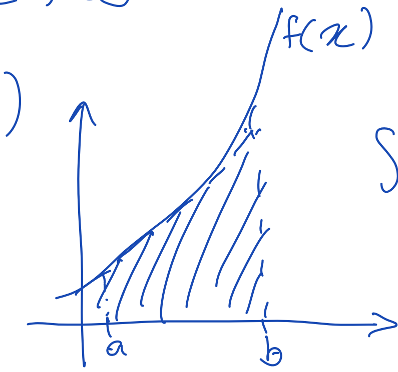
Q1. Show that $F(x) = 5x^3 + 2x^2 + 3x + 1$ is an antiderivative of $f(x) = 15x^2 + 4x + 3$.

Defn: Given a fcn f on $[a,b]$, a fcn F is an antiderivative of f if $F'(x) = f(x)$ $\forall x \in [a,b]$

$$F'(x) = \frac{d}{dx}(5x^3 + 2x^2 + 3x + 1)$$
$$= 15x^2 + 4x + 3$$
$$= f(x) \quad \blacksquare$$

QED end of proof



$\int_a^b f(x)\,dx$

Q2. Find another antiderivative of $f(x)$.

$$F(x) = 5x^3 + 2x^2 + 3x + C \quad \text{where } C \in \mathbb{R}$$

indefinite integral

Q3. Find all antiderivatives of $f(x) = e^x - 3x^2 + \sin(x)$.

Given fcn's f, g & const a

$$\int af(x) \pm g(x)\,dx = a\int f(x)\,dx \pm \int g(x)\,dx$$

$$\frac{d}{dx}(-\cos x) = -(-\sin x)$$

$$\int f(x)\,dx = \int e^x - 3x^2 + \sin x\,dx$$
$$= \int e^x\,dx - 3\int x^2\,dx + \int \sin x\,dx$$
$$= e^x - \frac{\cancel{3}}{\cancel{2+1}}x^{2+1} - \cos x + C$$
$$= e^x + x^3 - \cos x + C$$
$$= F(x)$$

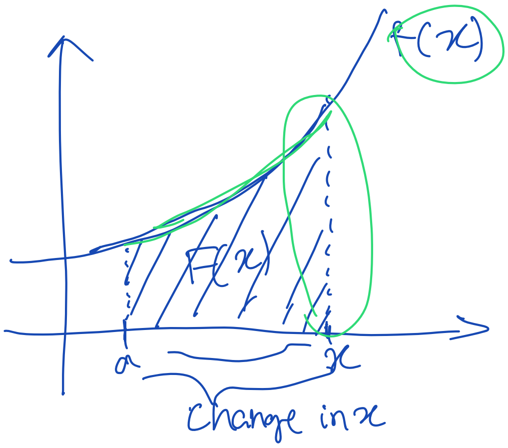
$$\frac{d}{dx}x^n = nx^{n-1}$$
$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$
$$\frac{d}{dx}\frac{x^{n+1}}{n+1} = \frac{n+1}{n+1}x^{\cancel{n+1-1}}$$

Fundamental Thm of Calculus (FTC)

x and a are fixed when integrating

#1: If $F(x) = \int_a^x f(t)\,dt$, then $F'(x) = f(x)$

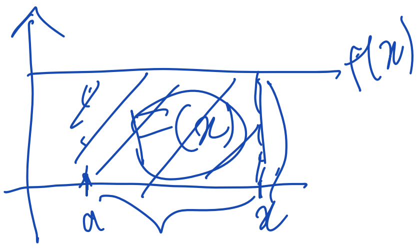
Do not confuse t w x



$F'(x) \longleftrightarrow$ rate of change of the area

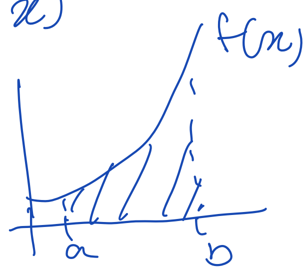
wrt to change in x

is the height of the shape i.e. $f(x)$



#2: If $F(x)$ is antiderivative of $f(x)$

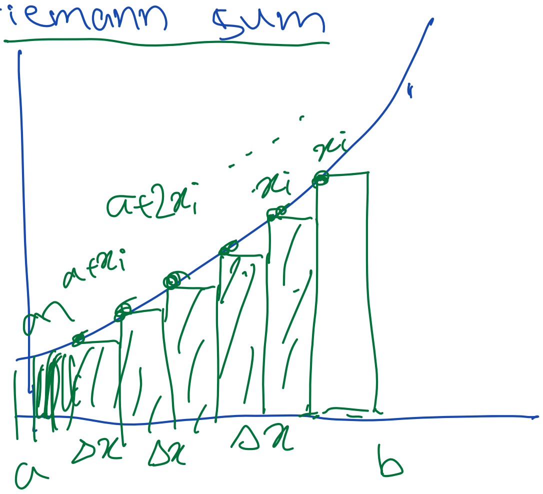
$$\int_a^b f(x)\,dx = F(b) - F(a)$$



No need to know:

