Scouting the Heights of the Kreuzer-Skarke Landscape

Andres Rios-Tascon (Cornell)

String Phenomenology Seminar Series 2020

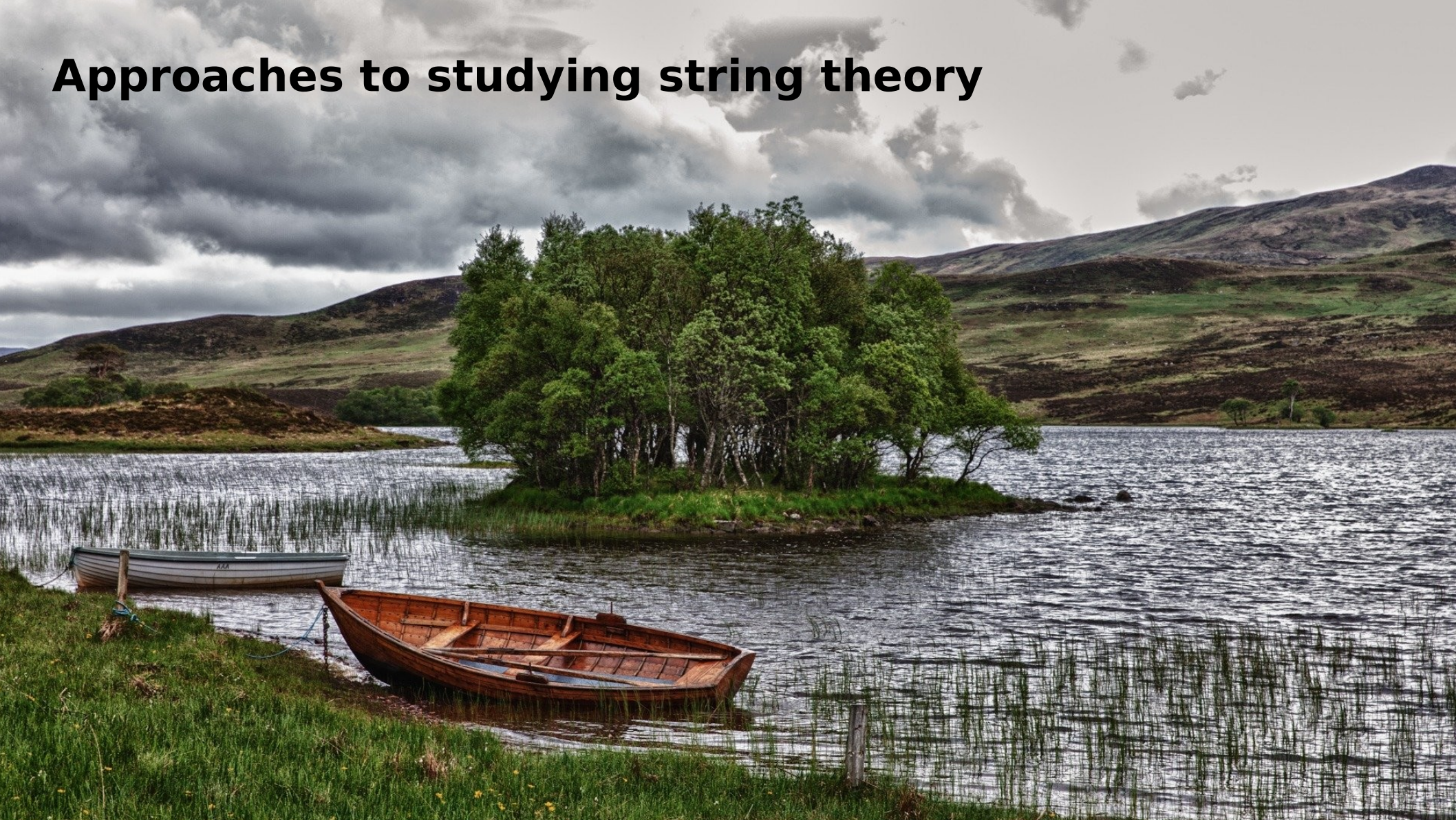
Outline

Goals: Explain our computational advancements for exploring KS.
Showcase our open-source package CYTools.

1. Motivation and background
2. Description of our new algorithms and computational advances
3. CYTools live demo
4. Conclusions

Based on: M. Demirtas, L. McAllister, A.R.T., 2008.01730, 201x.xxxxx

Approaches to studying string theory



Approaches to studying string theory

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Constructing Calabi-Yau 3-folds

Batyrev construction: [alg-geom/9310003]

1) Take 4D reflexive polytope.

2) Find fine, regular, star triangulation.

3) Interpret triangulation as a fan and construct the toric variety.

4) Anti-canonical hypersurface is a smooth CY threefold.

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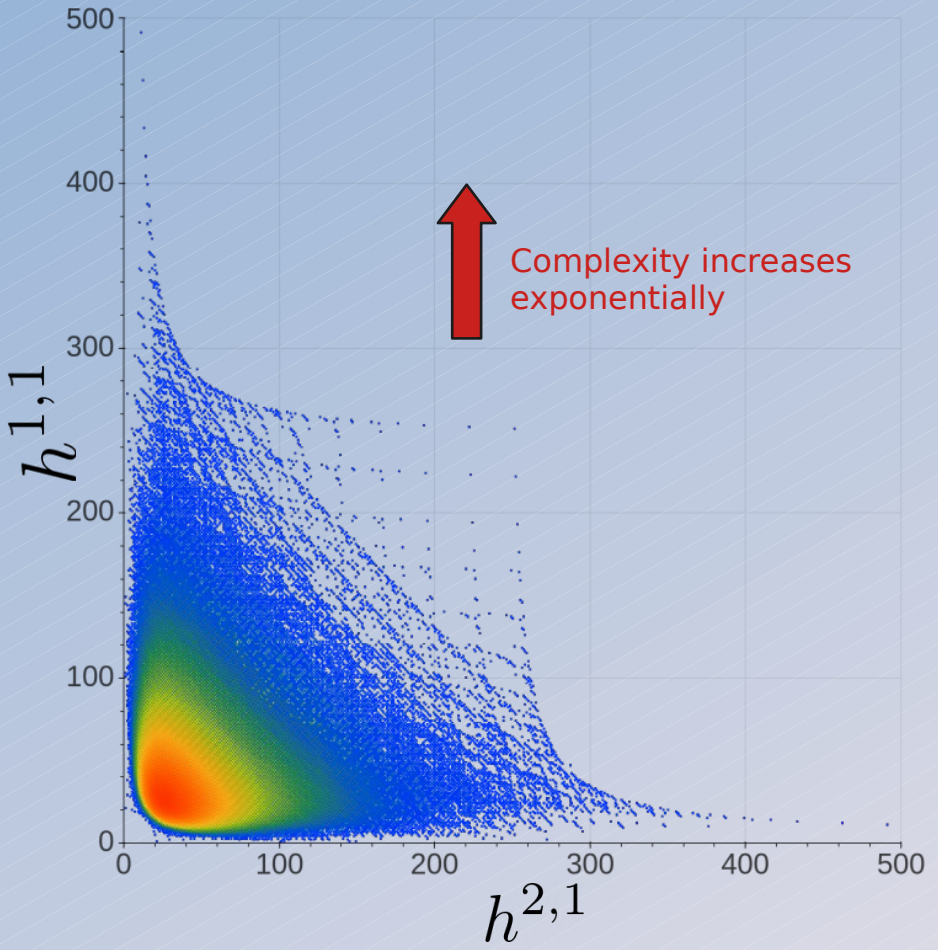
- They also provided “PALP: a Package for Analyzing Lattice Polytopes” to handle polytope computations. [math/0204356, arXiv:1205.4147]
- For the rest of the CY construction we are left on our own, but mathematical software like Sage can be used.

