Hi, Ryan!

Yes, your conjectured bound is true. The proof is quite simple. If you have looked at my proof of the $n^{\frac{1}{4}}$ upper bound (I believe I have sent you the corresponging TEX-file; if not, just tell me and I'll do it), you might have noticed that, among other things, it contains a proof of the estimate

$$\int_{\partial Q} \varphi(y) \frac{1}{h(y)+1}\, d\sigma(y) \leq 1$$

where Q is an arbitrary convex body containing the origin, $\varphi(y) = (2\pi)^{-\frac{n}{2}} e^{-\frac{|y|^2}{2}}$ is the density of the standard Gaussian measure in $\mathbb{R}^n$, $d\sigma(y)$ is the Lebesgue surface measure in $\mathbb{R}^n$, and $h(y)$ (which appears there as $|y|\alpha(y)$) is the distance from the origin to the tangent hyperplane of Q at $y \in \partial Q$.

Now suppose that Q is bounded by k hyperplanes $H_1, \dots, H_k$ whose distances from the origin are $h_1, \dots, h_k$ respectively. Let $(\partial Q)_j$ be the part of the boundary of Q contained in H_j. Then the Gaussian perimeter of Q equals

$$\int_{\partial Q} \varphi(y)\, d\sigma(y) = \sum_{j=1}^{k} \int_{(\partial Q)_j} \varphi(y)\, d\sigma(y) = \sum_{h_j > \sqrt{2\log k}} \dots + \sum_{h_j \leq \sqrt{2\log k}} \dots = \Sigma_1 + \Sigma_2 \,.$$

Now

$$\Sigma_1 \leq \sum_{h_j > \sqrt{2\log k}} \int_{H_j} \varphi(y)\, d\sigma(y) \leq k \frac{1}{\sqrt{2\pi}} e^{-\log k} < 1\,,$$

while

$$\Sigma_2 \leq (\sqrt{2\log k} + 1) \int_{\partial Q} \varphi(y) \frac{1}{h(y)+1}\, d\sigma(y) \leq \sqrt{2\log k} + 1.$$

Uniting these two estimates, we obtain that the Gaussian perimeter of Q does not exceed $\sqrt{2\log k} + 2$.

As to the Euclidean case, could you be more specific about what extremal problem you want to consider there: you want to bound the perimeter of a convex set Q from above assuming what? Or, maybe, you rather want a bound from below? Then, again, assuming what? Fixed volume? Fixed inradius? Something else?

By the way, I looked at your IAS web page and found that you are an expert in machine learning (actually your contribution to the subject seems quite impressive, at least to an outsider's eyes). That is a branch of mathematics that I am currently completely ignorant of. Do you know any good introductory book to the subject?

Yours, Fedja