# Adversarial Machine Learning

Machine Learning Course - CS-433
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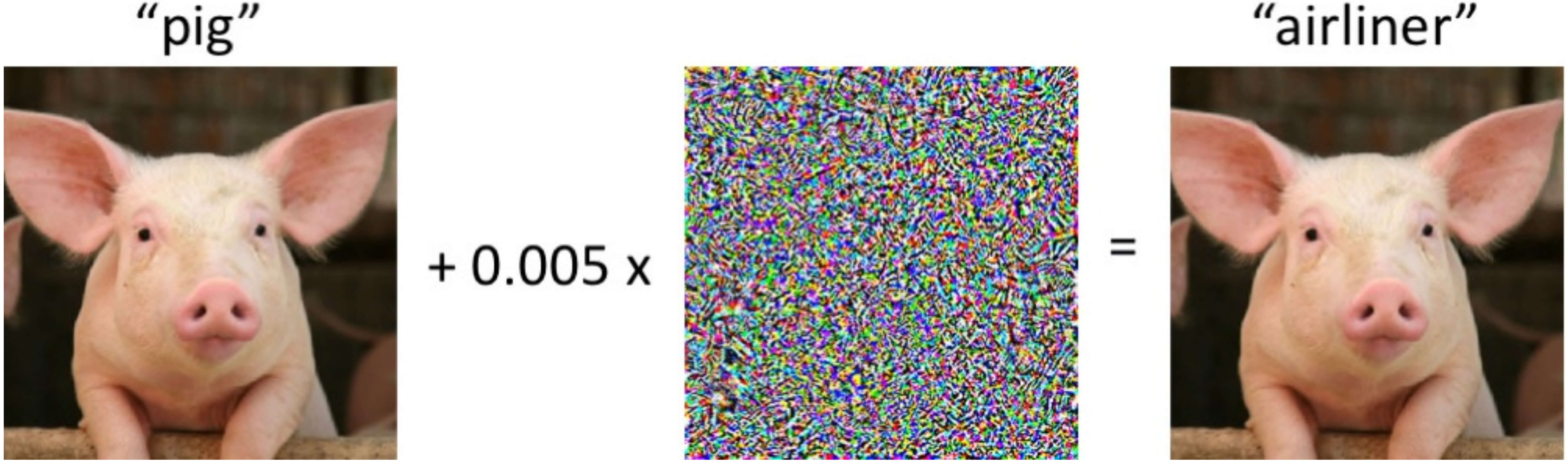
# Some input examples are hard for humans

Dog or mop?

- Some examples might be challenging for a human
- NNs typically have **no problem** with them
- However, NNs are not **robust** in their decisions

# Adversarial examples: small perturbations which cause **a misclassification with a high confidence**

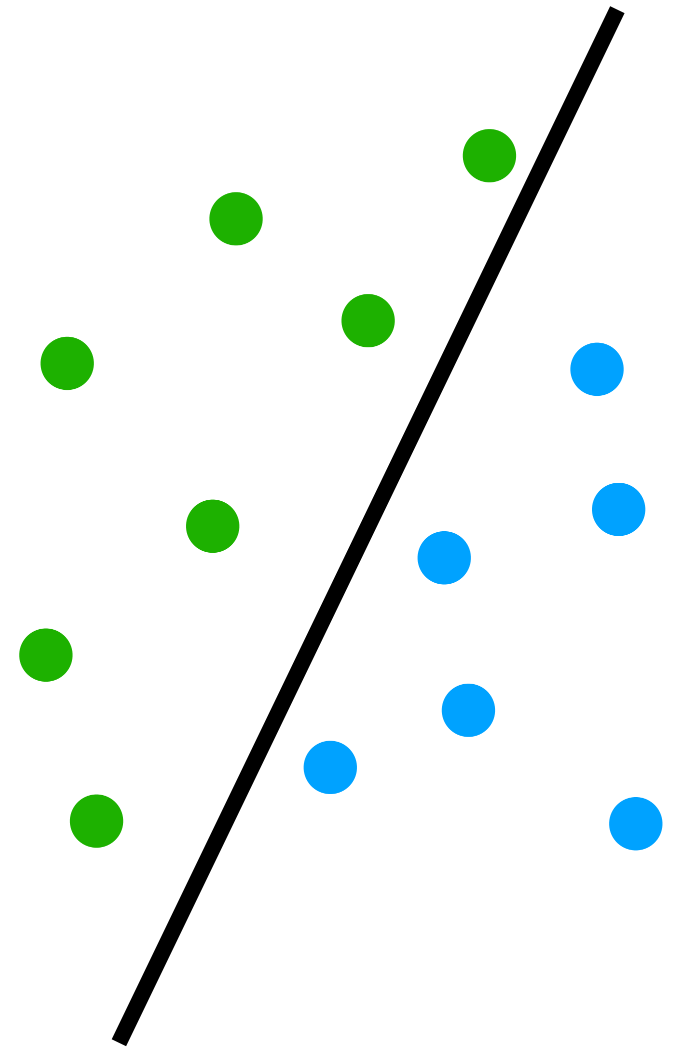



Source: Z. Kolter, A. Madry, NeurIPS'18 tutorial on adversarial robustness

NNs have difficulties with imperceptible but very specific input known as **adversarial examples**

- ➡ **Security problem**: consider a self-driving car and stop sign detection
- ➡ We don't understand how these models **generalize** and react to **distribution shifts**

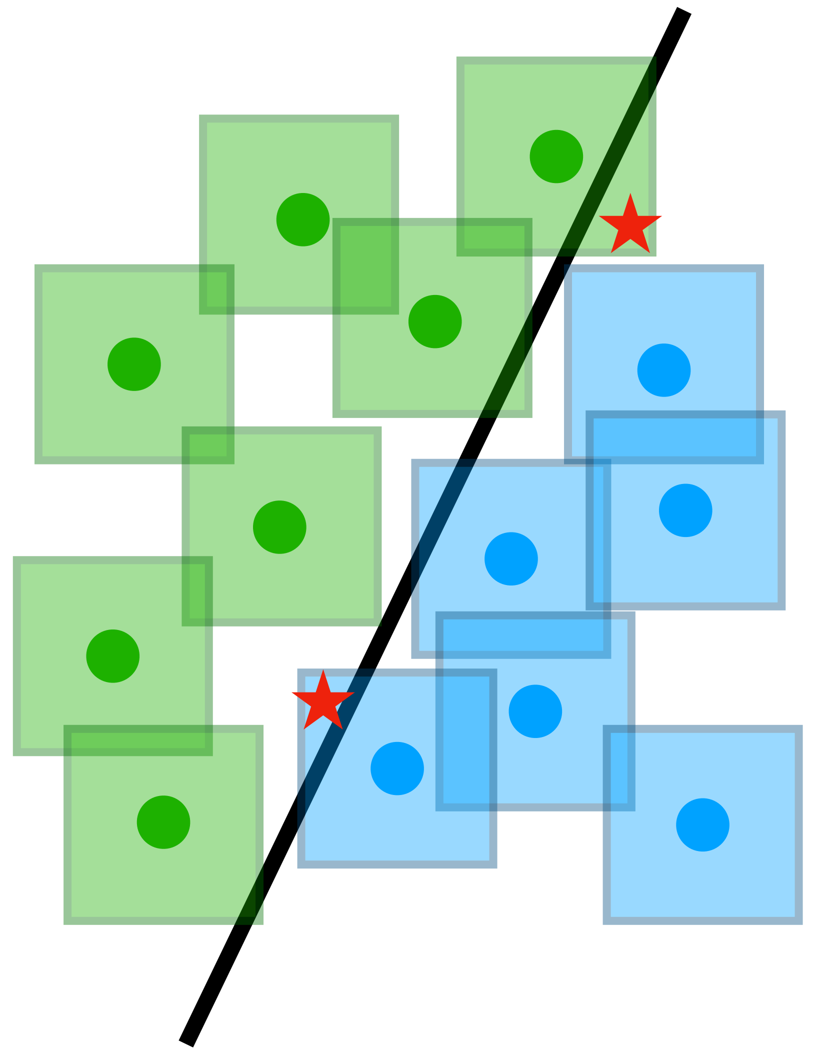
# Standard risk vs. adversarial risk



<u>Classification problem</u>: $(x, y) \sim \mathcal{D}, y \in \{-1, 1\}$

<u>Standard risk</u>: average zero-one loss over $x$

$$R(f) = \mathbb{E}_{\mathcal{D}}\left[1_{f(x) \neq y}\right] = \mathbb{P}_{\mathcal{D}}\left[f(x) \neq y\right]$$



<u>Adversarial risk</u>: average zero-one loss over **small, worst-case perturbations of** $x$

$$R_\varepsilon(f) = \mathbb{E}_{\mathcal{D}}\left[\max_{\hat{x}, \|\hat{x} - x\| \leq \varepsilon} 1_{f(\hat{x}) \neq y}\right]$$

# Adversarial vulnerability raises many questions

$$R_\varepsilon(f) = \mathbb{E}_{\mathcal{D}}\left[\max_{\hat{x}, \|\hat{x}-x\| \leq \varepsilon} 1_{f(\hat{x}) \neq y}\right]$$

- Threat model:
  - How should we define the adversary power?
  - What norm shall we consider? $\ell_\infty, \ell_2, \ell_1, \ell_0, \ldots$
  - Other set of perturbations?
- If $R(f) \leq \delta$, then how large can $R_\varepsilon(f)$ be?