Exploring the KS database

- The complexity of the database grows exponentially with the number of points on the polytope ($\sim h^{1,1}$).



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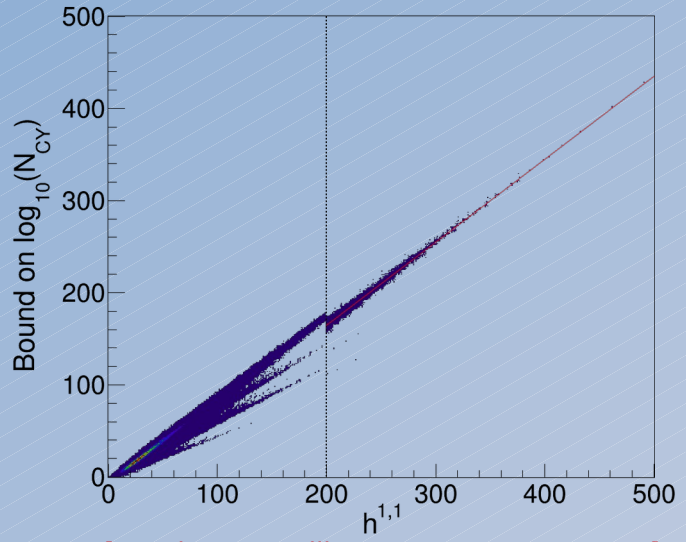
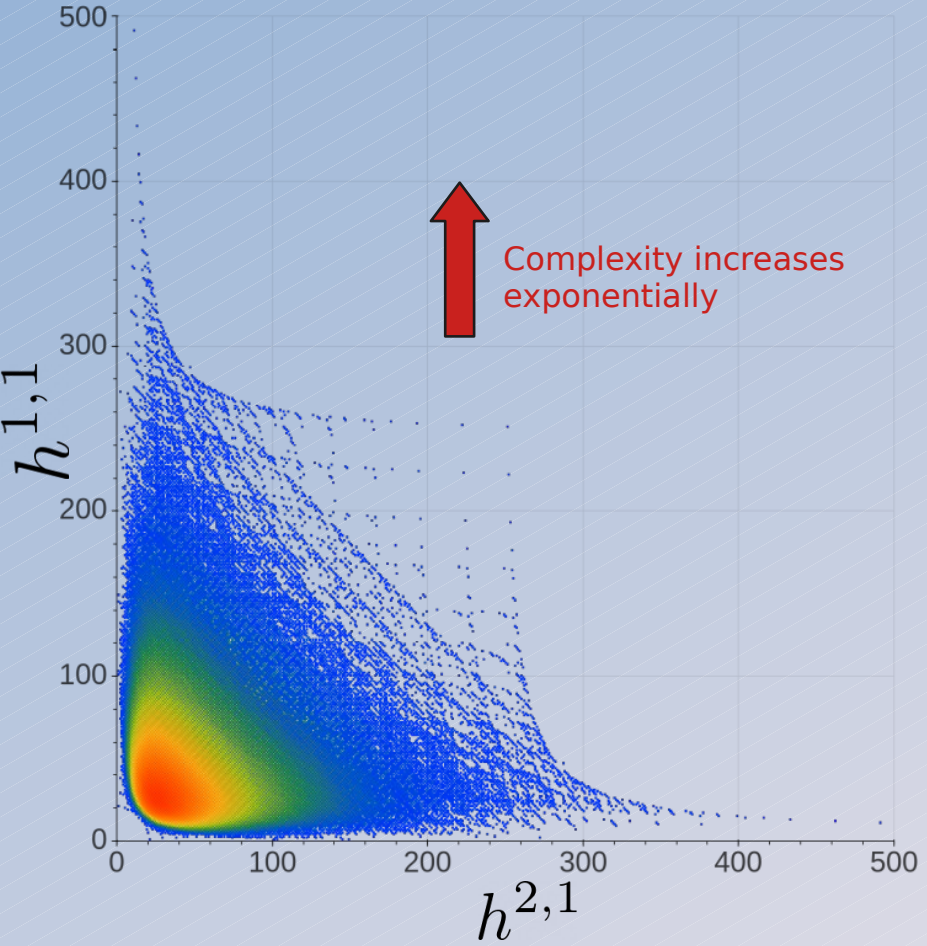


[Demirtas, McAllister, A.R.T., 2008.01730]



