If the prime factorization of a natural number n is

$$n = \prod_{i=1}^{k} p_i^{a_i}$$

then the number of divisors of n, $D(n)$, is

$$D(n) = \prod_{i=1}^{k} (a_i + 1)$$

Also, note that all triangle numbers can be represented as $\frac{n(n+1)}{2}$, and n and $n+1$ are coprime. Therefore

$$D\left(\frac{n(n+1)}{2}\right) = \begin{cases} D\left(\frac{n}{2}\right) D(n+1), & \text{if } 2 \mid n \\ D\left(\frac{n+1}{2}\right) D(n) & \text{otherwise} \end{cases}$$