1) $\lim\limits_{n\to\infty} \dfrac{3+n+7^n}{(\frac{1}{2})^n-2} = \lim \dfrac{7^n(\frac{3}{7^n}+\frac{n}{7^n}+1)}{(\frac{1}{2})^n-2} \overset{?}{=}$

$\overset{?}{=} \dfrac{\infty\cdot(0+0+1)}{0-2} = -\infty$

2) $\lim\limits_{n\to\infty} (\sqrt{2n^2+n}-n) = \lim (\sqrt{2n^2+n}-n)\dfrac{(\sqrt{2n^2+n}+n)}{(\sqrt{2n^2+n}+n)} =$

$= \lim \dfrac{2n^2+n-n^2}{\sqrt{2n^2+n}+n} = \lim \dfrac{n^2+n}{\sqrt{2n^2+n}+n} = \lim \dfrac{n^2(1+\frac{1}{n})}{n(\sqrt{2+\frac{1}{n}}+1)} \overset{?}{=}$

$\overset{?}{=} \dfrac{\infty\cdot(1+0)}{(\sqrt{2+0}+1)} = \infty$

3) $\lim\limits_{n\to\infty} \left(\dfrac{n+2}{3n+1}-2^n\right) = \lim\limits_{n\to\infty} \left(\dfrac{1+\frac{2}{n}}{3+\frac{1}{n}}-2^n\right) \overset{?}{=}$

$\overset{?}{=} \dfrac{1+0}{3+0} - \infty = -\infty$

4) určete a tak, aby platilo

$\lim(an^2-3n+n^2) = \infty$

$\lim(an^2-3n+n^2) =$ — pokud $a<-1$, potom

$an^2+n^2-3n = \underbrace{(a+1)}_{\text{je menší než } 0}n^2-3n$

potom $\lim\limits_{n\to\infty} (\underbrace{n^2}_{\to\infty}(\underbrace{(a+1)}_{\to a+1\neq 0} - \underbrace{\tfrac{3}{n}}_{\to 0})) =$

$= -\infty$

— pokud $a=-1$, potom

$an^2+n^2-3n = -n^2+n^2-3n = -3n$,

tedy $\lim\limits_{n\to\infty}(-3n) = -\infty$

— pokud $a>-1$, potom $a+1>0$

a tedy $\lim\limits_{n\to\infty}(n^2((a+1)-\frac{3}{n})) = \infty$

Tedy $\lim(an^2-3n+n^2) = \infty$ pro $a>-1$

5) $\lim_{n\to\infty}\left(\sqrt{n^2+2n+2}-n\right) \overset{\text{vynásobíme 1}}{=} \lim\left(\frac{n^2+2n+2-n^2}{\sqrt{n^2+2n+2}+n}\right) =$

$= \lim_{n\to\infty}\left(\frac{2n+2}{n\left(\sqrt{1+\frac{2}{n}+\frac{2}{n^2}}+1\right)}\right) = \lim_{n\to\infty}\left(\frac{2+\frac{2}{n}}{\sqrt{1+\frac{2}{n}+\frac{2}{n^2}}+1}\right) \overset{?}{=}$

$\overset{?}{=} \frac{2+0}{\underbrace{\sqrt{1+0+0}}_{1}+1} = 1$

6) $\lim_{x\to-3}\underbrace{\frac{x^2}{9-x^2}}_{f(x)}$

máme typ „$\frac{a}{0}$" pro $a \neq 0$ použijeme tedy patričnou větu;

pro $x \in (-\infty,-3)$; $\frac{x^2}{9-x^2} < 0$

pro $x \in (-3,0)$; $\frac{x^2}{9-x^2} > 0$

tedy $\lim_{x\to-3_-}\frac{x^2}{9-x^2} = -\infty$

a $\lim_{x\to-3_+}\frac{x^2}{9-x^2} = \infty$

tedy $\lim_{x\to-3_-} f(x) \neq \lim_{x\to-3_+} f(x) \Rightarrow \lim_{x\to-3}\frac{x^2}{9-x^2}$ neexistuje

7) Najděte limity funkce f v krajních bodech definičního oboru

a) $f(x) = \frac{\cos x}{x^2}$ Určíme $\mathcal{D}_f = \mathbb{R} \setminus \{0\}$ krajními body tedy jsou; $0_+, 0_-, +\infty, -\infty$

$\lim\limits_{x \to 0_+} \frac{\cos x}{x^2} = \infty$ $\lim\limits_{x \to 0_-} \frac{\cos x}{x^2} = \infty$, tedy $\lim\limits_{x \to 0} \frac{\cos x}{x^2} = \infty$

$\lim\limits_{x \to \infty} \frac{\cos x}{x^2} = 0$

jelikož máme, že $\frac{-1}{x^2} \leq \frac{\cos x}{x^2} \leq \frac{1}{x^2}$ pro $\forall x \in \mathcal{D}_f$
a zároveň $\lim\limits_{x \to \infty} \frac{-1}{x^2} = 0$ a $\lim\limits_{x \to \infty} \frac{1}{x^2} = 0$

stejný argument platí pro $x \to -\infty$ a tedy
$\lim\limits_{x \to -\infty} \frac{\cos x}{x^2} = 0$

b) $f(x) = \sqrt{x+5} - \sqrt{x+1}$ Určíme $\mathcal{D}_f = [-1, \infty)$. Krajními body tedy jsou -1 a ∞

$\lim\limits_{x \to -1_+} \sqrt{x+5} - \sqrt{x+1} = 2$

$\lim\limits_{x \to \infty} \left(\sqrt{x+5} - \sqrt{x+1}\right) = \lim\limits_{x \to \infty} \frac{x+5-x-1}{\sqrt{x+5} + \sqrt{x+1}} =$

$= \lim\limits_{x \to \infty} \frac{4}{\sqrt{x+5} + \sqrt{x+1}} = 0$