DISCRETE STRUCTURE (SECI1013)

SEM 1, 2024/2025

TITLE	CHAPTER 4 IN-SLIDE EXERCISE
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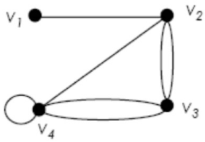
SECTION	03
LECTURER'S NAME	EN. MUHAMMAD ALIIF BIN AHMAD
SUBMISSION DATE	12 NOVEMBER 2024

UTM

Exercise 1

- Find the degree of each vertex in the graph.



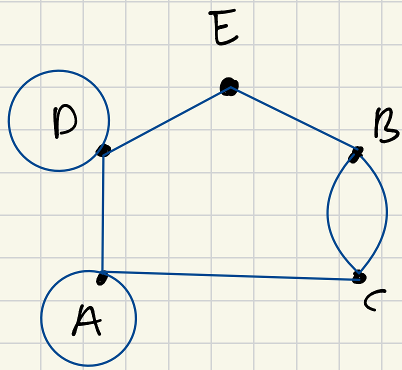
Vertex	V1	V2	V3	V4
Degree	1	4	4	5

UTM

Exercise 2

Draw the graph based on the following matrix:

$$A_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



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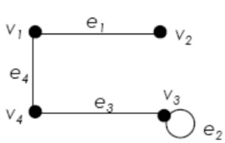
Exercise 3

- Find the adjacency matrix and the incidence matrix of the graph.



Adjacency matrix :

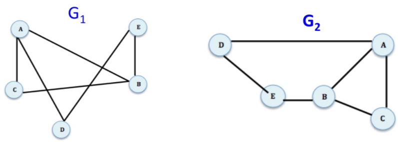
$$A_G = \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Incidence matrix :

$$I_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Exercise 4

Q: Show that the following two graphs are isomorphic.

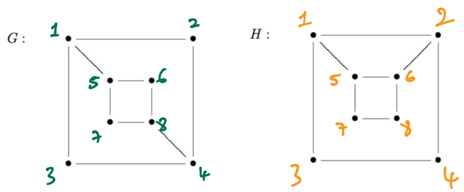


$$A_{G_1} = \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad A_{G_2} = \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Therefore, both are isomorphic because $A_{G_1} = A_{G_2}$

Exercise 5

Q: Is these two graphs are isomorphic?



$$G = \begin{array}{c|cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 4 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 5 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 6 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 7 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 8 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{array}$$

$$\neq$$

$$H = \begin{array}{c|cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 4 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 6 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 7 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 8 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}$$

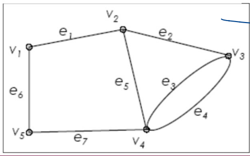
Therefore, both graphs are not isomorphic.

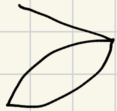
Exercise 6

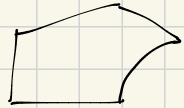
Tell whether the following is either a trail, path, circuit, simple circuit or none of these.

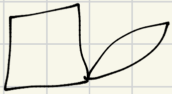
edge ✗ ✗ ✗ ✗
vertex ✓ ✗ ✓ ✗

- $(v_2, e_2, v_3, e_3, v_4, e_4, v_3)$
- $(v_4, e_7, v_5, e_6, v_1, e_1, v_2, e_2, v_3, e_3, v_4)$
- $(v_4, e_4, v_3, e_3, v_4, e_5, v_2, e_1, v_1, e_6, v_5, e_7, v_4)$



a)  = trail

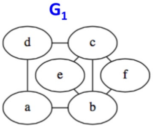
b)  = simple circuit or cycle

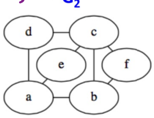
c)  = circuit

Exercise 7

Q: Which of the following graphs has Euler circuit? Justify your answer.

- start & end - same vertex
- all vertex & edge at least once
- graph connected, every vertex has even degree





G_1 :

vertex	a	b	c	d	e	f
Degree	2	4	4	2	2	2

G_2 :

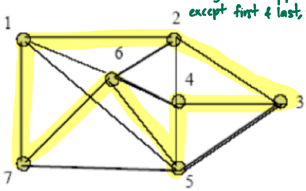
vertex	a	b	c	d	e	f
Degree	3	3	4	2	2	2

G_1 only because all vertex on G_1 only have even degree.

Exercise 8

Question: Is this graph has Hamiltonian cycle?

- every vertex appears exactly once, except first & last, which are the same

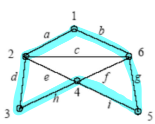


(1, 2, 3, 4, 5, 6, 7, 1)

Therefore, this graph has Hamiltonian cycle.

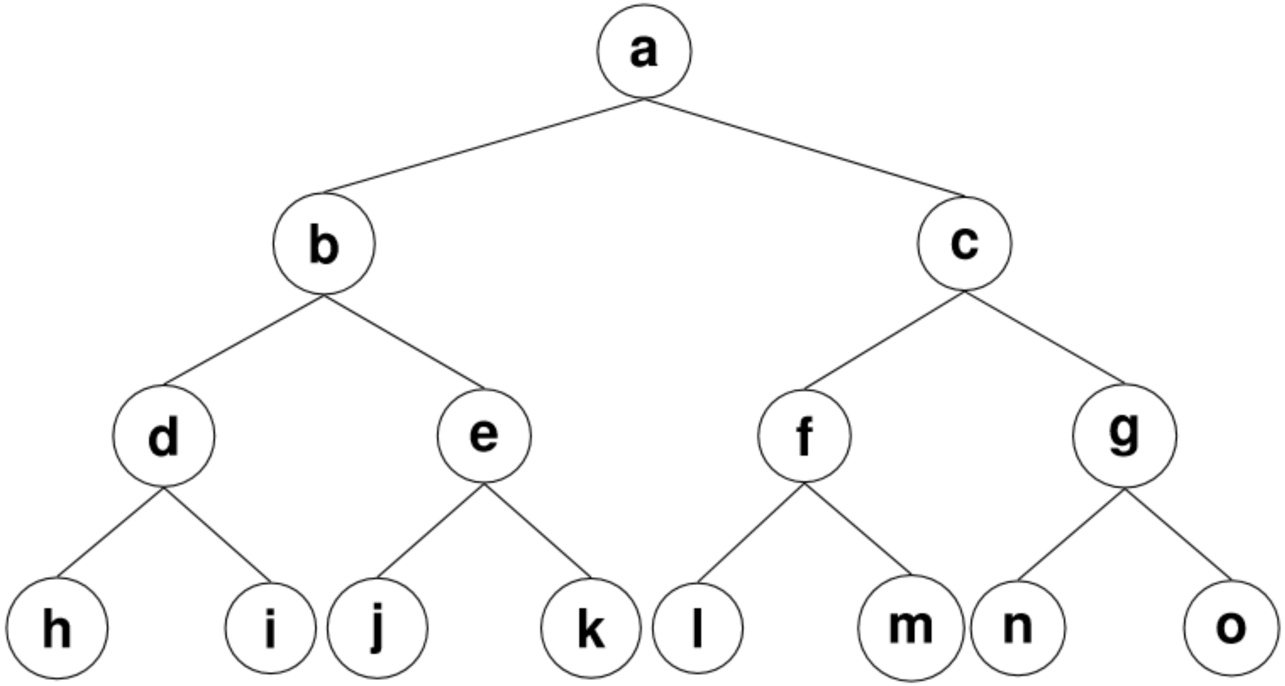
Exercise 10

Find a Hamiltonian circuit in this graph.



