

Let P_n be the set of prime numbers up to n , S_n be the set of all prime summation of n , a naive (and incorrect!) approach to count $|S_n|$ would be

$$|S_n| = \sum_{p \in P_n} |S_{n-p}|$$

Take 10 as example, the summation $3 + 7$ would be counted twice, hence the equation is not correct.

To avoid repeatedly counting the same summation, one would first divide S_n into several disjoint sets $S_{n,i}$, where $S_{n,i}$ represent the set of prime summation of n with largest part i . Summing up the size of all $S_{n,i}$ gives $|S_n|$. Doing so, however, would require $\Theta(n^2)$ space, and thus is suboptimal.

Now, let's look at a different perspective. Instead of counting $S_{n,i}$ for fixed n and different i , we count $S_{n,i}$ for fixed i and different n . In this scenario, only linear space is required, and the program is easier to implement as well.