

Thermal Systems

Saturday, October 24, 2020 8:58 PM



ModSim20
20_IntroT...

Introduction to Thermal Systems

Abstracting Thermal Systems

We've looked at how to abstract and model some epidemiological systems. To do this, we learned more about the system in order to understand its key elements and interdependencies. We then abstracted the system using a stock and flow diagram as a tool for visualizing the elements and interdependencies. Part of this process involved deciding what elements to leave out.

As we move toward abstracting other kinds of physical systems, we will again need to consider their elements and interdependencies, and will also need to make some decisions about what to put in and what to leave out of our model. We'll be looking specifically at thermal systems. One advantage of modeling systems that behave according to accepted physical laws is that it is often possible to make decisions on what factors to neglect through simple back of the envelope calculations that can provide quantitative justification for our abstraction decisions.

For thermal systems, we'll consider how a defined system changes state as a result of processes that affect it. That idea – that a system changes state as a result of processes – is simply describing stocks and flows. Stocks represent the state of the system, and flows represent the processes by which the system changes.

A thermal system interacts with its surroundings (the world around it) through flows of energy. A cup of hot coffee sitting on your desk is a good example. The system (coffee) is losing energy to the surroundings (the air around it) as heat is transferred from the coffee to the air. Another example is a car sitting in a hot parking lot on a sunny day. The air inside the car is gaining energy from the radiation of the sun shining through the car windows. These changes in energy correspond to changes in temperature – clearly the coffee is getting cooler as it loses energy. As we learn how to abstract thermal systems, we'll correlate energy and temperature so both can be tracked in your model.

While it is tempting to think about thermal systems as being all about changes in temperature, we will actually consider the more general question of how a system changes state as a result of various processes (flows of heat, work done on or by the system, phase change in the system, etc.).

Simple Thermal Abstraction Example

Let's think through a simple example to illustrate how we can abstract a thermal system and then create a stock and flow diagram that captures the interactions we have decided to include in our model.

Example: Pot of Water. Imagine we have a pot of water on a stove, and we're interested in knowing what will happen to it in the future. In abstracting this, you might choose to say:

"There is some quantity of water contained in a pan with no cover. The water is in thermal contact with the pan and with the air (free surface of the water only). The bottom of the pan is in thermal contact with a burner (which is on), and the sides of the pan are surrounded by air."

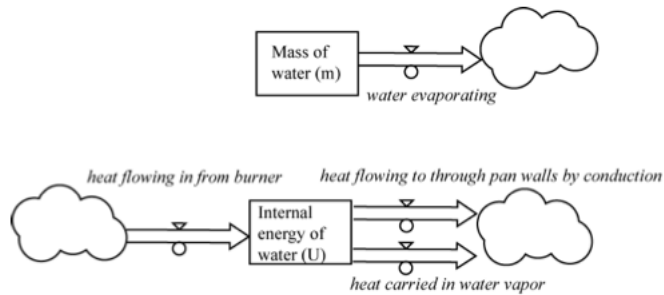
Let's say that we are really interested in what happens to the *water*, so we'll say that the *system* is the water in the pan, and the *surrounding* are everything else (pan, air, burner). (Of course, you could make a different choice here – e.g., you could decide that the system included the pan.)



Once you've decided what your system is, decide what stocks make sense to keep track of. There is a distinct possibility that some mass may leave the system by evaporating (assuming your pan is not covered, and the temperature of the water is allowed to reach the boiling point). So we'll consider mass to be a stock, and the rate of water leaving by evaporation to be a flow.

In order to know whether we do reach the boiling point temperature, it is useful to keep track of how the internal energy of the water changes (as you will find out oh so very soon, the internal energy is a function of temperature). How will the internal energy of the water change? It will increase as heat is transferred from the burner, to the pan, and finally to the water. It will decrease as heat is lost to the pan and then to the surrounding air. Finally, as the mass of the system decreases due to evaporation, the internal energy will also decrease – there will be less water in the system to actually have internal energy.¹ So, a simple stock and flow might look like this (if we left out a whole bunch of variables!):

¹ This is what is known as an *open* system; where mass can cross the system boundary. In a *closed* system, mass cannot cross the system boundary, but heat can.



