601a^4 \qquad J = 0.1154a^4 \tag{1}$$

For solid round beams with diameter D:

$$A = \frac{\pi}{4}D^2 \qquad I = \frac{\pi}{64}D^4 \qquad J = \frac{\pi}{32}D^4 \tag{2}$$

For round tubes with outside diameter D and thickness t:

$$A = \pi D \cdot t \qquad I = \frac{\pi}{8}D^3 t \qquad J = \frac{\pi}{4}D^3 t \tag{3}$$

For solid rectangular beams with width (perpendicular to the applied force) w and height (parallel to the applied force) h, and where a is the larger of w and h and b is the smaller:

$$A = w \cdot h \qquad I = \frac{1}{12}wh^3 \qquad J \approx \frac{1}{3}ab^3 - 0.21b^4 + 0.0175\frac{b^8}{a^4} \tag{4}$$

For rectangular tubes with width w, height h, and thickness t:

$$A = 2t(a+b) \qquad I = \frac{1}{3}wh^2 t \qquad J = \frac{2t(w-2)^2(h-t)^2}{w+h-t} \tag{5}$$

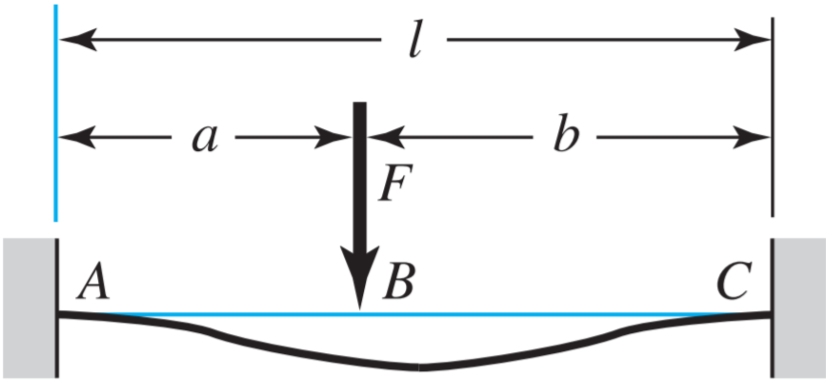
Deflection Equations

With the cross-sectional area and the beam's length l, we can calculate its mass m:

$$m = \ell \cdot A_c \cdot \rho \tag{6}$$

Based on the beam's material, cross-section, and relevant dimension we can calculate the amount it will deflect under a given load. We will use the equations from Shigley's Mechanical Engineering Design textbook, Appendix A-9.

We will define a force on a beam supported on both sides.



With a force F applied perpendicularly at distance a from the left side of the beam and b from the right side, the deflection y at distance x from the left side is:

$$\begin{aligned} y\left(x \leq a\right) &= F \cdot \frac{b^2 x^2}{6EI \cdot l^3}\left[3al - x\left(3a+b\right)\right] \\ y\left(x \geq a\right) &= F \cdot \frac{a^2\left(l-x\right)^2}{6EI \cdot l^3}\left[3bl - \left(l-x\right)\left(3b+a\right)\right] \end{aligned} \tag{7}$$

Substitute $b = l - a$, where $a \leq \frac{l}{2} \leq b$:

$$\begin{aligned} y\left(x \leq a\right) &= F \cdot \frac{x^2\left(l-a\right)^2}{6EI \cdot l^3}\left[3al - x\left(l+2a\right)\right] \\ y\left(x \geq a\right) &= F \cdot \frac{a^2\left(l-x\right)^2}{6EI \cdot l^3}\left[3xl - a\left(l+2x\right)\right] \end{aligned} \tag{8}$$

Taking the derivative of each and setting equal to zero, we find the local maxima of deflection:

$$x_{x\leq a} = 0,\ \frac{2al}{2a+l}\ ; \qquad\qquad x_{x\geq a} = \frac{l^2}{3l-2a},\ l \tag{9}$$

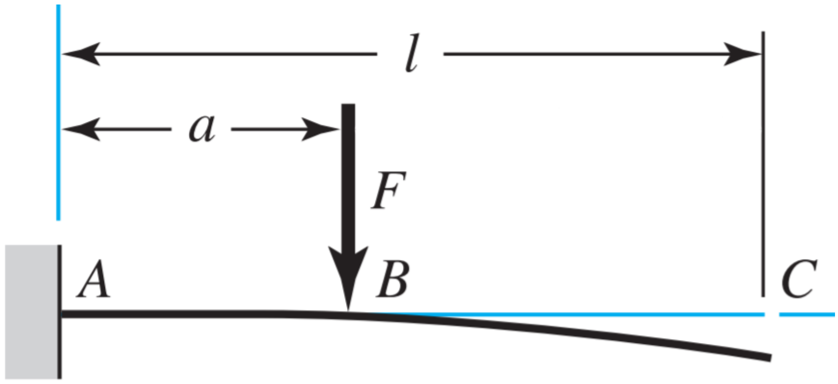
At $x = 0$ and $x = l$ the deflection is fixed at zero, so they cannot be the points of maxima. We must test the other solutions to make sure that they fulfil their domain requirements:

$$\begin{aligned} \frac{2al}{2a+l} \leq a &\implies a \geq \frac{l}{2}\ , \\ \frac{l^2}{3l-2a} \geq a &\implies a \leq \frac{l}{2}\ \cup\ l \leq a \leq \frac{3}{2}l \end{aligned} \tag{10}$$

