

# STAT394 Group Project Final Report

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## 1 Abstract

An exploratory data analysis (EDA) of the ‘diamonds’ dataset was undertaken. Based on the results of the EDA, the techniques of Principal Component Analysis (PCA), Linear Discriminant Analysis (LDA), Factor Analysis (FA) and Cluster Analysis (CA) were carried out. As these techniques were new to the project team and everyone was keen to learn, it was decided to experiment with each of them (with varying degrees of success).

The primary aim was to find the best predictor variables for ‘price’. A secondary aim was to use the new techniques of LDA and CA to see if simple classifications could be made. Other techniques used were single and multiple linear regression and testing to find the best regression model using the Akaike Information Criterion (AIC) and Bayes Information Criterion (BIC) tests.

## 2 Introduction

One of the key skills required of statisticians is the ability to explore a new and unfamiliar dataset and allow that exploration to guide their next steps. A second key skill is knowing which analytical methods to subsequently apply to the dataset. As students enrolled in the Victoria University of Wellington year three Multivariate Statistics paper (STAT394), we were tasked with undertaking a group project designed to give us practice in developing these skills.

We selected the ‘diamonds’ dataset, which was unfamiliar to us all. ‘Diamonds’ is available

in the `ggplot2` package in RStudio and is also freely available on Kaggle (“Diamonds Dataset, Kaggle.com” 2016). It contains information on ten different variables for 53940 diamonds. Of these ten variables three are categorical while seven are numerical. A full breakdown of the variables is given below.

This project provided the opportunity to face the challenges of working with a new and unfamiliar dataset and to gain experience in trouble-shooting the many issues that were encountered.

## 2.1 The variables

- Carat: a measure of the diamond’s weight. One carat equals 1/5 gram. In this data set there are diamonds with carat values ranging from 0.2 - 5.01.
- Cut: A diamond’s cut defines its proportions and its ability to reflect light. This variable has five levels: Fair (lowest quality), Good, Very Good, Premium and Ideal (highest quality).
- Color: A diamond’s color refers to how clear/colorless it is. This variable has seven levels: J (lowest quality), I, H, G, F, E, D (highest quality).
- Clarity: measures small imperfections on the surface and within the stone. This variable has eight levels: I1 (lowest clarity), SI2, SI1, VS2, VS1, VVS2, VVS1, IF (highest clarity).
- x: length in mm. In this data set there are diamonds with a length ranging from 0 - 10.74.
- y: width in mm. In this data set there are diamonds with a width ranging from 0 - 58.9.
- z: depth in mm. In this data set there are diamonds with a depth ranging from 0 - 31.8.
- Depth: total depth percentage. This is calculated by dividing the total width by the total depth ( $depth = \frac{2 \times z}{(x+y)}$ ). Depth percentage impacts how light reflects of the diamond. In this data set we have diamonds with a depth percentage ranging from 43% - 79%.
- Table: The flat facet on the top of a diamond is called its table. Table is calculated by dividing table width by the total width. Table percentage impacts how light reflects of the diamond. In this data set there are diamonds with a table percentage ranging from 43% - 95%.
- Price: price of the diamond in United States dollars (USD). In this data set there are diamonds with a price ranging from \$326 - \$18823.

### 2.1.1 Why ‘diamonds’?

We selected the diamonds dataset because it was easy to understand what the variables were measuring. This was in contrast to some of the other ones we looked at which required

domain specific familiarity, such as knowledge about biology or chemistry (or even worse, bio-chemistry!). This data set also allows us to undertake an investigation from which we glean insights about diamonds that have real-world application and use.

### 2.1.2 Aims

Our primary aim was to identify which variables best predict the *price* of a given diamond. We hypothesised that increased diamond size ('x', 'y' and 'z') and weight ('carat') would be positively correlated with diamond price. We also expected to see increased prices for diamonds with higher levels of the categorical variables, all of which reflect the quality of the diamond. We were unsure how diamond depth and table percentage would relate to diamond price.

We also sought to explore the techniques of Principal Component Analysis (PCA), Factor Analysis (FA), Linear Discriminant Analysis (LDA) and Cluster Analysis (CA) that we have learned in STAT394. The diamonds dataset has 280 unique possible interactions ( $5 \times 7 \times 8$ ) between different levels of the categorical variables. Our secondary aim was to discover whether one of these techniques could classify the data in a simpler manner, or predict into which level of a category a new observation would fall. Encouraged by our lecturer, we adopted an approach of 'playing' with the techniques with the aim of learning as much as possible, even if a more experienced statistician could see that they were not the correct tools to achieve our aims. We wanted to learn by doing, regardless of whether the results of these methods were successful for our project or not.

## 3 Methodology

The diamonds data set was accessed via the kaggle.com website ("Diamonds Dataset, Kaggle.com" 2016) while all statistical analysis and reporting was completed using the computer software package R (R Core Team 2017), in RStudio (RStudio Team 2022) and using RMarkdown (Allaire et al. 2020). Github ("Github.com" 2022) was used a repository for storing all relevant documents and code during this project.

Upon loading the data set into R we first used the following code to delete unnecessary columns, ensure categorical variables were treated as factors with set levels and created a subset data set containing only the numeric variables.

```
# Read the data set into R
diamonds <- read.csv("./diamonds.csv", encoding = "UTF-8")
# Remove the index column
diamonds$X <- NULL
# Set categorical variables as factors and set levels
diamonds$cut <- factor(diamonds$cut,
                        levels = c("Fair", "Good", "Very Good", "Premium", "Ideal"))
diamonds$color <- factor(diamonds$color,
                        levels = c("J", "I", "H", "G", "F", "E", "D"))
diamonds$clarity <- factor(diamonds$clarity,
```

```

levels = c("I1", "SI2", "SI1", "VS2", "VS1", "VVS2", "VVS1", "IF")
# Make data frame of just the numerical variables
diamonds_num <- subset(diamonds, select = c(carat, depth, table, price, x, y, z))

```

When scaling numerical values in our data set we used the `scale()` function.

When taking smaller samples from our data set to improve the interpretability of visual plots we used a seed consisting of either the student identification number of the team member who wrote the code or 1234567890, and the pseudo random number generator Mersenne Twister.

To begin with, we undertook an in-depth Exploratory Data Analysis (EDA) of the data. The key results from the EDA can be found in the results section below.

## 4 Results

### 4.1 Consolidated Exploratory Data Analysis (EDA)

The initial EDA was submitted previously as part of intermediary Milestones 3 and 4 for STAT34 and is not reproduced in full here. Key findings included:

- most numerical variables did not follow a Normal distribution. Some followed the beta distribution and some were undefined (according to the Cullen-Frey plots).
- strong positive correlations between a number of the numeric variables ('price', 'carat', 'x', 'y' and 'z'), and a strong negative correlation between 'depth' and 'table'.
- some of the density plots of the numerical variables showing multiple modes (peaks) suggesting that the effects of one or more of the levels of the categorical variables were having a strong influence on the distribution. These multiple modes were even more apparent in the density plots of the log-transformed variables.
- there were more extreme values for the Mahalanobis distances than would be expected for Normally distributed data.

In this section we present again some of the key findings from the consolidated Exploratory Data Analysis as well as findings discovered by team members subsequent to the previous submission of the EDA in Milestones 3 and 4.

## 4.2 Summaries of Data

Table 1: Summary statistics for 'diamonds' (2 d.p.)

	carat	depth	table	price	x	y	z
sample size	53940.00	53940.00	53940.00	53940.00	53940.00	53940.00	53940.00
minimum	0.20	43.00	43.00	326.00	0.00	0.00	0.00
first quartile	0.40	61.00	56.00	950.00	4.71	4.72	2.91
median	0.70	61.80	57.00	2401.00	5.70	5.71	3.53
mean	0.80	61.75	57.46	3932.80	5.73	5.73	3.54
third quartile	1.04	62.50	59.00	5324.25	6.54	6.54	4.04
maximum	5.01	79.00	95.00	18823.00	10.74	58.90	31.80
IQR	0.64	1.50	3.00	4374.25	1.83	1.82	1.13
standard deviation	0.47	1.43	2.23	3989.44	1.12	1.14	0.71
skewness	1.12	-0.08	0.80	1.62	0.38	2.43	1.52
kurtosis	4.26	8.74	5.80	5.18	2.38	94.21	50.08

A brief look at the summary statistics of the numerical variables (table 1) shows that there are a range of different scales. We also see high skewness values for carat, price, y and z and high kurtosis values for all variables, particularly y and z. Both of these indicate non-normality of our numeric variables. It is also interesting to note that at least one diamond has a zero value for x, y and z. These values are further investigated in the "Outliers and Unusual Points" section below.

### 4.2.1 Summaries of the Categorical Variables

These summaries are included to emphasise the fact that the categorical variables do not have an even distribution across their different levels. In some cases, this unevenness is quite pronounced.

### 4.2.2 Tables of counts for the categorical variables

Table 2: Count of 'color'

Level	Count
J	2808
I	5422
H	8304
G	11292
F	9542
E	9797
D	6775

Table 2 shows the number of diamonds in each level of the ‘color’ variable. As a reminder, ‘J’ is the lowest level and ‘D’ is the highest. Note that the most of the diamonds are in the middle of the range of levels.

Table 3: Count of ‘cut’

Level	Count
Fair	1610
Good	4906
Very Good	12082
Premium	13791
Ideal	21551

Table 3 gives a breakdown of the how many diamonds are in each level of the ‘cut’ variable. As a reminder, ‘Fair’ is the lowest level and ‘Ideal’ is the highest. We can see that most are in the ‘Ideal’, with a substantial number also in the ‘Premium’ and ‘Very Good’. The distribution increases markedly as the quality of ‘cut’ increases.

Table 4: Count of ‘clarity’

Level	Count
I1	741
SI2	9194
SI1	13065
VS2	12258
VS1	8171
VVS2	5066
VVS1	3655
IF	1790

Table 4 shows the number of diamonds in each level of the ‘clarity’ variable. As a reminder, ‘I1’ is the lowest level and ‘IF’ is the highest. Note that most of the diamonds are toward the middle of the range of levels.

#### 4.2.3 Distribution of Numeric Variables

While examining histograms, Cullen and Frey Plots, skewness and kurtosis of the numerical variables, we observed evidence that the numerical variables were not normally distributed. The assumption of normality underlies many statistical procedures so we investigated this in more depth using Normal Quantile plots and goodness-of-fit tests. The code for these tests is included below.

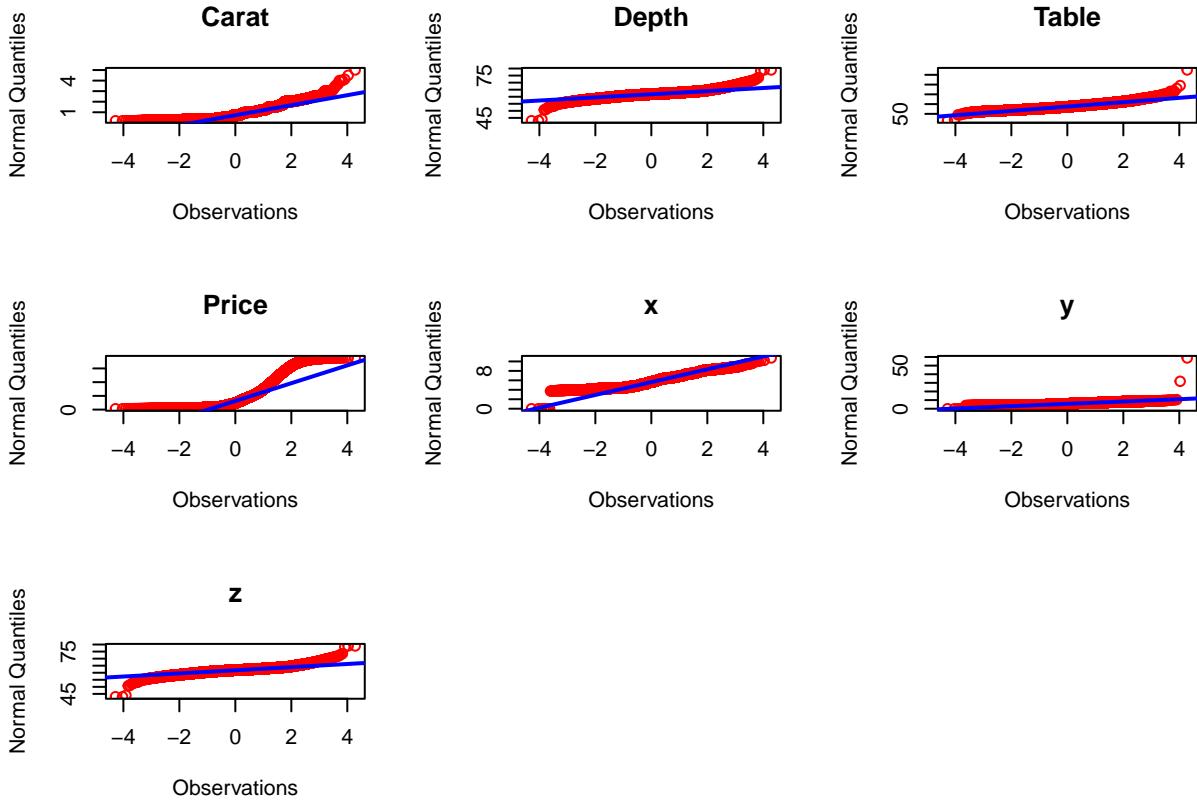


Figure 1: Normal QQ Plots of Numeric Variables

Despite the small size of the Normal QQ plots in figure 1, it is clear that each numeric variable deviates significantly from a normal distribution. We tested this using Kolmogorov-Smirnov goodness-of-fit test. The results from these tests are shown in the table below.

Table 5: Kolmogorov-Smirnov GOF results

Variable	Test_Statistic	p_value
Carat	0.122740	<2.2e-16
Depth	0.075871	<2.2e-16
Table	0.132250	<2.2e-16
Price	0.184670	<2.2e-16
x	0.093545	<2.2e-16
y	0.088528	<2.2e-16
z	0.089273	<2.2e-16

Table 5 shows that for all seven of the numeric variables, the Kolmogorov-Smirnov goodness-of-fit tests provided strong evidence for non-normality, as evidenced by the p-values which are all machine precision zero.

These tests supported our earlier observation that the data does not follow a normal distribution. We note this and proceed with our investigation anyway.

#### 4.2.4 Correlation Plot

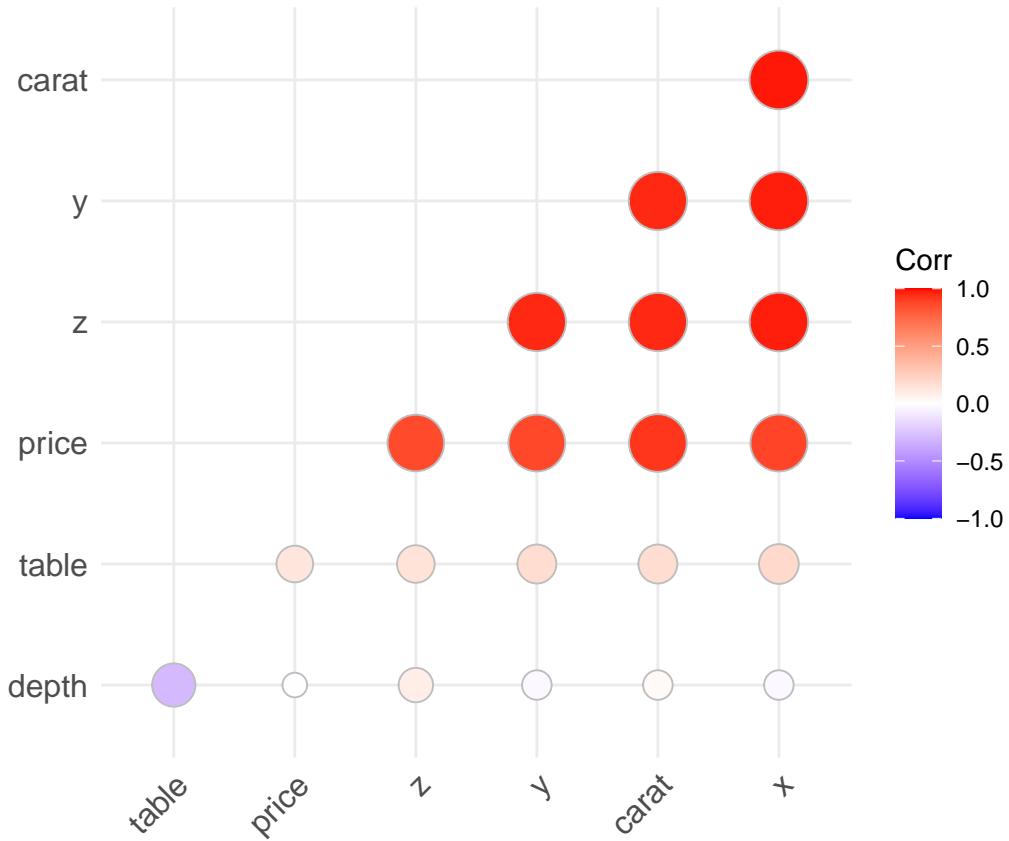


Figure 2: A Visualization of the Correlation Matrix

Figure 2 shows the correlation plot for the numerical variables in the diamonds data. ‘Carat’, ‘x’, ‘y’, ‘z’ and ‘price’ all show very strong correlations with each other, as evidenced by the large red dots. As ‘x’, ‘y’ and ‘z’ are all measures of size we should expect this and there may be some redundancy in these predictors. The ‘table’ variable is relatively uncorrelated with any of the others. ‘Depth’ and ‘table’ are negatively correlated (large purple dot), while depth is not correlated with any other variable. The strongest predictor of price is carat with length, width, depth also strongly correlated with price. Table is only very weakly correlated with price while depth is negatively correlated with price.

#### 4.2.5 Correlation Matrix

Table 6: Correlation matrix for the numerical variables

	carat	depth	table	price	x	y	z
carat	1.000	0.028	0.182	0.922	0.975	0.952	0.953
depth	0.028	1.000	-0.296	-0.011	-0.025	-0.029	0.095
table	0.182	-0.296	1.000	0.127	0.195	0.184	0.151
price	0.922	-0.011	0.127	1.000	0.884	0.865	0.861
x	0.975	-0.025	0.195	0.884	1.000	0.975	0.971
y	0.952	-0.029	0.184	0.865	0.975	1.000	0.952
z	0.953	0.095	0.151	0.861	0.971	0.952	1.000

Table 6 shows the correlation matrix. The strong positive correlations between price and ‘carat’ (0.922), price and ‘x’ (0.884), price and ‘y’ (0.865) and price and ‘z’ (0.861) supported one of our starting hypotheses, namely that these dimension variables would be positively correlated with ‘price’. We noticed that they were also highly correlated with each other (all values greater than 0.9), a fact we used subsequently to combine them into one of the factors for the Factor Analysis.

#### 4.2.6 Price differences across Categorical Variables

As our leading question is investigating which variables are most predictive or price we decided to investigate if there are significant differences in price across levels of each categorical variable.

##### 4.2.6.1 Cut

Here we get the mean price of diamonds of different levels of the ‘cut’ variable.

Table 7: Mean prices for the five levels of ‘cut’

	Mean Price
Fair	4359
Good	3929
Very Good	3982
Premium	4584
Ideal	3458

The mean price of diamonds differs for different levels of ‘cut’ (table 7). However, the higher levels of cut do not necessarily lead to higher prices.