# Adversarial vulnerability raises many questions

- How can we compute an adversarial example?
- Which access do we have to the model to attack it?
- How can we design a classifier $f$ so that it is robust?
  Related: given a non-robust classifier, can I somehow make it robust?
- Why are neural networks non-robust?

# Generating adversarial examples

<u>Task:</u> given an input $(x, y)$ and a model $f : \mathcal{X} \to \{-1,1\}$, find an input $\hat{x}$ , such that

(a) $\|x - \tilde{x}\| \leq \varepsilon$

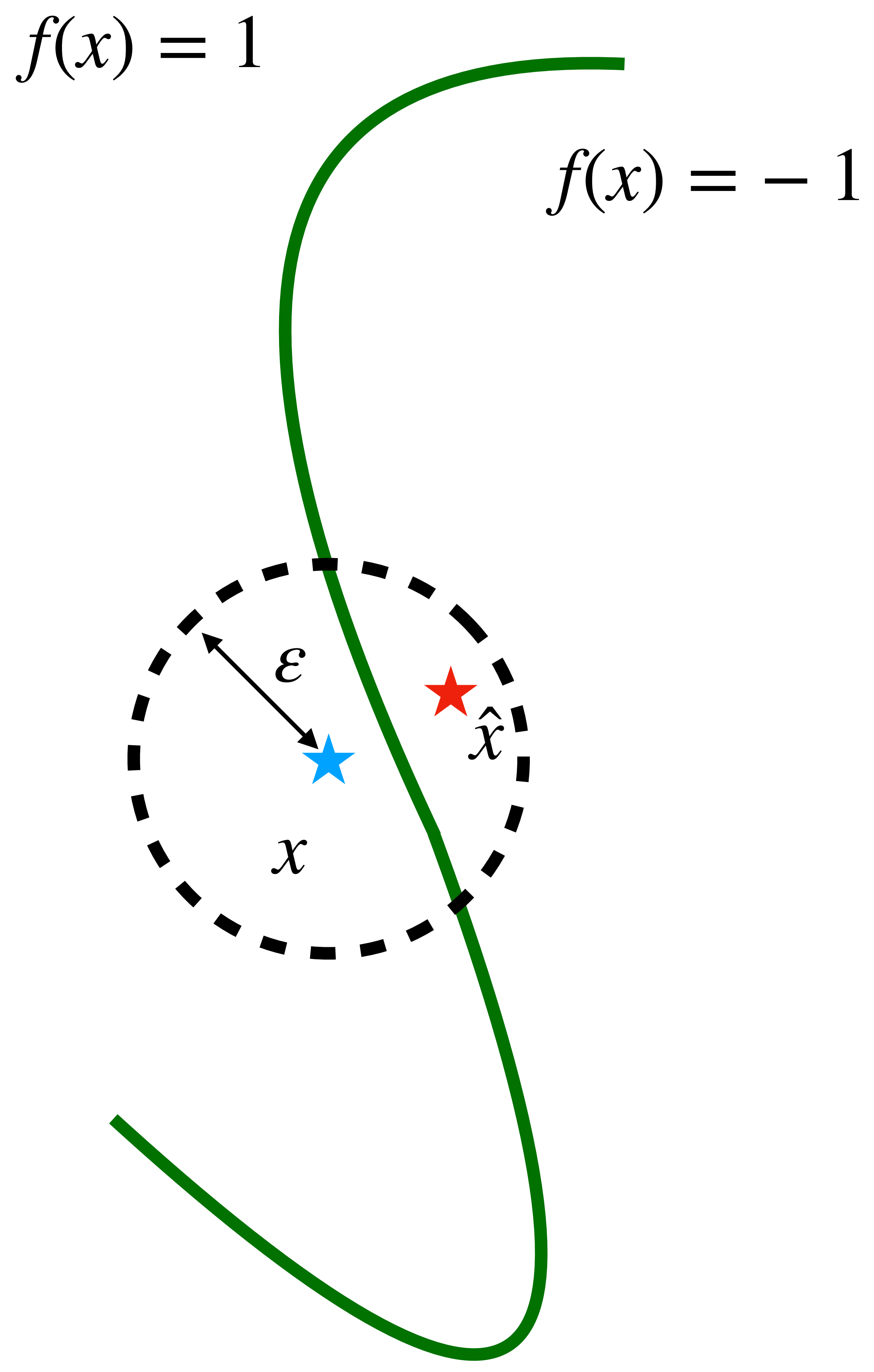
(b) the model $f$ makes a mistake on it

**Trivial case**: $x$ is already misclassified

➡ nothing to do

**General case**: $x$ is correctly classified

➡ find $\hat{x}$ such that $f(\hat{x}) \neq y$ and $\|\hat{x} - x\| \leq \varepsilon$
i.e., $\hat{x} \in B_x(\varepsilon) \cap \{x', f(x') = -y\}$



# Generating adversarial examples amounts to maximizing the classification loss w.r.t the inputs

Find an adversarial example by solving

$$\max_{\hat{x}, \|\hat{x}-x\| \leq \epsilon} 1_{f(\hat{x}) \neq y}$$

➡ Optimization problem with respect to the inputs

<u>Problem</u>**:** optimizing the indicator function $1_{f(\hat{x}) \neq y}$ is difficult:

1. The indicator function $1$ is not continuous
2. The NN prediction $f$ outputs the discrete class values $\{-1,1\}$

# Generating adversarial examples amounts to solving a constrained optimization problem

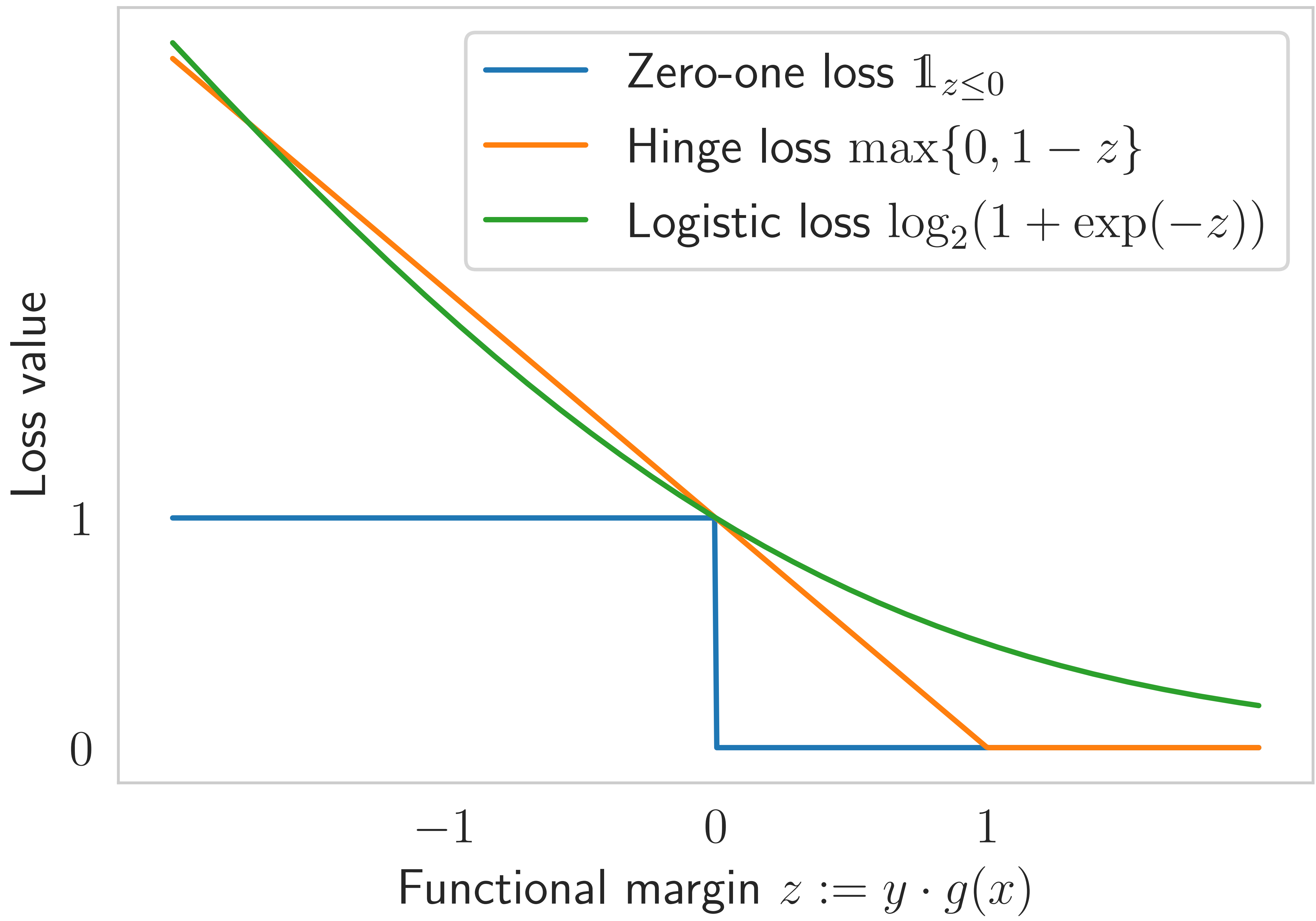
**Solution**:

1. Use instead a smooth classification loss $\ell$ (e.g., logistic or hinge loss)

2. Consider the output $g$ of the NN before classification (i.e., $f(x) = \text{sign}(g(x))$)

**Main idea:** replace the difficult problem over the indicator by a smooth problem

$$\max_{\hat{x}, \|\hat{x}-x\| \le \varepsilon} 1_{f(\hat{x}) \neq y} \quad \longrightarrow \quad \max_{\hat{x}, \|\hat{x}-x\| \le \varepsilon} \ell(yg(\hat{x}))$$




**Reminder**: decreasing, margin-based (i.e., dependent on $y \cdot g(x)$) classification losses

# Generating adversarial examples: white-box case

How to solve $\max_{\hat{x}, \|\hat{x}-x\| \leq \varepsilon} \ell(yg(\hat{x}))$ in the **white-box** case, i.e., if we know the model $g$?

Compute its gradient: $\nabla_x \ell(yg(x)) = \underbrace{y\ell'(yg(x))}_{\leq 0 \text{ since classification loss are decreasing}} \nabla_x g(x)$

We should move in the direction $\propto -y\nabla_x g(x)$

<u>Interpretation:</u> $f(x) = \text{sign}(g(x))$

- If $y = 1$, we want to decrease $g(x)$ and follow $-\nabla_x g(x)$
- If $y = -1$, we want to increase $g(x)$ and follow $\nabla_x g(x)$

⚠ Why using $\ell$, and not directly minimizing $yg(\hat{x})$?

$\rightarrow$ It won't extend to multi-class classification and to robust training.

# Generating adversarial examples: taking into account the constraints

We can linearize the loss $\tilde{\ell}(x) := \ell\left(yg(x)\right)$ to derive an iteration:

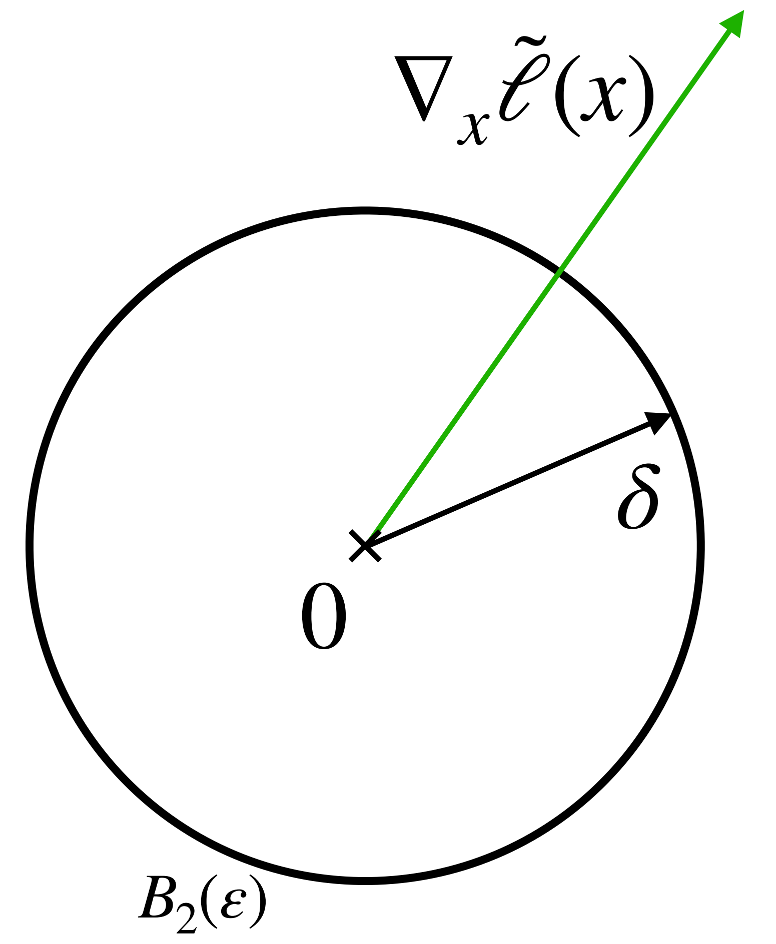
$$
\begin{aligned}
\max_{\|\hat{x}-x\|\leq\varepsilon} \tilde{\ell}(\hat{x}) &\approx \max_{\|\hat{x}-x\|\leq\varepsilon} \tilde{\ell}(x) + \nabla_x \tilde{\ell}(x)^T(\hat{x}-x) \\
&= \tilde{\ell}(x) + \max_{\|\hat{x}-x\|\leq\varepsilon} \nabla_x \tilde{\ell}(x)^T(\hat{x}-x) \\
&= \tilde{\ell}(x) + \max_{\|\delta\|\leq\varepsilon} \nabla_x \tilde{\ell}(x)^T \delta
\end{aligned}
$$

- We need to maximize the inner product under a norm constraint, i.e. find the optimal local update
- This is a simple problem for which we can get a closed-form solution depending on the norm used to measure the perturbation size $\|\delta\|$

# Generating adversarial examples: one-step attack

**Problem:**

$$\max_{\|\delta\|\leq\varepsilon} \nabla_x \tilde{\ell}(x)^T \delta$$



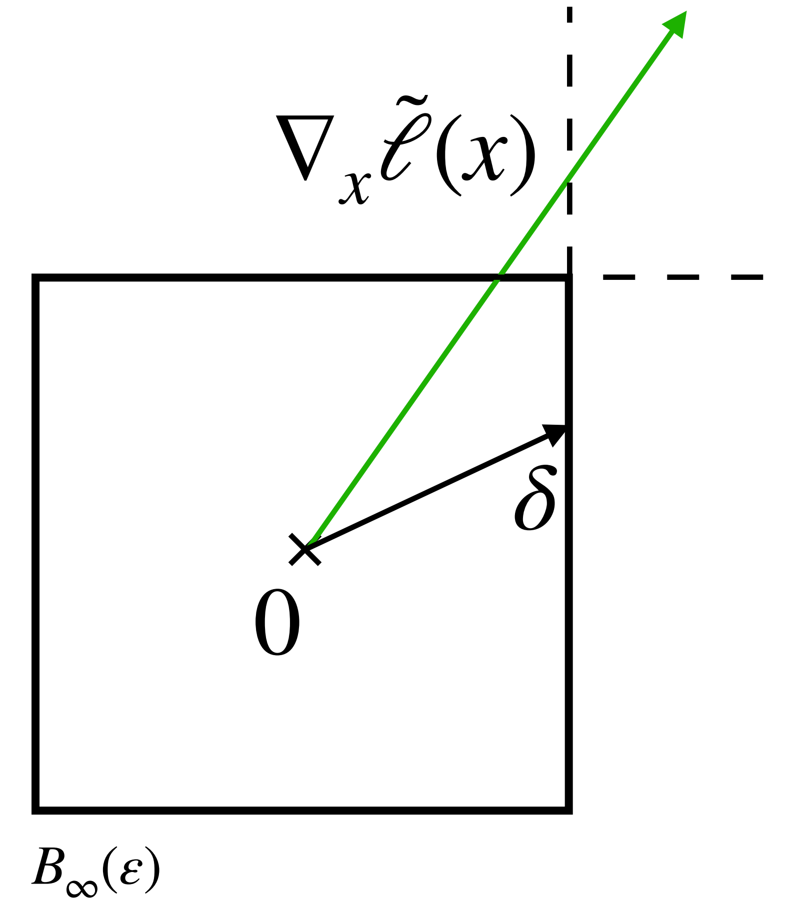

- Solution for the $\ell_2$ norm: $\delta_2^\star = \varepsilon \cdot \frac{\nabla_x \tilde{\ell}(x)}{\|\nabla_x \tilde{\ell}(x)\|_2} = -\varepsilon y \cdot \frac{\nabla_x g(x)}{\|\nabla_x g(x)\|_2}$

  ➡ $\hat{x} = x - \varepsilon y \cdot \frac{\nabla_x g(x)}{\|\nabla_x g(x)\|_2}$

- Solution for the $\ell_\infty$ norm: $\delta_\infty^\star = \varepsilon \cdot \text{sign}(\nabla_x \tilde{\ell}(x)) = -\varepsilon y \cdot \text{sign}(\nabla_x g(x))$

  ➡ $\hat{x} = x - \varepsilon y \cdot \text{sign}(\nabla_x g(x))$

  ➡ **Fast Gradient Sign Method** [Goodfellow et al., 2014]



# Generating adversarial examples: multi-step attack

These updates can be done iteratively and combined with a projection $\Pi$ on the feasible set (i.e., $\ell_2/\ell_\infty$ balls here)