The domain $a \geq \frac{l}{2}$ for the first solution contradicts our definition of $a \leq \frac{l}{2} \leq b$, therefore that solution is not valid. The second solution does not conflict, therefore it is valid and the maximum deflection occurs at $x = \frac{l^2}{3l-2a}$. Plugging this into the corresponding deflection equation gives the maximum deflection for a beam supported on both sides:

$$y_{max} = \frac{2Fa^2\left(l-a\right)^3}{3EI\left(2a-3l\right)^2} \tag{11}$$

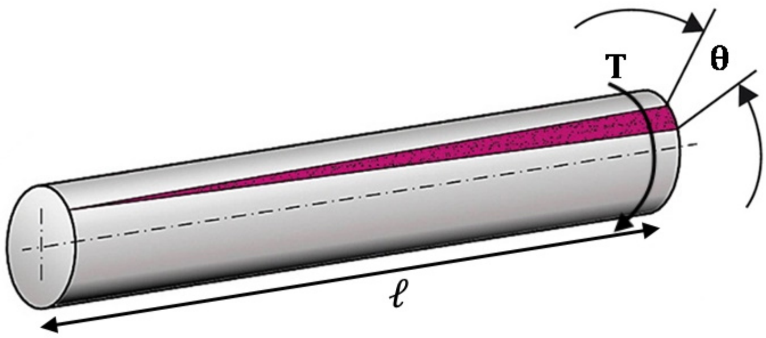
We will now define a force on a beam supported only on one side.



The beam is of length l, fixed on the left side, with a force F applied perpendicularly at a distance of a away from the fixture. It is clear that the maximum deflection will occur at the right end of the beam. The magnitude of this deflection is represented by:

$$y_{max} = \frac{Fa^2}{6EI}(3l - a) \tag{12}$$

Again we will define a beam of length l supported only on one side, but this time we apply a twisting torque T at the opposite end.



This beam will deflect angularly along its axis. It is clear that the maximum angular deflection occurs at the end where the force is being applied. The magnitude of this deflection is:

$$\theta_{max} = \frac{Tl}{GJ} \tag{13}$$

Lead Screw Calculator

AMB Calculator

This calculator is used to determine the properties of a lead screw based on its dimensions and materials, and to use it to convert between rotational and linear motion and force. The calculator also provides equivalent parameters that can be plugged into the Mechanism Calculator in order to use a lead screw as the final stage of a mechanism.

The basic transformation between rotational speed ω and linear speed v for a lead screw with n starts and pitch p is given by:

$$v = np \cdot \omega \tag{1}$$

This value np is called the "lead", and is the amount the screw advances linearly for every full rotation.

Because of inefficiencies in power transfer however, the torque : force ratio is not equal to the inverse of the lead as you might expect for a 100% efficient system. For a lead screw with outside diameter d, we will define the "mean diameter" of the lead screw d_m to be $d_m = d - \frac{1}{2}p$. The lead screw has a pitch angle of α and a coefficient of friction between the screw and nut of μ. Then the torque needed to oppose an applied force (e.g. to raise a mass) is given by the following equation:

$$T_R = F \cdot \frac{d_m}{2}\left(\frac{\pi\mu d_m + np\cos(\alpha)}{\pi d_m\cos(\alpha) - \mu np}\right) \tag{2}$$

Depending on the dimensions and materials of the lead screw, it may take a negative torque input in order to allow an applied force to spin the screw. This property is called being "backdrivable". The lead screw is backdrivable if the torque provided by an applied load is negative. If it is zero or positive, the lead screw is said to be non-backdrivable. For example, a mass is attached to a lead screw at height and released. If the lead screw begins to spin and the mass falls, the lead screw is backdrivable. If the mass stays in place and requires a negative torque input to lower it, the lead screw is not backdrivable.

$$T_L = F \cdot \frac{d_m}{2}\left(\frac{\pi\mu d_m - np\cos(\alpha)}{\pi d_m\cos(\alpha) + \mu np}\right) \tag{3}$$

The efficiency of the lead screw η is the ratio between the raise torque that would be required if there were no friction to the raise torque required with friction. This is equal to:

$$\eta = \frac{T_R|_{\mu=0}}{T_R} = \frac{np}{\pi d_m} \cdot \frac{\pi d_m\cos(\alpha) - \mu np}{\pi\mu d_m + np\cos(\alpha)} \tag{4}$$

Equivalent Radius and Load

In order to use a lead screw as the output in a mechanism for the Mechanism Calculator, we will define an equivalent radius and load so that the rotational output is translated correctly to linear movement. To find the equivalent radius r_{eq}, we equate the rotational to linear velocity transformation for both a wheel and a lead screw:

$$2\pi r \cdot \omega = v = np \cdot \omega \tag{5}$$

We can then solve for $r = r_{eq}$ to find the radius that will provide the same rotational to linear motion transformation as the given lead screw:

$$r_{eq} = \frac{np}{2\pi} \tag{6}$$

We calculated the raise and lower torques for the lead screw, but the Mechanism calculator takes a linear force L at the radius given. Since $T = F \cdot r$, we can say:

$$\begin{aligned} L_R &= \frac{T_R}{r_{eq}} = F \cdot \frac{\pi d_m}{np} \left(\frac{\pi \mu d_m + np\cos(\alpha)}{\pi d_m \cos(\alpha) - \mu np} \right) \\ L_L &= \frac{T_L}{r_{eq}} = F \cdot \frac{\pi d_m}{np} \left(\frac{\pi \mu d_m - np\cos(\alpha)}{\pi d_m \cos(\alpha) + \mu np} \right) \end{aligned} \tag{7}$$

If using the Mechanism Calculator with a lead screw, make sure to adjust for the large inefficiency it adds to the system by multiplying the existing efficiency by the lead screw efficiency calculated above.

Equations used in this calculator are based on Shigley's Mechanical Engineering Design textbook.