EXERCISES

6.1 Identify appropriate ARIMA models from the sample ACF below. Justify your choice using the knowledge of the theoretical ACF for ARIMA models.

(a) $n = 121$, data $= Z_t$

k	1	2	3	4	5	6	7	8	9	10
$\hat{\rho}_k$	.15	−.08	.04	.08	.08	.03	.02	.05	.04	−.11

(b) $n = 250$, data $= Z_t$

k	1	2	3	4	5	6	7	8	9	10
$\hat{\rho}_k$	−.63	.36	−.17	.09	−.07	.06	−.08	.10	−.11	.06

(c) $n = 250$, data $= Z_t$

k	1	2	3	4	5	6	7	8	9	10
$\hat{\rho}_k$	−.35	−.17	.09	−.06	.01	−.01	−.04	.07	−.07	.09

(d) $n = 100$, data $= Z_t$, $W_t = (1 - B)Z_t$, $\overline{W} = 2.5$, $S_W^2 = 20$.

k	1	2	3	4	5	6	7	8	9	10
$\hat{\rho}_Z(k)$	.99	.98	.98	.97	.94	.91	.89	.86	.85	.83
$\hat{\rho}_W(k)$	.45	−.04	.12	.06	−.18	.16	−.07	.05	.10	.09

(e) $n = 100$, data $= Z_t$, $W_t = (1 - B)Z_t$, $\overline{W} = 35$, $S_W^2 = 1500$

k	1	2	3	4	5	6	7	8	9	10
$\hat{\rho}_Z(k)$	.94	.93	.90	.89	.87	.86	.84	.81	.80	.80
$\hat{\rho}_W(k)$	.69	.50	.33	.19	.10	.08	.03	.01	.01	.00

6.2 Identify proper ARIMA models for the following data sets (read across):

(a)

−2.401	−.574	.382	−.535	−1.639	−.960	−1.118
−.719	−1.236	.117	−.493	−2.282	−1.823	.645
−.179	.589	1.413	.370	.082	−.531	−1.891
−.961	−.865	−.790	−1.476	−2.491	−4.479	−2.809
−2.154	−1.532	−2.119	−3.349	−1.588	.740	.907
1.540	.557	2.259	2.622	.701	2.463	2.714
2.089	3.750	4.322	3.186	3.192	2.939	3.263
3.279	.295	.227	1.356	1.912	1.060	.370
−.195	.340	1.084	1.237	.610	2.126	3.960
3.317	2.167	1.292	.595	.140	−.082	−.769
.870	1.551	2.610	2.193	1.353	−.600	−.455
.203	1.472	1.367	1.875	2.082	1.604	2.033
3.746	2.954	.676	1.163	1.368	.343	−.334
1.041	1.328	1.325	.968	1.970	2.296	2.896
1.918	1.569					

(b)

−1.453	.867	.727	−.765	−1.317	.024	−.542
−.048	−.805	.858	−.563	−1.986	−.454	1.738
−.566	.697	1.060	−.478	−.140	−.581	−1.572
.174	−.289	−.270	−1.002	−1.605	−2.984	−.122
.469	−.239	−1.200	−2.077	.421	1.693	.463

	.996	−.367	1.925	1.267	−.872	2.043	1.236
	.461	2.497	2.072	.593	1.281	1.023	1.500
	1.321	−1.673	.050	1.219	1.098	−.087	−.266
	−.417	.457	.880	.586	−.132	1.760	2.684
	.941	.177	−.008	−.180	−.217	−.165	−.720
	1.332	1.029	1.679	.627	.038	−1.412	−.095
	.476	1.350	.484	1.055	.957	.355	1.071
	2.526	.707	−1.096	.757	.670	−.477	−.540
	1.241	.704	.528	.173	1.389	1.115	1.519
	.180	.419					
(c)	3.485	5.741	5.505	3.991	3.453	4.773	4.142
	4.598	3.796	5.430	3.960	2.541	4.054	6.155
	3.778	5.066	5.422	3.908	4.302	3.876	2.888
	4.613	4.075	4.054	3.288	2.654	1.215	3.979
	3.452	3.569	2.523	1.584	3.998	5.135	3.842
	4.404	3.077	5.432	4.795	2.747	5.767	4.988
	4.311	6.456	6.114	4.785	5.646	5.516	6.121
	6.059	3.196	5.050	6.231	6.119	4.988	4.885
	4.777	5.666	6.081	5.801	5.126	7.067	8.015
	6.358	5.752	5.700	5.614	5.629	5.705	5.155
	7.204	6.871	7.555	6.565	6.081	4.719	6.090
	6.637	7.492	6.635	7.264	7.221	6.694	7.493
	9.012	7.274	5.622	7.593	7.533	6.432	6.424
	8.219	7.668	7.534	7.232	8.501	8.266	8.748
	7.501	7.856					
(d)	.315	−.458	−.488	−.170	.565	−.344	−1.176
	−1.054	−.826	.710	−.341	−1.809	−1.242	−.667
	−.999	2.812	1.286	−1.084	−1.505	−2.556	−.144
	−1.749	−3.032	−2.958	−2.827	−3.392	−2.431	−2.757
	−2.822	−3.314	−2.738	−1.979	−1.671	−2.977	−.709
	.718	.736	.879	1.642	2.180	1.963	.716
	.769	.973	.334	1.309	.878	.062	.169
	.677	1.851	.242	.828	−.317	−1.042	−2.093
	.653	.261	2.020	2.136	1.635	−.141	−1.747
	−2.047	−.752	−.211	−1.062	−1.565	.232	.015
	−.935	−.338	.853	.888	3.069	3.364	3.854
	4.419	2.145	2.291	1.753	1.058	1.048	.200
	1.424	.590	.356	.476	.684	−2.260	−.569
	−1.014	−.207	.638	−.664	−.469	−.215	−.296
	−1.561	.246					

