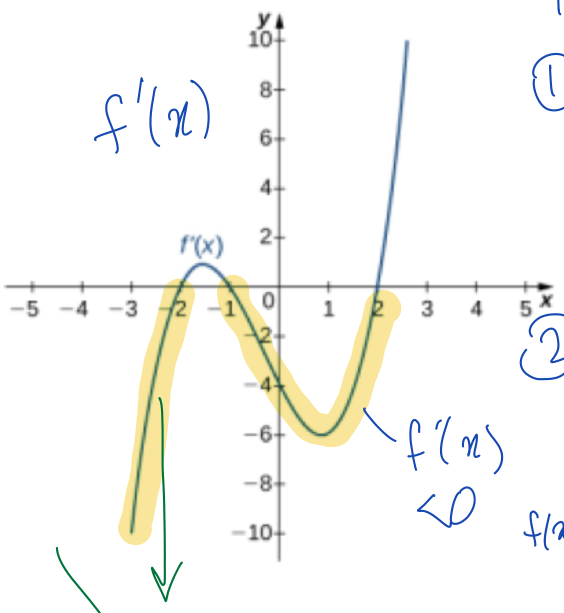
Q1. Analyze the given graph of $f'(x)$ and determine the intervals where $f(x)$ is increasing and decreasing.



Find:

① x s.t. $f(x)$ is increasing

$\iff f'(x) > 0$

$f(x)$ increasing on $\boxed{(-2,-1) \cup (2,+\infty)}$

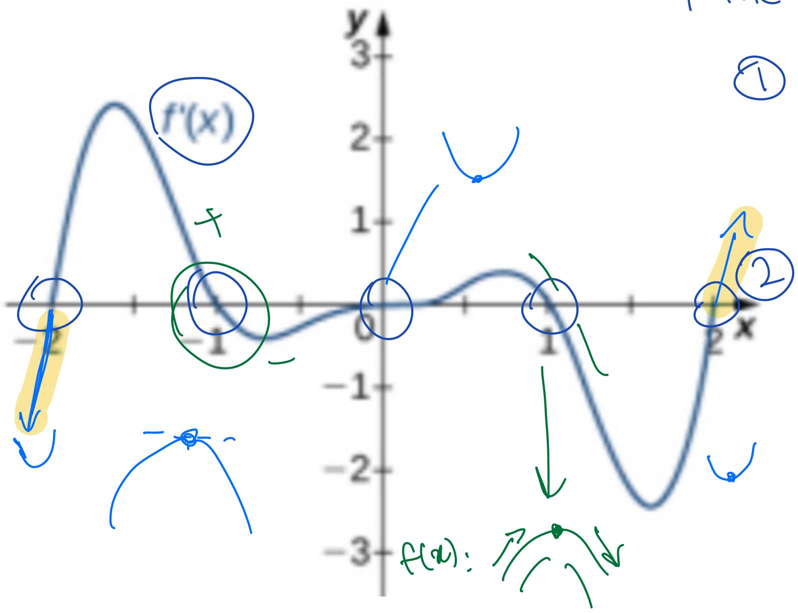
② x s.t. $f(x)$ is decreasing

$\iff f'(x) < 0$

$f(x)$ decreasing on $\boxed{(-\infty,-2) \cup (-1,2)}$

$f'(x) < 0 \iff$ slope of tangent to $f(x) < 0 \iff f(x)$ decreasing

Q2. Analyze the given graph of $f'(x)$ and determine the maximum and minima of $f(x)$.