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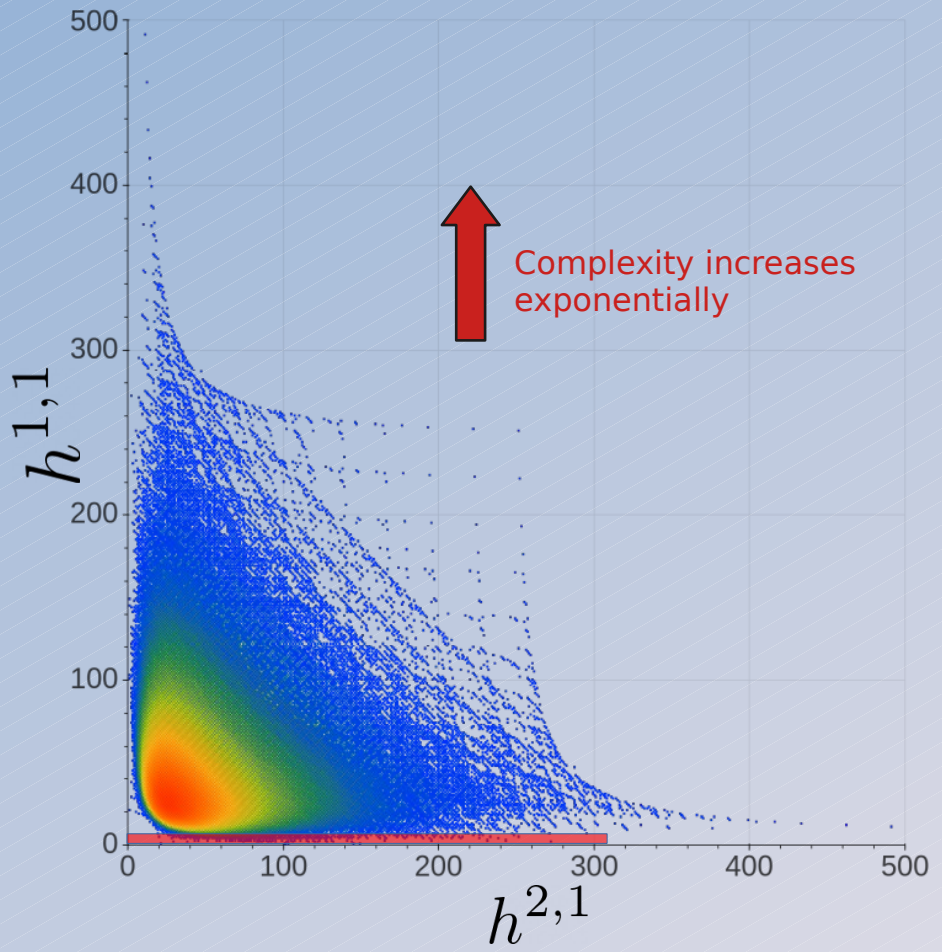

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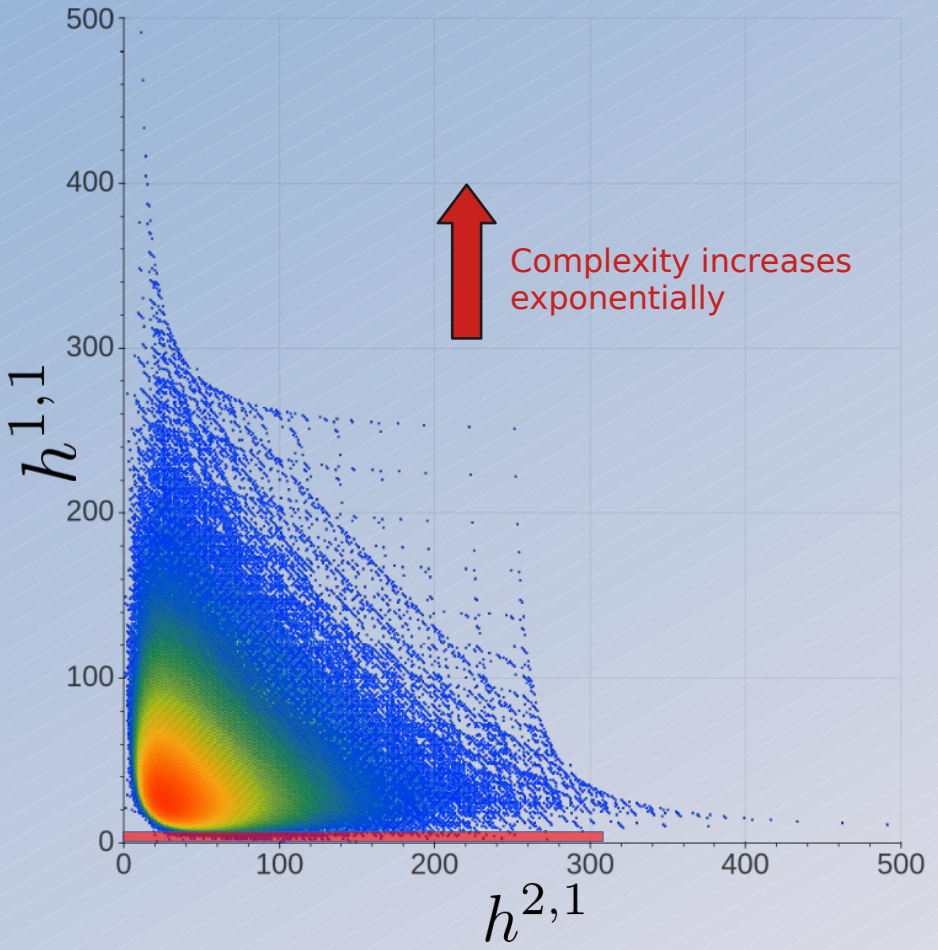
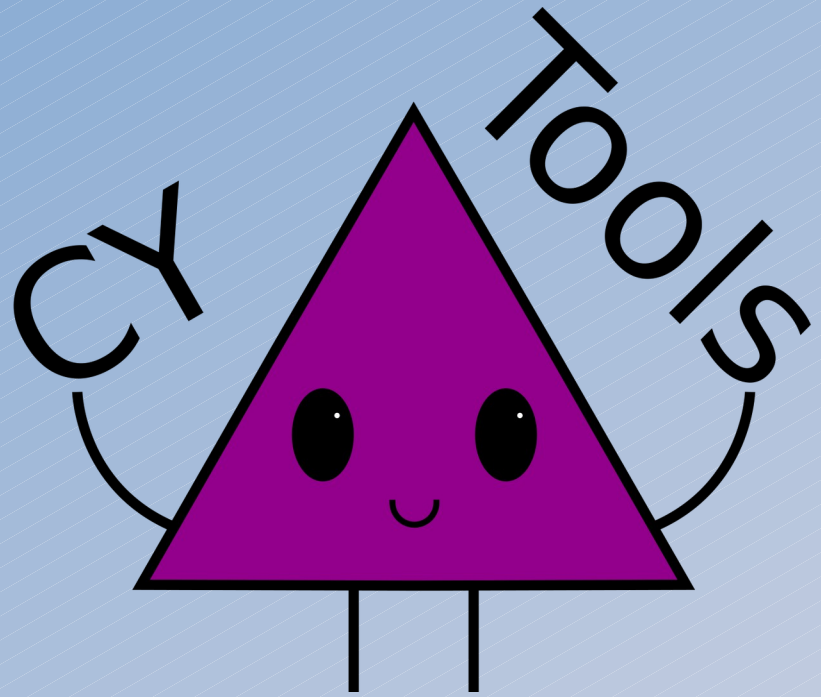
- Even though PALP and Sage are fairly powerful packages, we abandoned them and developed almost everything from scratch.