**Projected Gradient Descent**:

- $\ell_2$ norm:

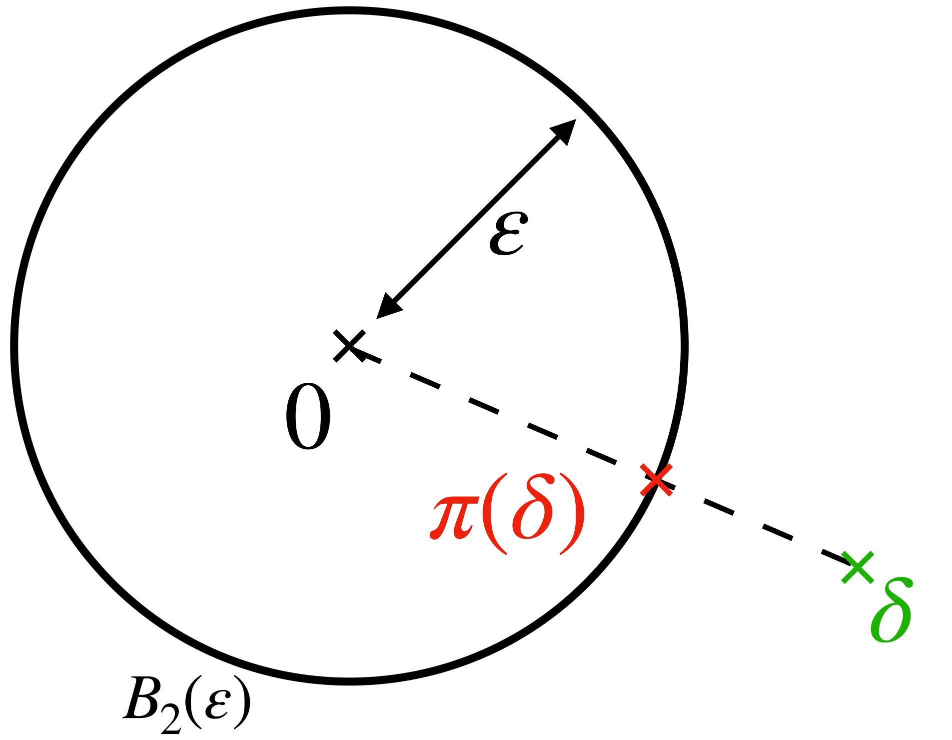
$$\delta^{t+1} = \Pi_{B_2(\varepsilon)}\left[\delta^t + \alpha \cdot \frac{\nabla \tilde{\ell}(x+\delta^t)}{\|\nabla \tilde{\ell}(x+\delta^t)\|_2}\right],$$

$$\text{where } \Pi_{B_2(\varepsilon)}(\delta) = \begin{cases} \varepsilon \cdot \delta/\|\delta\|_2, & \text{if } \|\delta\|_2 \geq \varepsilon \\ \delta, & \text{otherwise} \end{cases}$$

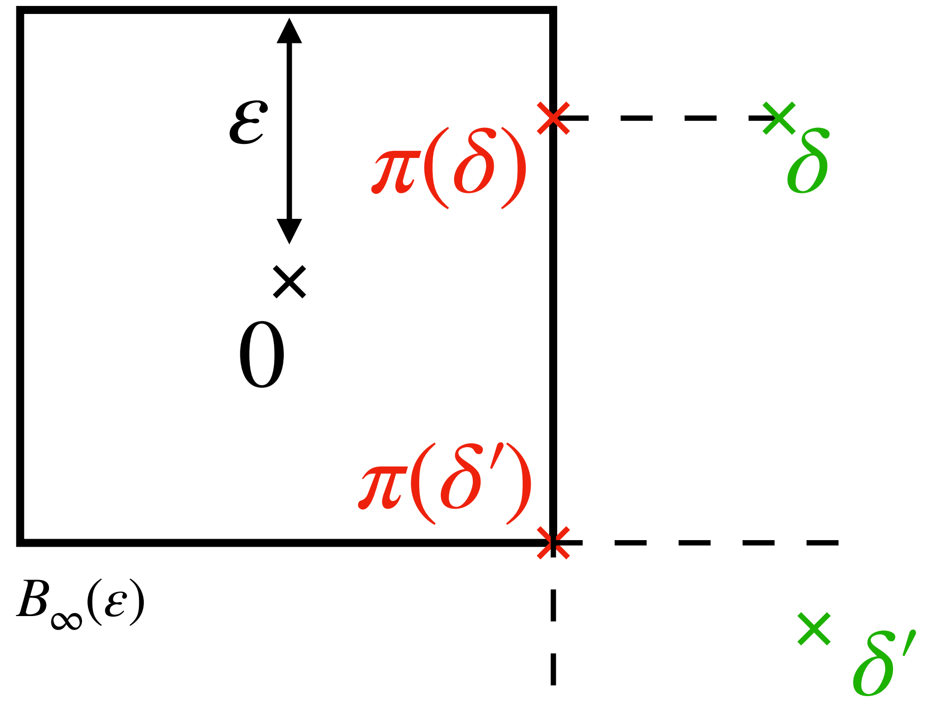
- $\ell_\infty$ norm:

$$\delta^{t+1} = \Pi_{B_\infty(\varepsilon)}\left[\delta^t + \alpha \cdot \text{sign}(\nabla \tilde{\ell}(x+\delta^t))\right],$$

$$\text{where } \Pi_{B_\infty(\varepsilon)}(\delta)_i = \begin{cases} \varepsilon \cdot \text{sign}(\delta_i), & \text{if } |\delta_i| \geq \varepsilon \\ \delta_i, & \text{otherwise} \end{cases}$$



