Welcome to Discussion :)

Logistics/Expectations

- cameras on
- email me before if can't make discussion
- >2 unexcused absence => dropped
  in same week
- watch lecture before discussion
- No mini-lectures, but will answer q's
  in ten minutes before Berkeley time
- Notes posted on my website afterward
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1 Implication

Which of the following implications are always true, regardless of P? Give a counterexample for each false assertion (i.e. come up with a statement $P(x,y)$ that would make the implication false).

(a) $\forall x \forall y P(x,y) \implies \forall y \forall x P(x,y)$.

"there's a y that works for every x"

(b) $\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$.

(c) $\exists x \forall y P(x,y) \implies \forall y \exists x P(x,y)$.

a) True - $\forall x \forall y$ says "for all x, for all y" i.e. "for all x and y" or "for all pairs (x,y)". So when $\forall x \forall y P(x,y)$ true, $\forall y \forall x P(x,y)$ true.

b) False - let $P(x,y) = x < y$ over universe of $\mathbb{Z}$
$\forall x \exists y P(x,y)$ true but $\exists y \forall x P(x,y)$ false.

c) True - $\exists x \forall y P(x,y)$ says there is an x' s.t. for all y, $P(x',y)$ is true. So set $x = x'$ for $\forall y \exists x P(x,y)$.

2 Truth Tables

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

* '$\wedge$' & '$\vee$' distribute

(a) $P \wedge (Q \vee P) \equiv P \wedge Q$

(b) $(P \vee Q) \wedge R \equiv (P \wedge R) \vee (Q \wedge R)$

(c) $(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$

a)

P	Q	$P \wedge (Q \vee P)$	$P \wedge Q$
T	T	T	T
T	F	T	F
F	T	F	F
F	F	F	F

← not equivalent

$P \wedge (Q \vee P) \not\equiv P \wedge Q$

- R false ⇒ false

b)

P	Q	R	$(P \vee Q) \wedge R$	$(P \wedge R) \vee (Q \wedge R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$(P \vee Q) \wedge R \equiv (P \wedge R) \vee (Q \wedge R)$

c)

P	Q	R	$(P \wedge Q) \vee R$	$(P \vee R) \wedge (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	F	F
F	F	T	T	T
F	F	F	F	F

~ R true
=> true

$$(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$$

3 Converse and Contrapositive

Consider the statement "if a natural number is divisible by 4, it is divisible by 2" (use "$x \mid y$" to denote x divides y).

(a) Write the statement in propositional logic. Prove that it is true or give a counterexample.

(b) Write the inverse of the implication in English and in propositional logic. Prove that it is true or give a counterexample. (The inverse of an implication $P \implies Q$ is $\neg P \implies \neg Q$.)

(c) Write the converse of the implication in English and in propositional logic. Prove that it is true

(d) Write the contrapositive of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

a) $(\forall x \in \mathbb{N})(4 \mid x \Rightarrow 2 \mid x)$

True: $4 \mid x \Rightarrow x = 4k, \; k \in \mathbb{Z}$

$= 2(2k), \; 2k \in \mathbb{Z}$

$\Rightarrow 2 \mid x$ □

b) inverse: if a natural number is not divisible by 4 it is not divisible by 2

$(\forall x \in \mathbb{N})(4 \nmid x \Rightarrow 2 \nmid x)$

False: 2 not divisible by 4 but divisible by 2

c) converse: any natural number divisible by 2 is divisible by 4

$(\forall x \in \mathbb{N})(2 \mid x \Rightarrow 4 \mid x)$

False: 2 divisible by 2, not by 4

d) contrapositive: any natural number not divisible by 2 is not divisible by 4

$(\forall x \in \mathbb{N})(2 \nmid x \Rightarrow 4 \nmid x)$

True: $2 \nmid x \Rightarrow \frac{x}{2}$ not integer

$\Rightarrow \left(\frac{x}{2}\right) \cdot \frac{1}{2} = \frac{x}{4}$ not integer

$\Rightarrow 4 \nmid x$ $\square$