Introducing CYTools



A software package for analyzing Calabi-Yau hypersurfaces in toric varieties.

Improvements for some key ingredients

1) 4D reflexive polytopes.

2) Fine, regular, star triangulations (FRSTs).

3) Intersection numbers.

4) Kähler cones.

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Handling 4D reflexive polytopes

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- End result is our Python code being ~4x faster than PALP.
- When transcribed to Julia, our code is ~40x faster than PALP. We were able to obtain all points and faces of all reflexive polytopes overnight on a personal computer while working on [2008.01730].

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Finding appropriate triangulations

Batyrev's construction requires finding a finding a fine, regular, star triangulation (FRST) of the polytope.

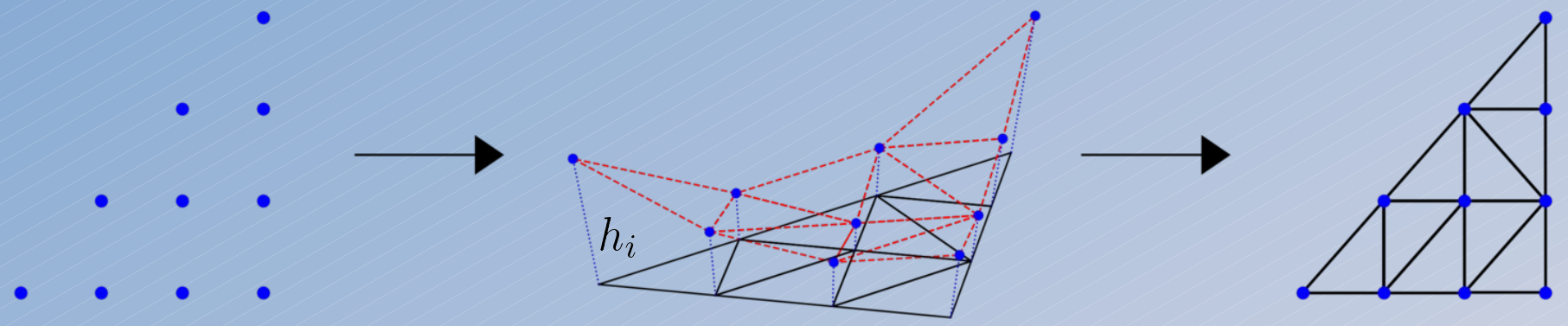
Star: Origin is vertex of all full-dim simplices → Simplices define cones of a fan

Fine: Uses all points (can exclude points interior to facets) → All singularities are resolved

Regular: Is needed so that the variety is Kähler, and will be key for our discussion. Let's see what it means more carefully.

Regular triangulations

A d-dimensional triangulation is regular if the points can be lifted by heights h_i to form a d+1-dimensional convex hull with the lower faces projecting down to the simplices of the triangulation.



This property is the hardest to check, and is partly responsible for the difficulty of studying CYs with large $h^{1,1}$.

Finding triangulations efficiently

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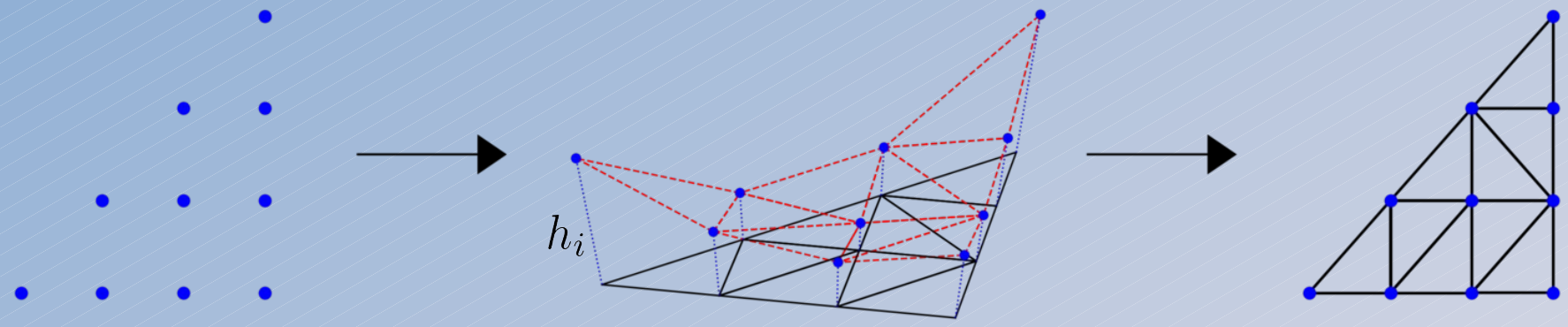

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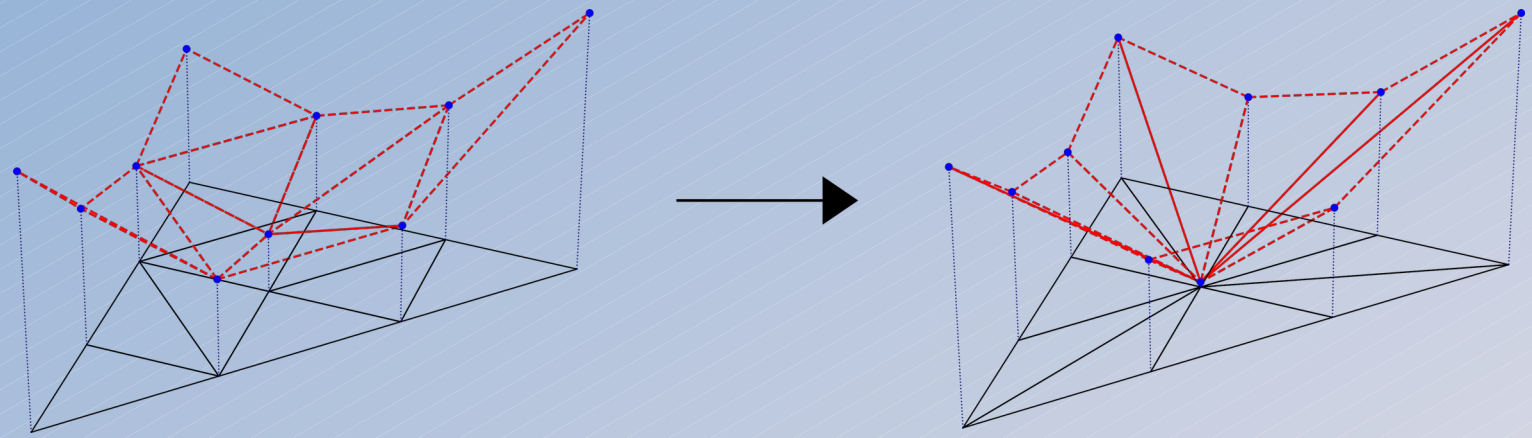

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- With appropriate choices of heights we can obtain FRSTs very efficiently.