Conservation tells us that the rate of change of water mass in the pot ($\frac{dm}{dt}$) is equal to the sum of the mass flows into or out of the system,

$$\frac{dm}{dt} = -\dot{m}_{\text{evap}},$$

where \dot{m}_{evap} is the rate of evaporation of water mass. Similarly, the rate of change of internal energy of water in the pot ($\frac{dU}{dt}$, where U is internal energy) is equal to the sum of the energy flows into and out of the system,

$$\frac{dU}{dt} = \dot{E}_{\text{burner}} - \dot{E}_{\text{evap}} - \dot{E}_{\text{cond}},$$

where \dot{E}_{burner} is the rate of energy transfer due to the burner, \dot{E}_{evap} is the rate of energy transfer due to evaporation, and \dot{E}_{cond} is the rate of energy transfer due to conduction to the walls of the pot.

Of course, we've not yet defined anything about how these processes work or how to model them: the flows in this stock and flow diagram are only defined in the most hand-wavy way. But we at least have a general picture of what is going on, and now can begin thinking about models for temperature and heat flow.

Existing Models for Energy Flows in Thermal Systems

One advantage in modeling thermal systems is that many phenomena we observe in these systems already have physical models. For example, there are models, laws and relationships that describe how heat flows through a solid, liquid or gas. In the section above, we abstracted a thermal system, but did not develop the terms in the differential equations much further than simply denoting the energy flows as some symbol (like \dot{E}). In this section, we'll introduce concepts and models that will be useful as you model and simulate simple thermal systems.

State Variables

We've talked about internal energy and mass as possible stocks for thermal systems. The internal energy and mass are all *extensive* state variables, meaning that they are dependent on the *extent*² (or size) of the system. If you decide to analyze a system that is twice as big, the mass will be higher, the volume will be higher, and the internal energy will be higher.

Contrast this with an *intensive* variable, which does not depend on the size of this system. This is a perfect time to discuss temperature, and to highlight that **temperature is not a stock**. This is because temperature is an *intensive* state variable, which means that it does not depend on the size of the system. Doubling the size of the system does not increase the temperature of the system, and it does not make sense to think about temperature "flowing". Of course, the fact that temperature is neither a stock nor a flow doesn't mean it's not important. Intensive state variables you are likely to encounter in thermal systems include:

- *Temperature*, denoted T , has units of Kelvins.³
- *Pressure*, denoted P , has units of Newtons/meter².
- *Density*, denoted ρ , has units of kg/meter³.

Processes for Changing Internal Energy: Heat and Work

The internal energy of a system can change due to heat being added to the system: put a pan on the stove, and when you turn on the flame, you will begin increasing the internal energy of the pan. Internal energy can also be changed by doing work on the system, or by having the system do work on something else. For example, in a car engine, expanding gas pushes up the pistons in the cylinders; this work done by the gas reduces the internal energy of the gas. In general,

$$\Delta U = Q - W$$

where ΔU is the change in the internal energy of the system, Q is the heat that is added, and W is the work done *by the system*.⁴ This is one form of the First Law of Thermodynamics, which says that energy must be conserved. The equation itself says that the internal energy of a system can be changed by heat transfer to or from the system (Q), or by work done on or by the system (W). Note that both heat and work have units of energy (Joules) here.

Heat is (rather circularly) defined as energy that is transferred by means other than doing work; examples of heat flow mechanisms include conduction (if you touch something hot, energy flows into

² This is an excellent mnemonic for remembering the difference between extensive and intensive variables.

³ In the metric system, temperature has units of Celsius. The Kelvin temperature scale is what is known as an absolute temperature scale (meaning that 0 K is the lowest temperature possible.) To convert from Celsius to Kelvin, simply add 273.15.

⁴ The typical sign convention for heat transfer is positive for heat transfer into the system, and negative for heat transfer out the system. Work typically has the opposite sign convention, hence the negative sign in the equation. Ask Jessica Townsend for an easy way to remember this.

you from the hotter object), radiation (e.g., energy radiating onto your skin from the sun as you lie on the beach), convection (energy lost to a surrounding fluid like air or water) and mass transfer (e.g., energy being carried away in evaporating water). Unfortunately this definition of heat tends to be at odds with our typical usage of the word – when we talk about “turning up the heat”, we typically mean that we wish to increase the *rate* of heat transfer. Thus, we often use the word “heat” to refer to a flow, which, while just fine in casual usage, is very wrong indeed in the world of physics and engineering.

Work includes both traditional mechanical work (i.e., exerting a force through a distance – as in moving a piston) and other forms of work (e.g., charging a battery).