(1, 2, 3, 4, 5, 6, 1)

Exercise 1



Find:

m, f, c, a

- Ancestors of m
- Descendents of g

g, n, o

- Parent of *e* b
- Children of *e* j, k
- Sibling of *h* i

Exercise 2

- How many matches are played in a tennis tournament of 27 players?

$m = 2$

$l = 27$

$n = (ml-1)/(m-1)$

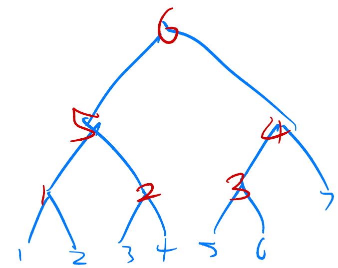
$= 53/1$

$= 53$

$i = 53 - 27$

$= 26$

Ex: 7 players



$n = (ml-1)/(m-1)$

$= 13$

$i = 13 - 7$

$= 6$

Exercise 3

Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one entrant has not lost. (Assume there are no ties.)

$m = 2 \quad l = 1000$

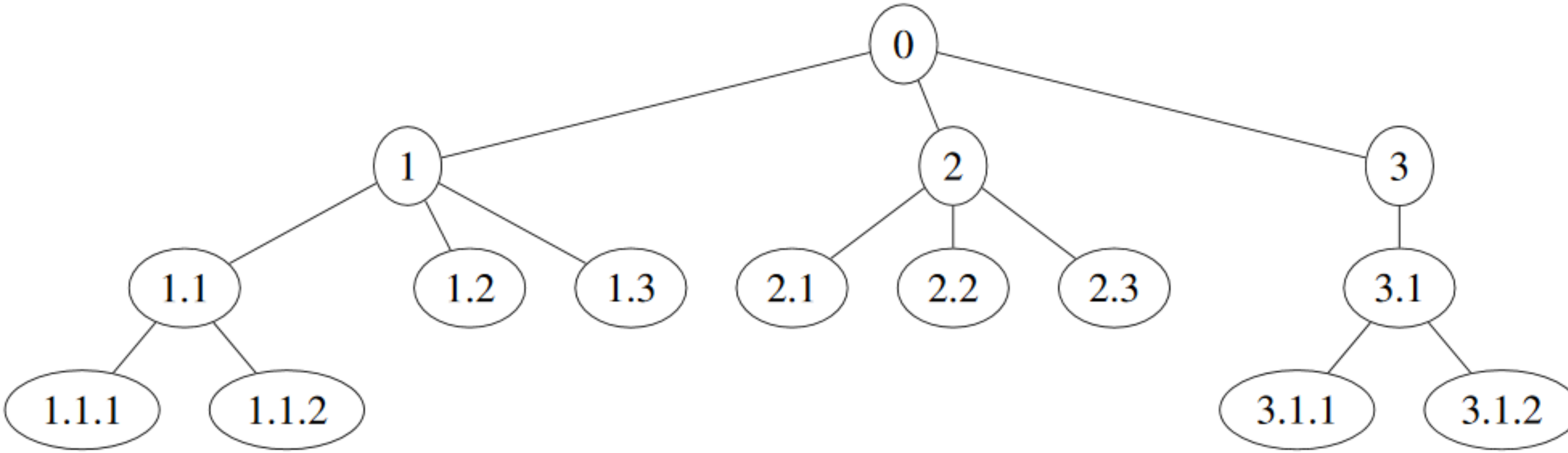
$n = (ml - 1)/(m-1)$

$= 1999$

$i = 1999 - 1000$

$= 999$

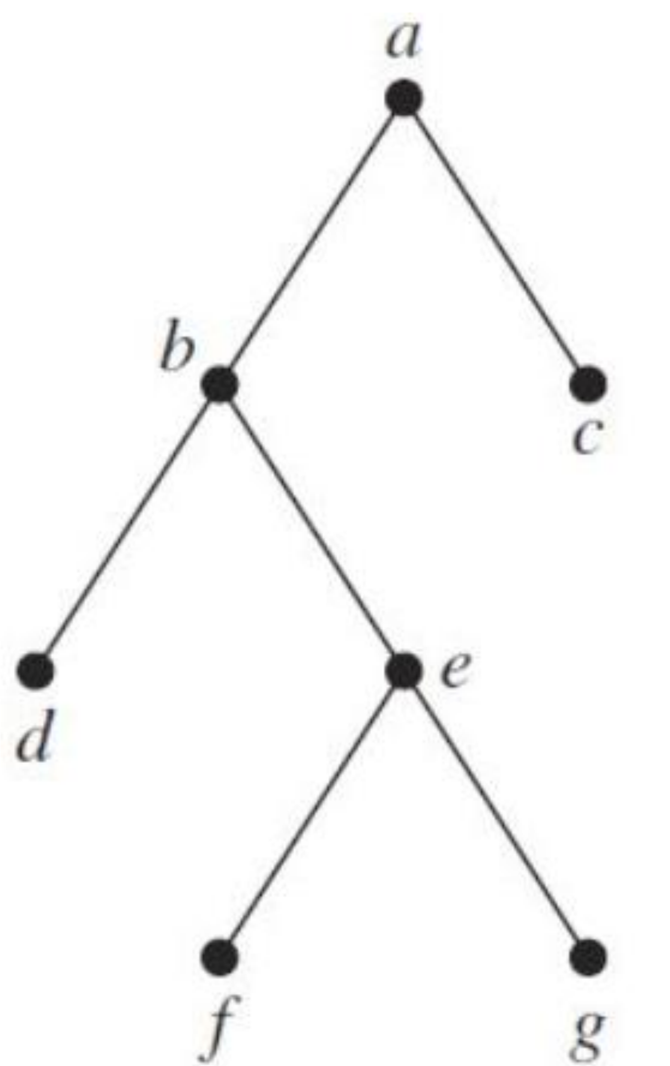
Exercise 4



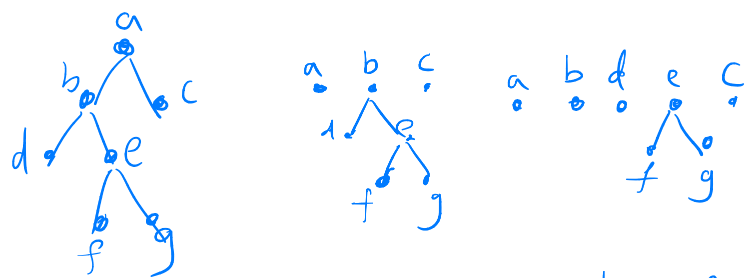
Find the lexicographic ordering of the above tree.