Obtain one triangulation

Year	$h^{1,1}$	CPU time
2014	25	a few hours
2017	491	2s
2019	491	20ms

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- The star condition can be enforced by using a much lower height for the origin.
- With appropriate choices of heights we can obtain FRSTs very efficiently.
- This method does not allow a complete enumeration of triangulations, but this is impossible at large $h^{1,1}$.

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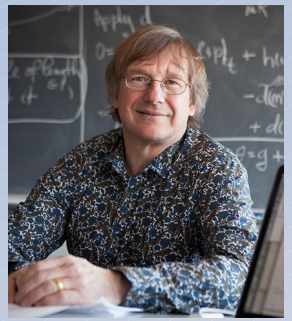
- There is a simple and computationally superior algorithm, as Mike Stillman explained to us: [c.f. e.g. 0803.1194]

  i. Compute intersection numbers of distinct divisors using full-dimensional cones in the fan.

  ii. Use linear relations of GLSM charge matrix to construct and solve equations for intersection numbers with self-intersections.



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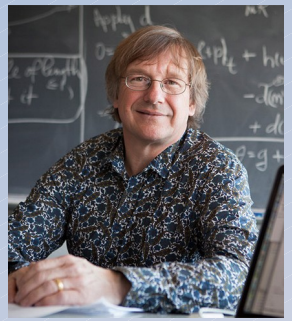

- We can find intersection numbers extremely quickly.

Compute intersection numbers

Year	$h^{1,1}$	CPU time
2017	100	30 mins
2018	491	30s
2019	491	3s

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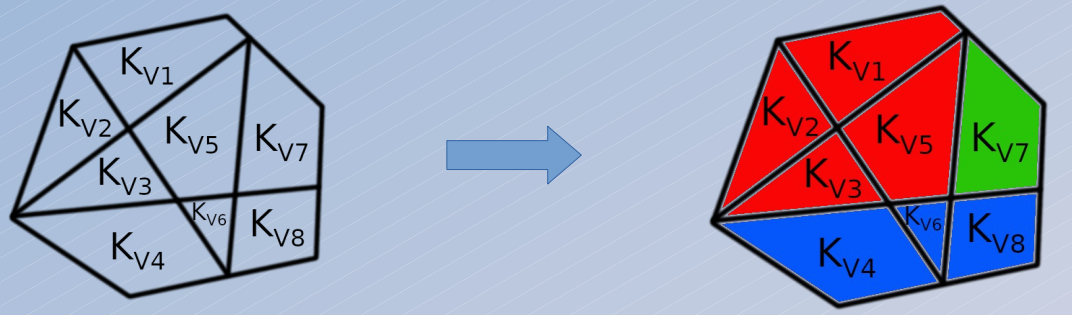

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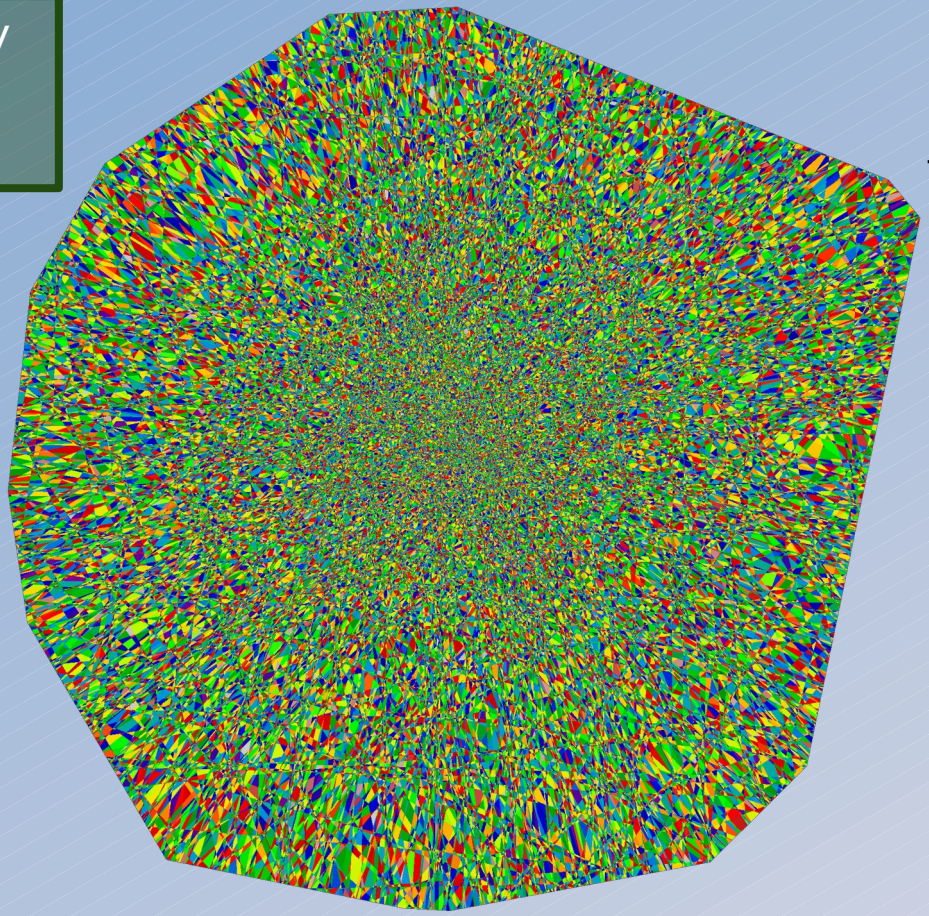
$$\mathcal{K}_\cup := \bigcup_{\{V|X\subset V_i\}} \mathcal{K}_{V_i} \qquad \mathcal{K}_{V_i} \subset \mathcal{K}_\cup \subset \mathcal{K}_X$$

Visualizing the gluing of Kähler cones

2D slice of secondary fan of the largest 4D reflexive polytope



Note: To render this image we computed 16 million triangulations of the largest polytope!