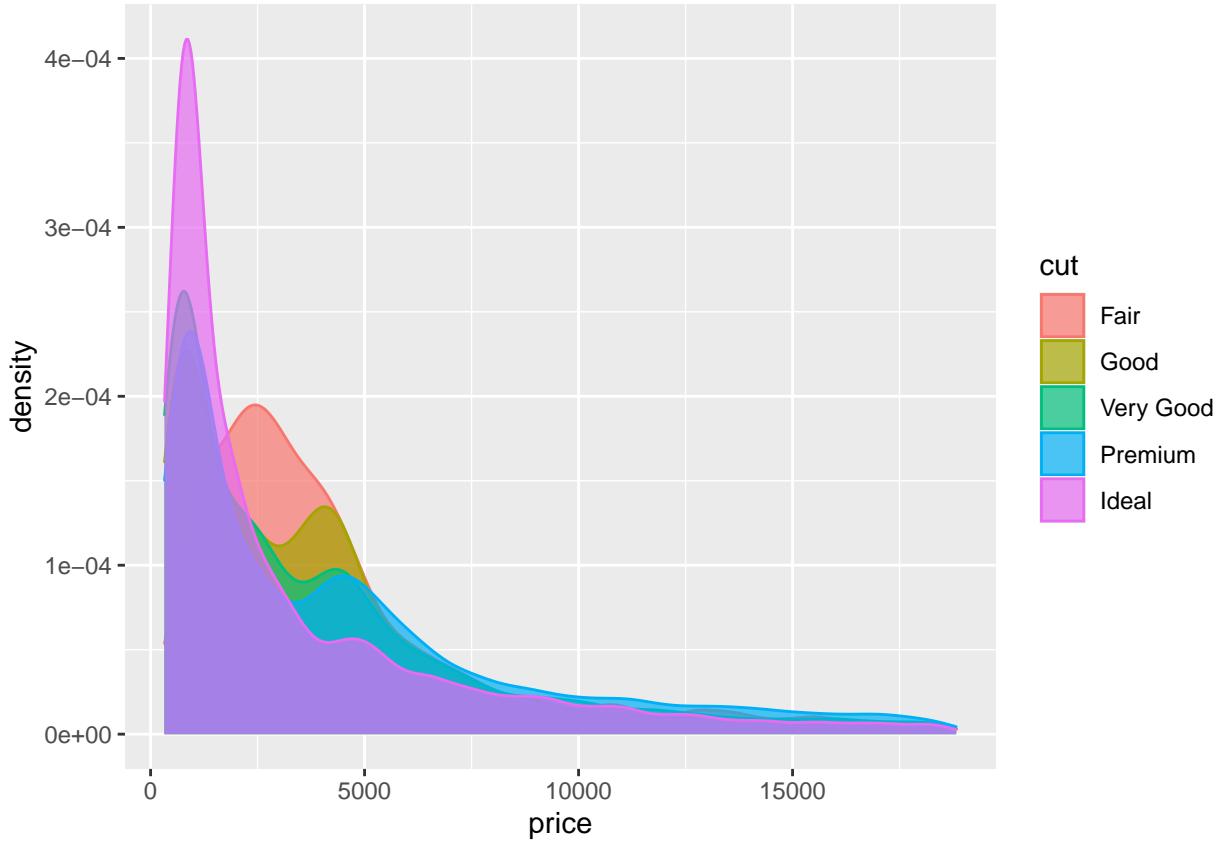


Figure 3: Densities of ‘price’ for different levels of ‘cut’

Figure 3 shows the density plots for the different levels of the ‘price’ variable. The legend on the right shows the level names and their order (‘fair’ = poorest, ‘ideal’ = best). Of note is the fact that the two best levels (premium and ideal) have significant peaks near the lower end of the price range compared with the other three. This is somewhat surprising, as intuitively one would imagine price to increase as the quality of cut increases.

We conducted a Analysis of Variance (ANOVA) for all three of the categorical variables to see if the observed differences in mean price across the various levels were significant. We also conducted a Levene’s test to verify if the assumption of homogeneity of variance was violated. If these tests showed violations of the ANOVA assumptions, we then implemented a non-parametric Kruskal Wallis Test. From the graph of the distributions of price for different levels of ‘cut’ we can see that not all of them have a shape consistent with being normally distributed. A one-way ANOVA is reasonably robust to departures from normality, particularly as we have a very large sample. The code for these tests may be found in the appendices.

Table 8: ANOVA of Price by Cut

Test	Test_Statistic	p_value
ANOVA	175.70	<2.2e-16
Levene's Test	123.60	<2.2e-16
Kruskal Wallis	978.62	<2.2e-16

Table 8 shows the results of the ANOVA, Levene's and Kruskal Wallis tests for the ‘cut’ variable. The tests are to check whether there are differences in price between the different levels of ‘cut’. The ANOVA test returned a p-value of < 2.2e-16 (machine precision zero), meaning that there is strong evidence to suggest that mean price differs across levels of “cut”.

The Tukey Test indicated that at the 5% significance level, the only pairs between which we do not see a significant difference in mean price are ‘Very Good’ and ‘Good’ as well as ‘Premium’ and ‘Fair’. Both the Levene’s test and Kruskal Wallis Test return significant results indicating that: a) the assumption of equal variance is violated and; b) that we have evidence of a significant difference in median prices for different levels of cut.

#### 4.2.6.2 Clarity

Here we get the mean price of diamonds of different levels of the ‘clarity’ variable.

Table 9: Mean prices for the 8 levels of ‘clarity’

	Mean Price
I1	3924
SI2	5063
SI1	3996
VS2	3925
VS1	3839
VVS2	3284
VVS1	2523
IF	2865

Table 9 shows that the mean price of diamonds varies across different levels of ‘clarity’. Interestingly the diamonds with the two highest clarities show the lowest mean price.

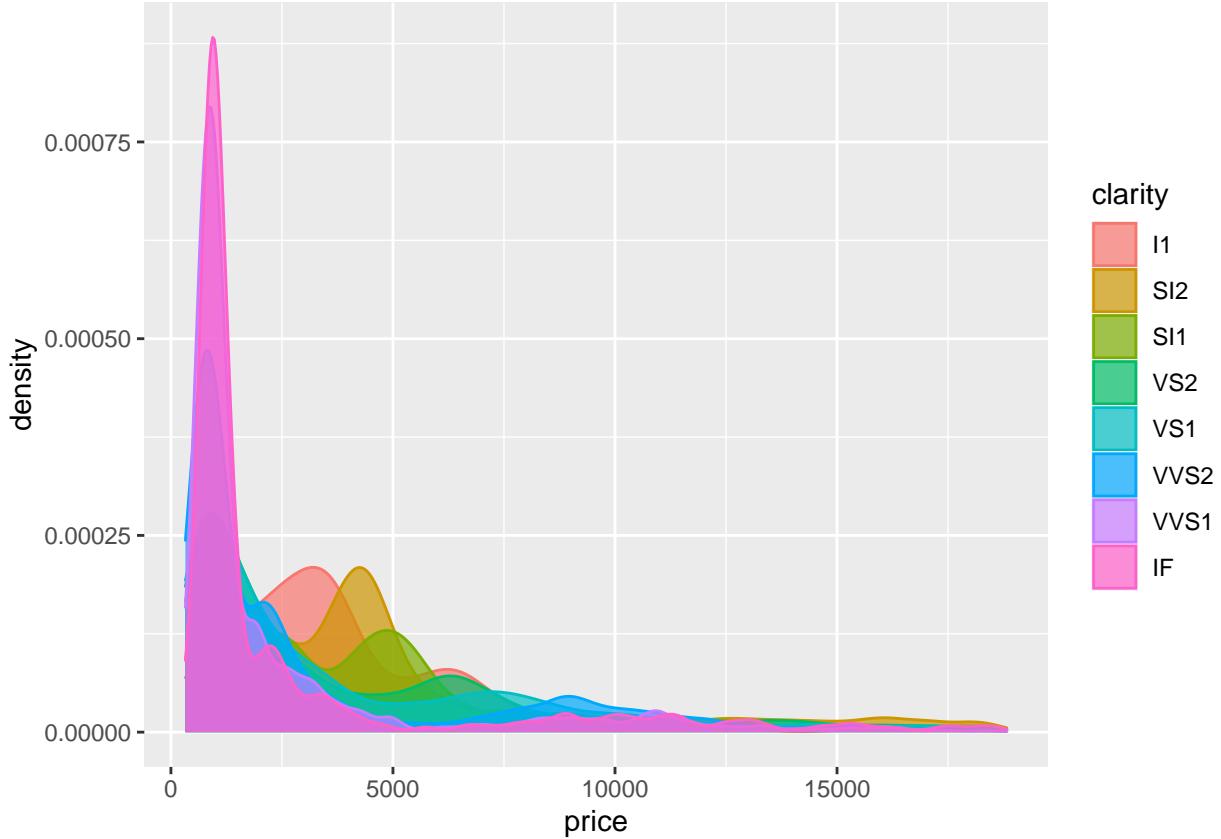


Figure 4: Densities of ‘price’ for different levels of ‘clarity’

Figure 4 shows the densities of the different levels of ‘clarity’ for price. A prominent peak is visible near the left for the better quality levels (IF, best, pink; WS1 second best, purple; WS2, third best, blue).

Table 10: ANOVA of Price by Clarity

Test	Test_Statistic	p_value
ANOVA	215.000	<2.2e-16
Levene’s Test	77.809	<2.2e-16
Kruskal Wallis	2718.200	<2.2e-16

Table 10 shows the table of results from the three tests on the variable ‘clarity’. The output from the ANOVA returns a p-value of  $< 2.2\text{e-}16$ , meaning there is strong evidence to suggest that mean price differs across levels of ‘clarity’. The output of the Levene’s test and Kruskal Wallis tests show that there is not equal variances between the levels, and that there is a significant difference between different levels of clarity. A Tukey test showed that most pairwise combinations show a significant difference. There are only six that do not show a difference and they are: SI1-I1; VS2-I1; VS1-I1; VS2-SI1; VS1-VS2; and IF-VVS1.

#### 4.2.6.3 Color

Here we get the mean price of diamonds of different levels of the ‘color’ variable.

Table 11: Mean prices for the 8 levels of ‘color’

	Mean Price
J	5324
I	5092
H	4487
G	3999
F	3725
E	3077
D	3170

Table 11 shows the mean prices for the different levels of the ‘color’ variable and it is clear that they vary. The diamonds with the two highest color quality show the lowest mean price.

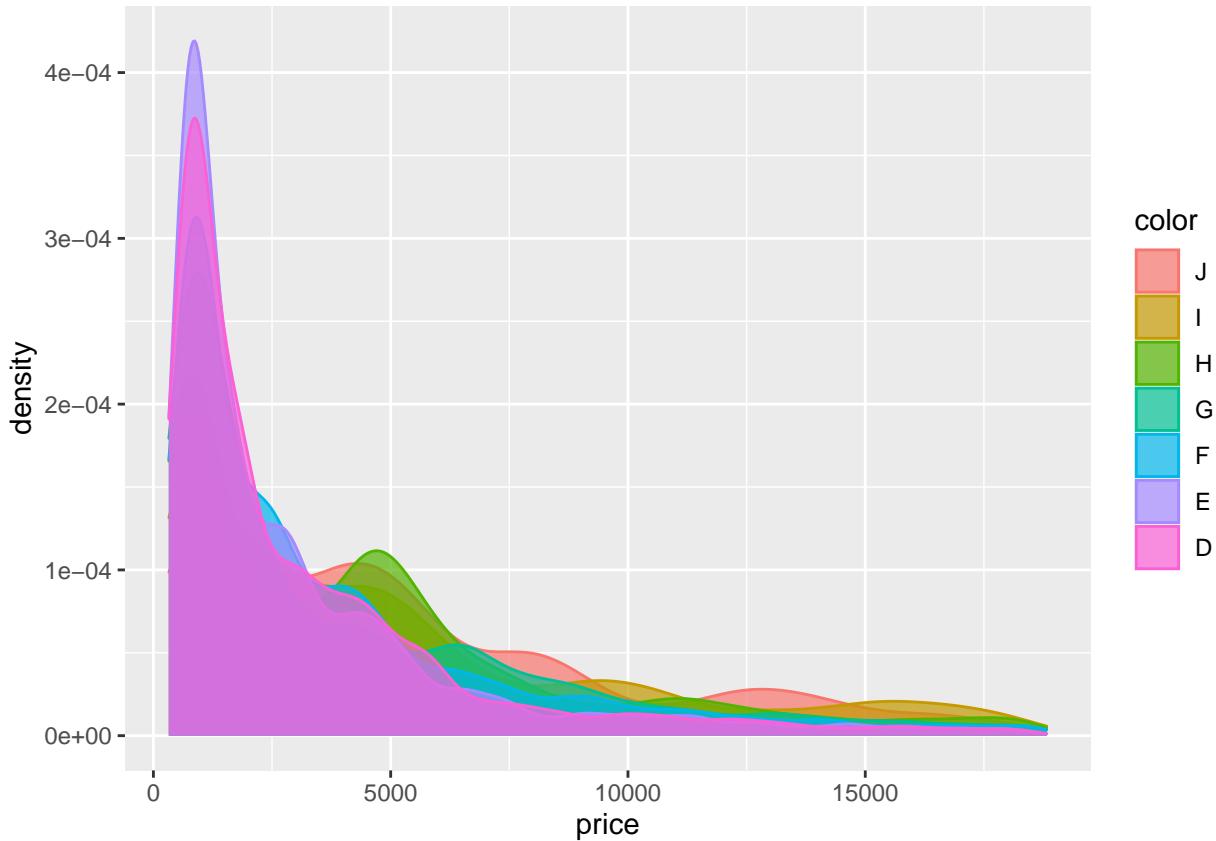


Figure 5: Densities of ‘price’ for the different levels of ‘color’

Figure 5 shows the density plots of ‘price’ for the different levels of the ‘color’ variable. There is a prominent peak on the left for the better quality levels of color (D in pink, E in purple, and F in blue), but otherwise the densities appear to be fairly similar.

Table 12: ANOVA of Price by Color

Test	Test_Statistic	p_value
ANOVA	175.70	<2.2e-16
Levene’s Test	219.12	<2.2e-16
Kruskal Wallis	1335.60	<2.2e-16

Table 12 shows the results of the tests for the ‘color’ variable. We have strong evidence to suggest that mean price differs across levels of ‘color’. The output of the Levene’s test and Kruskal Wallis test above show that the variances are not equal between the levels, and that there is a significant difference in median price across different levels of ‘color’. The output from the Tukey Test showed significant differences in mean price for nearly all pairwise comparisons of diamond colors.

#### 4.2.7 Outliers and Unusual Points

##### 4.2.7.1 Mahalanobis Distance

We decided to investigate outliers and unusual points in our dataset. These points may have increased leverage or influence when we come to using variables to predict price.

We began by using the Mahalanobis Distance to identify surprising and very surprising points. A ‘somewhat surprising’ point is one that is equal or greater than the 90th percentile, a ‘surprising’ point is one that is equal or greater than the 95th percentile, while a ‘very surprising’ point is one that is equal or greater than the 99th percentile. A ‘typical’ point is one that lies within the 90th percentile.

Table 13: Mahalanobis distances surprising values

Mahal Dist	Count
Typical	49611
Somewhat Surprising	1001
Surprising	1489
Very Surprising	1839

We see from table 13 while the vast majority of diamonds are typical in terms of their Mahalanobis distances, there are a reasonable amount of “Surprising” and “Very Surprising” points in this dataset. We can see how many very surprising points (as identified with the

Mahalanobis Distance) are members of each level of our categorical variables. This might help to indicate if any classes contain more surprising points than others.

Table 14: Mahalanobis distances for 'cut'

Mahal Dist	Count
Fair	517
Good	216
Very Good	286
Premium	412
Ideal	408

Table 14 shows the 'very surprising' Mahalanobis distances for the different levels of the variable 'cut'.

Table 15: Mahalanobis distances for 'clarity'

Mahal Dist	Count
I1	209
SI2	570
SI1	280
VS2	251
VS1	231
VVS2	103
VVS1	105
IF	90

Table 15 shows the 'very surprising' Mahalanobis distances for the different levels of the variable 'cut'.

Table 16: Mahalanobis distances for 'color'

Mahal Dist	Count
J	213
I	284
H	338
G	328
F	290
E	216
D	170

Table 16 shows the 'very surprising' Mahalanobis distances for the different levels of the variable 'color'.

#### 4.2.7.2 Zero Values

In this section we identify any zero values and determine whether or not they are errors. The code below shows the lowest values for each of the seven numerical variables. The number of elements to display was tweaked until the output showed all the zero values (if any). This was partially to discover whether the extreme values in the dataset were having undue influence on results.

```
sort(decreasing=F, diamonds$carat) [1:10]

## [1] 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2

sort(decreasing=F, diamonds$x) [1:10]

## [1] 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 3.73 3.73

sort(decreasing=F, diamonds$y) [1:10]

## [1] 0.00 0.00 0.00 0.00 0.00 0.00 0.00 3.68 3.71 3.71

sort(decreasing=F, diamonds$z) [1:22]

## [1] 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

## [16] 0.00 0.00 0.00 0.00 0.00 1.07 1.41

sort(decreasing=F, diamonds$depth) [1:10]

## [1] 43.0 43.0 44.0 50.8 51.0 52.2 52.3 52.7 53.0 53.1

sort(decreasing=F, diamonds$table) [1:10]

## [1] 43.0 44.0 49.0 49.0 50.0 50.0 50.1 51.0 51.0 51.0

sort(decreasing=F, diamonds$price) [1:10]

## [1] 326 326 327 334 335 336 336 337 337 338
```

The output above shows that x, y and z all have a number of zero values, which presumably are errors.

### 4.3 Identifying whether the zero observations are errors

We firstly make a dataframe version of the dataset to allow easy subsetting. Note that this version includes the ‘surprising’ Mahalanobis distance values (column name ‘surprise’). This is useful as it allows us to see which erroneous values have generated very large Mahalanobis distances.

```
diamonds.df <- data.frame(diamonds)
```

We now display the zero values for each of the variable, noting whether the other variables are consistent with zero readings, or whether they indicate that this is impossible. For example,

the diamond in row 1 below with zero mm length ('x'), but 6.62 mm width ('y').

```
# display the zero values for 'x'
```

```
diamonds.df[diamonds$x==0,]
```

```
##      carat      cut color clarity depth table price x     y z surprise
## 11183 1.07    Ideal     F    SI2  61.6    56 4954 0 6.62 0    Very
## 11964 1.00  Very Good    H    VS2  63.3    53 5139 0 0.00 0    Very
## 15952 1.14      Fair    G    VS1  57.5    67 6381 0 0.00 0    Very
## 24521 1.56    Ideal     G    VS2  62.2    54 12800 0 0.00 0    Very
## 26244 1.20   Premium    D   VVS1  62.1    59 15686 0 0.00 0    Very
## 27430 2.25   Premium    H    SI2  62.8    59 18034 0 0.00 0    Very
## 49557 0.71      Good    F    SI2  64.1    60 2130 0 0.00 0    Very
## 49558 0.71      Good    F    SI2  64.1    60 2130 0 0.00 0    Very
```

```
# display the zero values for 'y'
```

```
diamonds.df[diamonds$y==0,]
```

```
##      carat      cut color clarity depth table price x y z surprise
## 11964 1.00  Very Good    H    VS2  63.3    53 5139 0 0 0    Very
## 15952 1.14      Fair    G    VS1  57.5    67 6381 0 0 0    Very
## 24521 1.56    Ideal     G    VS2  62.2    54 12800 0 0 0    Very
## 26244 1.20   Premium    D   VVS1  62.1    59 15686 0 0 0    Very
## 27430 2.25   Premium    H    SI2  62.8    59 18034 0 0 0    Very
## 49557 0.71      Good    F    SI2  64.1    60 2130 0 0 0    Very
## 49558 0.71      Good    F    SI2  64.1    60 2130 0 0 0    Very
```

```
# display the zero values for 'z'
```

```
diamonds.df[diamonds$z==0,]
```

```
##      carat      cut color clarity depth table price x     y z surprise
## 2208   1.00   Premium    G    SI2  59.1    59 3142 6.55 6.48 0    Very
## 2315   1.01   Premium    H     I1  58.1    59 3167 6.66 6.60 0    Very
## 4792   1.10   Premium    G    SI2  63.0    59 3696 6.50 6.47 0    Very
## 5472   1.01   Premium    F    SI2  59.2    58 3837 6.50 6.47 0    Very
## 10168  1.50      Good    G     I1  64.0    61 4731 7.15 7.04 0    Very
## 11183  1.07    Ideal     F    SI2  61.6    56 4954 0.00 6.62 0    Very
## 11964  1.00  Very Good    H    VS2  63.3    53 5139 0.00 0.00 0    Very
## 13602  1.15    Ideal     G    VS2  59.2    56 5564 6.88 6.83 0    Very
## 15952  1.14      Fair    G    VS1  57.5    67 6381 0.00 0.00 0    Very
## 24395  2.18   Premium    H    SI2  59.4    61 12631 8.49 8.45 0    Very
## 24521  1.56    Ideal     G    VS2  62.2    54 12800 0.00 0.00 0    Very
## 26124  2.25   Premium    I    SI1  61.3    58 15397 8.52 8.42 0    Very
## 26244  1.20   Premium    D   VVS1  62.1    59 15686 0.00 0.00 0    Very
## 27113  2.20   Premium    H    SI1  61.2    59 17265 8.42 8.37 0    Very
## 27430  2.25   Premium    H    SI2  62.8    59 18034 0.00 0.00 0    Very
## 27504  2.02   Premium    H    VS2  62.7    53 18207 8.02 7.95 0    Very
```

```

## 27740 2.80      Good     G      SI2 63.8      58 18788 8.90 8.85 0      Very
## 49557 0.71      Good     F      SI2 64.1      60 2130 0.00 0.00 0      Very
## 49558 0.71      Good     F      SI2 64.1      60 2130 0.00 0.00 0      Very
## 51507 1.12      Premium   G      I1 60.4      59 2383 6.71 6.67 0      Very

```

The outputs above show that the zero values for ‘x’, ‘y’ and ‘z’ are all errors. We can tell this because these diamonds have carat, depth and price values, meaning that they cannot have zero length (x), width (y) and depth (z). Note that for a lot of the observations with zero values in one of these variables also have zero values in the others. The variable ‘y’ is a good example; the output shows that every observation with a zero value for ‘y’ also has zero values for ‘x’ and ‘z’. Note also that all of the zero values have ‘very surprising’ Mahalanobis distance results. This suggests that they are exerting undue influence or leverage on our results. However, this might be counterbalanced to a degree by the very large number of observations in the dataset.