Find :

① maximum of $f(x)$

$x = 1, -1$

↳ $f'(x)$ switches from ⊕ to ⊖, crossing x-axis

② minimum of $f(x)$

$x = -2, 0, 2$

↳ $f'(x)$ switches from ⊖ to ⊕

Q3. Produce a graph with the following properties: there is a local maximum at $x = 2$, a local mimimum at $x = 1$, and the graph is neither concave up nor concave down. + graph derivative

Given

1st derivative test

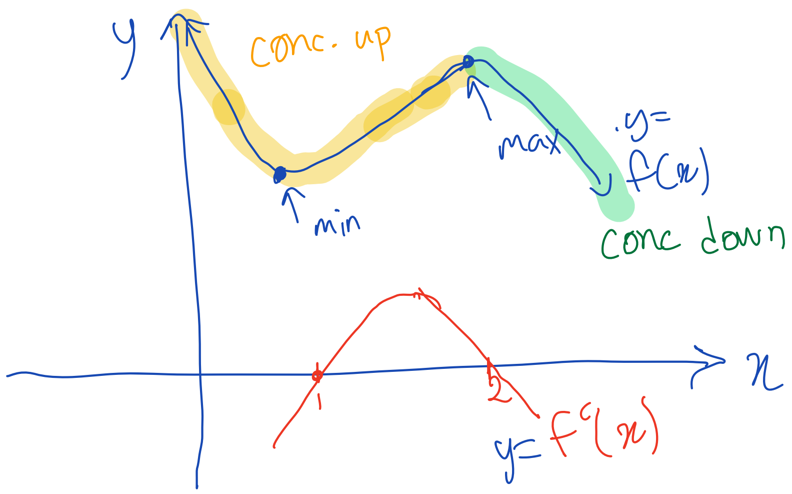
1) local max at $x=2$ $\longrightarrow$ $f'(x)=0$ at $x=2$

$f'(x)$ changes from $\oplus$ to $\ominus$

2) local min at $x=1$ $\longrightarrow$ $f'(x)=0$ at $x=1$

$f'(x)$ changes from $\ominus$ to $\oplus$

3) neither concave up/down $\rightarrow$ 2nd deriv



Q4. Consider $f(x) = x^3 - 6x^2$. $f'(x) = 3x^2 - 12x$ [power rule]

(a) Determine where $f(x)$ is increasing and decreasing. $f'(x) = 3x(x-4)$

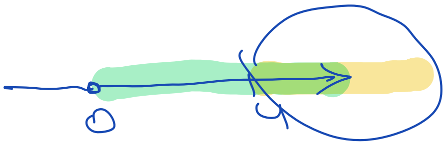
① $f(x)$ increasing $\longrightarrow f'(x) > 0$

