Vypočtěte derivaci

1) $f(x) = 12 + x^2 \cdot 2^x$

$f'(x) = 0 + (x^2 \cdot 2^x)' = x^2 2^x \ln 2 + 2^x \cdot 2x =$

$= 2^x(x^2 \ln 2 + 2x)$

2) $f(x) = \sqrt{1-x^2}$

$f'(x) = \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}}$

3) $f'(x) = \left(\sin^3\left(\frac{x}{4}\right)\right)' = 3 \cdot \sin^2\left(\frac{x}{4}\right) \cdot \frac{1}{4} \cdot \cos\left(\frac{x}{4}\right)$

4) $f(x)=\frac{1}{2}x-\frac{1}{2}\sin x\cos x$

$f'(x)=\frac{1}{2}-\frac{1}{2}(-\sin^2 x+\cos^2 x)=\frac{1}{2}(\sin^2 x-\cos^2 x+1)=$

$=\frac{1}{2}(\sin^2 x-\cos^2 x+(\sin^2 x+\cos^2 x))=\frac{1}{2}(2\sin^2 x)=\sin^2 x$

5) $f(x)=x\cdot\ln\frac{x-1}{x+1}$

$f'(x)=\ln\frac{x-1}{x+1}+x\cdot\left(\ln\frac{x-1}{x+1}\right)'=\underline{\underline{\ln\frac{x-1}{x+1}+\frac{2x}{x^2-1}}}$

$\left(\ln\frac{x-1}{x+1}\right)'=\frac{1}{\frac{x-1}{x+1}}\cdot\frac{(x+1)-(x-1)}{(x+1)^2}=\frac{x+1}{x-1}\cdot\frac{2}{(x+1)^2}=$

$=\frac{2}{x^2-1}$

6) $\lim\limits_{x \to -2} \frac{\sqrt{6+x} - \sqrt{2-x}}{x+2} = \lim\limits_{x \to -2} \frac{6+x-2+x}{(\sqrt{6+x}+\sqrt{2-x})(x+2)} =$

$= \lim\limits_{x \to -2} \frac{4+2x}{(\sqrt{6+x}+\sqrt{2-x})(x+2)} = \lim\limits_{x \to -2} \frac{2}{\sqrt{6+x}+\sqrt{2-x}} \overset{*}{=}$

$\overset{*}{=} \frac{2}{2+2} = \underline{\underline{\frac{1}{2}}}$

7) $\lim\limits_{x \to 1} \frac{x^3-2x^2+2x-1}{x^2-1}$

Nejdrive uprávime polynom: $x^3-2x^2+2x-1 = 2x(-x+1) +$
$+ x^3-1 = (-2x)(x-1) + (x-1)(x^2+x+1) = (x-1)(x^2-x+1)$

Potom tedy: $\lim\limits_{x \to 1} \frac{\cancel{(x-1)}(x^2-x+1)}{\cancel{(x-1)}(x+1)} = \lim\limits_{x \to 1} \frac{x^2-x+1}{x+1} =$

$= \underline{\underline{\frac{1}{2}}}$