#### 4.3.1 Upper-value outliers

The outputs below show the upper values for the seven numerical variables. This output guides further investigation as to which are likely to be genuine and which are probably errors.

```

# show ten largest values for each of the seven numerical variables
sort(decreasing=T, diamonds$carat)[1:10]

## [1] 5.01 4.50 4.13 4.01 4.01 4.00 3.67 3.65 3.51 3.50

sort(decreasing=T, diamonds$x)[1:10]

## [1] 10.74 10.23 10.14 10.02 10.01 10.00 9.86 9.66 9.65 9.54

sort(decreasing=T, diamonds$y)[1:10]

## [1] 58.90 31.80 10.54 10.16 10.10 9.94 9.94 9.85 9.81 9.63

sort(decreasing=T, diamonds$z)[1:10]

## [1] 31.80 8.06 6.98 6.72 6.43 6.38 6.31 6.27 6.24 6.17

sort(decreasing=T, diamonds$depth)[1:10]

## [1] 79.0 79.0 78.2 73.6 72.9 72.2 71.8 71.6 71.6 71.3

sort(decreasing=T, diamonds$table)[1:10]

## [1] 95 79 76 73 73 73 73 71 70 70

sort(decreasing=T, diamonds$price)[1:10]

## [1] 18823 18818 18806 18804 18803 18797 18795 18795 18791 18791

```

The output above shows the ten largest values for each of the seven numerical variables. At

a glance (informal inference) it appears that ‘carat’, ‘y’, ‘z’ and ‘table’ all have one or more values that are considerably higher than the rest, with y and z having maximums that are so extreme it is worth considering whether they are erroneous.

## 4.4 Determining whether the outliers are errors

In this section we go through the variables one at a time to determine whether the upper values are likely to be erroneous or genuine.

### 4.4.1 The ‘x’ variable: probably no upper value errors

```
# display the values of 'x' that are greater than or equal to 10
diamonds.df[diamonds$x>=10,]
```

	carat	cut	color	clarity	depth	table	price	x	y	z	surprise
## 25999	4.01	Premium	I	I1	61.0	61	15223	10.14	10.10	6.17	Very
## 26000	4.01	Premium	J	I1	62.5	62	15223	10.02	9.94	6.24	Very
## 26445	4.00	Very Good	I	I1	63.3	58	15984	10.01	9.94	6.31	Very
## 27131	4.13	Fair	H	I1	64.8	61	17329	10.00	9.85	6.43	Very
## 27416	5.01	Fair	J	I1	65.5	59	18018	10.74	10.54	6.98	Very
## 27631	4.50	Fair	J	I1	65.8	58	18531	10.23	10.16	6.72	Very

The output above shows that the largest ‘x’ values are not outrageously larger than the rest. Additionally, the values of the other variables for the largest ‘x’ value are reasonably aligned in terms of magnitude, suggesting that these large ‘x’ values are genuine.

### 4.4.2 The ‘y’ variable: probably 2 upper value errors

```
# display the values of 'x' that are greater than or equal to 11
diamonds.df[diamonds$y>=10,]
```

	carat	cut	color	clarity	depth	table	price	x	y	z	surprise
## 24068	2.00	Premium	H	SI2	58.9	57	12210	8.09	58.90	8.06	Very
## 25999	4.01	Premium	I	I1	61.0	61	15223	10.14	10.10	6.17	Very
## 27416	5.01	Fair	J	I1	65.5	59	18018	10.74	10.54	6.98	Very
## 27631	4.50	Fair	J	I1	65.8	58	18531	10.23	10.16	6.72	Very
## 49190	0.51	Ideal	E	VS1	61.8	55	2075	5.15	31.80	5.12	Very

The output above suggests that the two extreme values for ‘y’ are almost certainly errors. They are 58.90 and 31.80; both are far larger than the next highest value, which is 10.54. It is very unlikely that diamonds with such enormous width values do not also have extreme length and depth values, or that they did not sell for more money.

#### 4.4.3 The ‘z’ variable: probably 2 upper value errors

```
diamonds.df[diamonds$z>=8,]

##      carat      cut color clarity depth table price     x     y     z surprise
## 24068  2.00 Premium     H     SI2  58.9  57.0 12210 8.09 58.90  8.06    Very
## 48411  0.51 Very Good     E     VS1  61.8  54.7 1970  5.12  5.15 31.80    Very
```

Similarly with the extreme value of ‘z’ (31.80). The other dimensions of this diamond do not tally with the extreme depth value, nor does the relatively low price. The next highest value has been included for comparison (8.06). This is almost certainly an error.

#### 4.4.4 The ‘carat’ variable: probably no upper value errors

```
diamonds.df[diamonds.df$carat>4,]
```

```
##      carat      cut color clarity depth table price     x     y     z surprise
## 25999  4.01 Premium     I     I1   61.0    61 15223 10.14 10.10 6.17    Very
## 26000  4.01 Premium     J     I1   62.5    62 15223 10.02  9.94 6.24    Very
## 27131  4.13 Fair       H     I1   64.8    61 17329 10.00  9.85 6.43    Very
## 27416  5.01 Fair       J     I1   65.5    59 18018 10.74 10.54 6.98    Very
## 27631  4.50 Fair       J     I1   65.8    58 18531 10.23 10.16 6.72    Very
```

The output above shows that the highest value for ‘carat’ (5.01) is not outrageously higher than the next highest (4.50), suggesting that this is a genuine value. Also, the other variables are comparable between the highest and second highest, so it is likely that this value can be trusted.

#### 4.4.5 The ‘table’ variable: probably no upper value errors

```
# display the values of 'table' that are greater than or equal to 73
diamonds.df[diamonds.df$table>=73,]
```

```
##      carat      cut color clarity depth table price     x     y     z surprise
## 24933  2.01 Fair       F     SI1  58.6    95 13387 8.32 8.31 4.87    Very
## 49376  0.70 Fair       H     VS1  62.0    73  2100 5.65 5.54 3.47    Very
## 50774  0.81 Fair       F     SI2  68.8    79  2301 5.26 5.20 3.58    Very
## 51343  0.79 Fair       G     SI1  65.3    76  2362 5.52 5.13 3.35    Very
## 51392  0.71 Fair       D     VS2  55.6    73  2368 6.01 5.96 3.33    Very
## 52861  0.50 Fair       E     VS2  79.0    73  2579 5.21 5.18 4.09    Very
## 52862  0.50 Fair       E     VS2  79.0    73  2579 5.21 5.18 4.09    Very
```

The output above for the highest values of ‘table’ show that the diamond with the largest value (95) was also considerably larger in other ways and was much more expensive. This suggests that this is a genuine value, rather than an error.

Note that all the upper outliers above, whether genuine or erroneous, have generated ‘very

surprising’ values for their Mahalanobis distances.

#### 4.4.6 ‘depth’ and ‘price’: probably no upper value errors

```
sort(diamonds.df$depth, decreasing = TRUE)[1:10]  
  
## [1] 79.0 79.0 78.2 73.6 72.9 72.2 71.8 71.6 71.6 71.3  
sort(diamonds.df$price, decreasing = TRUE)[1:10]  
  
## [1] 18823 18818 18806 18804 18803 18797 18795 18795 18791 18791
```

The output above shows that neither ‘depth’ nor ‘price’ have any extreme values. We can probably safely conclude that these are all genuine upper values.

## 4.5 Colour-coded scatterplots

In these scatterplots we see the three categorical variables of ‘cut’, ‘color’ and ‘clarity’ all versus ‘price’, but further colour-coded so we can see the price of diamonds at the different levels of these variables. This exercise is useful to see why our subsequent linear regression model using the single predictor of ‘carat’ did not work as well as hoped. This is expanded upon in the commentary below under the heading “Why prediction without the categorical variables is not working well”.

#### 4.5.1 Scatterplot of ‘Carat’ vs ‘Price’, colour-coded by ‘Cut’

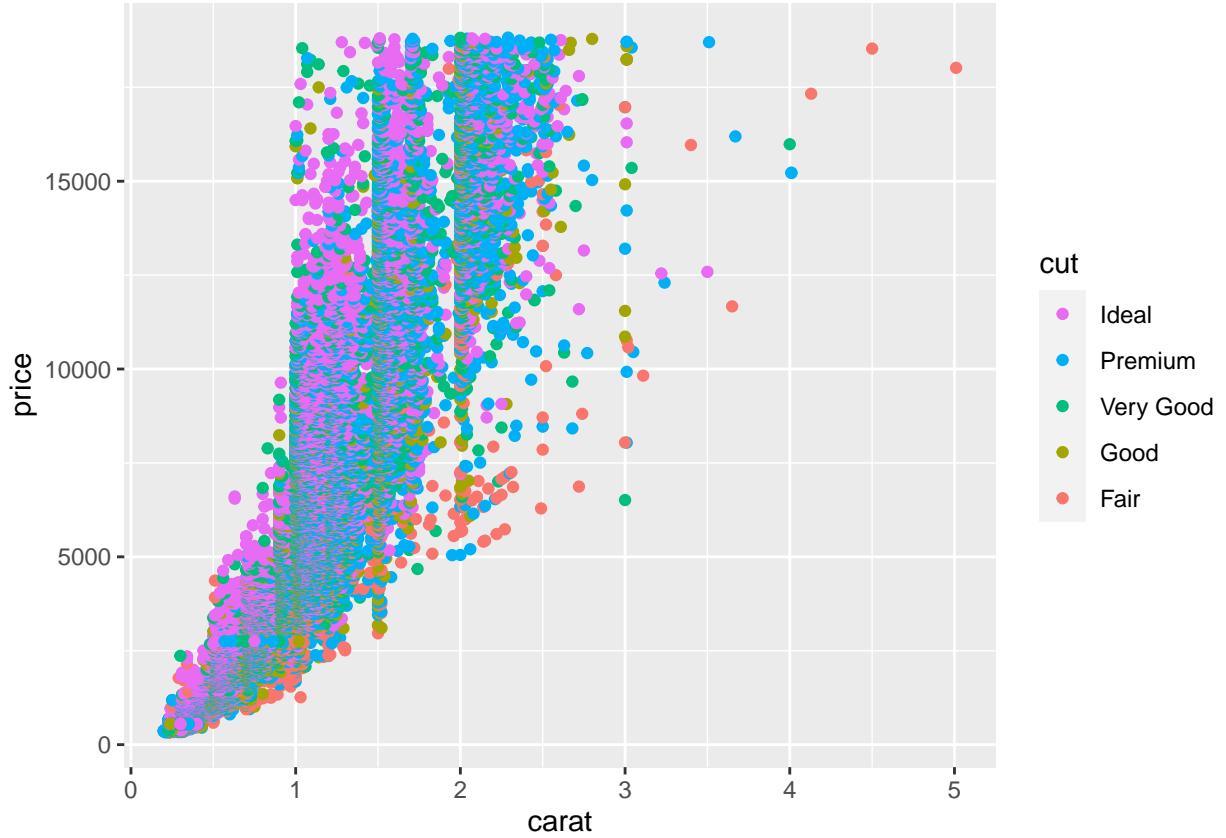


Figure 6: Carat vs Price, coloured by Cut

Figure 6 shows the scatterplot of ‘carat’ versus ‘price’ and coloured by ‘cut’. Notice the amount of overlap between diamonds from different levels of ‘cut’. There is no clear separation, meaning that a discriminant analysis or cluster analysis will struggle to distinguish between the levels.

#### 4.5.2 Scatterplot of ‘Carat’ vs ‘Price’, colour-coded by ‘Clarity’

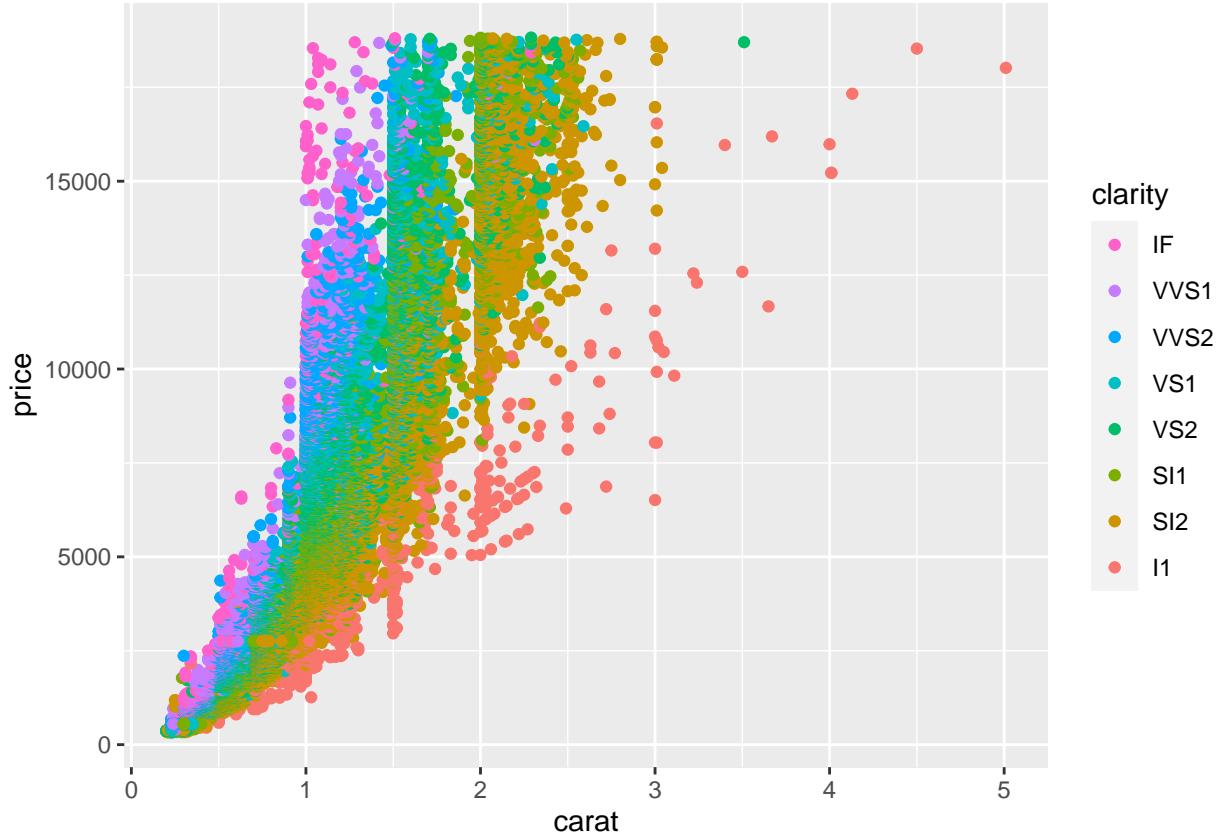


Figure 7: Carat vs Price coloured by Clarity

Figure 7 shows the scatterplot of ‘carat’ versus ‘price’ and coloured by ‘clarity’. In terms of separation of the different levels, this appears more promising. Note the clear colour bands running from the bottom left to the upper right. While not perfectly separated, we can see a clear orange band at the bottom, then a clearish yellow/brown band above that, and then a clear green band etc. However, these bands still run diagonally through a large range of ‘price’ values (for example, the band of crimson dots at the very left runs upward from a few hundred dollars all the way up past the 17,000 dollar mark), meaning that predicting price based solely on these levels will be problematic.

#### 4.5.3 Scatterplot of ‘Carat’ vs ‘Price’, colour coded by ‘Color’

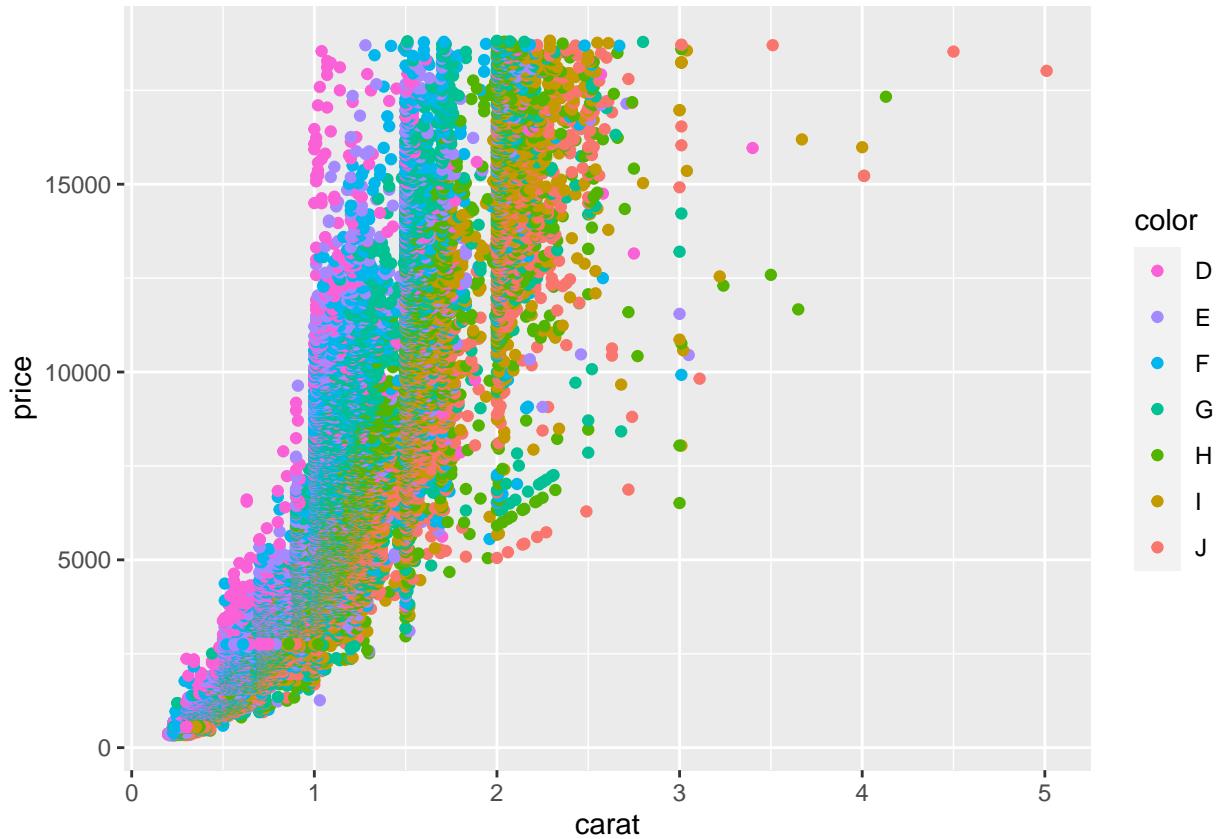


Figure 8: Carat vs Price coloured by Color

Figure 8 shows the scatterplot of ‘carat’ versus ‘price’ and coloured by ‘color’.

In the three scatterplots figures 6, 7 and 8 we see a clear trend of lighter diamonds (lower ‘carat’ values) that are most expensive (so, in the upper left of the plot) having the highest quality of ‘cut’, ‘clarity’ and ‘color’. This is no surprise, as we would expect the most expensive lighter diamonds to be of the best quality.

Conversely, we can also see some heavier diamonds which are of lower quality and lower price. Note in figure 6 that some of the lowest quality diamonds in terms of ‘cut’ (orange dots in the centre of the plot, on the vertical value of 2) are relatively cheap, despite being heavier than some of the lighter more expensive diamonds. But eventually the weight of the diamonds drives the price up regardless of the quality of ‘cut’, as we can see from the three orange dots in the top right of the plot. These are of the lowest quality cut, but are the three heaviest diamonds in the dataset, and so end up being expensive.

#### 4.5.4 Interactions

As we have 3 categorical variables: color, clarity and cut with 7, 8 and 5 levels respectively we have 280 separate interactions in our data set. This makes it likely that there will be unknown classes in our data.

## 5 Simple and Multiple Regression

As we are interested in predicting price we thought it would be appropriate to do an initial multiple regression including all variables to see which are thought to be significant in predicting price when all other variables are included in the model. We have done this firstly with the raw variables and then with scaled versions of the variables.