0 < 1 < 1.1 < 1.1.1 < 1.1.2 < 1.2 < 1.3 < 2 < 2.1 < 2.2 < 2.3 < 3 < 3.1 < 3.1.1 < 3.1.2

Exercise 5



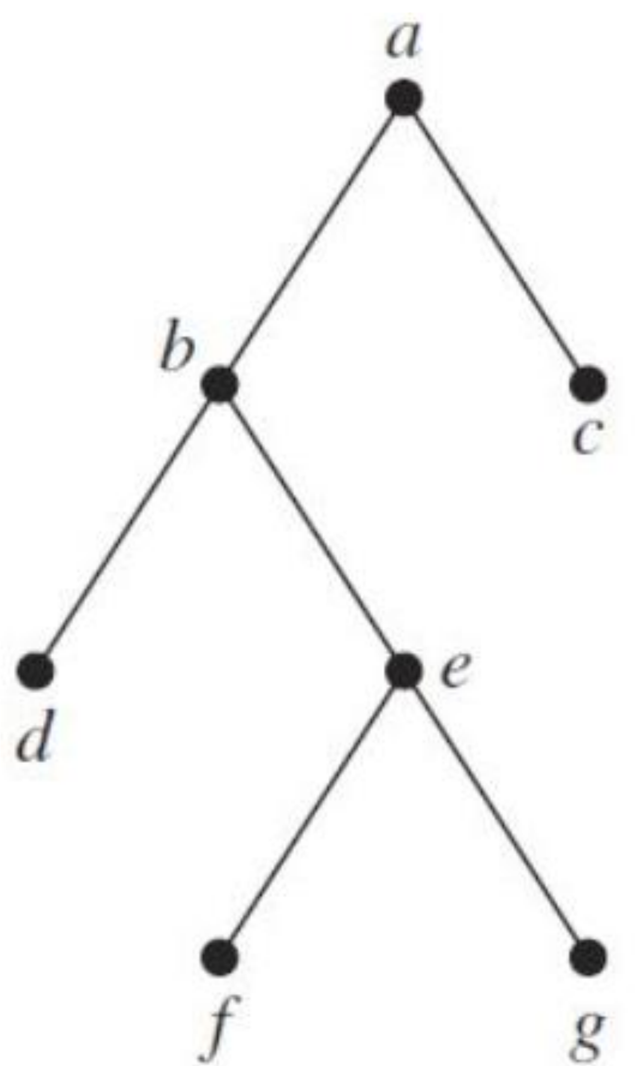
Determine the order in which a preorder traversal visits the vertices of the given ordered rooted tree.



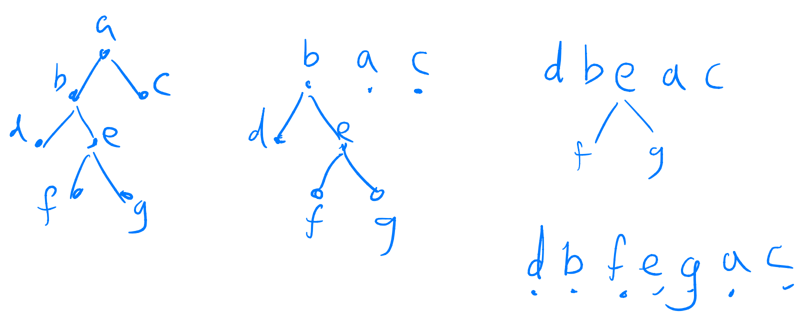
a b d e f g c

pre order : a, b, d, e, f, g, c

Exercise 6

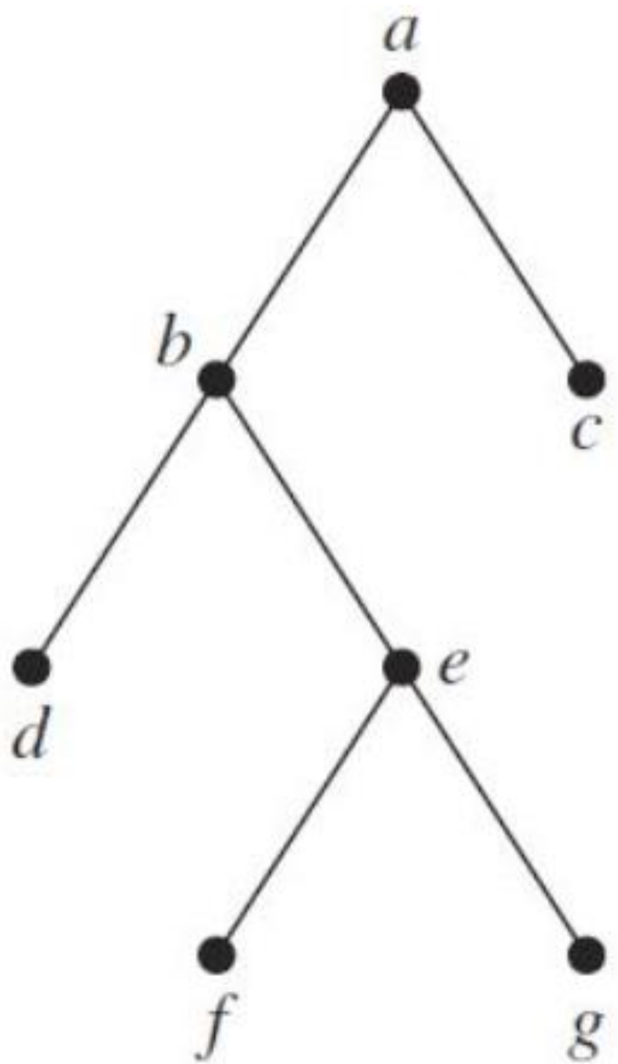


Determine the order in which a inorder traversal visits the vertices of the given ordered rooted tree.

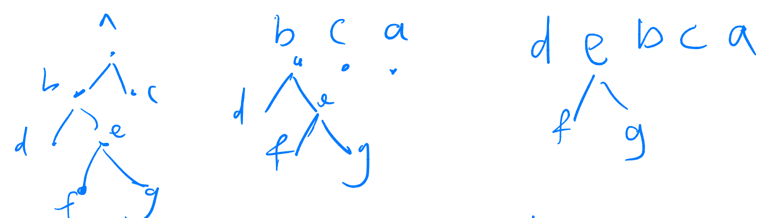


In order : d,b,f,e,g,a,c

Exercise 7



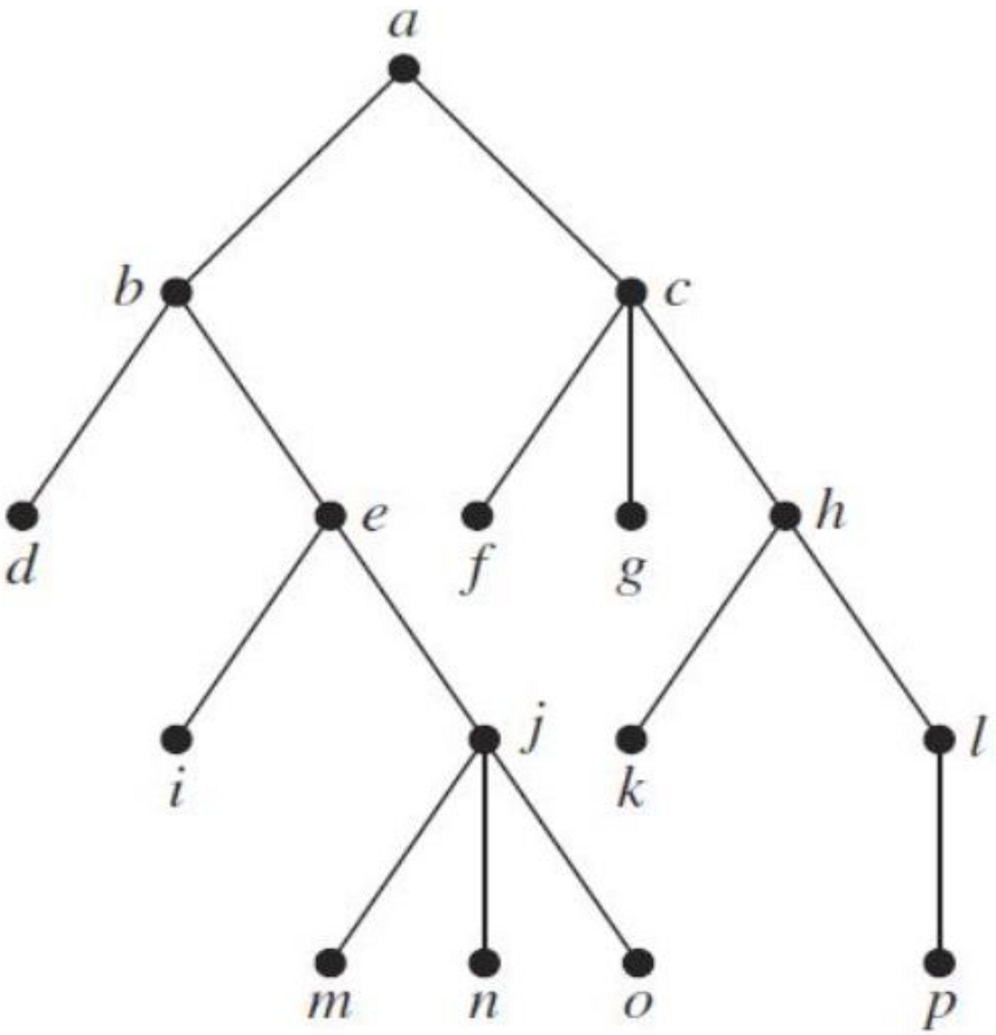
Determine the order in which a postorder traversal visits the vertices of the given ordered rooted tree.