6.3 Identify models for Series W1 through W7 using ESACF and compare them with the models suggested in the book using ACF and PACF.

7.9 Suppose that $(1 - \phi B)Z_t = (1 - \theta B)a_t$ is a tentatively entertained model for a process. Given

t	1	2	3	4	5	6	7	8	9	10	11	12
Z_t	−3.1	−.8	1.2	.6	2.8	−.9	.3	−1.4	−2.5	−1.1	.9	1.4

calculate the unconditional sum of squares for $\phi = .4$ and $\theta = .8$.

7.10 Consider the AR(1) model

$$(1 - \phi B)(Z_t - \mu) = a_t.$$

(a) For $\mu = 0$, find the maximum likelihood estimator for ϕ and its associated variance.

(b) Find the maximum likelihood estimators for ϕ and μ when $\mu \neq 0$.

(c) Discuss the relationship between the ordinary least square estimator and the maximum likelihood estimator for ϕ in the given model.

7.11 Illustrate the nonlinear estimation procedure discussed in Section 7.3 using the following models:

(a) The MA(1) model $Z_t = (1 - \theta B)a_t$.

(b) The AR(2) model $Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t$.

7.12 Derive the joint asymptotical distribution of the least squares estimators, $\left[\hat{\phi}_1, \hat{\phi}_2\right]'$, for the AR(2) model

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t.$$

7.13 Consider the yearly data of lumber production (in billions of board feet) in the United States given as follows:

Year	Production									
1921–1930	29.0	35.2	41.0	39.5	41.0	39.8	37.3	36.8	38.7	29.4
1931–1940	20.0	13.5	17.2	18.8	22.9	27.6	29.0	24.8	28.8	31.2
1941–1950	36.5	36.3	34.3	32.9	28.1	34.1	35.4	37.0	32.2	38.0
1951–1960	37.2	37.5	36.7	36.4	37.4	38.2	32.9	33.4	37.2	32.9
1961–1970	32.0	33.2	34 7	36.6	36.8	36.6	34.7	36.5	35.8	34.7
1071–1980	37.0	37.7	38.6	34.6	32.6	36.3	39.4	40.5	40.6	35.4
1981–1982	31.7	30.0								

(a) Plot the data and perform necessary analysis to construct an appropriate model for the series.

(b) Find and plot the forecasts for the next four years, and calculate 95% forecast limits.

(c) Update your forecasts when the 1983 observation became available and equaled 34.6.

8.6 Consider the following U.S. liquor sales (in millions of dollars):

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1970	580	514	555	563	627	596	632	639	577	611	639	875
1971	650	594	650	668	712	731	779	712	708	738	758	1073
1972	669	652	743	709	751	774	803	760	749	757	779	1066
1973	734	707	785	762	838	876	878	871	807	834	877	1236
1974	789	744	827	831	895	889	955	983	976	929	989	1294
1975	860	799	899	866	1016	978	1042	1026	944	1002	1009	1368
1976	908	849	916	958	1008	1033	1129	1019	984	1045	1049	1459
1977	910	927	981	1011	1041	1080	1138	1072	1033	1072	1111	1591
1978	950	932	1049	1021	1097	1151	1194	1174	1160	1135	1209	1692
1979	1071	1044	1158	1122	1209	1334	1360	1368	1297	1283	1375	1974
1980	1294	1258	1301	1297	1425	1378	1429	1452	1305	1377	1439	1958

(a) Build a seasonal ARIMA model for the series.

(b) Forecast the next 12 observations, and find their 95% forecast limits.

8.7 Consider the following U.S. personal consumption of gasoline and oil (in billions of dollars) between 1967 and 1982:

Year	I	II	III	IV
1967	16.6	16.9	17.1	17.5
1968	18.1	18.3	19.0	19.1
1969	19.8	20.8	20.9	21.4
1970	21.8	22.3	22.5	23.1
1971	23.5	23.4	24.0	24.6
1972	24.8	24.7	25.5	26.6
1973	27.6	27.9	28.4	30.6
1974	32.4	36.9	38.2	38.9
1975	38.6	39.3	41.3	42.4
1976	42.8	43.0	44.1	45.9
1977	47.2	48.6	48.3	48.5
1978	49.3	49.9	51.5	54.3
1979	57.7	62.2	70.5	75.9
1980	80.7	84.4	85.1	88.9
1981	94.1	95.1	94.6	94.7
1982	93.4	88.6	89.9	89.6

(a) Build a seasonal ARIMA model for the series.

(b) Forecast the next 4 observations and find their associated 95% forecast limits.