ETF 5500 Assignment 2

Arindom Baruah (32779267)

Question 1

Part A

Here, we have a data matrix Y with a dimension of $n \times 3$ which has a mean of 0.

For a vector Y which has been demeaned, the sample covariance matrix S can be obtained through the following computation.

$$S = \frac{1}{n-1}(Y^T Y) \tag{1}$$

To maintain matrix conformability, the dimension of S will be a 3×3 matrix.

In order to define the covariance matric S as defined by Equation 1, we require the following quanities:

1. A data matrix Y with a mean of 0.
2. The total number of observations n which is given by the number of rows in Y.

Part B

An Eigen Value problem is linear in nature and defined by Equation 2.

$$SW = \lambda W \tag{2}$$

Where,

S = Covariance Matrix
W = Eigen Vector
λ = Eigen Value

As the Eigen Vector is a column vector, hence, to satisfy matrix conformability in Equation 2, the dimension of W must be a 3×1 vector.

Part C

We are given the linear combination as stated through Equation 3.

$$X = \beta Y \tag{3}$$

We know that Y is a $n \times 1$ vector and β is stated as a 3×1 vector. To obtain the Variance-Covariance matrix, we perform the following computations.

$$\text{Var}(X) = \text{Var}(Y\beta)$$

$$\implies \text{Var}(X) = \sum_{i=1}^{n} \frac{(Y_i\beta)^2}{n-1}$$

$$\implies \text{Var}(X) = \beta^T \Big[\sum_{i=1}^{n} \frac{(Y_i)^2}{n-1}\Big] \beta$$

For a data matrix Y_i with mean $= 0$, the term $\sum_{i=1}^{n} \dfrac{(Y_i)^2}{n-1}$ is considered as the covariance and is denoted by S.

$$\boxed{\implies \text{Var}(X) = \beta^T S \beta} \tag{4}$$

Equation 4 provides us with the final form of the variance-covariance matrix.

We know, the dimension of β^T is 1×3 and that of *beta* is 3×1. In order to maintain matrix conformability, the value dimension of the variance covariance matrix $\text{Var}(X)$ will be 1×1. This suggests that the **resultant matrix is a scalar value.**