d f g e b c a

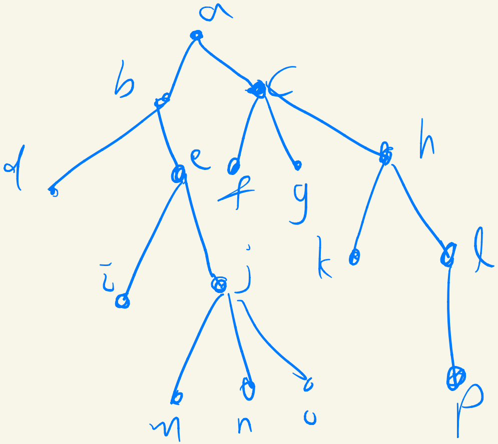
post order : d, f, g, e, b, c, a

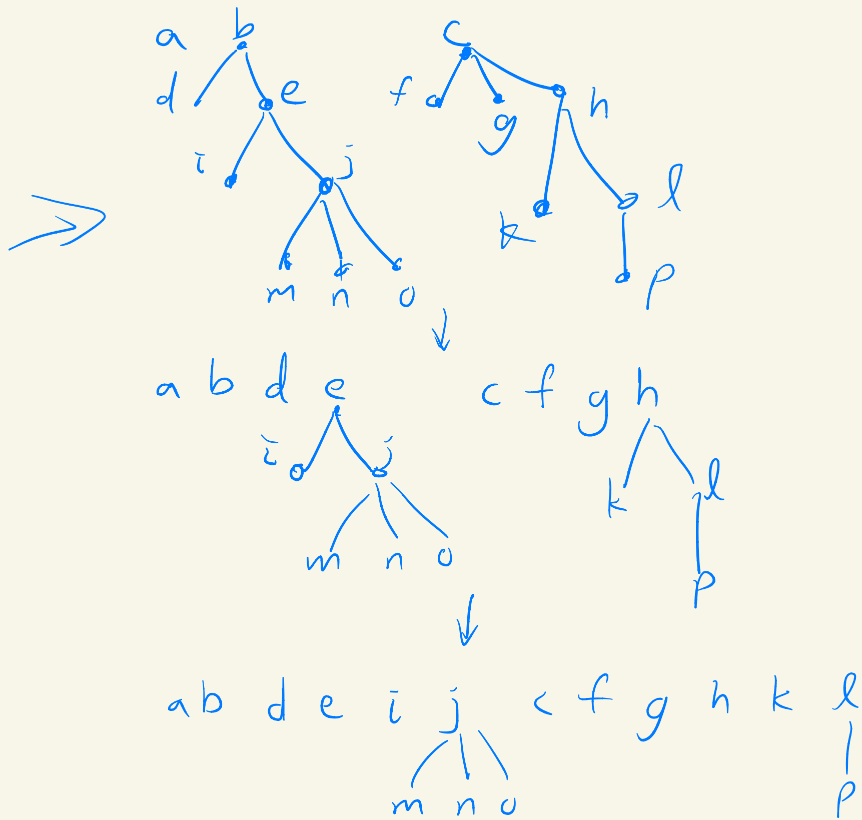
Exercise 8, 9, 10



Determine the order of preorder (8), inorder (9) and postorder (10) of the given rooted tree.

