H_0: $p_0(x) = \begin{cases} 1, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$

H_1: $p_1(x) = \begin{cases} \frac{e^{1-x}}{e-1}, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$

a) $n=1$

$$l = \frac{L_1}{L_0} = \frac{e^{1-x}}{e-1} \geq C$$

$$\Downarrow$$

$$e^{-x} \geq B$$

$$\Downarrow$$

$$x \leq A$$

$$P(x \leq A \mid H_0) = \alpha$$

$$\int_0^A dx = A = \alpha$$

$G: x \leq \alpha$ – наиболее мощный критерий

$$\alpha_1 = \sup_{H_0} P(x \in G \mid H_0) = \alpha$$

$$W = P(x \leq \alpha \mid H_1) = \int_0^{\alpha} \frac{e^{1-x}}{e-1} dx = \frac{e}{e-1} \int_0^{\alpha} e^{-x} dx =$$

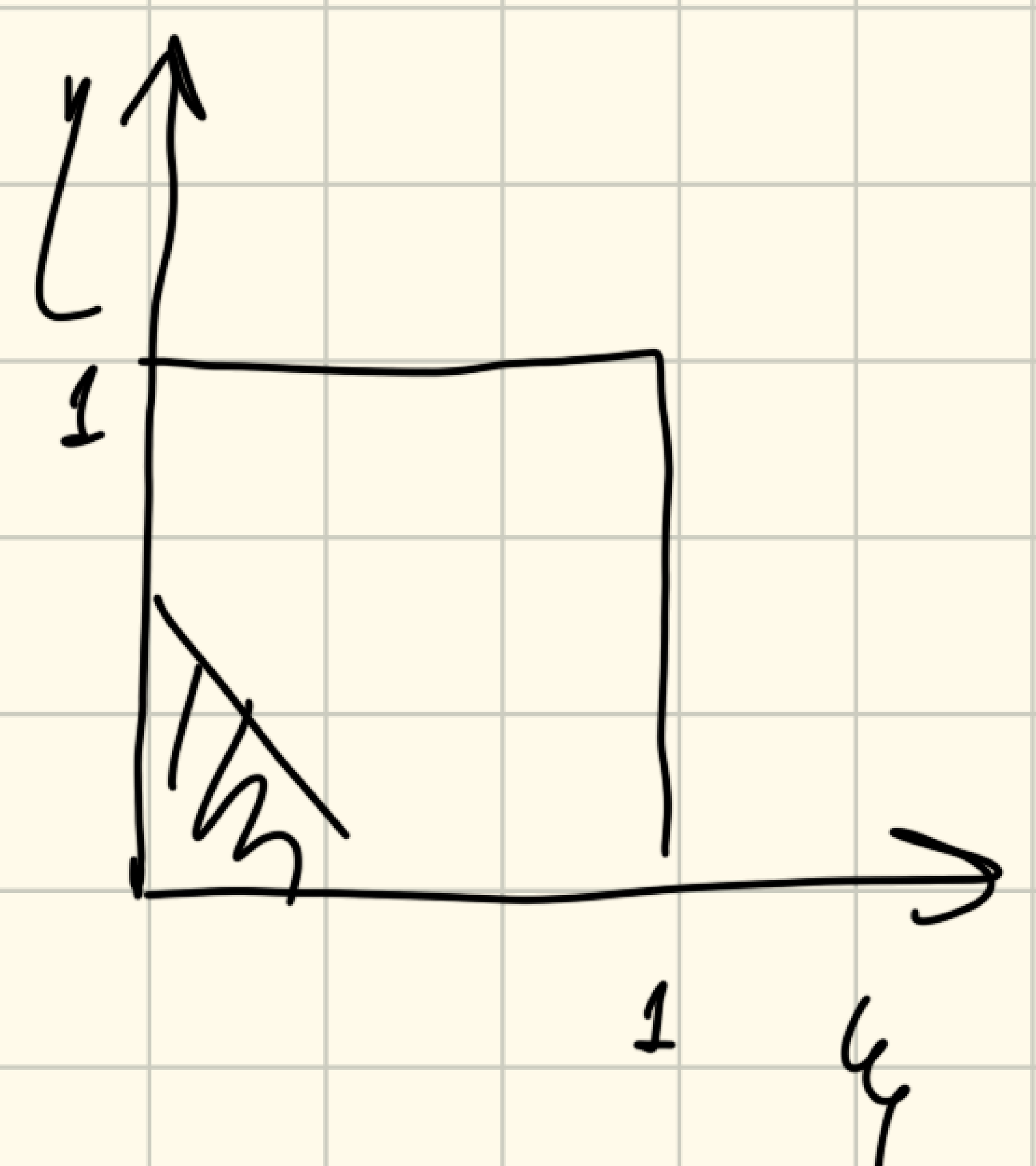
$$= \frac{e}{e-1}\left(-e^{-x}\right)\Big|_0^{\alpha} = \frac{e}{e-1}\left(1-e^{-\alpha}\right)$$