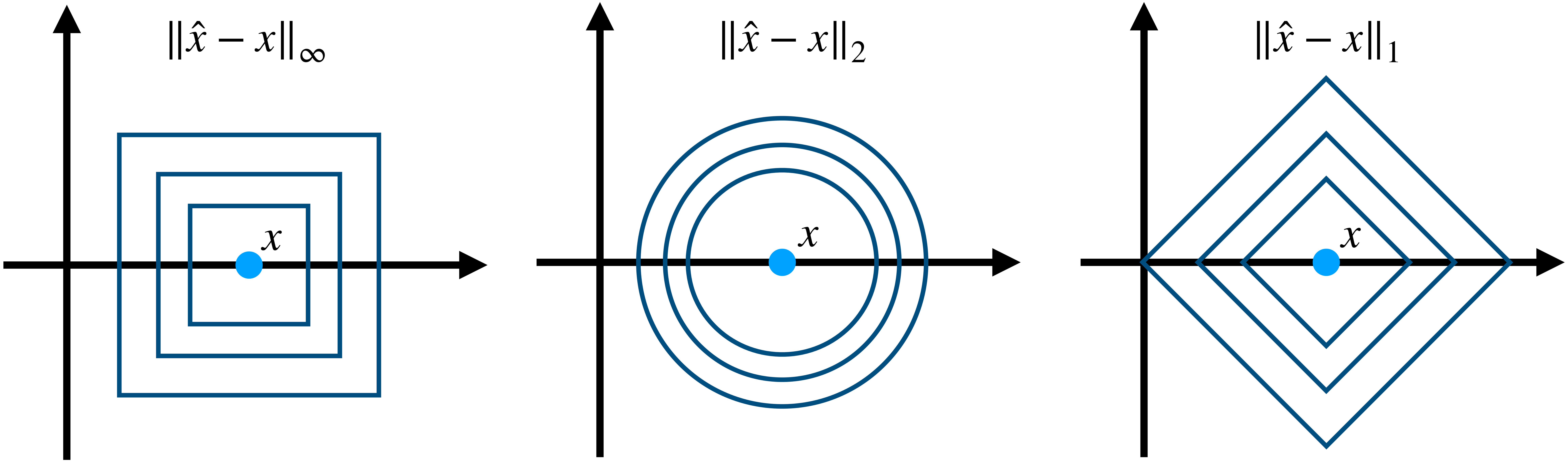

$\ell_2$−projection




$\ell_\infty$−projection

# Reminder: $\ell_p$ norms

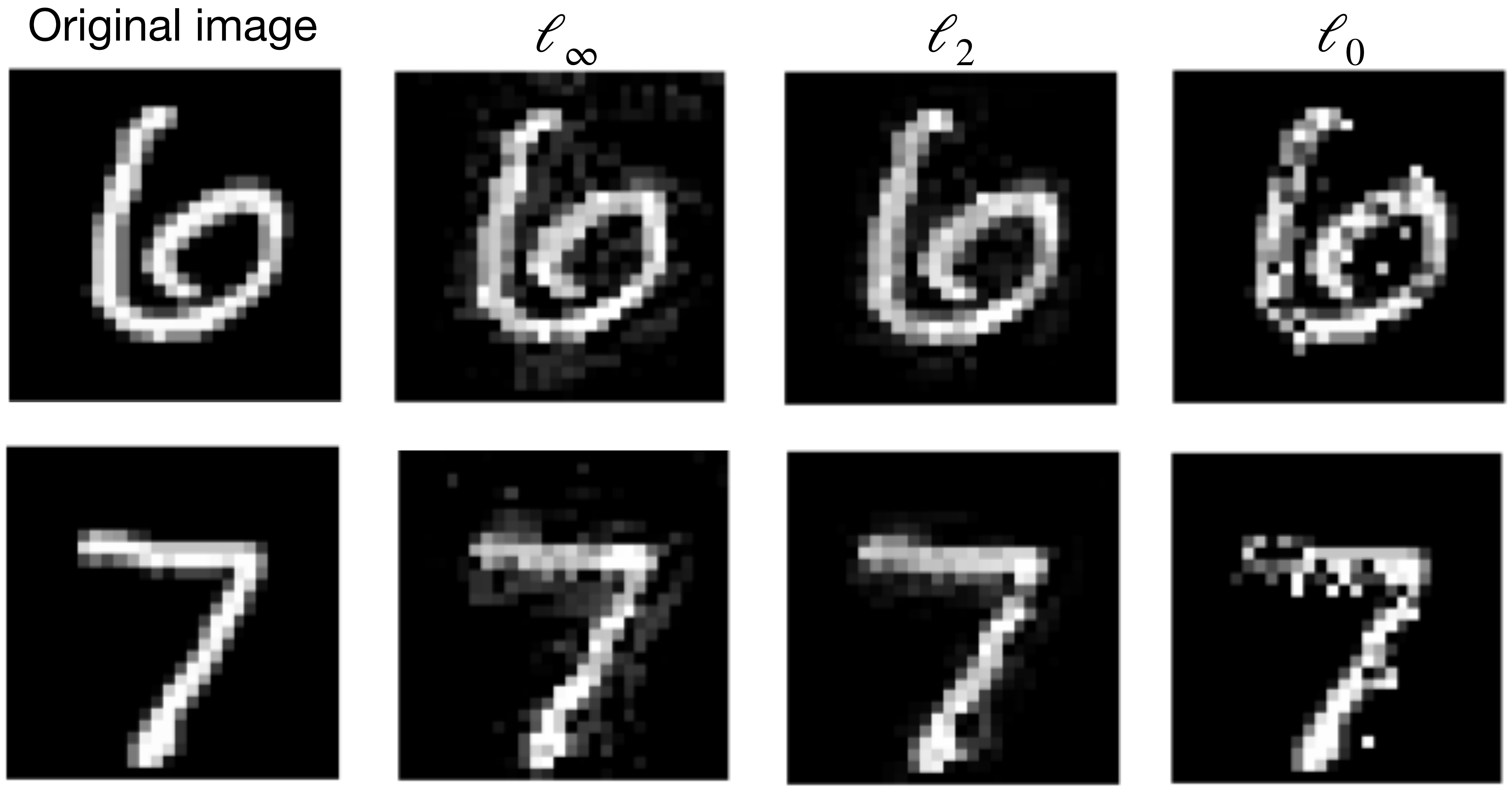
Different $\ell_p$ norms have different geometry




The difference is especially pronounced in high dimensions!

# Visualizations of different $\ell_p$ adversarial examples

The choice of the norm leads to different properties of the resulting adversarial perturbations: e.g. $\ell_\infty$ are **dense** and $\ell_0$ are **sparse**




Source: Towards Evaluating the Robustness of Neural Networks, Carlini et al., 2018

What perturbations do we even want to be robust to?

➡ a lot of research on formulating the "**right**" perturbation set!

# White-box attacks: implementation

- For a neural network, the gradients $\nabla_x g(x)$ can also be computed by **backpropagation** (note: they are taken w.r.t. **inputs**, not parameters!)

- Modern deep learning frameworks readily support this
  $\rightarrow$ **lab #10** (implement Fast Gradient Sign Method on MNIST in PyTorch)

- Now: what **if we don't know** $g(x)$**?** i.e., can we still run an attack if we don't know how to compute $\nabla_x g(x)$?

# Black-box attacks: query-based gradient estimation

There are different assumptions on the knowledge about the model $f$:

- **score-based**: we can query the model scores $g(x) \in \mathbb{R}$
- **decision-based**: we can query only the predicted class $f(x) \in \{-1,1\}$

In score-based case, we can approximate the gradient via a finite difference formula:

$$\nabla_x g(x) \approx \sum_{i=1}^{d} \frac{g(x+\alpha e_i) - g(x)}{\alpha} e_i$$

<u>Rmk</u>: similar techniques can be adapted to the decision-based case (if $x$ is close to the decision boundary)

# Black-box attacks via transfer attacks

Alternative approach: **transfer attacks**

1. train a **similar** surrogate model $\hat{f} \approx f$ on **similar** data
2. transfer the resulting white-box adversarial perturbation from $\hat{f}$ to $f$

- Success depends on how **similar** the model architecture and data are
- If we are allowed to query $f$ given some **unlabeled** inputs $\{x_i\}_{i=1}^n$ we can obtain $\{x_i, f(x_i)\}_{i=1}^n$ and learn $\hat{f}$ based on that (known as **model stealing**) $\rightarrow$ can facilitate **transfer attacks**

# Black-box attacks: summary

**General takeaway**: black-box attacks are of practical concern but:

- Query-based methods often require a lot of queries (10k-100k), particularly **decision-based** attacks → easy to restrict access for the attacker!
- Obtaining a surrogate model $\hat{f}$ can be costly and success is not guaranteed
- The final missing ingredient: **physically realizable attacks**

# Physically realizable attacks

To be applied in practice, adversarial examples need to satisfy some further requirements:

- invariance under JPEG compression (for images input directly in a digital format)
- invariance under photographic distortions (for real-world adversarial examples captured by a camera)
- invariance under different camera angles (for a moving camera, e.g., on a self-driving car)

→ a surge of papers on how to take these requirements into account


Source: Robust Physical-World Attacks on Deep Learning Visual Classification (CVPR 2018)

# How do we train robust models?

Now we know how to **generate** adversarial examples

We will see that we can just train on them to obtain robust models
→ known as **adversarial training**

- **Standard training**: the goal is to minimize the **standard risk**:

$$\min_{\theta} R(f_\theta) = \mathbb{E}_{\mathscr{D}} \left[ 1_{f(\hat{x}) \neq y} \right]$$

- **Adversarial training**: the goal is to minimize the **adversarial risk**:

$$\min_{\theta} R_\varepsilon(f_\theta) = \mathbb{E}_{\mathscr{D}} \left[ \max_{\hat{x}, \|\hat{x}-x\| \leq \varepsilon} 1_{f(\hat{x}) \neq y} \right]$$

# Adversarial training: formulation

**Goal:**

$$\min_{\theta} R_{\varepsilon}(f_{\theta}) = \mathbb{E}_{\mathscr{D}}\left[\max_{\hat{x}, \|\hat{x}-x\| \leq \varepsilon} 1_{f(\hat{x}) \neq y}\right]$$

- The data distribution $\mathscr{D}$ is unknown → approximate it by a **sample average**
- The classification loss is non-continuous → use a **smooth loss**

This results in the following **robust optimization** problem:

$$\min_{\theta} \frac{1}{n} \sum_{i=1}^{n} \max_{\hat{x}_i, \|x_i - \hat{x}_i\| \leq \varepsilon} \ell(y_i g_{\theta}(\hat{x}_i))$$

**Interpretation**: minimize the risk on adversarial examples

# Adversarial training: algorithm

$$\min_{\theta} \frac{1}{n} \sum_{i=1}^{n} \max_{\hat{x}_i, \|x_i - \hat{x}_i\| \leq \varepsilon} \ell(y_i g_\theta(\hat{x}_i))$$

**Adversarial training**: at each iteration $t$:

1. For each $x_i$, approximate $\hat{x}_i^\star \approx \arg \max_{\|x_i - \hat{x}_i\| \leq \varepsilon} \ell(y_i g_\theta(\hat{x}_i))$ via the **PGD attack**

2. Do a gradient descent step w.r.t. $\theta$ using $\frac{1}{n} \sum_{i=1}^{n} \nabla_\theta \ell(y_i g_\theta(\hat{x}_i^\star))$