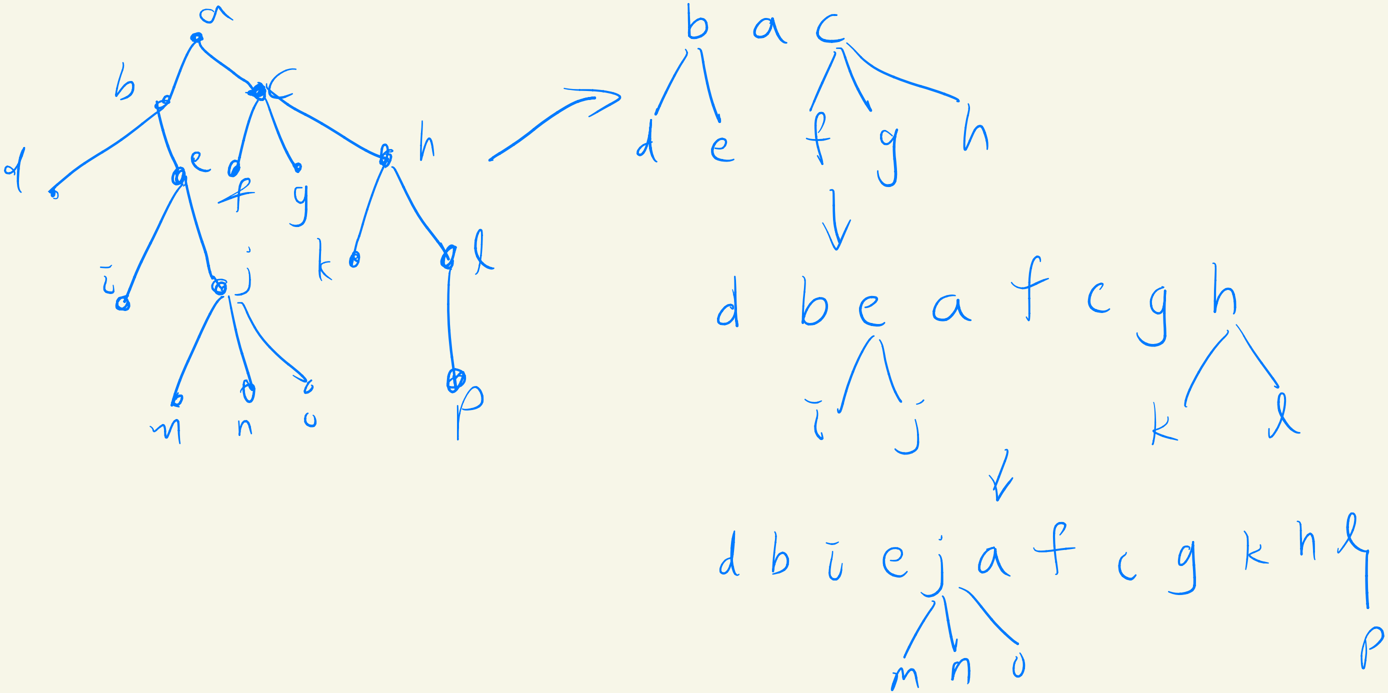
Exercise 8





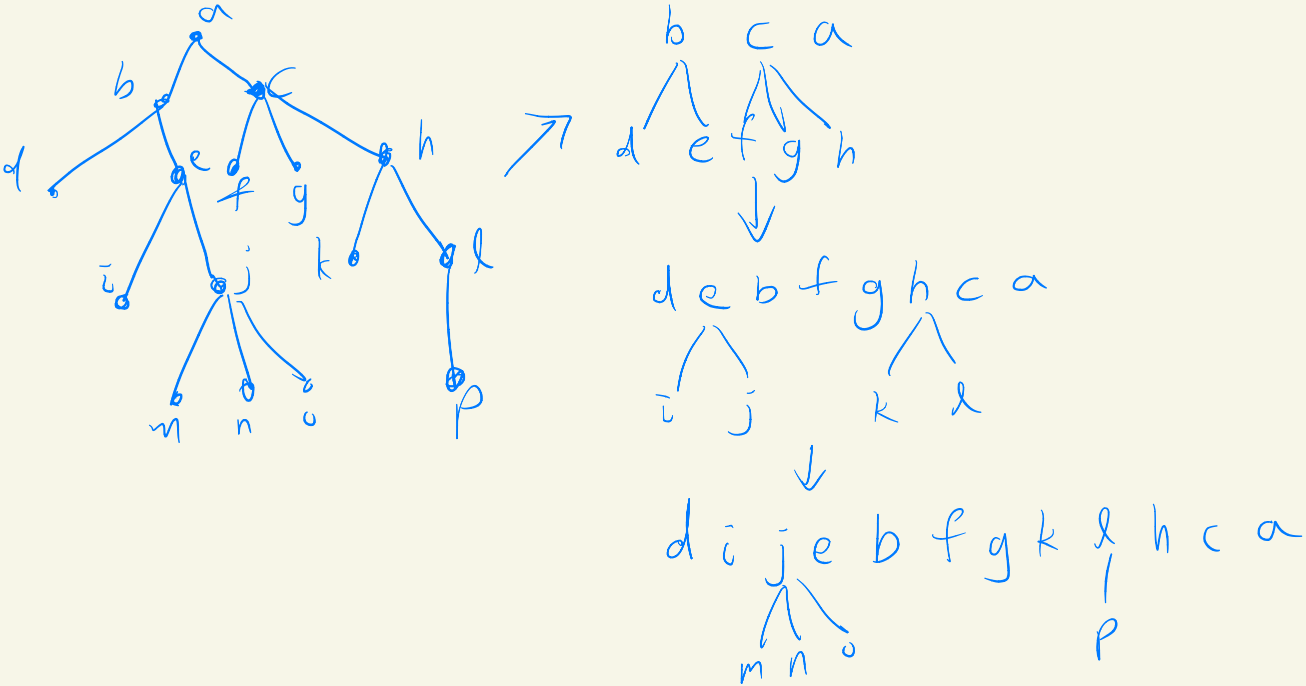
preorder: a, b, d, e, i, j, m, n, o, c, f, g, h, k, l, p

Exercise 9



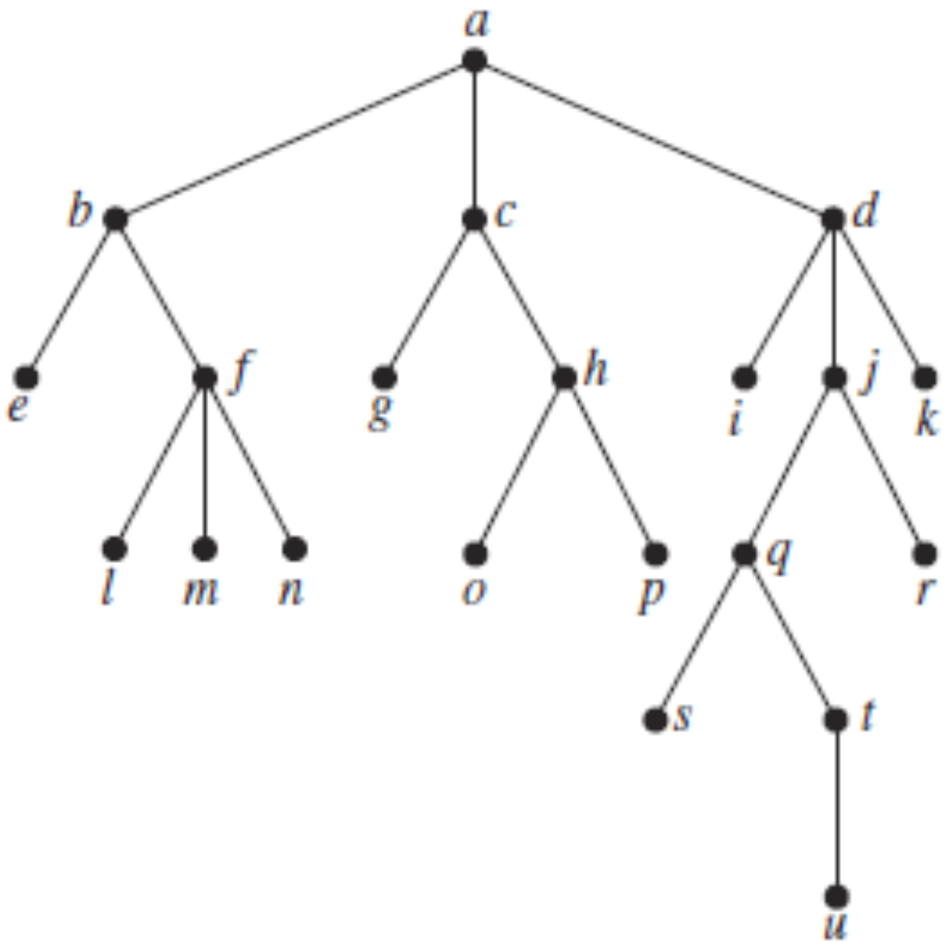
inorder: d, b, i, e, m, j, n, o, a, f, c, g, k, h, p, l