Each colored region is the Kähler cone of a different toric variety

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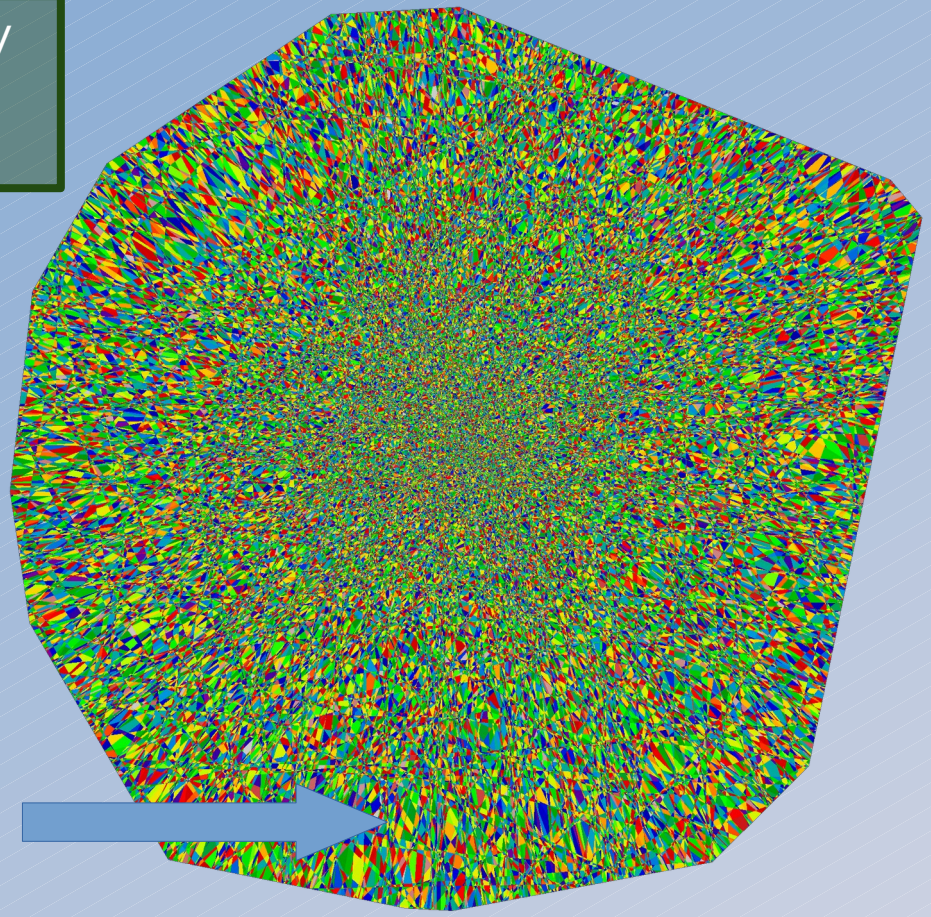

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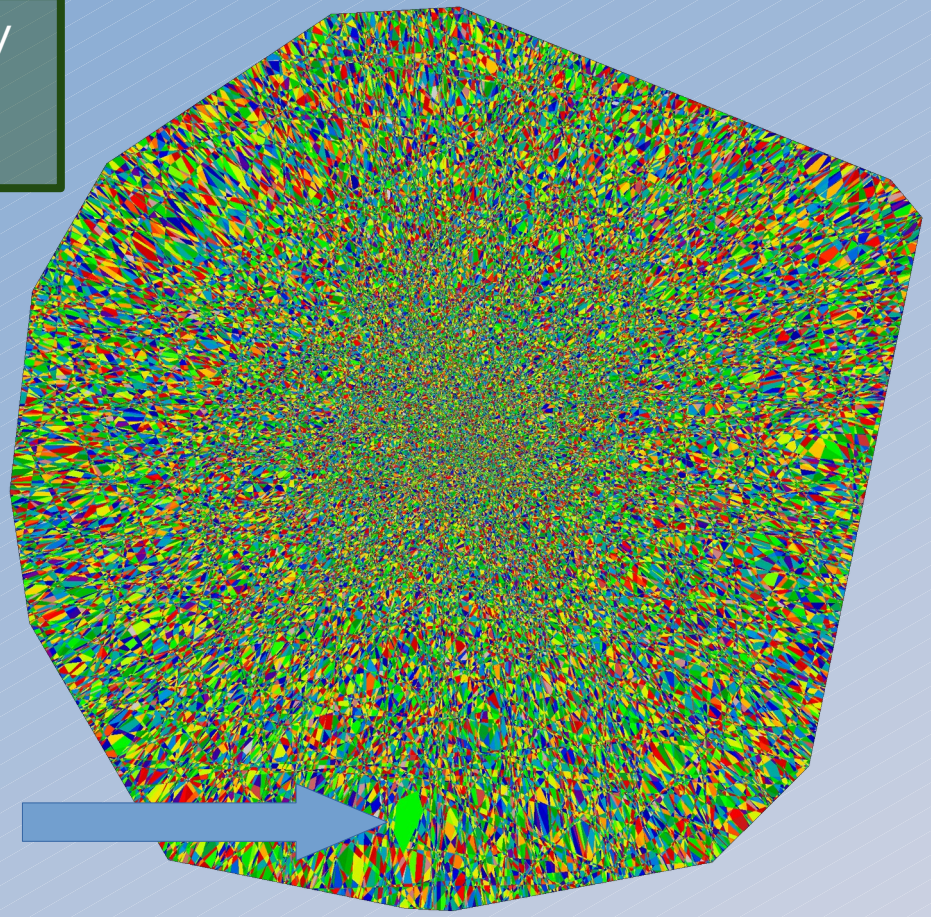