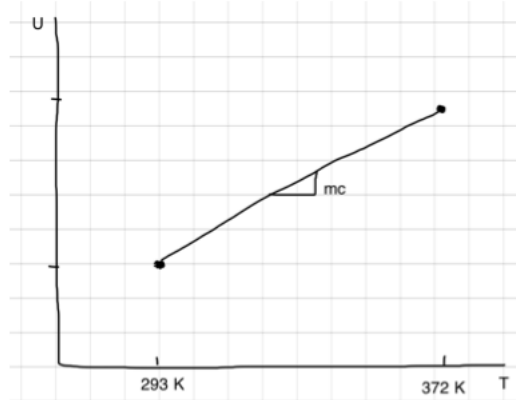
Thermodynamic Relationships

In order to model the flows in a thermal system, we need to deal with a couple of issues: first, *how does temperature relate to internal energy?*, and second, *what are the mechanisms for heat flow?* We deal with these two questions in the following sections.

Internal Energy, Temperature and Specific Heat

Although it might make perfectly good sense to talk about the amount of internal energy in a system, the reality is that this is not a quantity that we measure directly – rather, we tend to measure temperature.

Temperature, it turns out, is closely related to the internal energy of a system. What do you think would happen to the internal energy of a mass of water if heat were added to it? How do you think the temperature would change? Let's do the thought experiment and assume our water starts at room temperature (about 20 C or 293 K) and is heated to just below the boiling point (99 C or 372 K). The temperature is increasing (because we are adding heat) and the internal energy is also increasing. Here's what a plot of internal energy, U versus temperature, T might look like.



Note the slope of the line: the change in the internal energy is *nominally* (this is a model, after all) linearly proportional to the change in temperature:

$$\Delta U = mc\Delta T$$

where m is the mass of the system, c is the *specific heat* (units of J/K-kg), and ΔT is the change in temperature.

Specific heat is the amount of heat needed (in Joules) to raise 1 kg of mass by 1 Kelvin. Note that specific heat, c , is an intensive property. It is defined per unit mass (kg is in the denominator) and therefore is relevant for any size system. It's a property of a material (you can usually just look it up in a table for any material you are working with) and is not a function of mass or volume.

In thermodynamics, we often think about properties of a system both microscopically and macroscopically. Microscopically, temperature is related to the average kinetic energy $\langle KE \rangle$ of a particle in the system. For example, in a monatomic gas (like argon), the average kinetic energy of an atom in the gas is given by

$$\langle KE \rangle = \frac{3}{2} k_B T$$

where k_B is the Boltzmann constant, a fundamental physical constant with the value of $1.380 \times 10^{-23} \frac{\text{m}^2 \text{kg}}{\text{s}^2 \text{K}}$

The internal energy is a function of the average kinetic energy of the particles in the matter, which is a function of temperature, which allows us to finally state (although our thought experiment has already shown this), that:

$$\Delta U \propto \Delta T$$

Mechanisms for Heat Transfer

Just Tell Me the Power

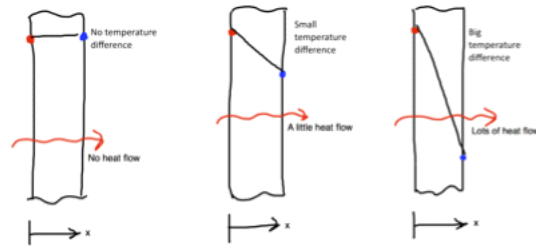
In some situations, heat transfer can be thought of simply as a constant power process. (Power is the amount of energy consumed per unit time. It is measured in Watts (W), where 1 Watt = 1 Joule/second.) For example, if you put a 500 W immersion heater into a cup of water, it's a pretty good model to simply assume that the 500 W heater is delivering 500 W to the water. Thus the rate of change of the internal energy would be:

$$\frac{dU}{dt} = 500 \text{ Watts}$$

Conduction

Conduction is a heat transfer mechanism that involves heat flow through matter, due to differences in temperature between one point and another. In conduction, the physical mechanism (on a microscopic scale) for heat transfer is vibration of particles, as opposed to bulk motion of particles. For example, if you hold a metal rod, and stick one end into a campfire, the atoms in the rod at that end absorb energy from the fire, and start to vibrate more. These vibrating atoms knock into their neighbors, causing them to vibrate, and so forth. Thus, the end that you are holding will eventually get hot as energy is carried down the rod by these vibrations: heat flows from the hot end to the cold end, which leads to the cold end getting warmer.