Exercise 10



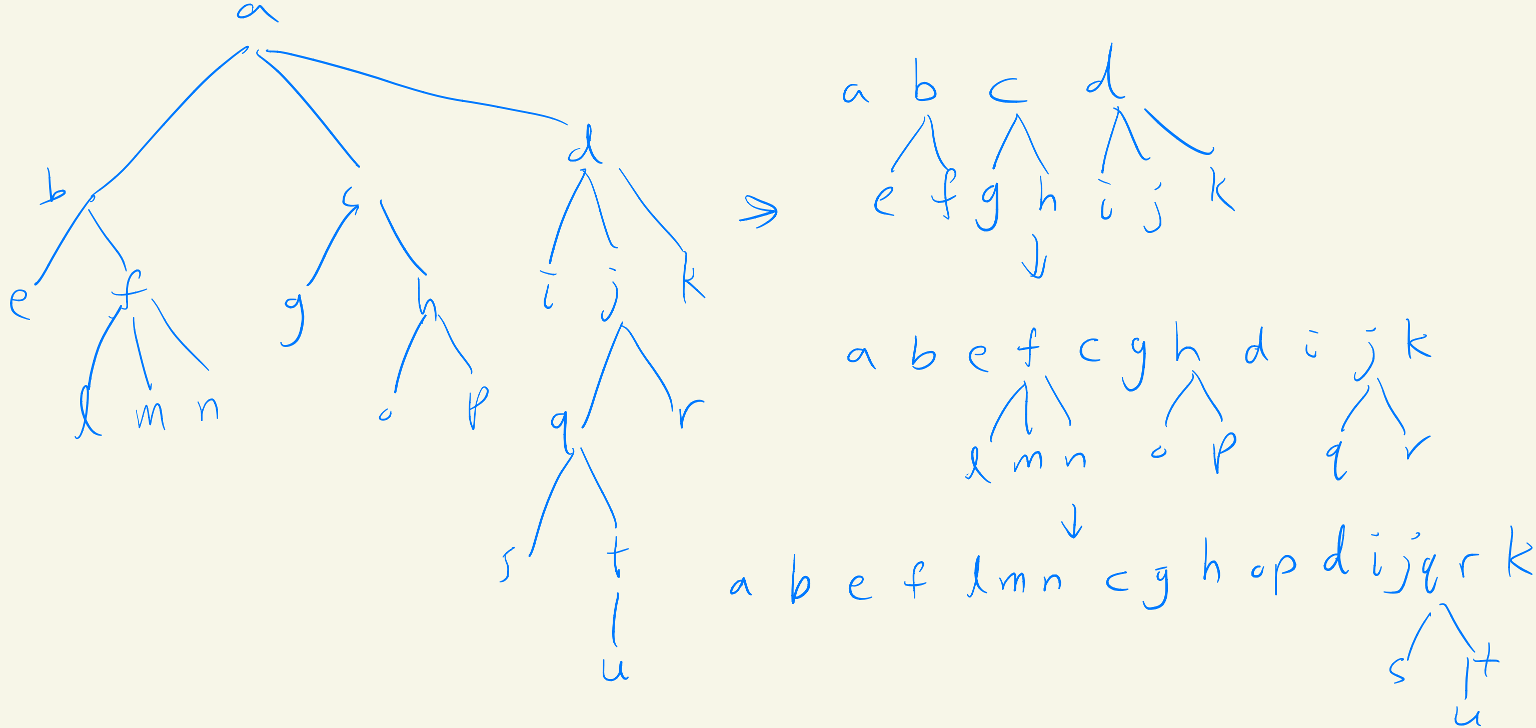
postorder : d, i, m, n, o, j, e, b, f, g, k, p, l, h, c, a

Exercise 11, 12, 13



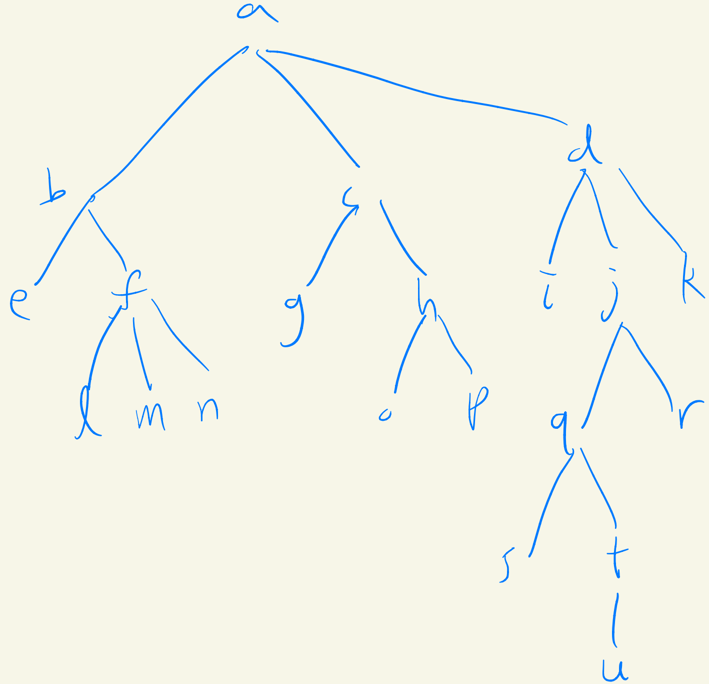
Determine the order of preorder (11), inorder (12) and postorder (13) of the given rooted tree.