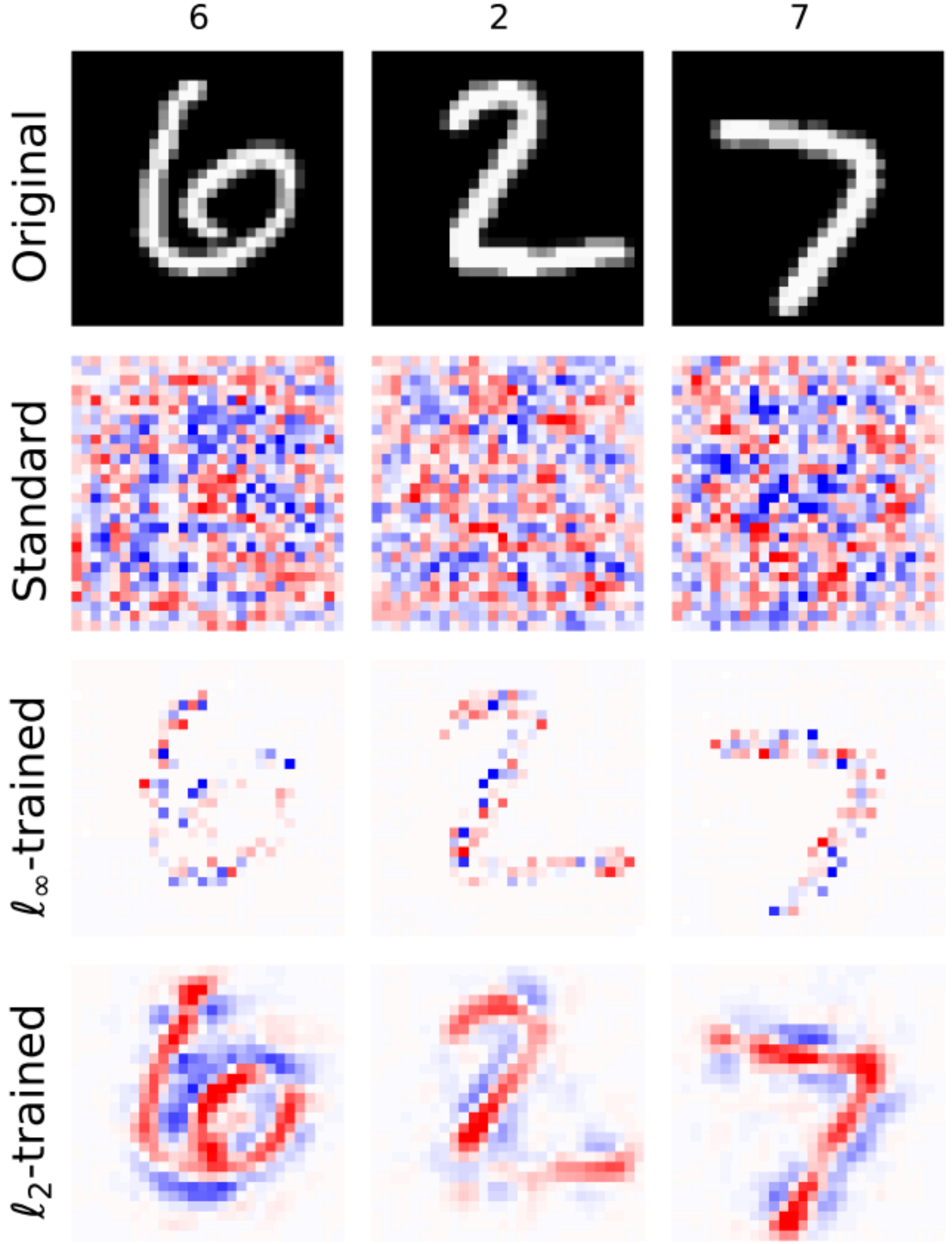
   ⚠ Note you are using $\hat{x}_i^\star$ and not $x_i$

# Adversarial training: discussion

**Good news**:

- Adversarial training is a state-of-the-art approach for robust classification!
- Adversarial training leads to **more interpretable gradients** $\nabla_x \ell(y g_\theta(x))$
- The algorithm is fully compatible with SGD → you will explore it in **lab #10** (adversarial training of a CNN on MNIST)

**Visualization of** $\nabla_x \ell(y g_\theta(x))$



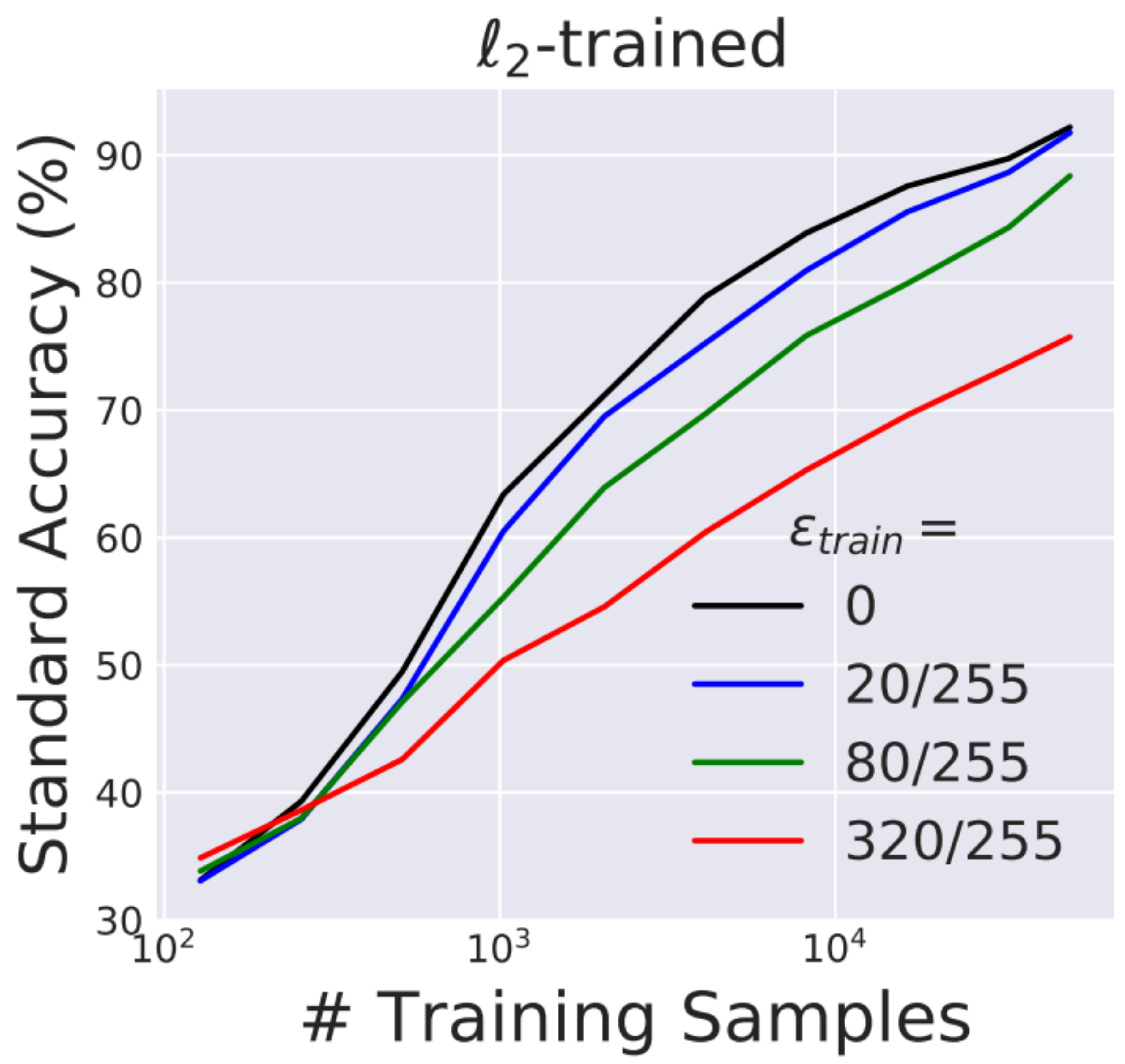

Source: Robustness May Be at Odds with Accuracy (ICLR 2019)

# Adversarial training: discussion

**Bad news**:

- Increased computational time: proportionally to the number of PGD steps
- **Robustness-accuracy tradeoff**: using a too large $\varepsilon$ leads to worse standard accuracy (right)

**Deep ConvNet on CIFAR-10**




Source: Robustness May Be at Odds with Accuracy (ICLR 2019)