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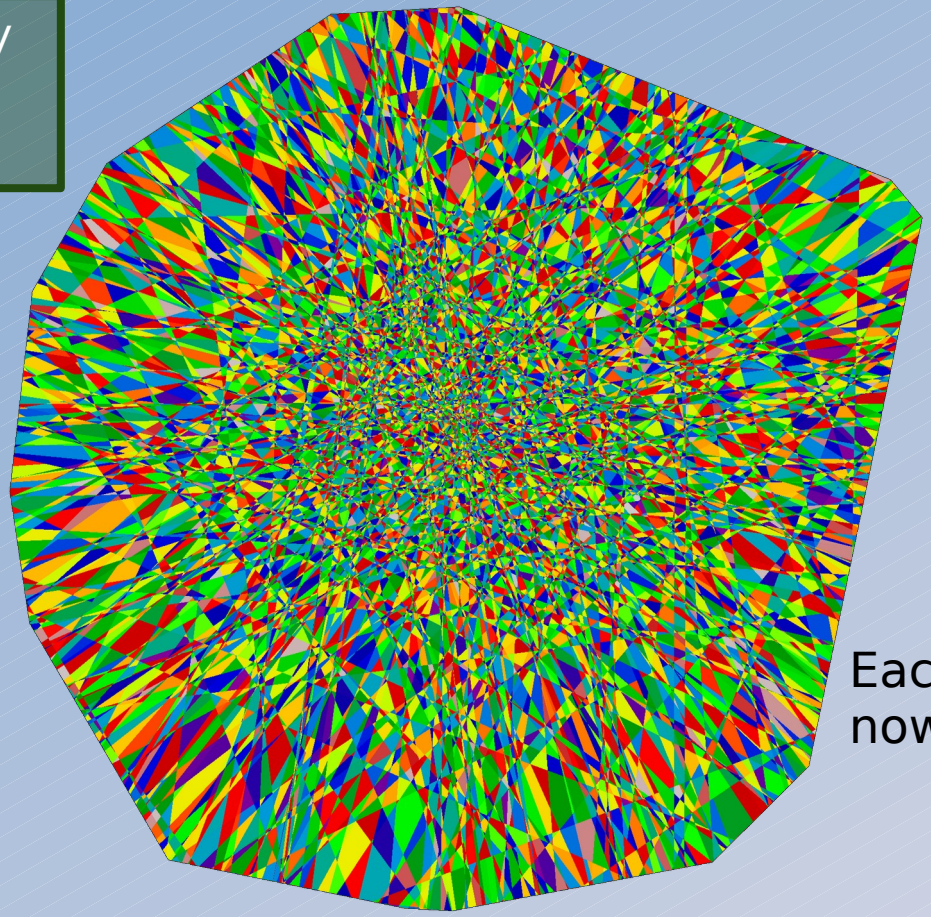

Each colored region is now $\mathcal{K}_{\cup}$ of a different CY.

Computing $\mathcal{K}_{\cup}$

- At small $h^{1,1}$ this can be done by finding all triangulations with the same CY and gluing the $\mathcal{K}_{V_i}$ of the ambient varieties.

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- With a good understanding of the secondary fan, we have managed to extend the Oda-Park/Berglund-Katz-Klemm algorithm so that $\mathcal{K}_{\cup}$ can be computed directly even at large $h^{1,1}$. (Work in progress)

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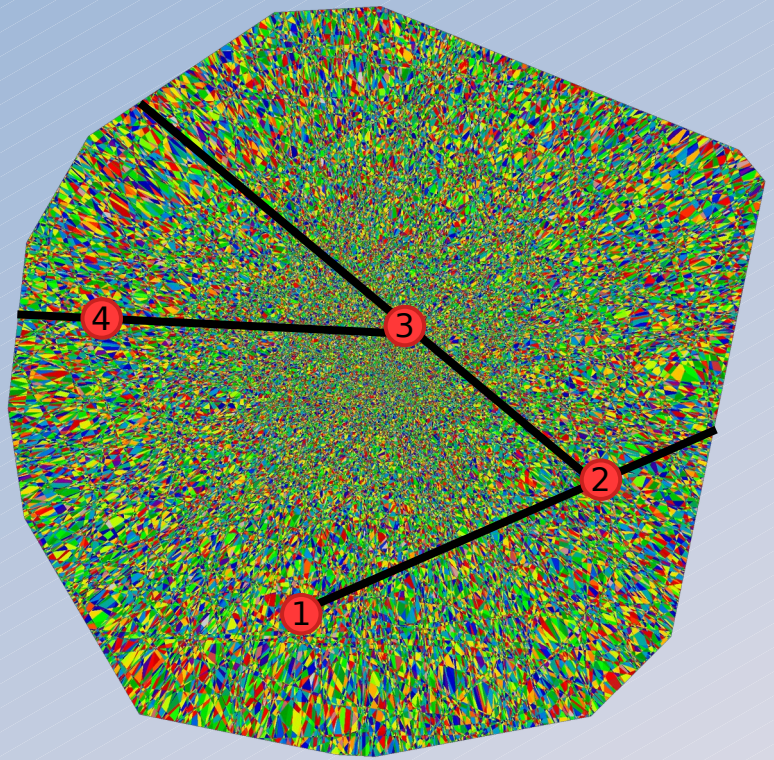
- The way we solve this is by combining random walks in the secondary fan with random bistellar flips. At small $h^{1,1}$ we verify that the sampled distribution is representative of the full set.
  [Demirtas, McAllister, A.R.T., 2008.01730]



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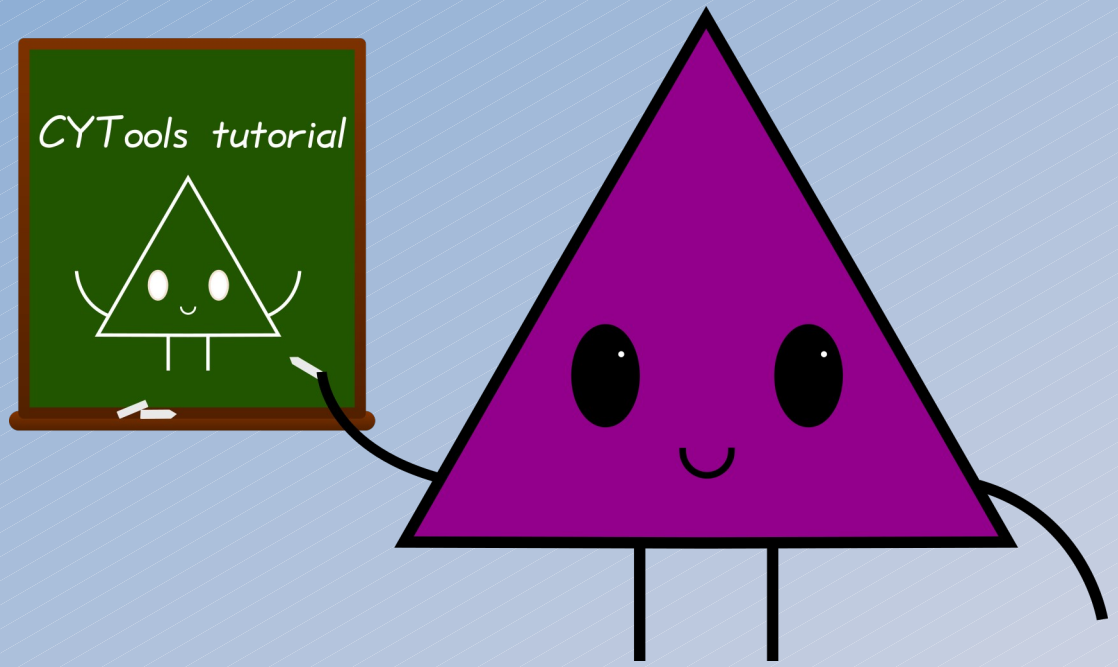
Live demo/tutorial