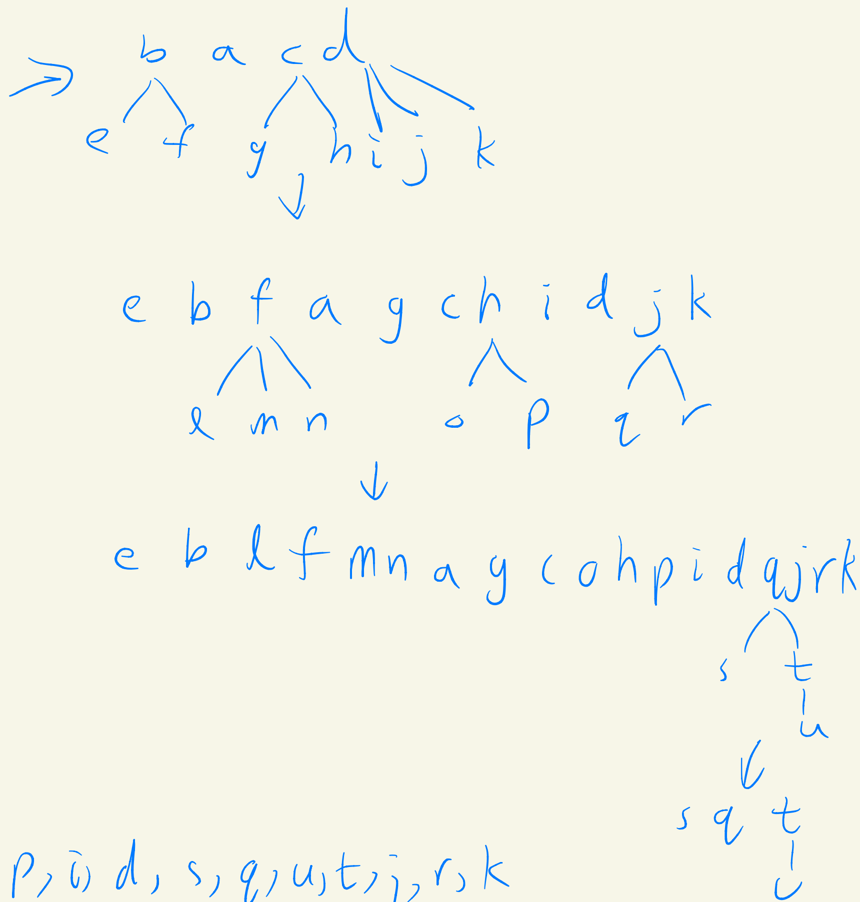
Exercise 11



preorder : a, b, e, f, l, m, n, c, g, h, o, p, d, i, j, q, s, t, u, r, k

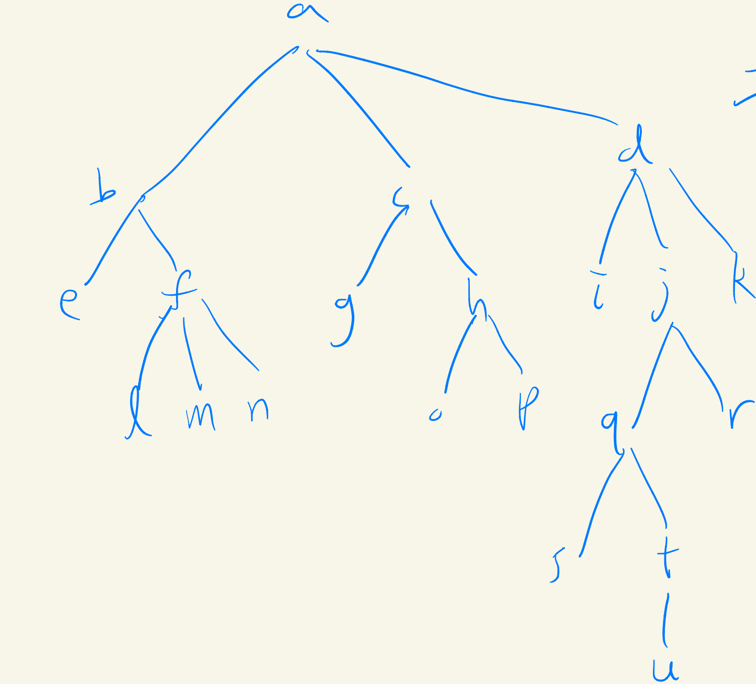
Exercise 12

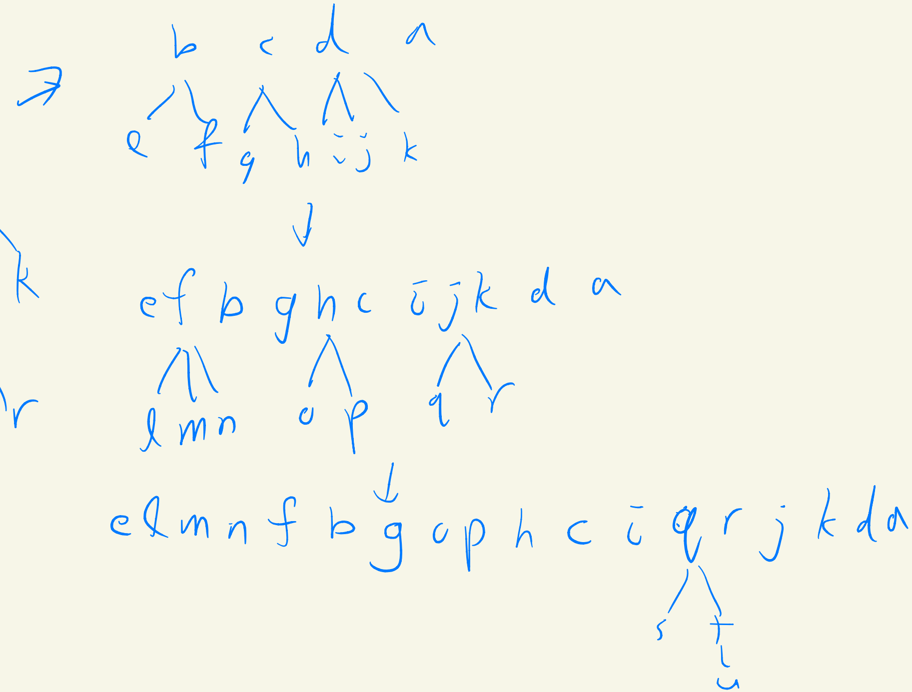


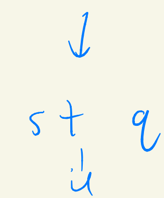


inorder : e, b, l, f, m, n, a, g, c, o, h, p, i, d, s, q, u, t, j, r, k

Exercise 13



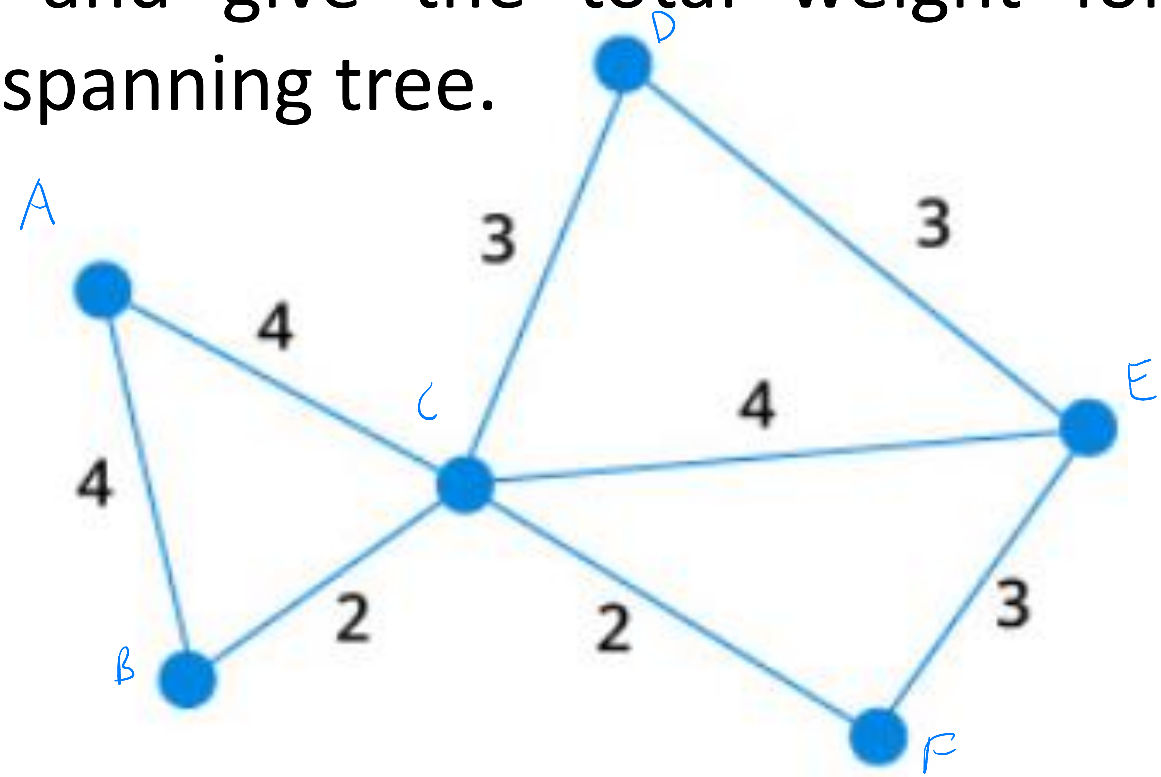




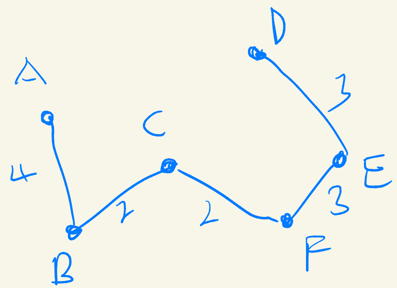
postorder: e, l, m, n, f, b, g, o, p, h, c, i, s, u, t, q, r, j, k, d, a

Exercise 14

Find the minimum spanning tree using Kraskal's Algorithm and give the total weight for the minimum spanning tree.