### 5.1 Simple linear regression using ‘carat’

We theorised that ‘carat’ would be a very good predictor of ‘price’ based on the fact that ‘carat’ was the variable most highly correlated with ‘price’, with a correlation of 0.9216 between the two variables. Here we fit a simple linear regression model using ‘carat’ as the sole predictor variable.

```
# fit the linear regression model
carat.lm <- lm(price~carat, data = diamonds, x=T)

# display summary
summary(carat.lm)

##
## Call:
## lm(formula = price ~ carat, data = diamonds, x = T)
##
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -18585.3   -804.8    -18.9    537.4  12731.7 
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -2256.36     13.06  -172.8   <2e-16 ***
## carat        7756.43     14.07   551.4   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1549 on 53938 degrees of freedom
## Multiple R-squared:  0.8493, Adjusted R-squared:  0.8493 
## F-statistic: 3.041e+05 on 1 and 53938 DF,  p-value: < 2.2e-16
```

```
# get anova table of linear regression model
anova(carat.lm)

## Analysis of Variance Table
##
## Response: price
##           Df   Sum Sq   Mean Sq F value    Pr(>F)
## carat       1 7.2913e+11 7.2913e+11 304051 < 2.2e-16 ***
## Residuals 53938 1.2935e+11 2.3980e+06
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The output from the linear regression model above shows that ‘carat’ is a good predictor of ‘price’. The t-test statistic is  $3.0405091 \times 10^5$ , which is an enormous number, and the p-value is 0. So, with a p-value of zero, we have very strong evidence to say that ‘carat’ is a good predictor of ‘price’.

## 5.2 Testing to find the best regression model

### 5.2.1 The Akaike Information Criterion (AIC) test

We now use the Akaike Information Criterion (AIC) stepwise regression test in R to find the best regression model that has ‘price’ as the response variable. We are particularly interested in whether any other models outperform the simple ‘carat’ model. The output for these tests is pages long, so for brevity the `results='hide'` command has been used in the code chunk to allow viewing of the code while not displaying the output.

```
# create version with the 'surprising' variable (Mahalanobis distance) removed
diamonds_reg <- diamonds[,-11]

# show code, but don't display results as they run to pages long
# fit the linear regression model with all predictors
full.lm <- lm(price ~ ., data = diamonds_reg)

# show code, but don't display results as they run to pages long
# display the model summary
summary(full.lm)

# use the step() function to perform the stepwise AIC test
step(full.lm, direction = "both")
```

Based on the output from the stepwise AIC regression comparison above, the best model which balances accuracy against parsimony is the one with predictors carat + cut + color + clarity + depth + table + x + z. In other words, the only predictor that was dropped is ‘y’. The AIC value for this model is 758425, which is the same for the model that additionally drops ‘z’. Therefore, following the principle of parsimony, we select the simplest model, which means selecting the one that drops the variables ‘y’ and ‘z’. This model is: price = carat +

cut + color + clarity + depth + table + x.

### 5.2.2 Bayes Information Criterion (BIC) model comparison

As a check, we also perform a model comparison using the BIC criteria.

```
# show code, but don't display results as they run to pages long
# store the sample size in a variable
n <- length(diamonds$price)

# repeat the step test with the added 'k' argument to perform the BIC
step(full.lm, direction = "both", k=log(n))
```

The model comparison using the Bayes Information Criterion (BIC) confirms that the best model is the one with variables carat + cut + color + clarity + depth + table + x. The BIC value for this model is 758621, compared with 758629 for the next best (which includes 'z').

## 5.3 Fitting the best model

The ‘best’ model (as found by the AIC test above) is fitted below.

```
best.lm <- lm(price ~ carat + cut + color + clarity +
                 depth + table + x, data = diamonds)
summary(best.lm)

##
## Call:
## lm(formula = price ~ carat + cut + color + clarity + depth +
##      table + x, data = diamonds)
##
## Residuals:
##       Min     1Q   Median     3Q    Max 
## -21385.0  -592.4  -183.7   376.5 10694.6 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -3.418     391.056  -0.009   0.993    
## carat        11256.968    48.600 231.626 <2e-16 *** 
## cutGood      580.240     33.572  17.283 <2e-16 *** 
## cutVery Good 726.820     32.212  22.564 <2e-16 *** 
## cutPremium   762.759     32.225  23.670 <2e-16 *** 
## cutIdeal     833.260     33.396  24.951 <2e-16 *** 
## colorI       903.322     26.337  34.299 <2e-16 *** 
## colorH       1389.382    24.890  55.820 <2e-16 *** 
## colorG       1887.561    24.313  77.637 <2e-16 *** 
## colorF       2096.670    24.813  84.497 <2e-16 *** 
## colorE       2160.267    24.922  86.682 <2e-16 ***
```

```

## colorD      2369.504    26.131   90.678   <2e-16 ***
## claritySI2 2702.077    43.812   61.674   <2e-16 ***
## claritySI1  3664.905    43.627   84.005   <2e-16 ***
## clarityVS2  4266.612    43.847   97.308   <2e-16 ***
## clarityVS1  4577.589    44.535   102.786  <2e-16 ***
## clarityVVS2 4950.168    45.847   107.972  <2e-16 ***
## clarityVVS1 5007.061    47.152   106.190  <2e-16 ***
## clarityIF    5344.338    51.015   104.761  <2e-16 ***
## depth        -66.769     4.091   -16.322  <2e-16 ***
## table        -26.457     2.911   -9.089   <2e-16 ***
## x            -1029.478    20.549  -50.098   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1130 on 53918 degrees of freedom
## Multiple R-squared:  0.9198, Adjusted R-squared:  0.9198
## F-statistic: 2.944e+04 on 21 and 53918 DF,  p-value: < 2.2e-16

```

The summary output for the ‘best’ model above shows how the categorical variables, all of which measure diamond quality, add a lot to the accuracy of the model. For example, ‘Clarity’, which has 8 levels, adds from 2702.077 (lowest quality clarity) to 5344.338 (highest quality clarity) to price depending on which level the diamond is categorised at. This is a wide range and cannot be obtained with these categorical variables dropped from the model.

## 5.4 Check Regression Assumptions

Here we check the regression assumptions by examining the diagnostic plots.

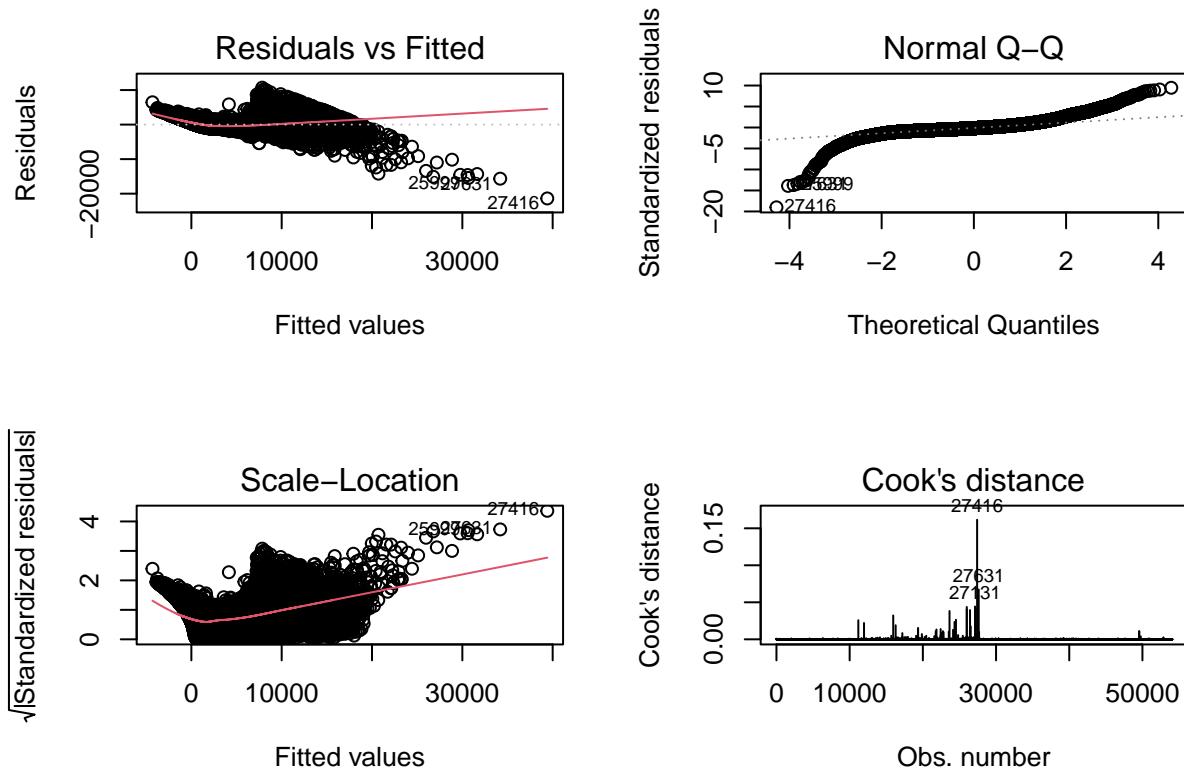


Figure 9: Diagnostic plots to check regression assumptions

By examining the residual plots (figure 9) we see that it is likely that the assumptions underlying a multiple linear regression such as linearity, homogeneity of the variance and normality of the residuals are violated. There also appears to be a number of points with high leverage and influence. We should therefore be cautious in concluding much from this regression model.

## 5.5 Scaled multiple regression model

In this version we scale the numerical variables and include the three categorical variables in the model.

```
diamonds.lm <- lm(price ~ scale(carat) + scale(x) + scale(y) +
                     scale(z) + scale(depth) + scale(table) +
                     cut + clarity + color, data = diamonds)
summary(diamonds.lm)

## 
## Call:
## lm(formula = price ~ scale(carat) + scale(x) + scale(y) + scale(z) +
##     scale(depth) + scale(table) + cut + clarity + color, data = diamonds)
```

```

## 
## Residuals:
##      Min       1Q   Median      3Q     Max 
## -21376.0    -592.4   -183.5    376.4 10694.2 
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) -2564.403   53.433 -47.993 <2e-16 *** 
## scale(carat) 5335.934  23.050 231.494 <2e-16 *** 
## scale(x)     -1131.028  36.903 -30.648 <2e-16 *** 
## scale(y)      10.975   22.081   0.497   0.619  
## scale(z)     -35.369   23.631  -1.497   0.134  
## scale(depth) -91.410   6.496 -14.071 <2e-16 *** 
## scale(table) -59.156   6.506  -9.092 <2e-16 *** 
## cutGood       579.751  33.592  17.259 <2e-16 *** 
## cutVery Good  726.783  32.241  22.542 <2e-16 *** 
## cutPremium    762.144  32.228  23.649 <2e-16 *** 
## cutIdeal      832.912  33.407  24.932 <2e-16 *** 
## claritySI2    2702.586  43.818  61.677 <2e-16 *** 
## claritySI1    3665.472  43.634  84.005 <2e-16 *** 
## clarityVS2    4267.224  43.853  97.306 <2e-16 *** 
## clarityVS1    4578.398  44.546 102.779 <2e-16 *** 
## clarityVVS2   4950.814  45.855 107.967 <2e-16 *** 
## clarityVVS1   5007.759  47.160 106.187 <2e-16 *** 
## clarityIF     5345.102  51.024 104.757 <2e-16 *** 
## colorI        903.154  26.337  34.292 <2e-16 *** 
## colorH       1389.131  24.891  55.809 <2e-16 *** 
## colorG       1887.359  24.313  77.628 <2e-16 *** 
## colorF       2096.544  24.813  84.492 <2e-16 *** 
## colorE       2160.280  24.922  86.683 <2e-16 *** 
## colorD       2369.398  26.131  90.674 <2e-16 *** 
## --- 
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 
## 
## Residual standard error: 1130 on 53916 degrees of freedom 
## Multiple R-squared:  0.9198, Adjusted R-squared:  0.9198 
## F-statistic: 2.688e+04 on 23 and 53916 DF,  p-value: < 2.2e-16

```

This multiple regression indicates that all variables but y and z are significant in the model (once the effect of the other predictors has been accounted for). This model accounts of 91.98% of the variation in the data.

## 6 Principal Component Analysis

We will perform a PCA with the aim of being able to explain the variation in the data set with fewer dimensions. We also hope to then create a parsimonious model for predicting diamond price through principal components regression.

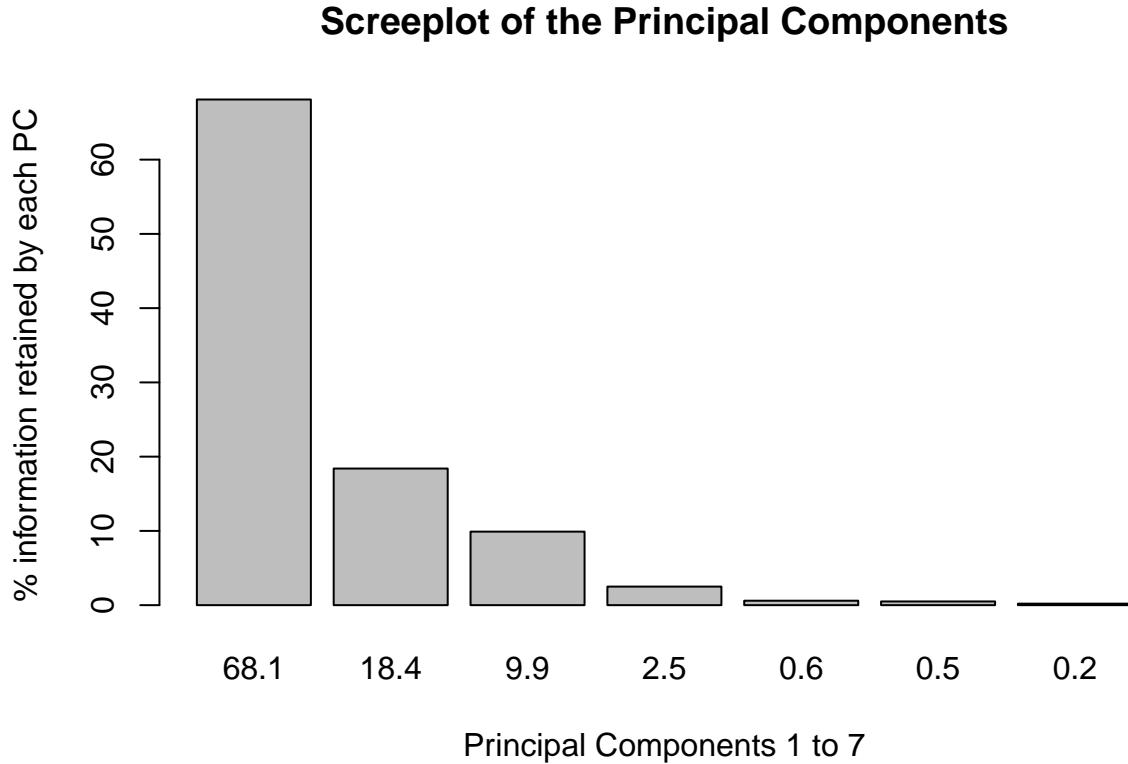


Figure 10: Scree plot of the Principal Components

Figure 10 shows the scree plot for the Principal Components of the diamonds dataset. PC1 contains 68.1% of the variance, PC2 contains 18.4%, while PC3 contains 9.9%. Between them they contain 96.4%, which is clearly very good. Components beyond the third principal component explain minimal variance (less than 4%). This suggests we can describe the data using only the first two principal components without losing much information. We will now examine which variables contribute most strongly to each Principal Component with a particular focus on the first two.

```
pca.diamonds$rotation
```

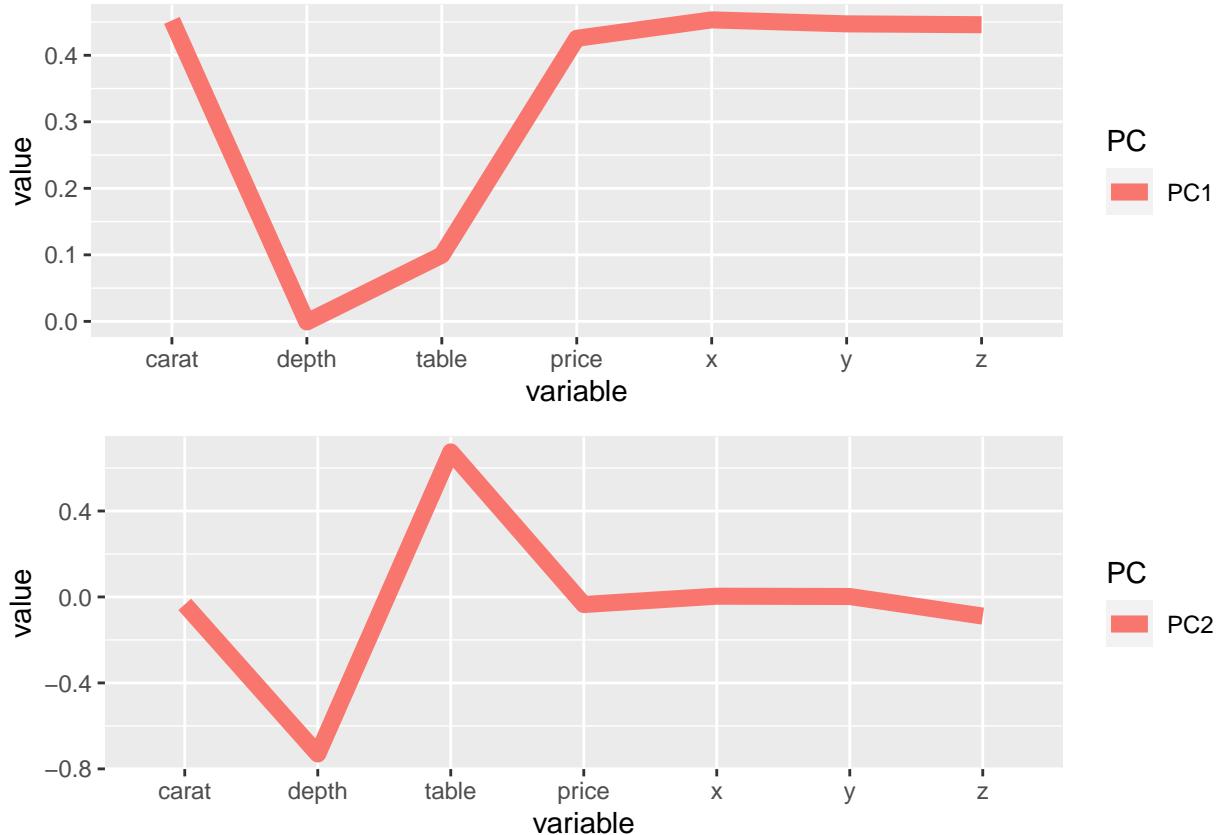
	PC1	PC2	PC3	PC4	PC5
## carat	0.4524454941	-0.034696011	0.005494814	-0.06835945	0.13399948
## depth	-0.0009161301	-0.730679714	-0.672829294	-0.04724800	-0.08873829
## table	0.0995160875	0.675067376	-0.728069469	-0.05954060	-0.01037614

```

## price  0.4255192667 -0.035257945  0.105449477 -0.84977817 -0.05377206
## x      0.4532125054  0.003512550  0.039508824  0.24299509  0.08898016
## y      0.4472649035  0.002157912  0.054188788  0.32846061 -0.77405793
## z      0.4459536619 -0.089035176 -0.039603439  0.31700727  0.60339656
##          PC6          PC7
## carat   0.76815114  0.425880295
## depth   0.01445027 -0.055600264
## table  -0.02526831 -0.002049255
## price  -0.27330947 -0.082814286
## x      0.19846061 -0.828658219
## y      -0.21526655  0.208857094
## z      -0.49867040  0.279957944

```

Looking at the eigenvectors we see that PC1 is primarily defined by carat, price, x, y and z. PC1 seems to be defined by dimension variables and, as we saw in the correlation matrix, price is closely associated with these variables. The variables with the most weight in PC2 are depth and table. We see a similar pattern in PC3 with depth and table having the largest weighting. This makes sense as in our examination of correlations between variables we saw that price, x, y, z and carat were all strongly correlated. The opposite loadings of depth and table in PC2 and PC3 reflect their negative correlation. A visualization of the eigenvectors of the first two principal components is shown in figure ??



A biplot of the data projected over the first two principal components is shown below.

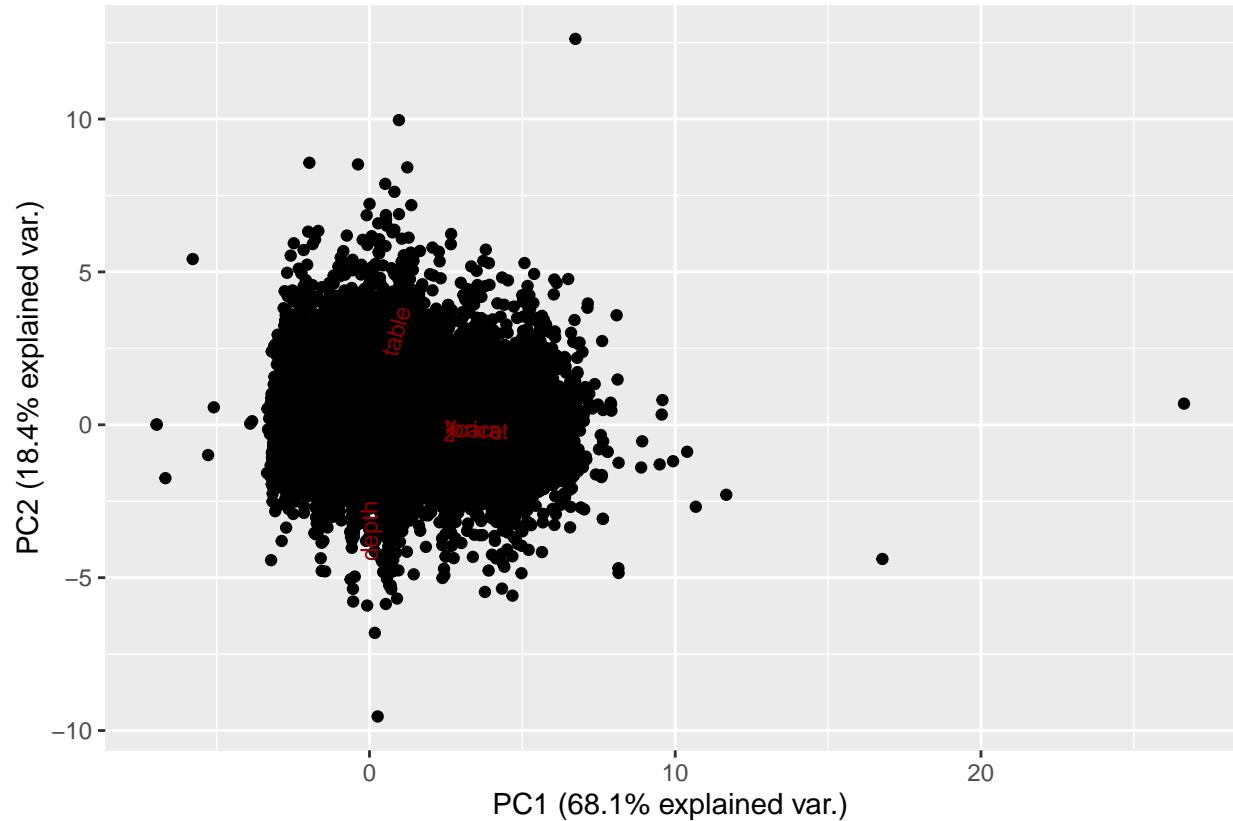


Figure 11: Biplot of Principal Components

The large number of observations in the biplot of PC1 versus PC2 in figure 11 and consequent overlapping of the dots makes interpretation difficult. We will take a smaller sample of 1000 from the diamonds data and redo the PCA after first checking that the smaller sample has similar properties to the full sample. Also of note in this biplot are two significant outliers to the far right, which are extreme values of PC1. We shall identify those before creating the smaller sample.

## 6.1 Identify the extreme PC1 values