b) $n=2$, ур-нь значимости α

$$L = \frac{e^{2-x_1-x_2}}{(e-1)^2} \geq C$$

$$x_1 + x_2 \leq A$$

$$P(x_1 + x_2 \leq A \mid H_0) = \alpha$$



Пусть α мало

Тогда область примимае. вид 

$$\int_0^{A} dx_1 \int_0^{A-x_1} dx_2 = \frac{A^2}{2} = \alpha$$

$$A = \sqrt{2\alpha}$$

$$G: x_1 + x_2 \leq \sqrt{2\alpha}$$

$$\alpha_1 = \iint_G dx_1\, dx_2 = \alpha$$

$$= \int_0^A dx_1 \int_0^{A-x_1} \frac{e^{2-x_1-x_2}}{(e-1)^2}\, dx_2 =$$

$$= \frac{e^2}{(e-1)^2} \int_0^A e^{-x_1} dx_1 \int_0^{A-x_1} e^{-x_2} dx_2 =$$

$$= \frac{e^2}{(e-1)^2} \int_0^A e^{-x_1}\left(1-e^{-A+x_1}\right) dx_1 =$$

$$= -\frac{A e^2 e^{-A}}{(e-1)^2} + \frac{e^2}{(e-1)^2}\left(1-e^{-A}\right) =$$

$$= \frac{e^2}{(e-1)^2}\left(1-e^{-A}-Ae^{-A}\right)$$

c) $$L = \prod_{i=1}^n \frac{e^{1-x_i}}{e-1} \geq C$$

$$\ln L = \sum_{i=1}^n \ln \frac{e^{1-x_i}}{e-1} = n \cdot \ln \frac{e}{e-1} - \sum_{i=1}^n x_i \geq \tilde{C}$$

$$\sum x_i < A$$

$$\frac{\sum x_i - n M x_i}{\sqrt{n D x_i}} \sim N(0,1)$$

$$P\left(\frac{\sum x_i - nMx}{\sqrt{nDx}} \le \frac{A - nMx}{\sqrt{nDx}} \,\Big|\, H_0\right) = \alpha$$

$$Mx = \frac{1}{2}$$

$$Dx = \frac{1}{12} \qquad \frac{A - n\cdot\frac{1}{2}}{\sqrt{n\cdot\frac{1}{12}}} = u_\alpha$$

$$A = \frac{n}{2} + u_\alpha\sqrt{\frac{n}{12}}$$

$$G:\ \sum x_i \le \frac{n}{2} + u_\alpha\sqrt{\frac{n}{12}}$$

$$\alpha_1 = \alpha$$

$$W = P\left(\frac{\sum x_i - Mx}{\sqrt{nDx}} \le \frac{A - Mx}{\sqrt{nDx}} \,\Big|\, H_1\right) =$$

$$Mx = \int_0^1 \frac{x e^{1-x}}{e-1}\,dx = \frac{e}{e-1}\int_0^1 x e^{-x}\,dx =$$

$$= \frac{e}{e-1}\left(-xe^{-x}\Big|_0^1 + \int_0^1 e^{-x}\,dx\right) =$$

$$= \frac{e}{e-1}\left(-e^{-1} + (1 - e^{-1})\right) = \frac{e-2}{e-1}$$

$$Mx^2 = \int_0^1 \frac{x^2 e^{1-x}}{e-1}\,dx = \frac{e}{e-1}\left(-\int_0^1 x^2 de^{-x}\right) =$$

$$= \frac{e}{e-1}\left(-x^2 e^{-x}\Big|_0^1 + \int_0^1 2x e^{-x} dx\right) =$$

$$= \frac{e}{e-1}\left(-e^{-1} - 2e^{-1} + 2 - 2e^{-1}\right) = \frac{2e-5}{e-1}$$

$$Dx = Mx^2 - (Mx)^2 = \frac{e^2 - 3e + 1}{(e-1)^2}$$

$$P\left(\frac{\Sigma x_i - Mx}{\sqrt{nDx}} \leq \frac{\frac{n}{2} + u_\alpha\sqrt{\frac{n}{12}} - n\frac{e-2}{e-1}}{\sqrt{n \cdot \frac{e^2-3e+1}{(e-1)^2}}}\right)$$

$\sim N(0,1)$ $\qquad \underbrace{}_{B}$

$$W = \int_{-\infty}^{B} \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{x^2}{2}} dx \to 1$$

$B \to +\infty$
$n \to \infty$

$\Rightarrow$ критерий является состоятельным

d) $G: X_{min} \leq C$

$$P(\vec{x_n} \in G \mid H_0) = \alpha$$

$$H_0: \xi \sim R(0,1)$$

$$\eta_{min} = 1 - (1 - F(x))^n =$$

$$P(\eta_{min} \le C) = 1 - (1 - F(C))^n =$$

$$= 1 - (1 - C)^n = \alpha$$

$$(1 - C)^n = -\alpha + 1$$

$$1 - C = \sqrt[n]{1 - \alpha}$$

$$\boxed{C = 1 - \sqrt[n]{1 - \alpha}}$$

$\alpha = \alpha_1$

$$W = P(x_{min} \le C) = \dots$$

$$F_1(z) = \int_0^z \frac{e^{1-x}}{e-1}\, dx = \frac{e}{e-1}(1 - e^{-z})$$

$W =$

$$P(x_{min} \le c) = 1 - \left(1 - \frac{e}{e-1}(1 - e^{-c})\right)^n =$$

~~состоятельный~~

$$e^{-1+\sqrt[n]{1-\alpha}} = e^{-1} e^{e^{-\frac{1}{n}\ln(1-\alpha)}} =$$

$$= e^{-1} e^{\left(1-\frac{1}{n}\ln(1-\alpha)+o\left(\frac{1}{n}\right)\right)}$$

$$1-\left(1-\frac{e}{e-1}\cdot\left(1-e^{-\frac{1}{n}\ln(1-\alpha)}\right)\right)^n =$$

$$= 1-\left(1-\frac{e}{e-1}\left(1-1+\frac{1}{n}\ln(1-\alpha)+o\left(\frac{1}{n}\right)\right)\right)^n =$$

$$= 1-\left(1-\frac{1}{n}\cdot\frac{e\ln(1-\alpha)}{e-1}\right)^n \to$$

$$\to 1-e^{\frac{e}{e-1}\ln(1-\alpha)} \to 1-(1-\alpha)^{\frac{e}{e-1}} \neq 1$$

Критерий не состоятельный