Integrály

① $\int \frac{3+xe^x}{x}dx = \int \frac{3}{x} + e^x dx = \int \frac{3}{x}dx +$
$+ \int e^x dx = 3\ln|x| + e^x + c \qquad c \in R$

② Per partes

(a) $\int x^2 \cdot \sin x \, dx \overset{\text{použijeme per partes}}{=} -x^2\cos x + \int 2x\cos x \, dx \overset{**}{=}$

1. definujeme $f = x^2 \qquad f' = 2x$
$g' = \sin x \qquad g = -\cos x$

2. spočítame $\int 2x\cos x \, dx$

$\int 2x\cos x \, dx \overset{\text{per partes}}{=} 2x\sin x - \int 2\sin x \, dx \overset{*}{=}$

definujeme $f = 2x \qquad f' = 2$
$g' = \cos x \qquad g = \sin x$

spočítame $\int 2\sin x \, dx = -2\cos x + c'$

potom $\overset{*}{=} 2x\sin x + 2\cos x + c$
potom $\overset{**}{=} -x^2\cos x + 2x\sin x + 2\cos x + c$

(b) $\int \cos^4 x \cdot \sin x \, dx = -\cos^5 x - \int 4\cos^4 x \sin x \, dx \overset{*}{=}$

1. definujeme $f = \cos^4 x \qquad f' = 4\cos^3 x \cdot (-\sin x)$
$g' = \sin x \qquad g = -\cos x$

$\stackrel{*}{=} -\cos^5 x - 4\int \cos^4 x \sin x\,dx$

dále postupujeme jako s obyč. rovnicí;

$$\int \cos^4 x \sin x\, dx = -\cos^5 x - 4\int \cos^4 x \sin x\, dx$$

$$5\int \cos^4 x \sin x\, dx = -\cos^5 x$$

$$\int \cos^4 x \sin x\, dx = \frac{-\cos^5 x}{5} + c$$

③ Substituce

$$\int \sin(4x-1)\, dx = \int \sin(y)\frac{dy}{4} = \frac{-1}{4}\cos(y) + c =$$

označme: $f(y) = \sin(y)$

$y = 4x - 1$

$dy = 4\,dx \Rightarrow dx = \frac{dy}{4}$

$$= \frac{-1}{4}\cos(4x-1) + c$$

④

$$\int_2^6 \sqrt{x-2}\, dx = \left[\frac{2}{3}(x-2)^{3/2}\right]_2^6 = \frac{2}{3}\left(\sqrt{(6-2)^3} \stackrel{*}{-}\right.$$

pro $f(x) = \sqrt{x-2}$ máme $D_f = \langle 2; \infty)$

pak máme $\int \sqrt{x-2}\, dx = \left| \begin{matrix} \text{sub.} & \\ f(y) = \sqrt{y} & dy = 1dx \\ y = x-2 & dx = dy \end{matrix} \right| =$

$$= \int \sqrt{y}\, d = y^{3/2} \cdot \frac{2}{3} + c = (x-2)^{3/2} \cdot \frac{2}{3} + c$$

$$\stackrel{*}{-} \left.\sqrt{(2-2)^3}\right) = \frac{2}{3}\left(\sqrt{4^3} - 0\right) = \frac{16}{3}$$

(5) $\int_0^1 \frac{1}{x^2}\,dx = \lim_{c\to 0+} [-x^{-1}]_c^1 = \lim_{c\to 0+} -1^{-1} + c^{-1} \overset{*}{=}$

pro $f(x) = \frac{1}{x^2}$ máme $\mathcal{D}_f = \mathbb{R}\setminus\{0\}$, tedy jde o nevlastní integrál

$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = x^{-1}\cdot(-1) + c$$

$\overset{*}{=} \lim_{c\to 0+}\left(1+\frac{1}{c}\right) = \infty$