# Key question: so why do adversarial examples exist?

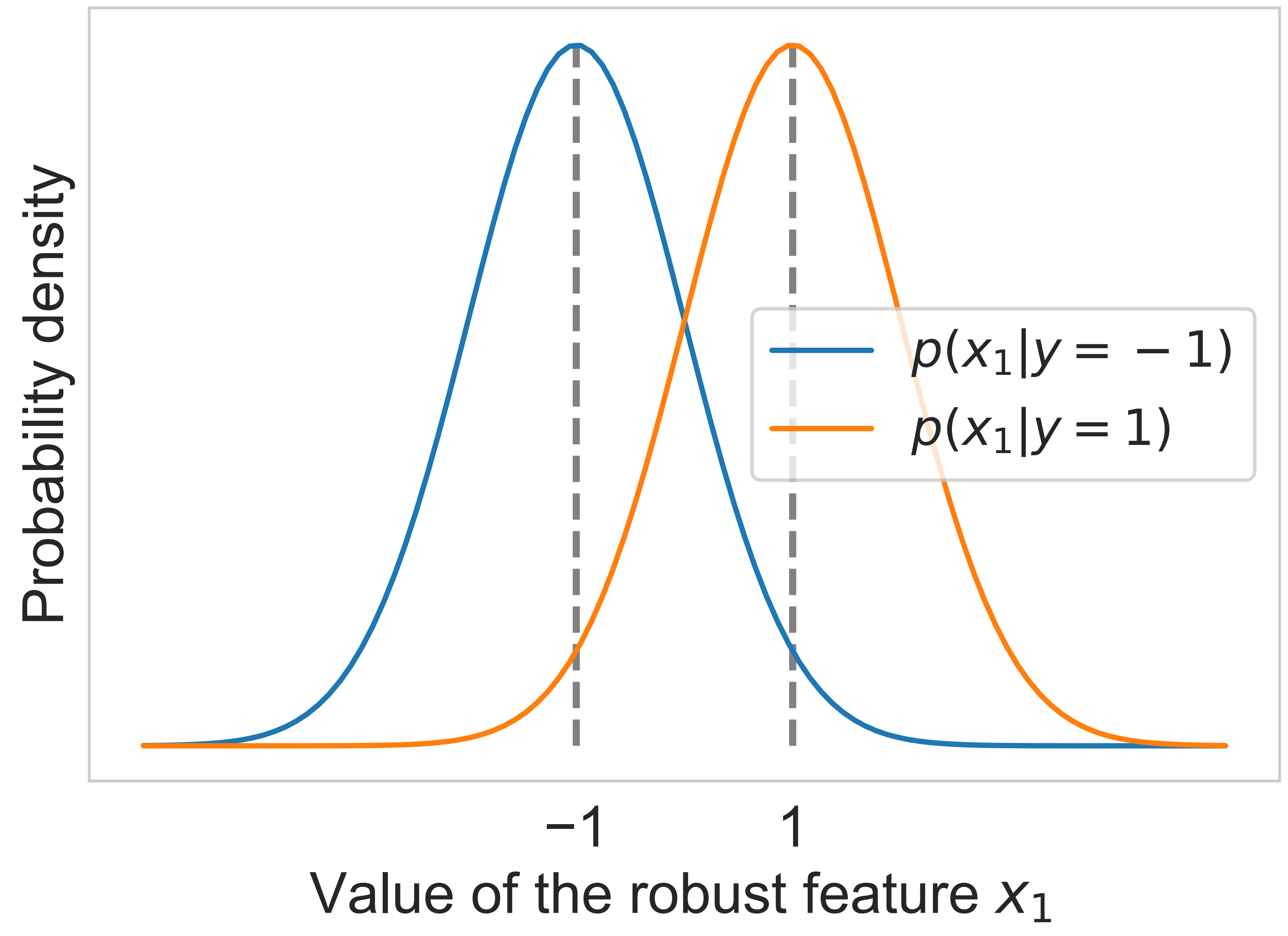
We can conceptualize it with a simple model

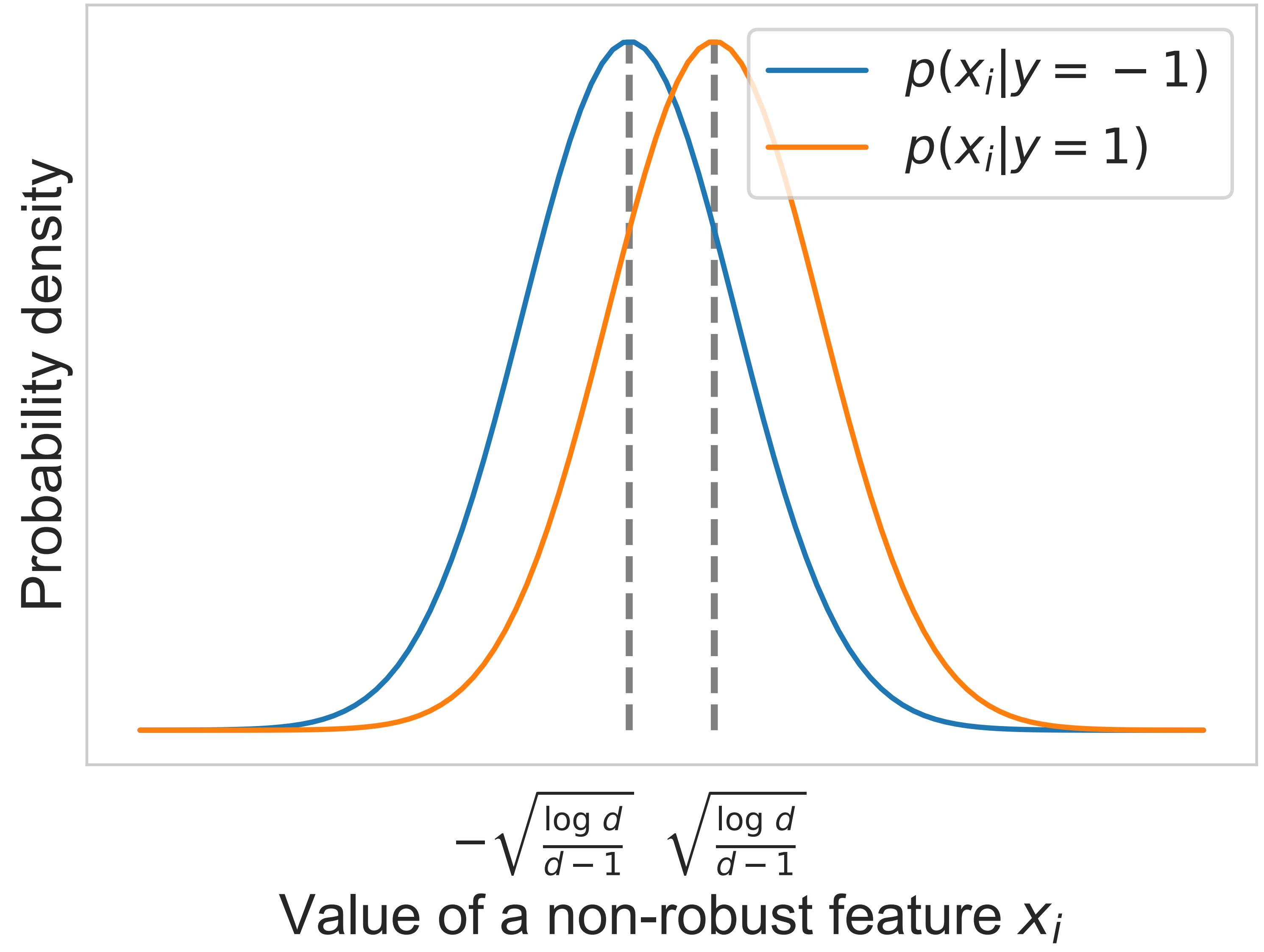
Consider $x \in \mathbb{R}^d$, $y \sim \mathcal{U}(\{-1,1\})$, $Z_i \sim \mathcal{N}(0,1)$:

- **Robust features**: $x_1 = y + Z_1$
- **Non-robust features:** $x_i = y\sqrt{\frac{\log d}{d-1}} + Z_i$ for $i \in \{2,\ldots,d\}$

We'll see that when $d \to \infty$:

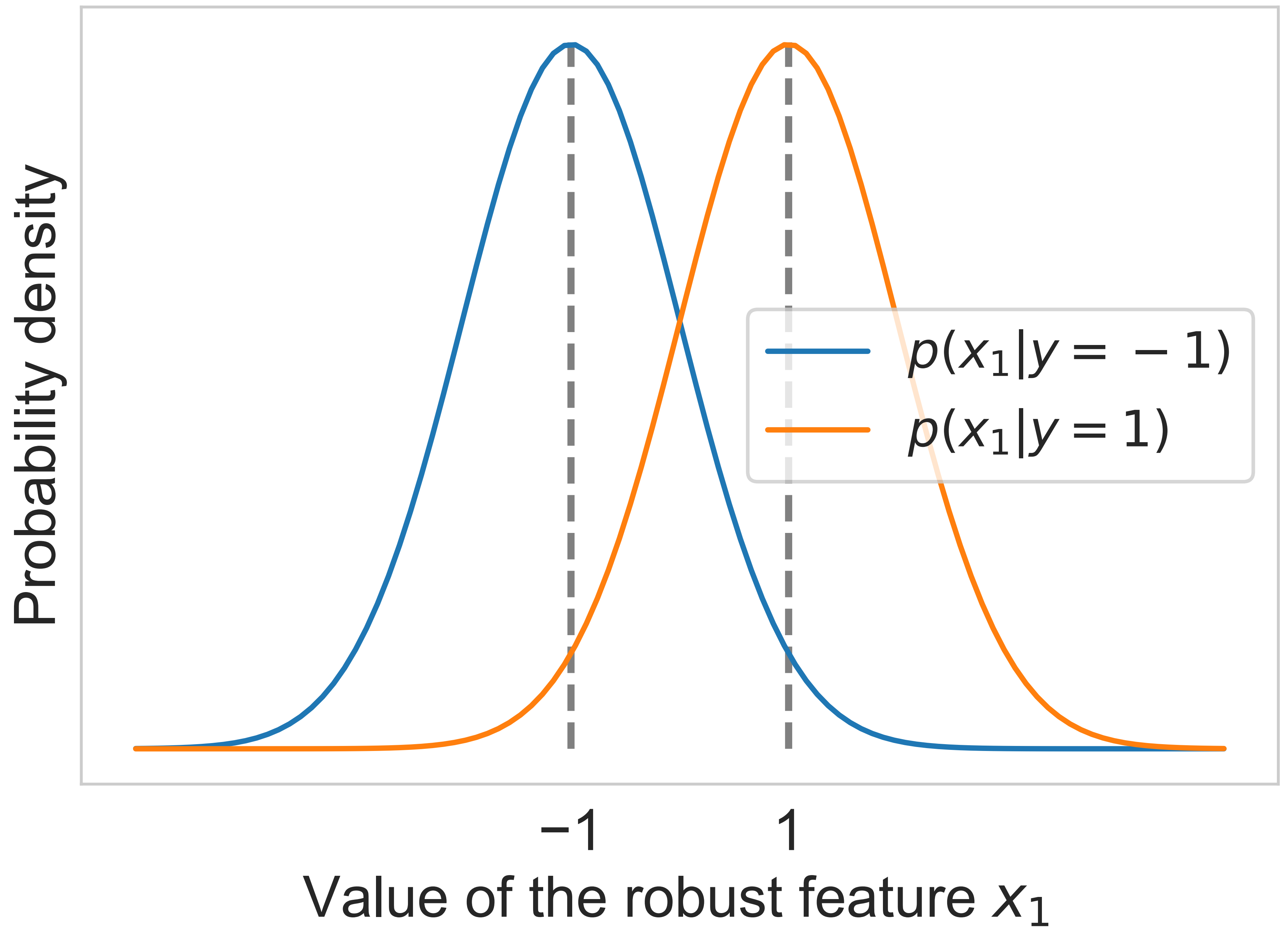
- **standard risk** can be arbitrarily **small**
- **adversarial risk** can be arbitrarily **large**