```
sort(decreasing=T,(pca.diamonds$x[,1]))[1:10]
```

```
##      24068      48411      27416      27631      49190      27131      25999      26000
## 26.641670 16.776354 11.669245 10.673276 10.389346 9.928569 9.581991 9.558664
##      26445      27680
## 9.491827 8.919222
```

## 6.2 PCA with Smaller Sample

**Screeplot of the Principal Components (reduced dataset)**

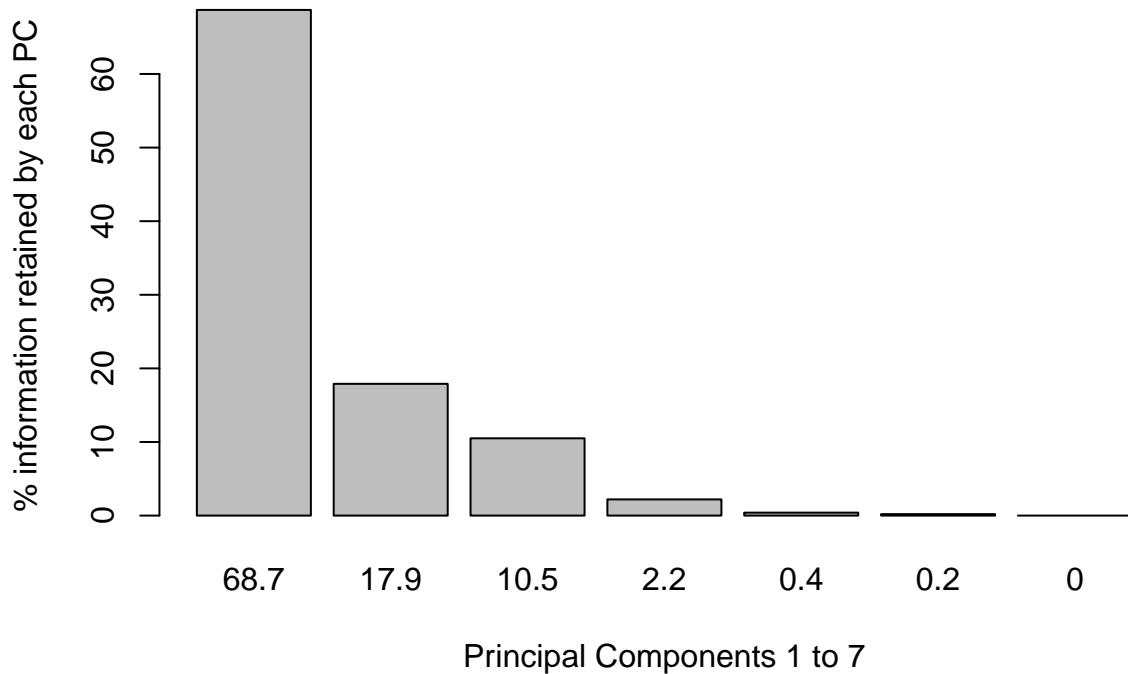


Figure 12: Scree plot of the Principal Components (reduced dataset)

Figure 12 shows the scree plot of the Principal Components for the reduced dataset. PC1 contains 68.7% of the variance, PC2 contains 17.9%, while PC3 contains 10.5%. Between them they contain 97.1%.

```
summary(reducedpca)
```

```
## Importance of components:  
##          PC1    PC2    PC3    PC4    PC5    PC6    PC7  
## Standard deviation 2.1937 1.1191 0.8565 0.39676 0.16404 0.12767 0.03195  
## Proportion of Variance 0.6875 0.1789 0.1048 0.02249 0.00384 0.00233 0.00015  
## Cumulative Proportion 0.6875 0.8664 0.9712 0.99368 0.99753 0.99985 1.00000
```

We see from the summary output above that the PCA with a sample size of 1000 has almost identical properties to the PCA of the whole dataset, which allows us to proceed.

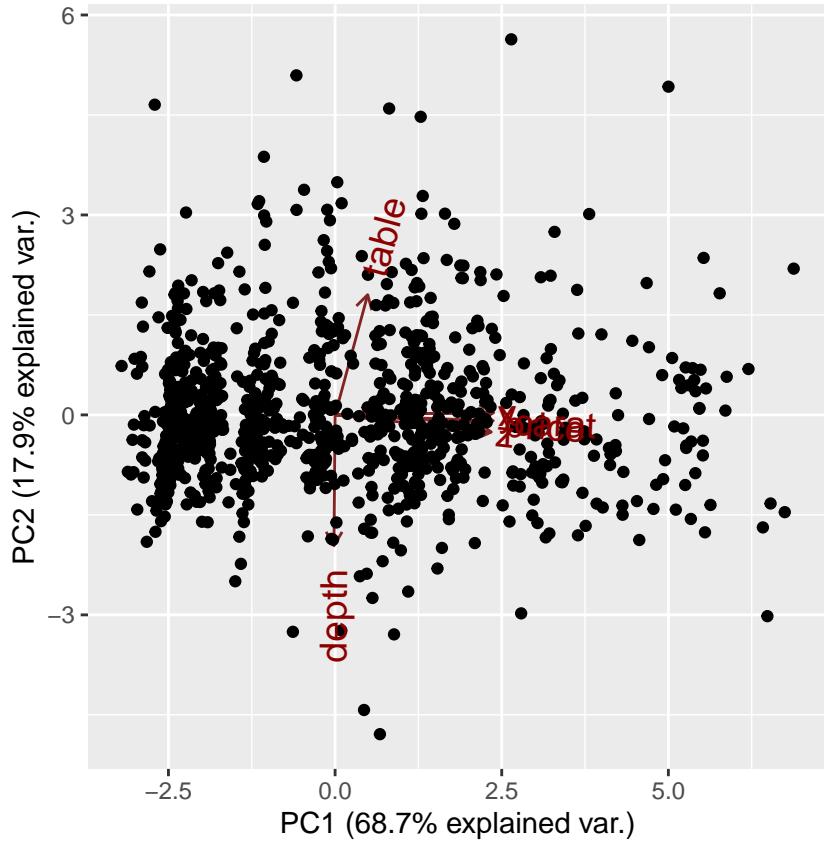


Figure 13: Biplot of the reduced dataset PC1 vs PC2

Producing a biplot using the reduced dataset allows us to interpret the biplot much better (figure 13). We see that carat, x, y, z and price are strongly influence the first PC. While depth and table strongly (and reasonably equally) influence the second PC. We see evidence of the negative correlation between table and depth and a close to zero correlation between these two variables and the variables that strongly influence PC1. We also have an indication that there is redundancy between carat, x, y, z and price. The percentage of variation retained by each Principal Component is included in the axis labels.

We also thought it would be interesting to examine the biplots with observations projected onto the first two principal components coded for different levels of our categorical variables and to examine if any patterns emerge. Unfortunately in figure ?? we don't see any clear clustering or pattern emerge with through the categorical variables. This indicates it may be hard to classify the data using the categorical variables.

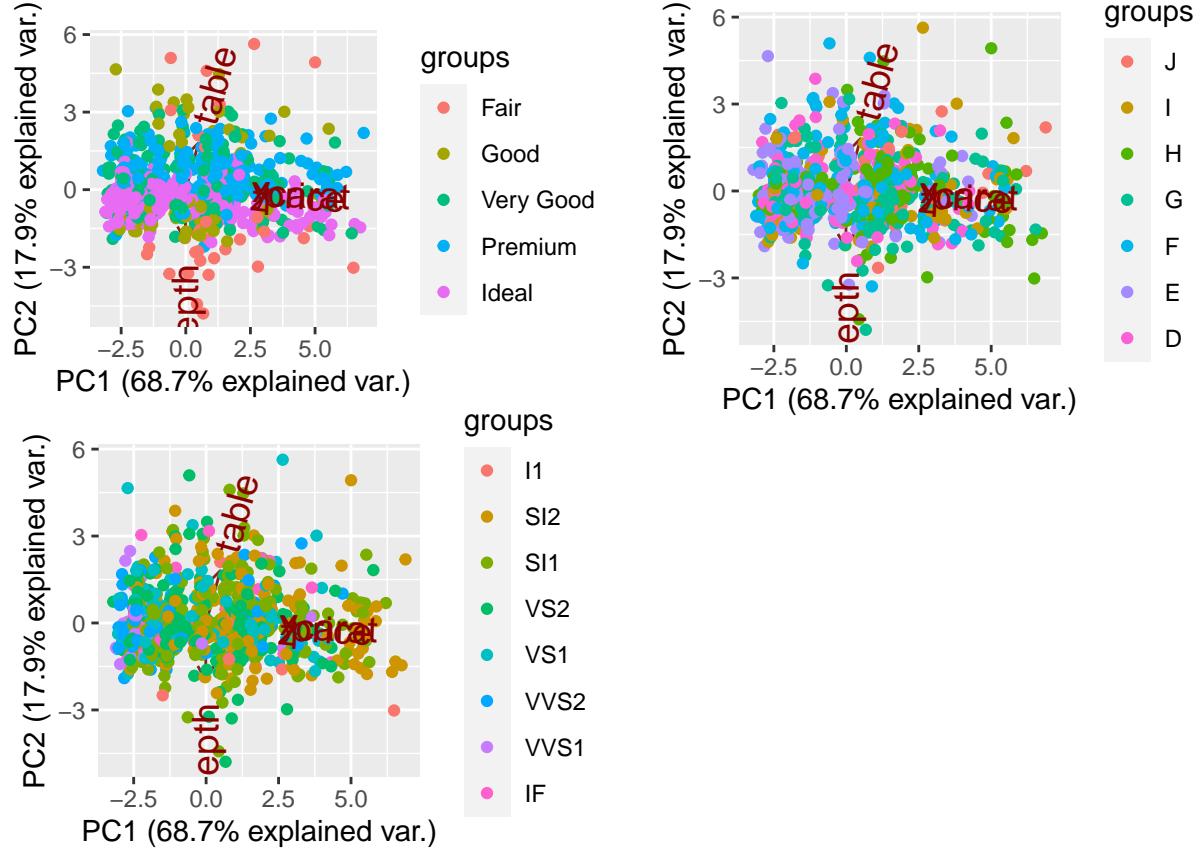


Figure 14: Colour Coded Biplot

In the above analysis we have shown we are able to use principal components analysis to reduce the number of dimensions in the dataset while still retaining a large proportion of its variation. In this investigation we are interested in predicting the price of diamonds. To do this we will perform a regression with principal components with price as the dependent variable and the other principal components as explanatory variables.

### 6.3 Principal Components Regression

We will now perform a principal regression analysis with price as the response variable and all other numerical variables as the explanatory variables. We Will again use the smaller sample to increase the ease of interpretation of the visual plots. Below we see an initial multiple regression using only the numeric variables

```
numericlmlm <- lm(scale(price) ~ scale(carat) + scale(table) + scale(depth) + scale(x) + s
summary(numericlmlm)
```

```
##  
## Call:  
## lm(formula = scale(price) ~ scale(carat) + scale(table) + scale(depth) +  
##     scale(x) + scale(y) + scale(z), data = diamonds_num)
```

```

## 
## Residuals:
##   Min     1Q Median     3Q    Max
## -5.9853 -0.1542 -0.0127  0.0872  3.1982
## 
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 1.359e-14 1.616e-03 0.000 1.00000  
## scale(carat) 1.270e+00 7.509e-03 169.085 < 2e-16 ***
## scale(table) -5.738e-02 1.727e-03 -33.216 < 2e-16 ***
## scale(depth) -7.295e-02 1.976e-03 -36.910 < 2e-16 ***
## scale(x)      -3.699e-01 1.211e-02 -30.547 < 2e-16 ***
## scale(y)       1.899e-02 7.307e-03  2.599 0.00937 ** 
## scale(z)       7.364e-03 7.837e-03  0.940 0.34744  
## --- 
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 0.3752 on 53933 degrees of freedom
## Multiple R-squared:  0.8592, Adjusted R-squared:  0.8592 
## F-statistic: 5.486e+04 on 6 and 53933 DF,  p-value: < 2.2e-16

```

In this model all but one (z) of the seven numeric predictors are found to be significant in the model. This model explains 86% of variation in price. We will now attempt to create a more parsimonious model using the principal components.

## Screeplot of the regression Principal Components

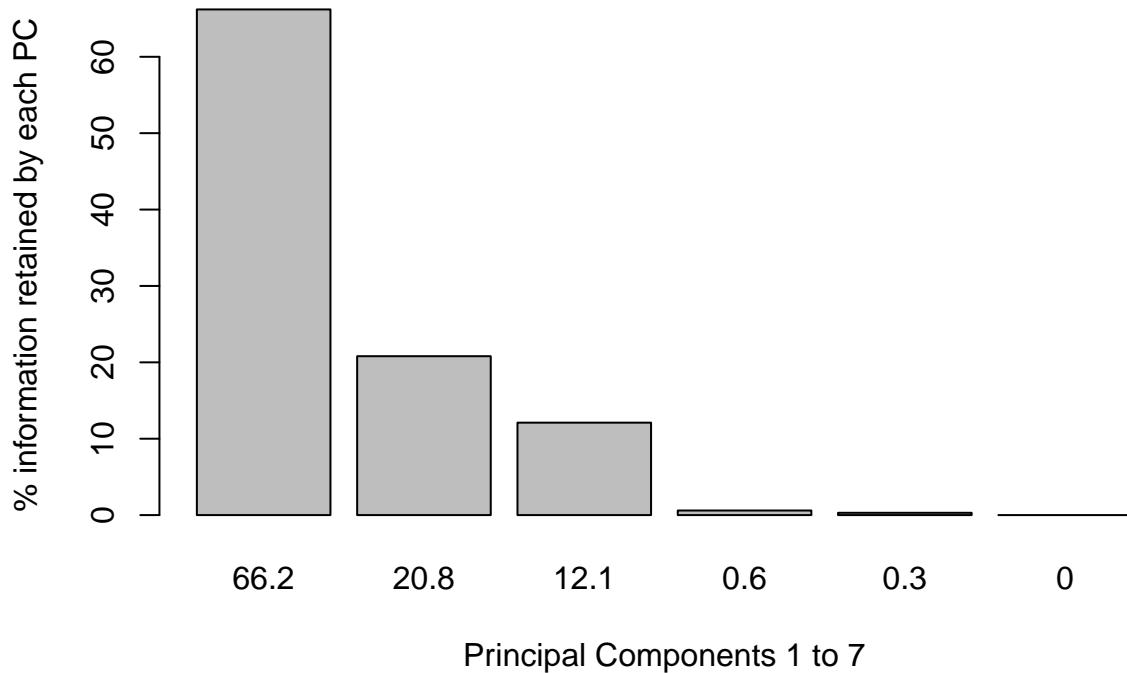


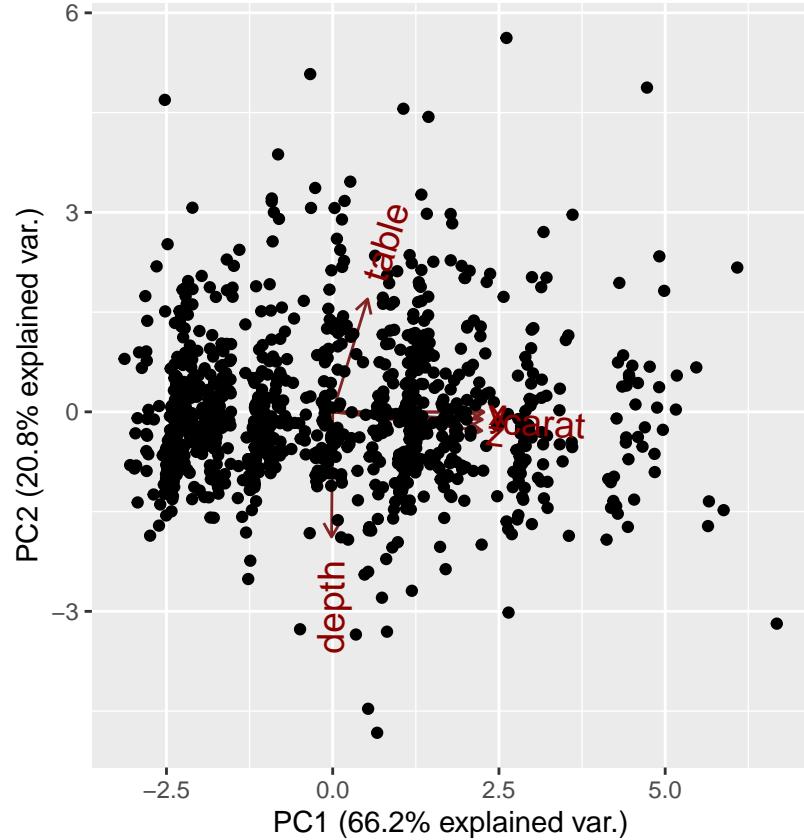
Figure 15: Scree plot of the regression Principal Components

Figure 15 shows the scree plot of the proportion of variance explained by each of the Principal Components of the PC regression. The first three principal components explain nearly all (99%) the variation in the data.

```
PCAprice$rotation
```

```
##          PC1          PC2          PC3          PC4          PC5
## carat  0.494554831 -0.0461150758  0.04445202 -0.806136953 -0.318086228
## depth -0.003656238 -0.7370594608 -0.66518876 -0.040970793  0.112128623
## table  0.115148753  0.6649799315 -0.73788433 -0.006134332 -0.004793799
## x      0.499625318 -0.0008101947  0.07042242  0.107199954  0.477863033
## y      0.499442121 -0.0022545940  0.07443970  0.134779378  0.482475245
## z      0.493026395 -0.1114123072 -0.02395964  0.564596906 -0.651989189
##          PC6
## carat  0.016652366
## depth  0.001569924
## table  0.002926009
## x      -0.711034219
## y      0.702896417
## z      -0.008867829
```

We see that in the first PC carat, x, y and z all have approximately equal weighting. Again this reflects the strong positive correlation between these two variables. In the second PC depth and table have the strongest weighting. We see that removing price from the PCA does not change much in terms of how much variation the first few components explain as well as which variables contribute most strongly to each component. This is because of how strongly price is correlated with the dimension variables.



Even with price not included we still see the same general structure as the PCA with price.