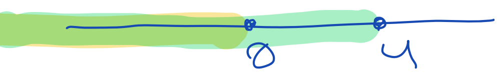
$\longrightarrow 3x^2 - 12x > 0$

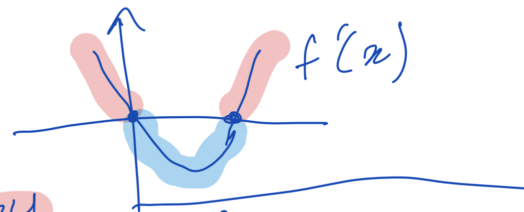
$\longrightarrow 3x(x-4) > 0$

$x>0$ ⊕ $\times$ ⊕ $x>4$ $=$ ⊕ $\longrightarrow x>4$

$x<0$ ⊖ $\times$ ⊖ $x<4$ $=$ ⊕ $\longrightarrow x<0$







$\therefore$ $f(x)$ increasing on $x \in (-\infty, 0) \cup (4, \infty)$

② $f(x)$ decreasing $\longrightarrow f'(x) < 0$

$\longrightarrow 3x(x-4) < 0$

$\longrightarrow$ ⊖ $\times$ ⊕ $=$ ⊖

$3x<0$ $\quad x-4>0$

$x<0$ AND $x>4$

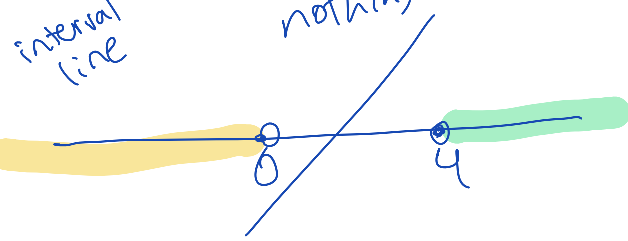
⊕ $\times$ ⊖ $=$ ⊖

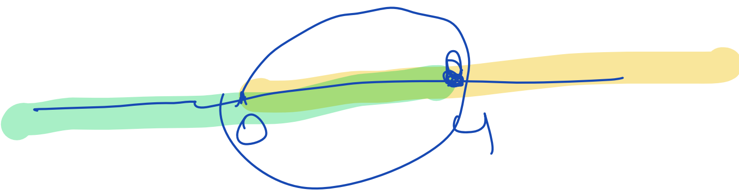
$3x>0$ $\quad x-4<0$

$x>0$ $\quad x<4$

interval line

nothing in common





$\Longrightarrow 0 < x < 4$

$\therefore$ $f(x)$ decreasing on $x \in (0, 4)$

(b) Determine the local minima and maxima of $f(x)$. $f'(x) = 3x^2 - 12x$
$= 3x(x-4)$

$f'(x) = 0$ at $x=0$ & $x=4$

① loc. min

$\Longleftrightarrow f'(x)$ changes from ⊖ to ⊕

x	$-\infty$	0		4		$+\infty$
$3x$	$-$		$+$		$+$	
$x-4$	$-$		$-$		$+$	
$f'(x)$	$+$		$-$		$+$	

$\therefore$ local max at $x=0$ local min at $x=4$

② local max

$\Longleftrightarrow f'(x)$ changes from ⊕ to ⊖

$f''(x) < 0$ $\quad$ $f''(x) > 0$

(c) Determine where $f(x)$ is concave up and concave down.

$$\boxed{\text{If need to find concavity sign or inflection pt} \rightarrow \text{need 2nd deriv}}$$

$\rightarrow f'(x) = 3x^2 - 12x$

$\rightarrow f''(x) = (3x^2 - 12x)'$

$= 6x - 12$

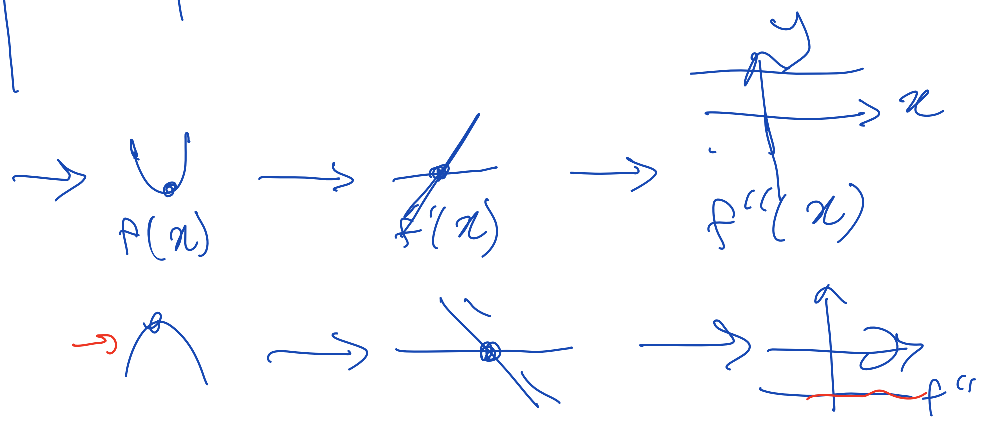
$\therefore f''(x) < 0$ for $x < 2$

$\therefore$ conc. down $x < 2$

x	$-\infty$ $\quad$ 2	2 $\quad$ $+\infty$
$6x-12$	$-$	$+$

$f(x)$ conc-up $\rightarrow$

" down



$\therefore f''(x) > 0$ for $x > 2$

$\therefore$ conc. up for $x > 2$

$f''(x) = 0$

(d) Locate any inflection points of $f(x)$.

$f''(x) = 0$

$x = 2$

Q5. Suppose that $f(t)$ represents the size of a population at time t. Express the sentence

increasing conc. down

"the population is growing more slowly"

using mathematical notation and $f(t)$, $f'(t)$, and $f''(t)$.

growing → f increasing → $f'(t) > 0$

growing more slowly → rate of growth decreasing

$f'(t)$ decreasing

→ $f''(t) < 0$

$\therefore$ $f'(t) > 0$ and $f''(t) < 0$

represents size of population