Meaning: We can find the area under the curve if we have the indefinite integral

riemann sum



$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=0}^{n} f(x_i)\Delta x$$

n is # of rectangles

$$\Delta x = \frac{b-a}{n}$$

$$x_i = a + \Delta x \cdot i$$

a

$a + \Delta x$

$a + 2\Delta x$

Q4. Find all antiderivatives of $f(x) = (\sqrt[2]{x})^3$. $= x^{\frac{3}{2}}$

$n = \frac{3}{2}$

$\int x^n \, dx = \frac{1}{n+1} x^{n+1}$

$\int f(x)dx = \int x^{\frac{3}{2}} dx$

$\frac{3}{2} + 1 = \frac{5}{2}$

$= \frac{1}{\frac{3}{2}+1} \cdot x^{\frac{3}{2}+1} + C$

$= \frac{2}{5} x^{\frac{5}{2}} + C$

THE FUNDAMENTAL THEOREM OF CALCULUS

1. If $F(x) = \int_a^x f(t)dt$ then $F'(x) = f(x)$

Q5. Use the Fundamental Theorem of Calculus (Part 1) to find the derivative:

$$\frac{d}{dx} \int_4^x \frac{\overbrace{dt}^{f(t)}}{\sqrt{16 - t^2}}$$

Let $F(x) = \int_4^x \overbrace{\frac{1}{\sqrt{16-t^2}}}^{f(t)} dt$

$\frac{d}{dx} \int_4^x \frac{dt}{\sqrt{16-t^2}} dt = \underbrace{\frac{d}{dx} F(x)}_{F'(x)} = f(x) = \frac{1}{\sqrt{16-x^2}}$

[FTC 1]

THE FUNDAMENTAL THEOREM OF CALCULUS

1. If $F(x) = \int_a^x f(t)dt$ then $F'(x) = f(x)$.

Q6. Use the Fundamental Theorem of Calculus (Part 1) to find the derivative:

$$\frac{d}{dx}\int_0^{\sqrt{x}} t\,dt \qquad f(t)=t$$

Let $F(x) = \int_a^x \cancel{f(t)}\,dt$ ($f(t) = t$) . Then $F(\sqrt{x}) = \int_a^{\sqrt{x}} t\,dt$

then $F'(x) = f(x)$

$$\frac{d}{dx} F(\sqrt{x}) = \frac{d}{dx}\int_0^{\sqrt{x}} t\,dt$$

$$= F'(\sqrt{x}) \cdot (\sqrt{x})' \qquad \text{[chain rule]}$$

$$= F'(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$$= f(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} \qquad \text{[FTC 1]}$$

$$= \cancel{\sqrt{x}} \cdot \frac{1}{2\cancel{\sqrt{x}}} = \frac{1}{2}$$

THE FUNDAMENTAL THEOREM OF CALCULUS

2. $\int_a^b f(t)dt = F(b) - F(a)$ for any antiderivative $F(x)$ of $f(x)$.

Q7. Use the Fundamental Theorem of Calculus (Part 2) to find the derivative:

$$\frac{d}{dx}\int_0^{\sqrt{x}} t\,dt$$

$f(t) = t$

$F(x) = \frac{x^2}{2} + C$

Find antiderivative of $f(t) = t$

$\int f(t)\,dt = \int t\,dt = \frac{t^2}{2} + C = F(t)$

Find $\int_0^{\sqrt{x}} t\,dt = F(\sqrt{x}) - F(0)$ [FTC2]

$= \frac{(\sqrt{x})^2}{2} + \cancel{C} - \left(\frac{0^2}{2} + \cancel{C}\right)$

0

$= \frac{x}{2}$

$\frac{d}{dx}\left(\int_0^{\sqrt{x}} t\,dt\right) = \frac{d}{dx}\left(\frac{x}{2}\right) = \boxed{\frac{1}{2}}$

Q8. Calculate the following:

$$\int_{-2}^{3} x^2 + 3x - 5dx$$

$\int x^2 dx = \frac{x^{2+1}}{2+1}$

$\int 3x dx = 3\int x dx = \frac{3}{2}x^2$

$$\int_{-2}^{3} x^2 + 3x - 5 \, dx = \left[\frac{x^3}{3} + \frac{3}{2}x^2 - 5x + \cancel{C}\right]_{x=-2}^{x=3}$$

$$[FTC2] = \frac{(3)^3}{3} + \frac{3}{2}(3)^2 - 5(3) + \cancel{C} - \left(\frac{(-2)^3}{3} + \frac{3}{2}(-2)^2 - 5(-2) + \cancel{C}\right)$$

$$= \frac{-35}{6}$$

Check

$$\frac{d}{dx}\left(\frac{x^3}{3} + \frac{3}{2}x^2 - 5x\right)$$

$$= x^2 + 3x - 5$$

$$F(x) = \frac{x^3}{3} + \frac{3}{2}x^2 - 5x + C$$