```
PCApricevars <- prcomp(smallerSample[, -c(2:4, 7, 11)],
                        center = TRUE, scale = TRUE)$x
diamondPricePCA <- lm(smallerSample$price ~ PCApricevars[, 1] + PCApricevars[, 2]
                       + PCApricevars[, 3] + PCApricevars[, 4] + PCApricevars[, 5] +
                         PCApricevars[, 6])
summary(diamondPricePCA)

##
## Call:
## lm(formula = smallerSample$price ~ PCApricevars[, 1] + PCApricevars[, 2] +
##     PCApricevars[, 3] + PCApricevars[, 4] + PCApricevars[, 5] +
##     PCApricevars[, 6])
## 
## Residuals:
```

```

##      Min       1Q    Median       3Q      Max
## -14748.2   -604.6   -109.9    401.3    7965.2
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)            3967.32     46.55  85.229 < 2e-16 ***
## PCAPricevars[, 1]     1835.78     23.37  78.555 < 2e-16 ***
## PCAPricevars[, 2]    -208.03     41.66 -4.993 7.01e-07 ***
## PCAPricevars[, 3]     444.25     54.69  8.122 1.35e-15 ***
## PCAPricevars[, 4]   -4228.97    249.04 -16.981 < 2e-16 ***
## PCAPricevars[, 5]   -2619.38    349.95 -7.485 1.58e-13 ***
## PCAPricevars[, 6]   10444.39   1418.67  7.362 3.79e-13 ***
##
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1472 on 993 degrees of freedom
## Multiple R-squared:  0.8703, Adjusted R-squared:  0.8695
## F-statistic: 1110 on 6 and 993 DF,  p-value: < 2.2e-16

```

We see that every single principal component is found to be significant in this model when the effects of the other principal components is taken into account. Therefore initially it seems we have failed to create a more parsimonious model for predicting price through using regression with principal components. However if we refit the model with just the first two principal components (see below) we still create a model that is able to explain over 81% of the variance in price with only two components. While this is not as good as the model with all the principal components (which explains 85%) it is still reasonably good and is much more simple. The original numeric multiple regression model explained 86% of the variation in price with 6 explanatory variables while our PCR model explains 81% with just the first two principal components. Principal components regression has allowed us to explain nearly the same amount of variation in price in a much more parsimonious way.

```

diamondpricePCA2 <- lm(smaller.sample$price ~ PCAPricevars[,1] + PCAPricevars[,2])
summary(diamondpricePCA2)

##
## Call:
## lm(formula = smaller.sample$price ~ PCAPricevars[, 1] + PCAPricevars[, 2])
##
## Residuals:
##      Min       1Q    Median       3Q      Max
## -8846.0  -1162.0   -139.3    877.8   8890.8
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)            3967.32     56.28   70.49 < 2e-16 ***

```

```

## PCApricevars[, 1] 1835.78      28.26   64.97 < 2e-16 ***
## PCApricevars[, 2] -208.03      50.37   -4.13 3.94e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1780 on 997 degrees of freedom
## Multiple R-squared:  0.8096, Adjusted R-squared:  0.8092
## F-statistic:  2119 on 2 and 997 DF,  p-value: < 2.2e-16

```

It is likely that the remaining, unexplained variation could be explained by the categorical variables in the dataset. As Principal Components Analysis/Regression can only be used for numerical variables we had to leave out the variables of cut, clarity and color.

## 7 Factor Analysis

In our investigation of the principal components of the dataset we observed that the first two principal components explained the vast majority of the variation in the data. The first was dominated by the strong positive correlation between price and the dimension variables (x, y, z, carat) which we may call price + dimension. The second was the negative correlation between depth and table which we might combine into light performance as both variables are crucial in giving a diamond its “sparkle”. We hypothesise that these two factors (1. “Price + Dimension” 2. “Light Performance”) may explain the unobserved variable of “overall diamond quality”. To investigate this we will conduct an factor analysis with two factors.

```

factanal(diamonds[,-c(2:4,11)], factors =2)

##
## Call:
## factanal(x = diamonds[, -c(2:4, 11)], factors = 2)
##
## Uniquenesses:
## carat depth table price      x      y      z
## 0.041 0.005 0.877 0.205 0.006 0.045 0.039
##
## Loadings:
##          Factor1 Factor2
## carat    0.976
## depth    0.114   0.991
## table    0.152  -0.316
## price    0.885  -0.113
## x        0.987  -0.140
## y        0.967  -0.141
## z        0.980
##
##          Factor1 Factor2

```

```

## SS loadings     4.641   1.141
## Proportion Var 0.663   0.163
## Cumulative Var 0.663   0.826
##
## Test of the hypothesis that 2 factors are sufficient.
## The chi square statistic is 22188.2 on 8 degrees of freedom.
## The p-value is 0

```

As we hypothesized, ‘carat’, ‘price’, ‘x’, ‘y’ and ‘z’ are well explained by factor 1. ‘Depth’ is very well explained by factor 2, but ‘table’ is explained only to a lesser degree. We see some evidence of the factor structure in our hypothesis with the ‘price + dimension’ variables strongly explained in Factor 1 and ‘Light Performance’ variables most strongly explained by factor 2. However, we also see other original variables we did not expect in each of our factors. If our hypothesis was correct we would see a negligible loading of depth and table in factor 1 and a negligible loading for the price + dimension variables in factor 2.

Factor 1 explains 66% of the variance in the data and Factor 2 explains an additional 16%. Together are two factors manage to explain 83% of the variation in the data. We also see high uniqueness ratings for ‘table’ indicating that the two factors do not well account for it’s variance.

Overall the hypothesis test gives a highly significant result meaning that two factors are not sufficient to capture the full dimensionality of the data set.

## 8 Linear Discriminant Analysis (LDA)

### 8.1 LDA Predicting Price

The secondary aim of our project was to investigate methods of classifying the diamonds dataset more simply. We decided to investigate whether, if we split diamond price into three subgroups (low, medium and high) we could accurately predict which of these groups a diamond should belong too. We split the diamonds into the three subgroups using the upper and lower quartiles. Diamonds were classed as having “low” price if they were in the LQ and classed as “high” if they were in the UQ. Diamonds were classed as having “medium” price if they were in between. We then used an LDA with the other numeric variables to see how well we could classify diamonds into there price levels.

```

## Call:
## lda(pricerank$pricerank ~ carat + depth + table + x + y + z,
##      data = diamonds.Train)
##
## Prior probabilities of groups:
##       high      low      med
## 0.2489554 0.2504178 0.5006267
##
## Group means:

```

```

##          carat      depth      table        x        y        z
## high 1.4250825 61.72694 57.78159 7.158513 7.152218 4.413184
## low  0.3306303 61.75514 56.98001 4.432106 4.445780 2.740933
## med  0.7185482 61.75910 57.53199 5.668705 5.670010 3.501461
##
## Coefficients of linear discriminants:
##          LD1        LD2
## carat  2.54046402 -11.73607571
## depth -0.04812048  0.24034962
## table  0.03440418  0.07200277
## x     -2.47599887  4.39072481
## y     -0.28389419  0.23815436
## z     -0.12082748  0.09133016
##
## Proportion of trace:
##       LD1       LD2
## 0.8784 0.1216

```

The first thing we notice from the LDA output is the prior probabilities are not equal but match up very closely with the expected probabilities given we took the upper and lower quartiles to rank diamond price. Examining the group means tells us that depth and table do not vary much between levels of diamond price whereas the other variables do. We also see that the first linear discriminant function retains a large proportion of the variability. We can visualize the LDA in the graph below.

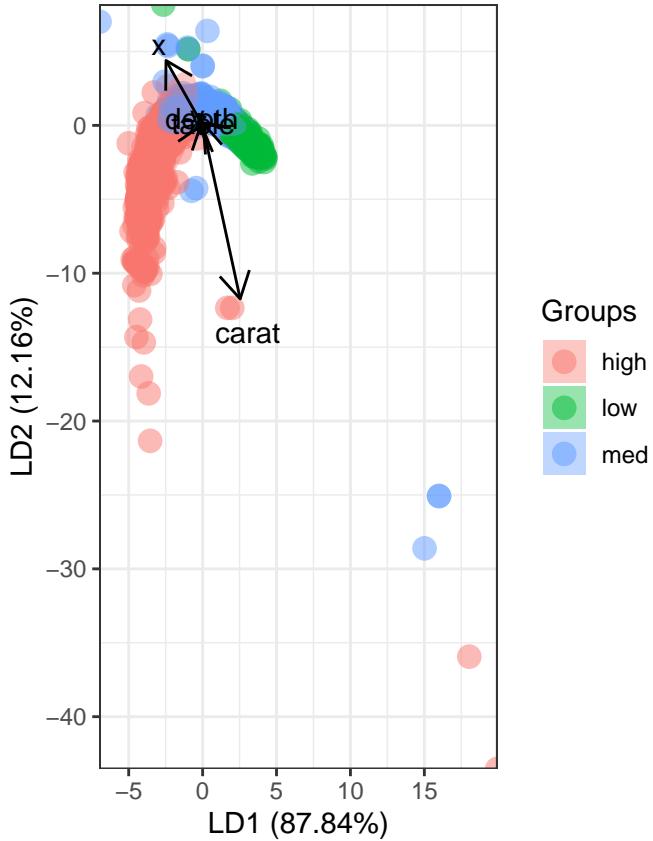


Figure 16: Ordination Plot for LDA of Price

The ordination plot (figure 16) shows some evidence of the three groups being separable with a linear function. There is still some overlap as well as some points (or groups of points) that seem to be far removed from the rest of the data. We will now see how this LDA performs as a classifier.

```
## Confusion Matrix and Statistics
##
##          Actual
## RealisticPredictedPR high  low  med
##                 high 2030    0  221
##                 low   2 2611  677
##                 med  728   91 4500
##
## Overall Statistics
##
##          Accuracy : 0.8417
## 95% CI : (0.8347, 0.8485)
## No Information Rate : 0.4971
## P-Value [Acc > NIR] : < 2.2e-16
```

```

##                                     Kappa : 0.7482
##
##  Mcnemar's Test P-Value : < 2.2e-16
##
## Statistics by Class:
##
##          Class: high Class: low Class: med
## Sensitivity           0.7355    0.9663    0.8336
## Specificity           0.9727    0.9168    0.8501
## Pos Pred Value        0.9018    0.7936    0.8460
## Neg Pred Value        0.9152    0.9880    0.8379
## Prevalence            0.2541    0.2488    0.4971
## Detection Rate        0.1869    0.2404    0.4144
## Detection Prevalence  0.2073    0.3029    0.4898
## Balanced Accuracy     0.8541    0.9415    0.8418

```

We see that LDA was reasonably successful at classifying diamonds into the three price levels of low, medium and high with an overall accuracy of 84%. As the prior probabilities of classes were not equal we should also check the within-class accuracy. We see that all classes performed well with “low” having the highest balanced accuracy. We can conclude that we are able to predict the value of a diamond reasonably accurately with an LDA.

## 8.2 LDA predicting categorical variables

As discussed previously the diamonds data set has three categorical variables each with many levels. This means we have the possibility of 280 unique interactions between the levels of these categorical variables. If we could easily classify a diamond into the correct factor level based on the numeric variables this would be ideal however it is likely to be difficult to classify diamonds using this many factors. We will begin by verifying whether predicting the factor level (out of 280) that a diamond will belong too is as complex a task as we expect by using an LDA.

We will not include the output from the LDA using the interactions as it is very long given there are 280 classes. Instead we will report the key finding; the overall accuracy of using the interactions as a classifier.

```
## [1] 0.06546961
```

An overall accuracy of 0.065 is very low and therefore we can conclude that an LDA with all other variables does a poor job at predicting which factor level a diamond should be classified as. It’s likely the data is not linearly separable into 280 classes. We will now see if a Linear Discriminant Analysis using the numerical variables can be used to accurately predict what level of a categorical variable a diamond belongs too. We will attempt this for each categorical variable. If successful we will have found a simple way to classify diamonds based on the numeric variables in the data set. This could be useful as the categorical variables are likely to be the most time consuming to assess, likely needing expert evaluation. If we can

accurately predict what level of a categorical variable a diamond will belong to based on the other, more easily measured, variables this could have real-world, practical benefits.

```
LDAcut <- lda(cut ~ carat + depth + table + price
               + x + y + z, data = diamonds.Train2)
LDAcolor <- lda(color ~ carat + depth + table + price
                  + x + y + z, data = diamonds.Train2)
LDAclarity <- lda(clarity ~ carat + depth + table + price
                   + x + y + z, data = diamonds.Train2)
```

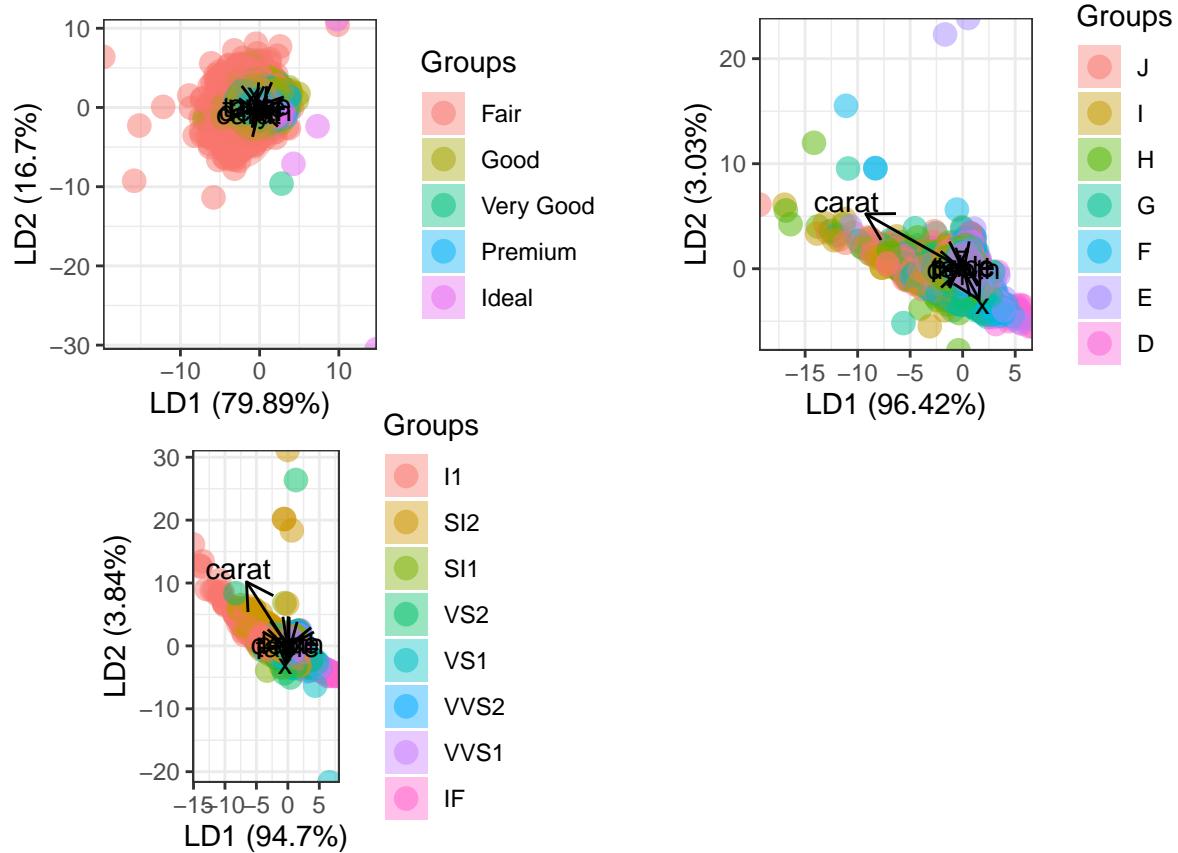


Figure 17: Ordination plots

All three ordination plots (figure 17) show a lot of overlap between different levels of the categorical variables. It seems unlikely that a linear discriminant function would have much success in partitioning the data according to these classes. We shall compute the confusion matrix for each LDA and assess the overall accuracy of each model.

Table 17: Classification Accuracy for Different LDAs

Class	Accuracy
Cut	0.6258
Color	0.1738
Clarity	0.2039

Table 17 displays the accuracy of the classification achieved by the LDA for the different categorical variables. The only variable that was classified with any success was ‘cut’. For ‘color’ and ‘clarity’ the LDA performs poorly. As “cut” was the only class to perform moderately well as classifier we will investigate its performance further.

LDAcut

```
## Call:
## lda(cut ~ carat + depth + table + price + x + y + z, data = diamonds.Train2)
##
## Prior probabilities of groups:
##      Fair       Good  Very Good    Premium      Ideal
## 0.02973538 0.09083101 0.22437326 0.25680130 0.39825905
##
## Group means:
##           carat     depth     table     price        x        y        z
## Fair      1.0378845 64.06230 59.02217 4324.327 6.236620 6.172709 3.976776
## Good      0.8474930 62.35461 58.68827 3921.449 5.835173 5.846637 3.637005
## Very Good 0.8047383 61.82327 57.94319 3976.481 5.736677 5.765891 3.558380
## Premium   0.8920031 61.26609 58.75017 4585.801 5.974316 5.941453 3.646883
## Ideal      0.7026392 61.71047 55.94881 3456.299 5.506723 5.519916 3.400944
##
## Coefficients of linear discriminants:
##           LD1        LD2        LD3        LD4
## carat -1.1399770736 -0.7654500021 -1.6344087398 6.1979429288
## depth -0.4313836891 -0.6258113096 -0.0222994014 -0.3772585315
## table -0.5535698046  0.1099286561  0.1453488495 -0.0892954001
## price  0.0001287086  0.0001352239  0.0001759807 -0.0001005275
## x      -0.5721476535  1.2446100820 -6.7713856573 -4.3794312244
## y      0.4971894042 -1.1101163035  5.9652716094  0.2853573062
## z      0.0399048498 -0.3226799256  1.0868425469  3.8779408496
##
## Proportion of trace:
##    LD1     LD2     LD3     LD4
## 0.7989 0.1670 0.0332 0.0009
```

The first thing we notice is that the prior probabilities for each class are very different. We see small amounts of variation in group means for different variables across different classes

but there is nothing to striking. As the data are on different scales it is harder to interpret the group mean differences. The variables with strongest weight in LD1 are carat, x, y and table while the variables with the most weight in LD2 are x, y, carat and depth. LD1 retains almost 80% of the variation in the data while LD2 retains only 17%.

```
confusionMatrix(RCMcut)
```

```
## Confusion Matrix and Statistics
##
##           actual
## RPcut      Fair Good Very Good Premium Ideal
##   Fair       193   109      27      4      2
##   Good        15   137      95     25      0
##   Very Good   16   223     611     369    101
##   Premium     61   262     691    1746    182
##   Ideal       44   262     992     584   4109
##
## Overall Statistics
##
##           Accuracy : 0.6258
##           95% CI  : (0.6166, 0.6349)
##           No Information Rate : 0.4046
##           P-Value [Acc > NIR] : < 2.2e-16
##
##           Kappa : 0.4484
##
## McNemar's Test P-Value : < 2.2e-16
##
## Statistics by Class:
##
##           Class: Fair Class: Good Class: Very Good Class: Premium
## Sensitivity          0.58663   0.13797   0.25290   0.6400
## Specificity          0.98652   0.98632   0.91604   0.8529
## Pos Pred Value       0.57612   0.50368   0.46288   0.5935
## Neg Pred Value       0.98708   0.91915   0.81080   0.8760
## Prevalence           0.03029   0.09144   0.22247   0.2512
## Detection Rate       0.01777   0.01262   0.05626   0.1608
## Detection Prevalence 0.03085   0.02505   0.12155   0.2709
## Balanced Accuracy    0.78657   0.56214   0.58447   0.7465
##
##           Class: Ideal
## Sensitivity          0.9351
## Specificity          0.7089
## Pos Pred Value       0.6859
## Neg Pred Value       0.9415
## Prevalence           0.4046
```

```

## Detection Rate          0.3784
## Detection Prevalence    0.5517
## Balanced Accuracy       0.8220

```

The confusion matrix clearly shows a large amount of errors for each individual class. We again get the overall accuracy measurement of 0.6258 with a 95% confidence interval of (0.6166,0.6349). In terms of the accuracy of classification for individual classes Ideal, Fair and Premium had higher accuracy ratings than Good or Very Good. It was important to check individual class accuracy as the prior probabilities of an observation belonging to each class were very different.

We conclude that LDA is not particularly effective at correctly classifying diamonds into different levels of the categorical variables. Using the levels of the categorical variables as classes was more successful than using the interactions with the cut variable returning the highest accuracy. Even this was not particularly successful. We therefore conclude the data is not linearly separable by using the levels of categorical variables as classes.

### 8.3 Why prediction without the categorical variables is not working well

#### 8.3.1 Create modified version of dataset only containing carat values between 1 and 1.1

```

library(dplyr)
diamonds_car2 <- diamonds[,c(1,4,7)]
diamonds_car2[1:4,]

##   carat clarity price
## 1  0.23     SI2   326
## 2  0.21     SI1   326
## 3  0.23     VS1   327
## 4  0.29     VS2   334

diamonds_car19_21 <- filter(diamonds_car2, between(carat, 1, 1.1))
diamonds_car19_21[1:4,]

