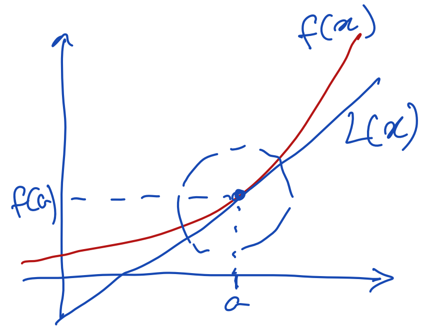
Q1. Find the linear approximation to $y = \underbrace{1/x}_{f(x)}$ near $a = 2$.

$f(x) = \frac{1}{x} = x^{-1}$




Def: Linear approx. $L(x)$ to $f(x)$ at $x=a$ is

$$L(x) = f'(a)(x-a) + f(a) \quad *$$

Intuitively, approximating the fcn using a linear fcn near $x=a$

$f(x) \approx L(x)$ near $x = a$

Soln 1) Find $f'(a) = f'(x)\Big|_{x=a}$

$$= \frac{-1}{x^2}\Big|_{x=2}$$

$$f'(2) = \frac{-1}{4}$$

2) Apply (*) $L(x) = f'(2)(x-2) + f(2)$

$$= \frac{-1}{4}(x-2) + \frac{1}{2}$$

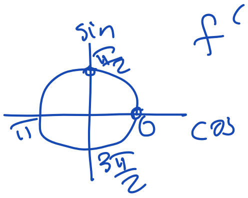
$-\frac{x}{4} + \frac{1}{2} + \frac{1}{2}$

$$\boxed{L(x) = \frac{-x}{4} + 1}$$ — linear approx. of $f(x) = \frac{1}{x}$ near $x = 2$

Q2. Find the linear approximation to $y = \underbrace{\sin(x)}_{f(x)}$ near $a = \frac{\pi}{2}$.

Soln 1) Find $f'(a)$

$f'\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$




2) $L(x) = f'\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right)$

$= 0\left(x - \frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right)$

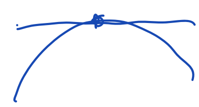
$$\boxed{L(x) = 1}$$ linear approx. of $f(x) = \sin x$ near $x = \frac{\pi}{2}$

Q3. When is the linear approximation constant?

$L(x) = f'(a)(x - a) + f(a)$

$L(x)$: constant; $f'(a) = 0$; $f(a)$: constant

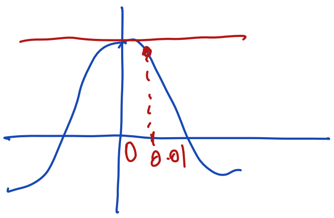
$\because f'(a) = 0$

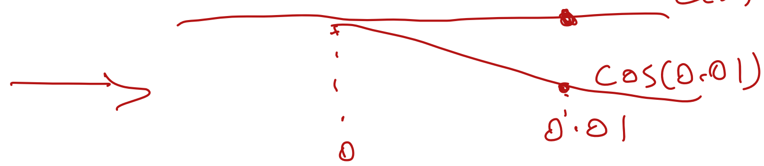


$\therefore$ The lin. appr. is const. when slope at $x$ is 0

Q4. Approximate cos(0.01) and determine the numerical error of the approximation.
In this question, be sure to use radians on your calculator.

If we find $L(x)$ at a given pt, $x=a$ then value of $f$ near any $b$ very close to $a$ can be approximated using $L(x)$




$\cos(x) \approx L(x)$ near $x=a$

1) Find a pt (a) for approximation

Let $a=0$

2) $L(x) = f'(a)(x-a) + f(a)$ $\qquad f(x) = \cos x$

$f'(x) = -\sin x$

$= -\underbrace{\cancel{\sin(0)}}_{0}(x-0) + \underbrace{\cancel{\cos(0)}}_{1}$

$L(x) = 1 \qquad L(0.01) = 1 \approx \cos(0.01)$

Calculate $|1 - \cos(0.01)| = 4.99 \times 10^{-5}$

$0.00\ldots$

Q5. Find the differential $dy$ of $y = \frac{1}{x+1}$ for $x = 1$ and $dx = 0.25$.

$a = 1$

$f(x) = \frac{1}{x+1} \rightarrow f'(x) = \frac{-1}{(x+1)^2}$

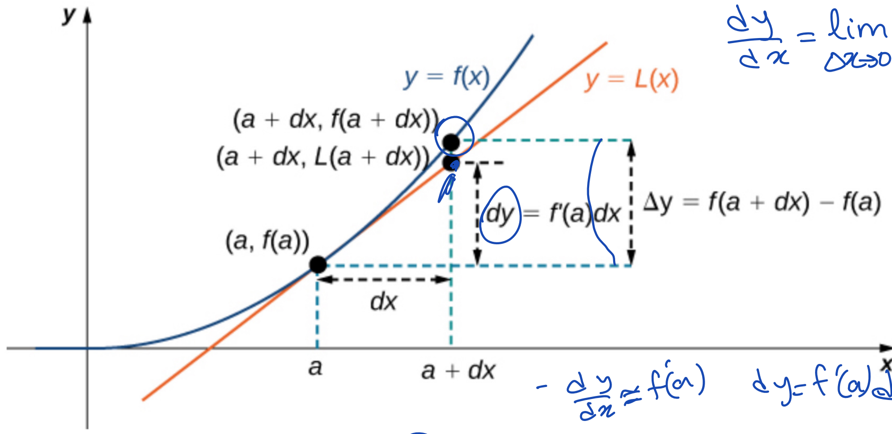
$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

1) Find $dy = f'(a)\,dx$

$= f'(x)\big|_{x=1}\,dx$

$= \frac{-1}{(1+1)^2} \cdot 0.25$

$\boxed{= \frac{-1}{16}}$




$-\ \frac{dy}{dx} \approx f'(a) \quad dy = f'(a)\,dx$

OpenStax Fig 4.11: The differential $dy = f'(a)dx$ is used to approximate $\Delta y$ if: $x$ increases from $x = a$ to $x = a + dx$.

Approximation of change in y (i.e. differential)

as $x$ moves from 1 to $1 + dx = 1 + 0.25 = 1.25$

$h = 3$ $\quad r = 2$ is initial

Q6. Find the differential $dV$ if a circular cylinder of height 3 changes from $r = 2$ to $r = 1.9$.

$a = 2 \qquad a + dr = 1.9$

$2 + dr = 1.9$

$\boxed{dr = -0.1}$

$V(r,h) = \pi r^2 h^{=3}$

$V(r) = \pi r^2 (3) \rightarrow V'(r) = 6\pi r$

$dV = V'(r)\,dr$

$= 6\pi r\,dr$

$= 6\pi(2)(-0.1)$

$= \frac{-6}{5}\pi$