BC	CF	FE	ED	CD	AB	AC	CE
2	2	3	3	3	4	4	4

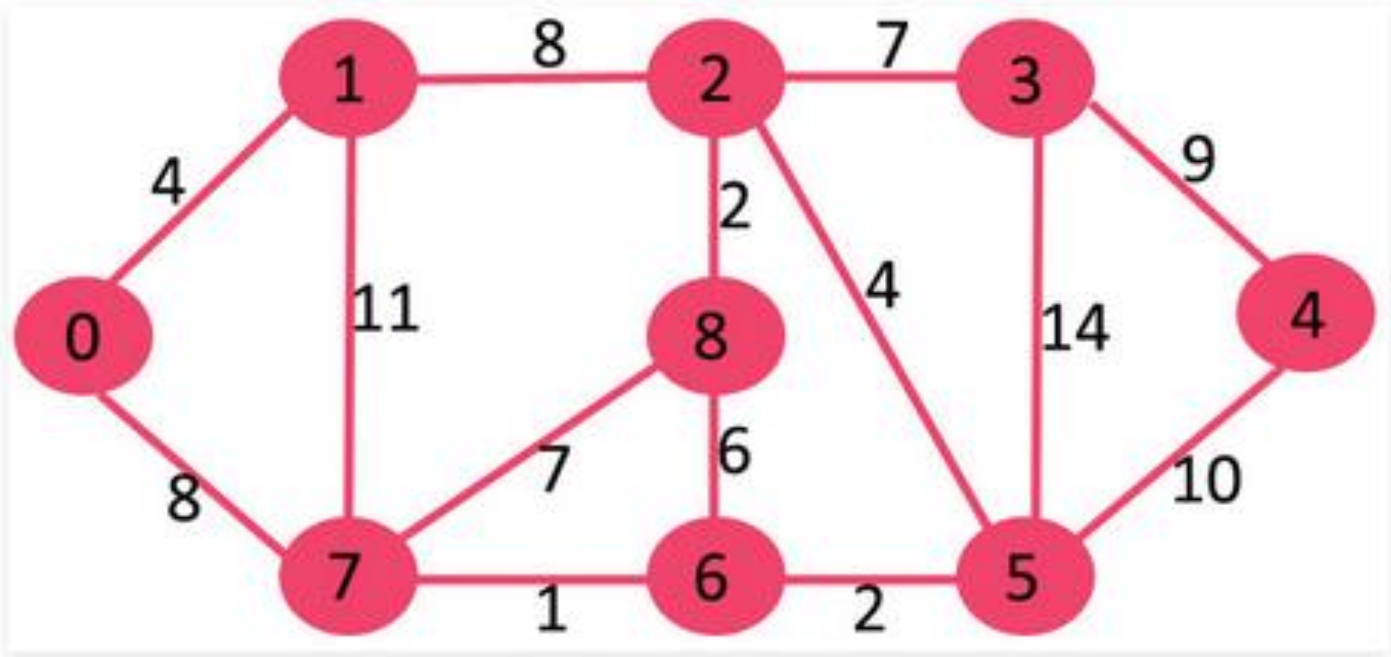


$4+2+2+3+3=14$

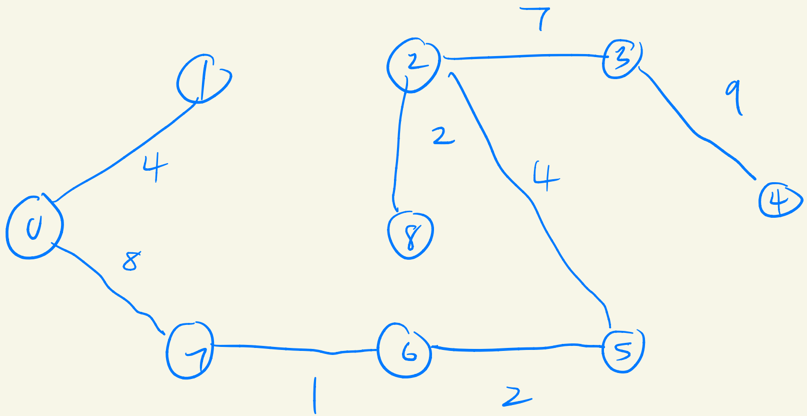
Total weight = 14

Exercise 15

Find the minimum spanning tree using Kraskal's Algorithm and give the total weight for the minimum spanning tree.



edge	67	56	28	01	25	68	23	78	12	07	34	45	17	35
weight	1	2	2	4	4	6	7	7	8	8	9	10	11	14

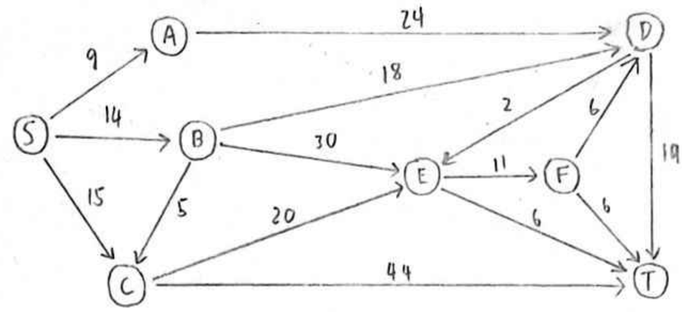


Total weight $= 1+2+2+4+4+7+8+9$

$= 37$

CH4P3 Exercise 1

Q: Given a weighted digraph, find the shortest path from S to T, using Djikstra Algorithm.



Iteration	S	N	L(S)	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(T)
0	{}	{S,A,B,C,D,E,F,T}	0	∞	∞	∞	∞	∞	∞	∞
1	{S}	{A,B,C,D,E,F,T}	0	9	14	15	∞	∞	∞	∞
2	{S,A}	{B,C,D,E,F,T}		9	14	15	33	∞	∞	∞
3	{S,A,B}	{C,D,E,F,T}			14	15	32	44	∞	∞
4	{S,A,B,C}	{D,E,F,T}				15	32	35	∞	∞
5	{S,A,B,C,D}	{E,F,T}					32	34	∞	51
6	{S,A,B,C,D,E}	{F,T}						34	45	40
7	{S,A,B,C,D,E,T}	{F}								40

Shortest Path: S → B → D → E → T

Exercise 2

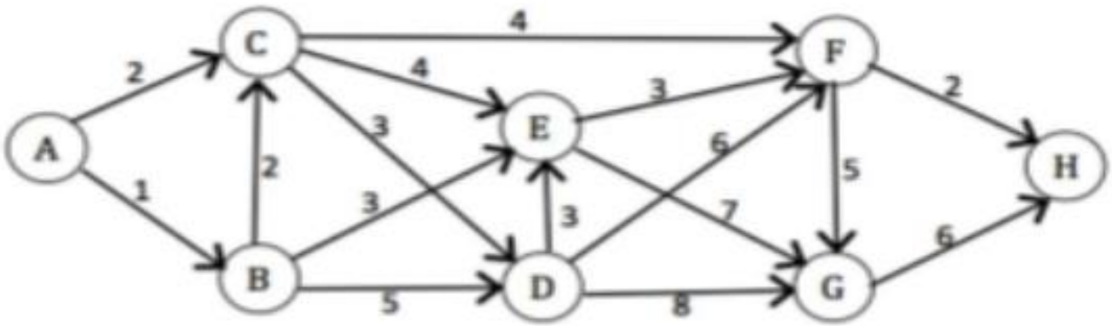


Figure 5

Based on Dijkstra's algorithm, complete Table 1 to find the shortest path from city A to city H. (Note: Copy Table 1 into your answer booklet).

(8 marks)

Iteration	S	N	L(a)	L(b)	L(c)	L(d)	L(e)	L(f)	L(g)	L(h)
0	{ }	{a, b, c, d, e, f, g, h}	∞	∞	∞	∞	∞	∞	∞	∞
1	{a}	{b, c, d, e, f, g, h}	0	1	2	-	-	-	-	-
2	{a, b}	{c, d, e, f, g, h}		1	2	6	4	-	-	-
3	{a, b, c}	{d, e, f, g, h}			2	5	4	6	-	-
4	{a, b, c, e}	{d, f, g, h}				5	4	6	11	-
5	{a, b, c, e, d}	{f, g, h}				5		6	11	-
6	{a, b, c, e, d, f}	{g, h}						6	11	8
7	{a, b, c, e, d, f, h}	{g}								8

∴ The shortest path { A, C, F, H } #