##   carat clarity price
## 1  1.01      I1   2781
## 2  1.01      I1   2788
## 3  1.01     SI2   2788
## 4  1.05     SI2   2789

```

'Carat' vs 'price', coloured by 'clarity'

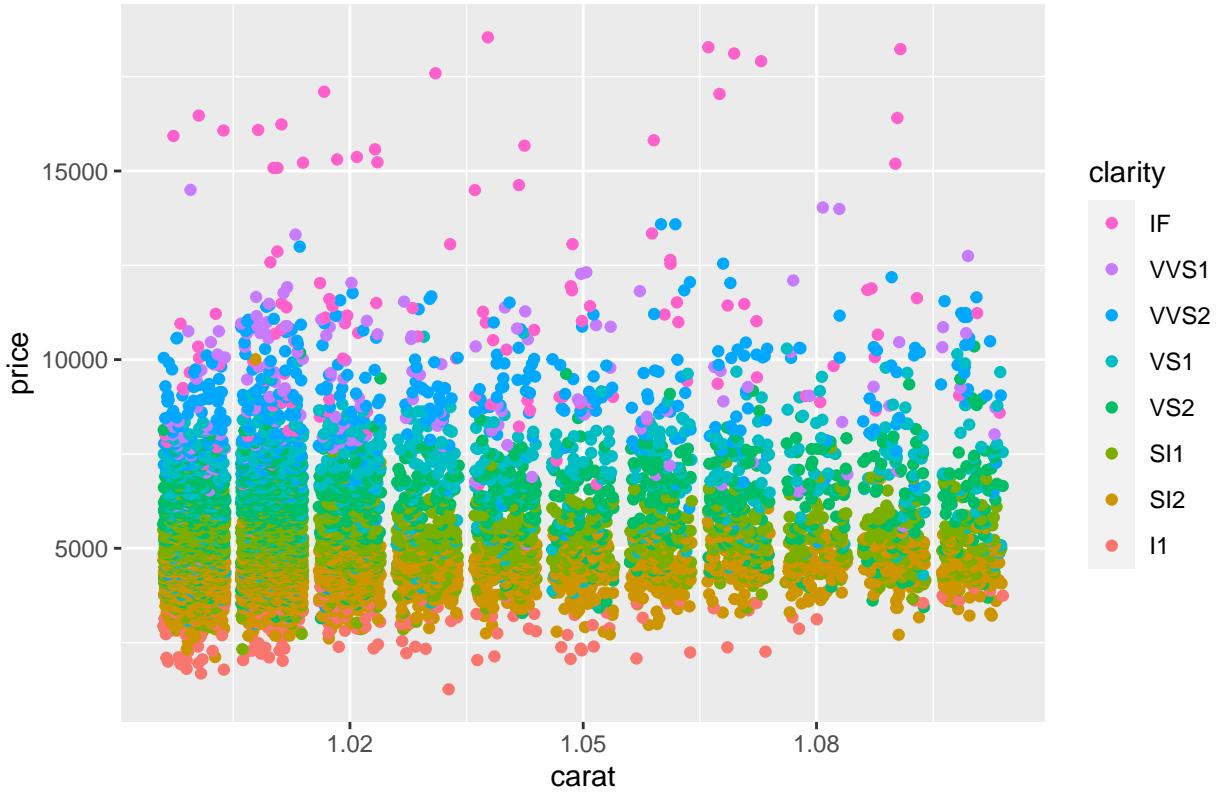


Figure 18: Carat (limited values) vs Price coloured by Clarity

Figure 18 shows a the scatterplot of ‘price’ vs ‘carat’ vs ‘clarity’, but just for values of ‘carat’ between 1 and 1.1, which helps us see the pattern more clearly. Firstly, note the vertical spread of the data points; most of the values begin at around 2500 USD and generally have extreme upper values larger than 15,000 USD, so a spread of approximately 10,000 USD. There are clear colour bands running horizontally, with the lowest quality ‘clarity’ diamonds near the bottom of the price range, while the highest values are near the top.

So, using ‘carat’ alone to predict these prices clearly is not enough; given the spread of price for any given value of carat, the categorical variables play an important in determining where in the range any given diamond will be. This was the reason our regression models that excluded the categorical variables did not perform nearly as well as those that included them.

## 8.4 Discriminant analysis using the reduced dataset (price, carat and clarity)

To address the issue above, we attempted another discriminant analysis using the reduced dataset above, that is, for ‘carat’ values between 1.0 and 1.1. The hope was that the relatively clear bands seen in figure 18 would enable the LDA to this time, as there is less overlap between levels in the reduced dataset than there is in the original.

So, we repeat the Linear Discriminant Analysis process using the reduced dataset.

```
# split dataset into 'train' and 'test'  
set.seed(1234567890, kind = "Mersenne-Twister")  
ind <- sample(c("Tr","Te"),  
             nrow(diamonds_car19_21),replace = TRUE,prob = c(0.6,0.4))  
  
# display first few elements to check  
ind[1:15]  
  
## [1] "Te" "Tr" "Te" "Te" "Te" "Te" "Tr" "Te" "Te" "Te"  
## [11] "Te" "Tr" "Tr" "Te" "Te" "Tr" "Tr" "Te" "Tr"  
  
# check that the proportions have come out as expected  
nrow(diamonds_car19_21)  
  
## [1] 7568  
  
table(ind)  
  
## ind  
##   Te      Tr  
## 3008 4560  
  
prop.table(table(ind))  
  
## ind  
##       Te      Tr  
## 0.397463 0.602537
```

The output above confirms that the random allocation of values to either the training or testing datasets produced the desired result of approximately 60% in the training set and 40% in the testing set.

#### 8.4.1 Create training and test subsets

```
Train <- diamonds_car19_21[ind=="Tr",]  
Test <- diamonds_car19_21[ind=="Te",]
```

#### 8.4.2 Create LDA and display summary

```
library(MASS)  
LDA.diam <- lda(clarity~carat+price,  
                 data=Train)
```

#### 8.4.3 Boxplots of the LDA means

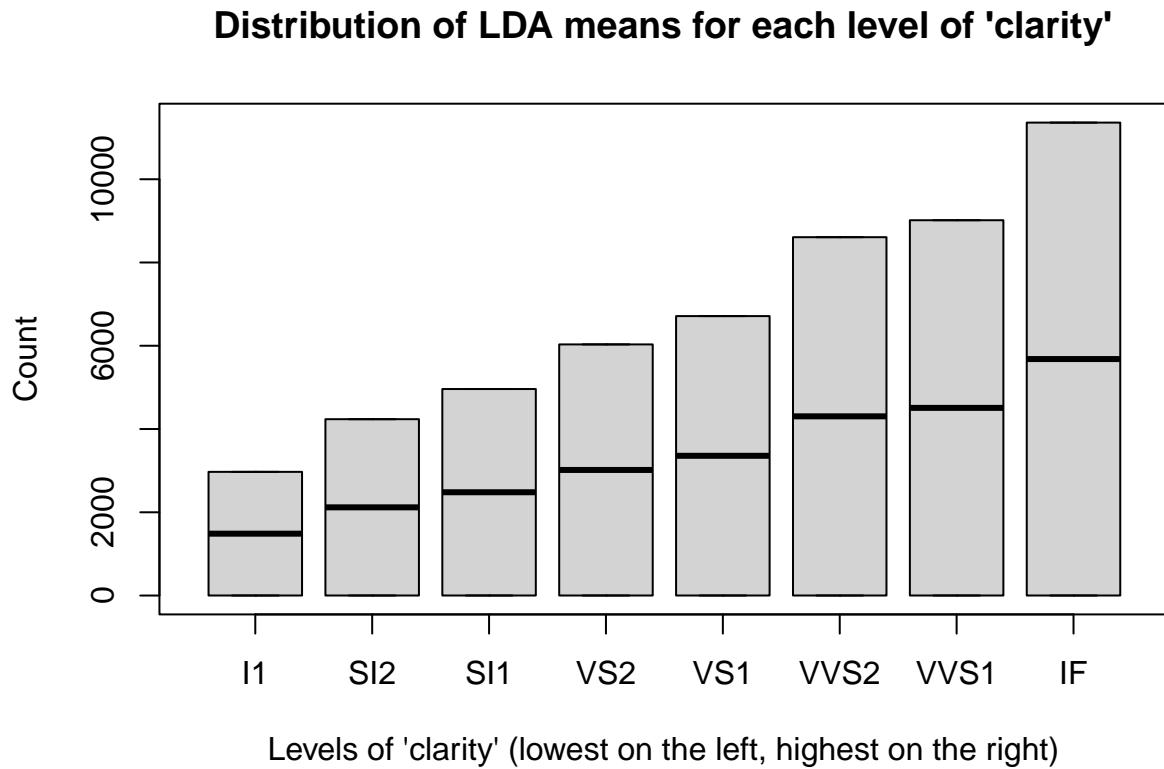


Figure 19: Distribution of LDA means

Figure 19 shows the distribution of the LDA means. It is obvious that there is an increasing number of diamonds of higher ‘clarity’ levels.

#### 8.4.4 Create Prediction object

```
Pred <- predict(LDA.diam)
```

#### 8.4.5 Biplot

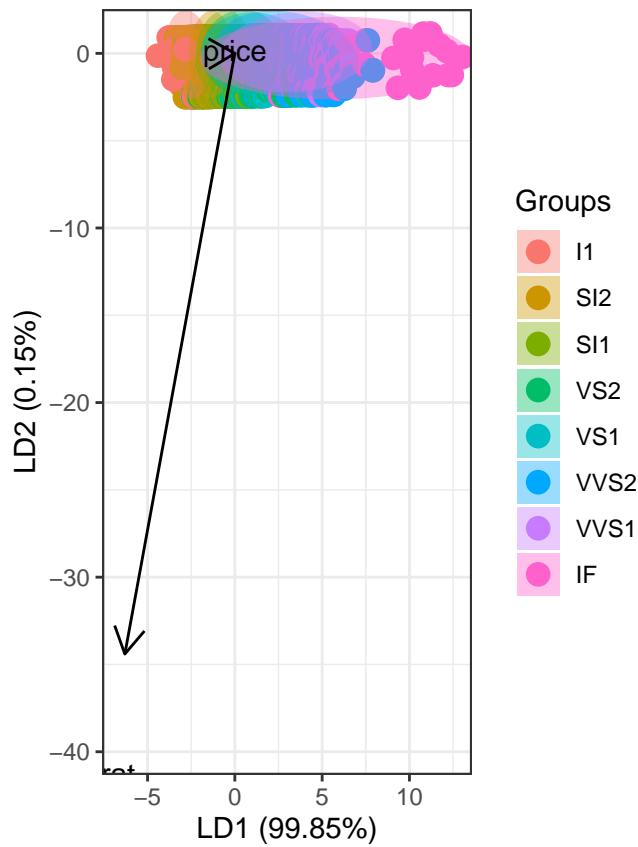


Figure 20: Biplot of ‘carat’, ‘price’ against ‘clarity’

Figure 20 shows the biplot for ‘carat’ versus ‘price’ against the different levels of ‘clarity’. There is still a large amount of overlap visible near the top of the plot (the different coloured dots), meaning that separation of the levels is not going to be easy.

#### 8.4.6 Partition plot

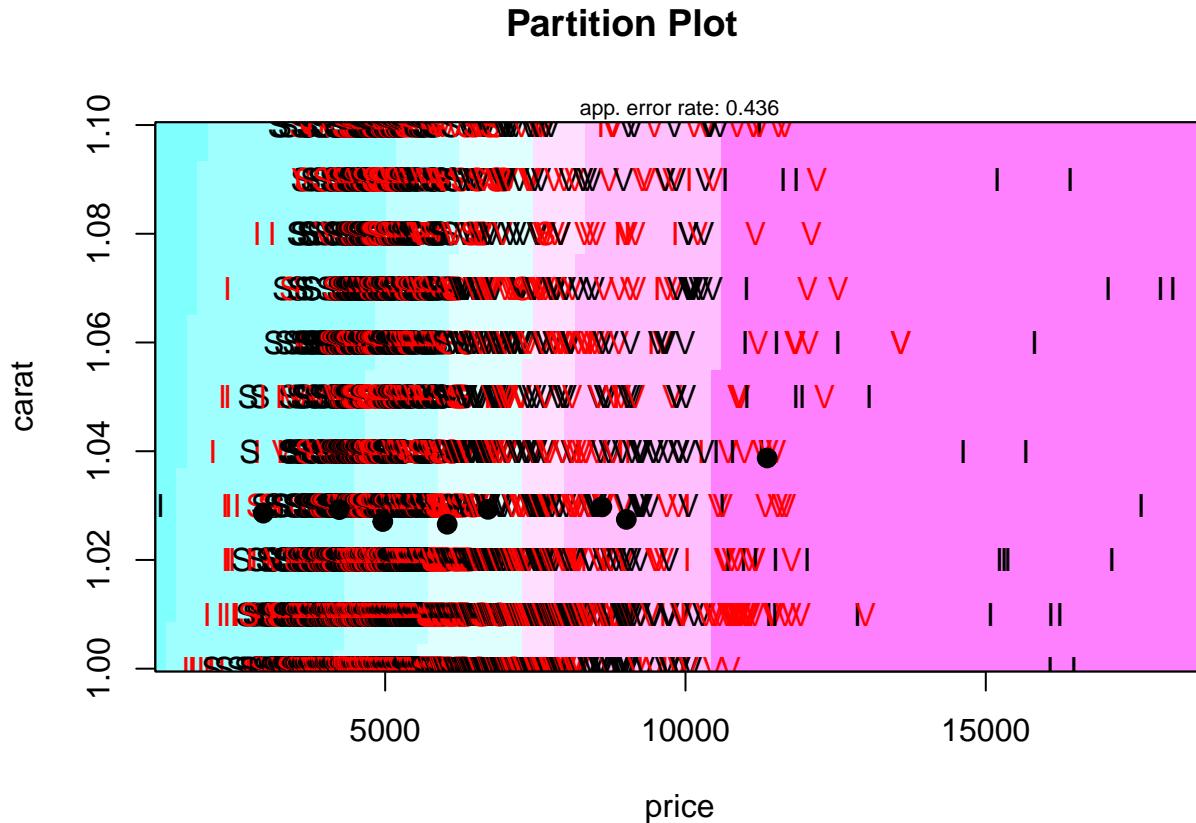


Figure 21: Partition plot for levels of ‘clarity’ as predicted by ‘carat’ and ‘price’

Figure 21 shows the partition plot for the discriminant analysis (DA) of the different levels of the ‘clarity’ variable as predicted by ‘carat’ and ‘price’. As a reminder, the levels are: I1 (lowest clarity), SI2, SI1, VS2, VS1, VVS2, VVS1, IF (highest clarity). We can see from the number of red font values that there are many mis-categorised observations, suggesting that the LDA has not worked particularly well.

Actual										
##	Realistic	Predicted	I1	SI2	SI1	VS2	VS1	VVS2	VVS1	IF
##		I1	0	0	0	0	0	0	0	0
##		SI2	65	545	185	40	24	4	0	0
##		SI1	0	260	578	158	57	5	4	0
##		VS2	0	3	76	366	155	30	3	1
##		VS1	0	0	1	45	81	28	9	1
##		VVS2	0	1	0	12	43	118	31	21
##		VVS1	0	0	0	0	0	0	0	0
##		IF	0	0	0	0	1	14	17	26

The confusion matrix above shows that there were many mis-categorisations. There is a

pattern evident. The leading diagonal shows the correct predictions. There are many incorrect predictions for values immediately above and below the leading diagonal, meaning that the DA partitions struggled to separate adjacent categories.

```
prop_succ <- sum(diag(RCM))/sum(RCM)
prop_succ
```

```
## [1] 0.5698138
```

```
per_succ <- signif(prop_succ*100,4)
per_succ
```

```
## [1] 56.98
```

The output above shows that the percentage of successful predictions was 56.98%, indicating that this LDA is a mediocre predictor.

#### 8.4.7 Summary of LDA

The relatively poor performance of LDA in this instance is perhaps not surprising given the amount of overlap we saw in figure 18 earlier (the scatterplot of ‘price’ vs ‘carat’ vs ‘clarity’ for ‘carat’ values between 1 and 1.1); there simply isn’t enough geometrical separation between the different levels of ‘clarity’ for the algorithm to accurately categorise the boundary cases.

## 9 Cluster Analysis

While LDA returned some positive results in terms of classification were were interested to investigate how a Cluster Analysis would perform with the diamonds data. We used a K-means algorithm to split the data into distinct clusters. Therefore centroids are computed as the mean of all observations belonging to each class. The Euclidean Distance was used for intra/inter class distances. As we were initially unsure of how many clusters we should attempt to divide the data into we will first examine the sum of the squares of the intra-class disatances for different numbers of clusters. We will again use the smaller sample to aid in the interpretation of visual displays.

## 9.1 Optimal cluster number

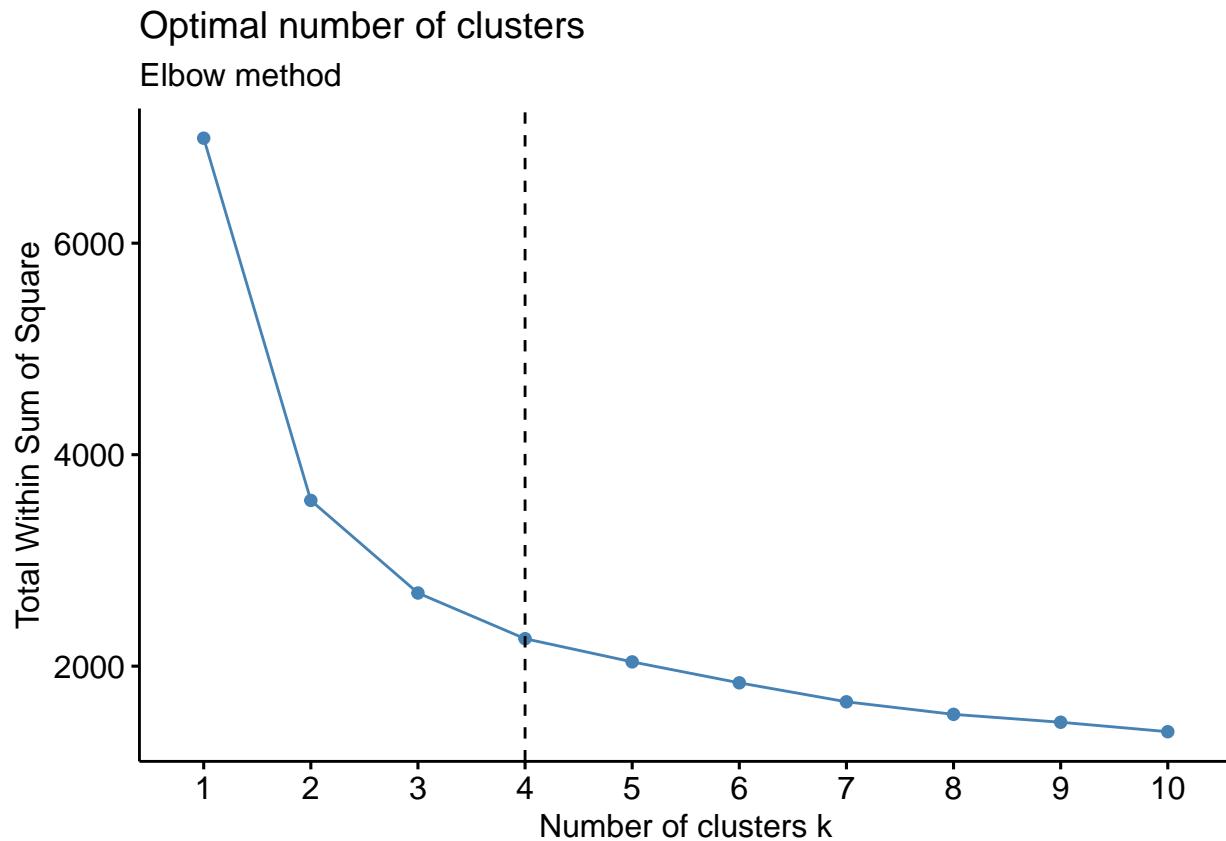


Figure 22: Optimal number of clusters

Figure 22 shows that when the elbow method is used, we see that 4 clusters is the optimum number. It is worth noting that the total within sum of squares is large for any selected number of clusters.

## 9.2 kmeans function

```
clusters <- kmeans(scale(smaller.sample[,-c((2:4),11)]),
                     algorithm = "MacQueen", centers = 4,
                     iter.max = 100, nstart = 50)

clusters$centers

##          carat        depth       table      price         x         y
## 1  0.3436636  0.4625156 -0.19171931  0.17367610  0.4846585  0.4876979
## 2 -0.8302592  0.0195466 -0.31324680 -0.71288391 -0.9107135 -0.9109156
## 3  1.9509740  0.1578009  0.05304506  2.09674493  1.7073677  1.7102105
```

```
## 4 0.2424997 -1.1838989 1.40775657 0.06611287 0.4472637 0.4388989
##          z
## 1 0.5477885
## 2 -0.9007307
## 3 1.7128313
## 4 0.2761387
```

We create four clusters with sizes of 127, 455, 137 and 281 respectively. We see clear differences in the mean of each numeric variable across the four clusters. We see that cluster 3 contains diamonds with a higher mean price, carat, x, y and z. While cluster 2 contains smaller, less valuable diamonds. Diamonds in cluster 4 have larger absolute mean scores for depth and table compared to the other clusters.