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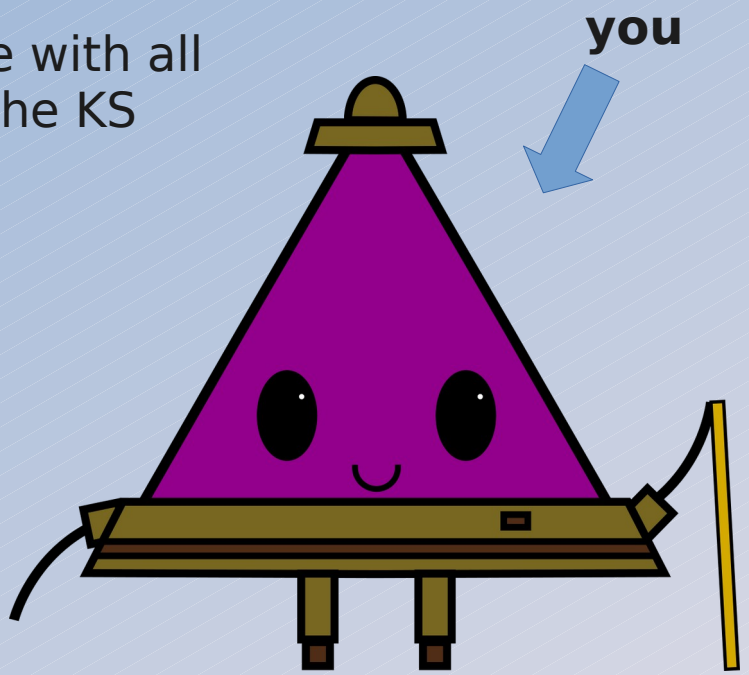

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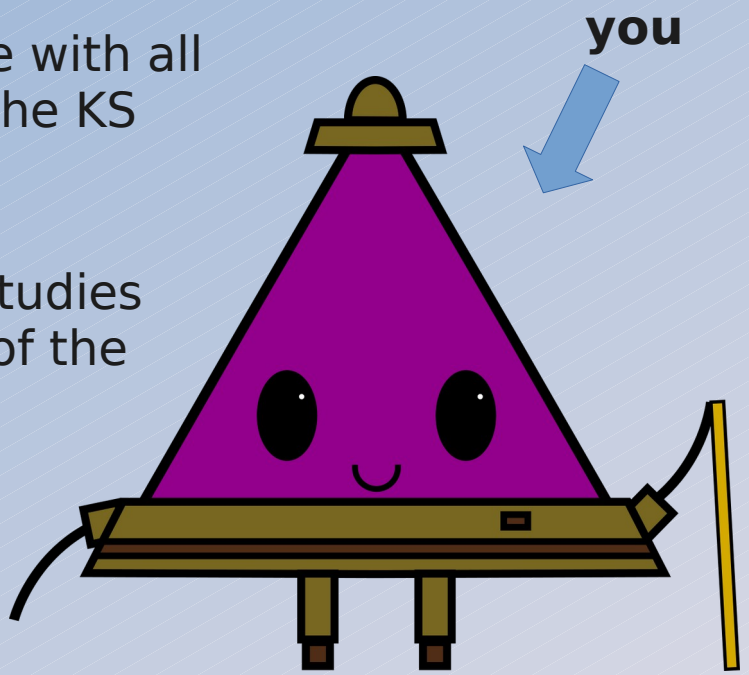

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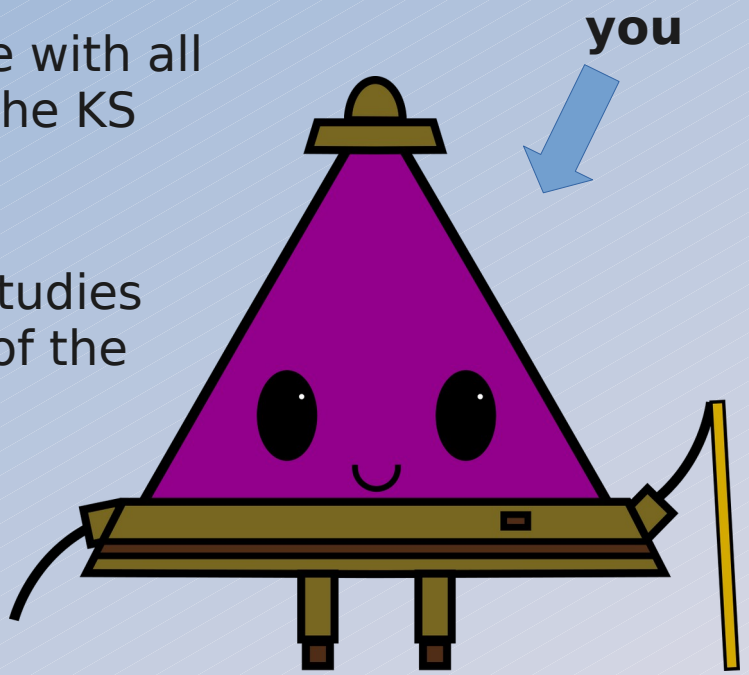
If you want to be a beta tester you can contact either Mehmet Demirtas or me.



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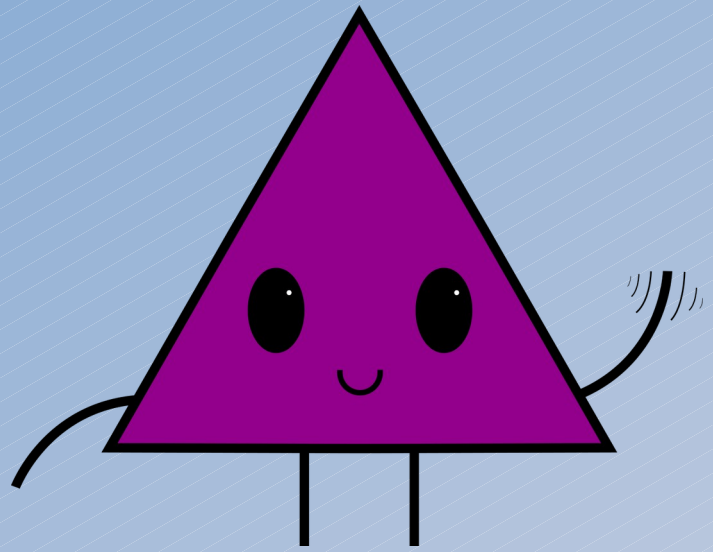

Questions?