

$$2. \quad L = \sum_{i=1}^I \omega_i \log \left[\frac{1}{1 + \exp[-\phi^T x_i]} \right] + \sum_{i=1}^I (1 - \omega_i) \log \left[\frac{\exp[-\phi^T x_i]}{1 + \exp[-\phi^T x_i]} \right]$$

Here, $\text{sig}(\phi^T x_i) = \frac{1}{1 + \exp[-\phi^T x_i]}$

$$1 - \text{sig}(\phi^T x_i) = \frac{\exp[-\phi^T x_i]}{1 + \exp[-\phi^T x_i]}$$

Now,

$$L = \sum_{i=1}^I \omega_i \log [\text{sig}(\phi^T x_i)] + \sum_{i=1}^I (1 - \omega_i) \log [1 - \text{sig}(\phi^T x_i)]$$

$$\frac{\partial L}{\partial \phi} = \frac{\partial}{\partial \phi} \left(\sum_{i=1}^I \omega_i \log [\text{sig}(\phi^T x_i)] + \sum_{i=1}^I (1 - \omega_i) \log [1 - \text{sig}(\phi^T x_i)] \right)$$

$$= \sum_{i=1}^I \omega_i \frac{1}{\text{sig}(\phi^T x_i)} \cdot \frac{\partial \text{sig}(\phi^T x_i)}{\partial \phi} + \sum_{i=1}^I (1 - \omega_i) \frac{1}{1 - \text{sig}(\phi^T x_i)} \cdot - \frac{\partial \text{sig}(\phi^T x_i)}{\partial \phi}$$

$$= \sum_{i=1}^I \left(\frac{\omega_i}{\text{sig}(\phi^T x_i)} - \frac{(1 - \omega_i)}{1 - \text{sig}(\phi^T x_i)} \right) \frac{\partial \text{sig}(\phi^T x_i)}{\partial \phi}$$

$$= \sum_{i=1}^I \left(\frac{\omega_i - \omega_i \text{sig}(\phi^T x_i) - \text{sig}(\phi^T x_i) + \omega_i \text{sig}(\phi^T x_i)}{\text{sig}(\phi^T x_i) (1 - \text{sig}(\phi^T x_i))} \right) \text{sig}(\phi^T x_i) (1 - \text{sig}(\phi^T x_i)) \cdot x_i$$

$$= \sum_{i=1}^I (\omega_i - \text{sig}(\phi^T x_i)) x_i$$

$$= - \sum_{i=1}^I (\text{sig}(\phi^T x_i) - \omega_i) x_i$$

$$\frac{\partial L}{\partial \phi} = - \sum_{i=1}^I \left(\frac{1}{1 + \exp[-\phi^T x_i]} - \omega_i \right) x_i$$