```
clusters$withinss
```

```
## [1] 733.8363 680.2943 417.9465 427.4378
```

The within cluster sum of squares indicates that the clusters are not very compact. We shall now visualize the clusters with a scatterplot using the first two principal components of our dataset as the two dimensions. The first two principal components account for over 86% of the variance so this should be an appropriate visualization.

### 9.3 Cluster plot

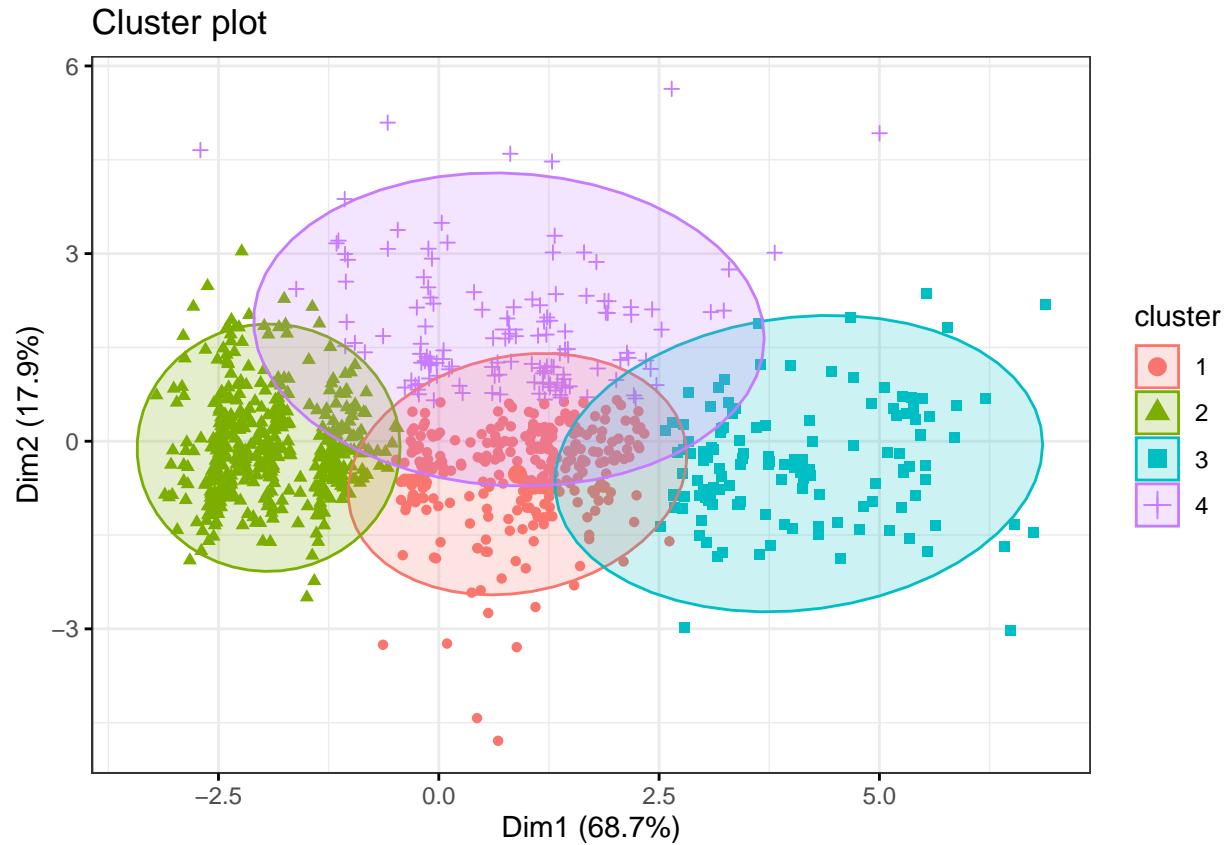


Figure 23: Cluster plot

In the cluster plot (figure 23) we see significant overlap between the four clusters as well as many observations that do not seem to fit well in any of the clusters. It seems that the diamond dataset does not easily split into distinct clusters.

It is worth mentioning that ‘diamonds’ already has categories and levels within those categories, and that the clustering above does not take the pre-existing categories into account, but attempts to find naturally occurring classes within the data.

### 9.4 Silhouette plot

```
##   cluster size ave.sil.width
## 1       1  283      0.29
## 2       2  455      0.51
## 3       3  127      0.34
## 4       4  135      0.21
```

Clusters silhouette plot  
Average silhouette width: 0.38

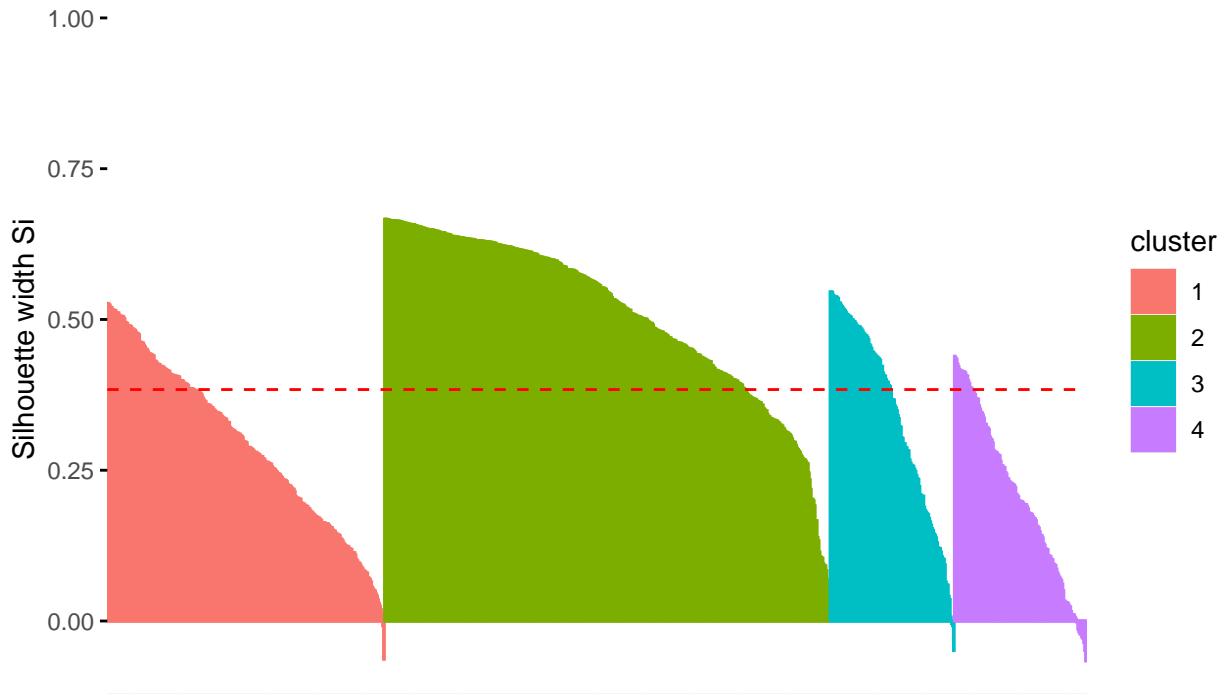


Figure 24: Clusters silhouette plot

The silhouette plot (figure 24) helps us to visualize how similar points each observation is to the cluster that it is assigned too. We see very few negative values indicating that very few observations have been assigned to the wrong cluster. However we do have a large proportion of values with low positive values indicating that these observations were close to the borderline between two clusters. We see that the average silhouette width is 0.38.

## 10 Conclusion

During this investigation we applied a number of the new analytical methods that we learned in STAT394. As this project was principally a learning task we attempted them all, even though an experienced statistician may have concluded beforehand that they were inappropriate or of dubious value. We mostly performed these techniques on a subset of the data that only included the numerical variables. Not all of the analytical techniques worked as we had hoped, and when this occurred we investigated why.

The primary aims of our multivariate statistical investigation on the diamonds data set were as follows

- To find the best way of predict diamond price using the other variables in the data set

- To attempt to classify observations, in particular the different levels of the three categorical variables
- To explore the new statistical techniques of PCA, LDA, FA and CA in relation to our data set

We had the most success with our first aim; predicting diamond price. This was first achieved using the multiple regression model with all variables except for ‘y’ and ‘z’, in other words the model;  $\text{price} = \text{carat} + \text{cut} + \text{color} + \text{clarity} + \text{depth} + \text{table} + \text{x}$ . The best model was obtained using stepwise AIC and BIC tests. The diagnostics for the multiple regression model showed that all the assumptions were violated, meaning the model might not produce accurate predictions. Secondly, a principal components analysis using the numerical variables was successful in reducing the dimensionality of the dataset while still retaining a large proportion of the variance. We were able to use the first two principal components in a model using principal components regression and this performed almost as well as the “best” model identified through stepwise multiple regression despite being much more parsimonious. We did however note that a model including all principal components was more effective at predicting price.

An examination of the key principal components inspired us to conduct a factor analysis with the hypothesis that the factors “price + dimension” and “light conductance” might be sufficient to explain variation in the dataset. While we saw evidence of our hypothesized structure in the factor loadings, two factors was not found to be sufficient to capture all the variation in the data.

We attempted to address our second aim through the techniques of LDA and CA. We had success in predicting diamond price level using an LDA of the other numeric variables. An LDA was also undertaken attempting to correctly classify diamonds into levels of each categorical variable. This failed to reliably discriminate observations. As using known classes in the dataset had mixed success we attempted a cluster analysis. The cluster analysis identified four clusters as the optimal number to describe the data however there was significant overlap between these. We conclude that the diamonds dataset presents a difficult classification problem and that the reason for this is the high number of possible interactions between levels of our three categorical variables.

## 11 Appendices

### 11.1 Code for Producing KS Tests

```
ks.test(diamonds$carat, "pnorm", mean=mean(diamonds$carat), sd=sd(diamonds$carat))

## Warning in ks.test.default(diamonds$carat, "pnorm", mean =
## mean(diamonds$carat), : ties should not be present for the Kolmogorov-Smirnov
## test

##
```

```

##  Asymptotic one-sample Kolmogorov-Smirnov test
##
## data: diamonds$carat
## D = 0.12274, p-value < 2.2e-16
## alternative hypothesis: two-sided

ks.test(diamonds$depth, "pnorm", mean=mean(diamonds$depth), sd=sd(diamonds$depth))

## Warning in ks.test.default(diamonds$depth, "pnorm", mean =
## mean(diamonds$depth), : ties should not be present for the Kolmogorov-Smirnov
## test

##
##  Asymptotic one-sample Kolmogorov-Smirnov test
##
## data: diamonds$depth
## D = 0.075871, p-value < 2.2e-16
## alternative hypothesis: two-sided

ks.test(diamonds$table, "pnorm", mean=mean(diamonds$table), sd=sd(diamonds$table))

## Warning in ks.test.default(diamonds$table, "pnorm", mean =
## mean(diamonds$table), : ties should not be present for the Kolmogorov-Smirnov
## test

##
##  Asymptotic one-sample Kolmogorov-Smirnov test
##
## data: diamonds$table
## D = 0.13225, p-value < 2.2e-16
## alternative hypothesis: two-sided

ks.test(diamonds$price, "pnorm", mean=mean(diamonds$price), sd=sd(diamonds$price))

## Warning in ks.test.default(diamonds$price, "pnorm", mean =
## mean(diamonds$price), : ties should not be present for the Kolmogorov-Smirnov
## test

##
##  Asymptotic one-sample Kolmogorov-Smirnov test
##
## data: diamonds$price
## D = 0.18467, p-value < 2.2e-16
## alternative hypothesis: two-sided

ks.test(diamonds$x, "pnorm", mean=mean(diamonds$x), sd=sd(diamonds$x))

## Warning in ks.test.default(diamonds$x, "pnorm", mean = mean(diamonds$x), : ties
## should not be present for the Kolmogorov-Smirnov test

```

```

##  

##  Asymptotic one-sample Kolmogorov-Smirnov test  

##  

##  data: diamonds$x  

##  D = 0.093545, p-value < 2.2e-16  

##  alternative hypothesis: two-sided  

ks.test(diamonds$y, "pnorm", mean=mean(diamonds$y), sd=sd(diamonds$y))  

## Warning in ks.test.default(diamonds$y, "pnorm", mean = mean(diamonds$y), : ties  

## should not be present for the Kolmogorov-Smirnov test  

##  

##  Asymptotic one-sample Kolmogorov-Smirnov test  

##  

##  data: diamonds$y  

##  D = 0.088528, p-value < 2.2e-16  

##  alternative hypothesis: two-sided  

ks.test(diamonds$z, "pnorm", mean=mean(diamonds$z), sd=sd(diamonds$z))  

## Warning in ks.test.default(diamonds$z, "pnorm", mean = mean(diamonds$z), : ties  

## should not be present for the Kolmogorov-Smirnov test  

##  

##  Asymptotic one-sample Kolmogorov-Smirnov test  

##  

##  data: diamonds$z  

##  D = 0.089273, p-value < 2.2e-16  

##  alternative hypothesis: two-sided

```

## 11.2 Code for producing ANOVA/Levene's Test/Kruskal Wallis/Tukey of Price by Cut

```

cutanova <- aov(price ~ cut, data = diamonds)
summary(cutanova)  

##           Df   Sum Sq   Mean Sq F value Pr(>F)  

## cut          4 1.104e+10 2.760e+09   175.7 <2e-16 ***  

## Residuals  53935 8.474e+11 1.571e+07  

## ---  

## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  

leveneTest(price ~ cut, data= diamonds)  

## Levene's Test for Homogeneity of Variance (center = median)  

##           Df F value    Pr(>F)

```

```

## group      4   123.6 < 2.2e-16 ***
##          53935
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
kruskal.test(price ~ cut, data = diamonds)

##
## Kruskal-Wallis rank sum test
##
## data: price by cut
## Kruskal-Wallis chi-squared = 978.62, df = 4, p-value < 2.2e-16
TukeyHSD(cutanova, conf.level = 0.95)

## Tukey multiple comparisons of means
## 95% family-wise confidence level
##
## Fit: aov(formula = price ~ cut, data = diamonds)
##
## $cut
##              diff      lwr      upr      p adj
## Good-Fair    -429.89331 -740.44880 -119.3378 0.0014980
## Very Good-Fair -376.99787 -663.86215  -90.1336 0.0031094
## Premium-Fair   225.49994  -59.26664  510.2665 0.1950425
## Ideal-Fair     -901.21579 -1180.57139 -621.8602 0.0000000
## Very Good-Good    52.89544  -130.15186  235.9427 0.9341158
## Premium-Good    655.39325   475.65120  835.1353 0.0000000
## Ideal-Good     -471.32248  -642.36268 -300.2823 0.0000000
## Premium-Very Good  602.49781   467.76249  737.2331 0.0000000
## Ideal-Very Good   -524.21792  -647.10467 -401.3312 0.0000000
## Ideal-Premium   -1126.71573 -1244.62267 -1008.8088 0.0000000

```

### 11.3 Code for producing ANOVA/Levene's Test/Kruskal Wallis/Tukey of Price by Clarity

```

clarityanova <- aov(price ~ clarity, data = diamonds)
summary(clarityanova)

##             Df Sum Sq Mean Sq F value Pr(>F)
## clarity          7 2.331e+10 3.330e+09     215 <2e-16 ***
## Residuals      53932 8.352e+11 1.549e+07
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```

leveneTest(price ~ clarity, data= diamonds)

## Levene's Test for Homogeneity of Variance (center = median)
##          Df F value    Pr(>F)
## group      7 77.809 < 2.2e-16 ***
##             53932
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

kruskal.test(price ~ clarity, data = diamonds)

## 
## Kruskal-Wallis rank sum test
##
## data: price by clarity
## Kruskal-Wallis chi-squared = 2718.2, df = 7, p-value < 2.2e-16

TukeyHSD(clarityanova, conf.level = 0.95)

## Tukey multiple comparisons of means
## 95% family-wise confidence level
##
## Fit: aov(formula = price ~ clarity, data = diamonds)
##
## $clarity
##           diff       lwr      upr     p adj
## SI2-I1  1138.8599147  683.395891 1594.32394 0.0000000
## SI1-I1   71.8324571 -378.570901  522.23582 0.9997320
## VS2-I1   0.8207037 -450.377702  452.01911 1.0000000
## VS1-I1  -84.7132999 -542.298929  372.87233 0.9992819
## VVS2-I1 -640.4316203 -1109.531923 -171.33132 0.0009165
## VVS1-I1 -1401.0540535 -1881.569711 -920.53840 0.0000000
## IF-I1   -1059.3295848 -1580.334655 -538.32451 0.0000000
## SI1-SI2 -1067.0274575 -1229.386830 -904.66808 0.0000000
## VS2-SI2 -1138.0392109 -1302.591274 -973.48715 0.0000000
## VS1-SI2 -1223.5732146 -1404.907129 -1042.23930 0.0000000
## VVS2-SI2 -1779.2915349 -1987.983831 -1570.59924 0.0000000
## VVS1-SI2 -2539.9139681 -2773.136347 -2306.69159 0.0000000
## IF-SI2  -2198.1894995 -2506.318797 -1890.06020 0.0000000
## VS2-SI1  -71.0117534 -220.988718    78.96521 0.8410824
## VS1-SI1  -156.5457571 -324.764949   11.67343 0.0899007
## VVS2-SI1 -712.2640774 -909.667681 -514.86047 0.0000000
## VVS1-SI1 -1472.8865106 -1696.064436 -1249.70859 0.0000000
## IF-SI1  -1131.1620420 -1431.760399 -830.56369 0.0000000
## VS1-VS2  -85.5340037 -255.870471   84.80246 0.7958312
## VVS2-VS2 -641.2523240 -840.463263 -442.04138 0.0000000

```

```

## VVS1-VS2 -1401.8747572 -1626.652874 -1177.09664 0.0000000
## IF-VS2 -1060.1502885 -1361.938605 -758.36197 0.0000000
## VVS2-VS1 -555.7183203 -769.001243 -342.43540 0.0000000
## VVS1-VS1 -1316.3407535 -1553.679770 -1079.00174 0.0000000
## IF-VS1 -974.6162849 -1285.873083 -663.35949 0.0000000
## VVS1-VVS2 -760.6224332 -1019.466585 -501.77828 0.0000000
## IF-VVS2 -418.8979645 -746.848084 -90.94785 0.0027364
## IF-VVS1 341.7244687 -2.356168 685.80510 0.0531204

```

## 11.4 Code for producing ANOVA/Levene's Test/Kruskal Wallis/Tukey of Price by Color

```

coloranova <- aov(price ~ color, data = diamonds)
summary(coloranova)

##           Df Sum Sq Mean Sq F value Pr(>F)
## color       6 2.685e+10 4.475e+09   290.2 <2e-16 ***
## Residuals 53933 8.316e+11 1.542e+07
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

leveneTest(price ~ color, data= diamonds)

## Levene's Test for Homogeneity of Variance (center = median)
##           Df F value Pr(>F)
## group      6 219.12 < 2.2e-16 ***
##             53933
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

kruskal.test(price ~ color, data = diamonds)

## 
## Kruskal-Wallis rank sum test
##
## data: price by color
## Kruskal-Wallis chi-squared = 1335.6, df = 6, p-value < 2.2e-16

TukeyHSD(coloranova, conf.level = 0.95)

## Tukey multiple comparisons of means
## 95% family-wise confidence level
##
## Fit: aov(formula = price ~ color, data = diamonds)
##
## $color

```

```

##          diff      lwr      upr    p adj
## I-J   -231.94307  -501.11666   37.23053 0.1449244
## H-J   -837.14882 -1089.88345  -584.41420 0.0000000
## G-J   -1324.68235 -1568.82093 -1080.54376 0.0000000
## F-J   -1598.93162 -1847.48867 -1350.37458 0.0000000
## E-J   -2247.06554 -2494.88601 -1999.24508 0.0000000
## D-J   -2153.86392 -2413.70535 -1894.02250 0.0000000
## H-I   -605.20576  -807.34909  -403.06243 0.0000000
## G-I   -1092.73928 -1284.02646  -901.45210 0.0000000
## F-I   -1366.98856 -1563.88381 -1170.09331 0.0000000
## E-I   -2015.12248 -2211.08707 -1819.15789 0.0000000
## D-I   -1921.92086 -2132.88224 -1710.95948 0.0000000
## G-H   -487.53352  -654.89884  -320.16821 0.0000000
## F-H   -761.78280  -935.53004  -588.03556 0.0000000
## E-H   -1409.91672 -1582.60860 -1237.22484 0.0000000
## D-H   -1316.71510 -1506.25419 -1127.17600 0.0000000
## F-G   -274.24927  -435.23673  -113.26182 0.0000106
## E-G   -922.38320 -1082.23107  -762.53532 0.0000000
## D-G   -829.18158 -1007.09708  -651.26607 0.0000000
## E-F   -648.13392  -814.65208  -481.61576 0.0000000
## D-F   -554.93230  -738.86403  -371.00057 0.0000000
## D-E    93.20162   -89.73351   276.13675 0.7437450

```

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