D. Tsipras, S. Santurkar, L. Engstrom, A. Turner, and A. Madry. Robustness May Be at Odds with Accuracy, ICLR 2019

# Model is only using the robust feature $x_1$




assuming
$p(y=1)=p(y=2)$

**MLE**: $\arg\max_{\hat{y}\in\{\pm 1\}} p(\hat{y} \mid x_1) = \arg\max_{\hat{y}\in\{\pm 1\}} \frac{p(x_1 \mid \hat{y})p(\hat{y})}{p(x_1)} = \arg\max_{\hat{y}\in\{\pm 1\}} p(x_1 \mid \hat{y})$

**Standard risk**: $\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-0.5(x+1)^2} dx \approx 0.16 \rightarrow$ good but not perfect!

# Model is using both robust and non-robust features (I)

Let's derive MLE using **all** features using the shortcut notation $x_i = ya_i + Z_i$:

$$\begin{aligned}
\arg\max_{\hat{y}\in\{\pm 1\}} p(\hat{y} \mid x) &= \arg\max_{\hat{y}\in\{\pm 1\}} \prod_{i=1}^{d} p(x_i \mid \hat{y}) \\
&= \arg\max_{\hat{y}\in\{\pm 1\}} \sum_{i=1}^{d} \log p(x_i \mid \hat{y}) \\
&= \arg\max_{\hat{y}\in\{\pm 1\}} \sum_{i=1}^{d} \log \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x_i - \hat{y}a_i)^2} \\
&= \arg\max_{\hat{y}\in\{\pm 1\}} \sum_{i=1}^{d} (x_i - \hat{y}a_i)^2 \\
&= \arg\max_{\hat{y}\in\{\pm 1\}} \sum_{i=1}^{d} (x_i^2 - 2x_i\hat{y}a_i + \hat{y}^2 a_i^2) = \arg\max_{\hat{y}\in\{\pm 1\}} \hat{y} \sum_{i=1}^{d} x_i a_i
\end{aligned}$$

# Model is using both robust and non-robust features (II)

The MLE expression we maximize over $\hat{y} \in \{-1,1\}$ becomes:

$$\hat{y}\sum_{i=1}^{d} x_i a_i = \hat{y}y(\sum_{i=1}^{d} a_i^2) + \hat{y}\sum_{i=1}^{d} a_i Z_i = \hat{y}y(1 + \log(d)) + \hat{y}Z,$$

where $Z := \sum_{i=1}^{d} a_i Z_i \sim \mathcal{N}(0,1 + \log d)$

Scaling by $1/(1 + \log d)$, the MLE expression results in:

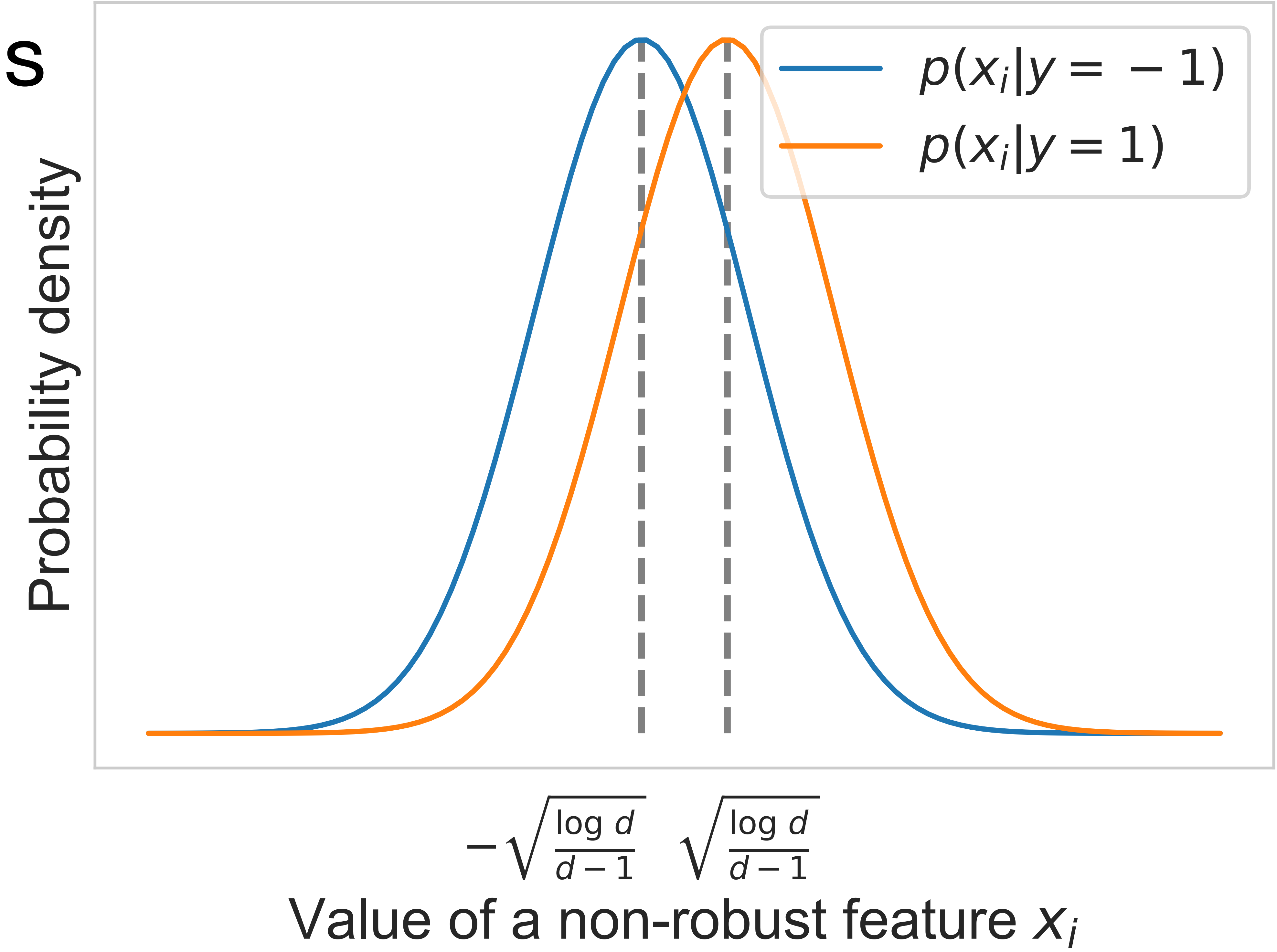
$$y\hat{y} + \hat{y}Z \text{ with } Z \sim \mathcal{N}(0,1/(1 + \log d))$$

**Conclusion**: when the dimension $d \to \infty$, $\hat{y}Z \to 0$ and standard risk $\to 0$

**Interpretation**: using the non-robust features improves standard risk!

# What about adversarial risk?

- The adversary can use tiny $\ell_\infty$- perturbations $\varepsilon = 2\sqrt{\frac{\log d}{d-1}}$ ( $\rightarrow 0$ when $d \rightarrow \infty$)
- Optimal adversarial strategy:
  $\hat{x}_1 = \left(1 - 2\sqrt{\frac{\log d}{d-1}}\right)y + Z_1$ (almost unaffected)
  $\hat{x}_i = -\sqrt{\frac{\log d}{d-1}}y + Z_i$ (completely flipped!)




- **Adversarial risk** $R_\varepsilon(f)$ will become $\approx 1$ (due to non-robust $x_i$) although **standard risk** $R(f)$ is $0$!
- **But**: only using the robust feature $x_1$ leads to $R_\varepsilon(f) \approx R(f) = 0.16$

➡ **tradeoff** between accuracy and robustness