One common physical system for which the heat conduction model comes in very handy is heat loss through a wall. Let's imagine you build a house of blue foam, with 4 inch thick blue foam walls. How much heat might be lost through a wall? What if the internal and external walls were at the same temperature? Well, the Second Law of Thermodynamics (and perhaps your intuition) tells us that heat flows from hot to cold, so if there is no temperature difference, no heat will flow. What if the internal wall is just a little hotter than the external wall? One might venture to guess that a little bit of heat will flow. What if the internal wall were much hotter than the external wall? A lot of heat will flow. You get the picture.



However, what if I was trying to save money and gave you 2 inch thick blue foam to use instead? You might expect that you would lose *more* heat through the thinner wall. What if I gave you 4 inch thick concrete instead of blue foam? Now, this could get interesting. It would depend on how well concrete conducts heat as compared to blue foam. This is typically captured in another material property called the *thermal conductivity*, k (units of W/m-K).⁵ Finally, you might expect that you would lose more heat through a larger wall (higher area) than a smaller wall.

All of these factors are captured in Fourier's law (which is, of course, a very good model, not a law), which tells us that the heat flow \dot{Q} is proportional to the temperature difference across the wall, ΔT , the thermal conductivity of the wall material k , and the area A of the wall, and inversely proportional to the thickness of the wall d :

$$\dot{Q} = -\frac{kA}{d}\Delta T$$

You might expect a wall with a larger surface area to lose more heat than a wall with a smaller surface area (i.e., bigger houses lose more heat through the walls than smaller houses). Furthermore, the thicker the wall, the more resistant the wall is to letting heat through, and the less heat will travel through the wall.

There is one other interesting thing about Fourier's Law: the minus sign. Heat flows from hot to cold (left to right in the figures above, which is the positive x direction), but the temperature gradient is defined as $T(x+d) - T(x)$ which would be negative in the examples above. Hence the minus sign in Fourier's Law, to indicate that the direction of heat flow is opposite to the temperature gradient.

Convection

The second way to transfer heat is by the actual motion of particles. *Convection* heat transfer occurs between a fluid in motion (usually

⁵The units of thermal conductivity have a "per unit time" built in since 1 Watt = 1 Joule/s. Hence, thermal conductivity relates a temperature difference to a *rate* of heat transfer.

a liquid or gas like water or air) and a bounding surface when the two are at different temperatures. You might imagine wearing shorts on a day with seasonal Arctic breezes (lots of heat transfer between your warm skin and the cold air). Convection can also occur in still air – the fact that the air molecules can move around on their own (unlike the molecules in a solid which just vibrate in place) means that a still (or quiescent ⁶) gas or liquid can be spurred into motion in the presence of a temperature gradient. A hot surface (like a pan just pulled from the oven) heats the air around it, which causes the air to flow upwards, which brings cooler air into contact with the pan, effectively setting up a natural convection current around the pan. At a larger scale, most of the earth's weather is driven by convection processes.

Although it is easy to understand how convection leads to heat transport, there is not one uniform physical model for convective processes, as they often involve complex fluid dynamics. Having said that, there are many situations in which you can build useful models for convective heat loss without getting into the weeds of fluid dynamics. The simplest model is called Newton's Law of Cooling⁷ (again, not a law, but a model):

$$\dot{Q} = hA(T_{\infty} - T_s)$$

where A is the area of the surface exposed to the convective currents, T_{∞} is the temperature of the surrounding fluid (oftentimes the surrounding air), T_s is the temperature of the surface where the heat transfer is happening, and h is known as the heat transfer coefficient, which is an important parameter describing a given convective heat transfer physical system. The simplest approach to determining the heat transfer coefficient is to start with an estimate (using the table below) and then refining that estimate using empirical relationships found in any heat transfer text.

Typical Values of the Convection Heat Transfer Coefficient

Process	$h(W/m^2K)$
Natural convection – Gases	2 – 25
Natural convection – Liquids	50 – 1000
Forced convection – Gases	25 – 250
Forced convection – Liquids	50 – 20,000

Radiation: Heat Transfer by Light

Radiation, as a heat transfer mechanism, is the transport of heat by emitted photons (i.e., light) due to the thermal motion of charged

⁶ This is very fun word to say out loud. Go ahead, try it. Quiescent. Quiescent. Quiescent.

⁷ This is a restatement of the same Law of Cooling in the Robot Book. It looks different because it finds heat flow rather than temperature change. The constant r used in the book relates to h and A , used here.

particles in a body. In contrast with conduction, where the energy is carried by mechanical vibrations, in radiation, the energy is carried by light. For example, when you turn on a space heater or an electric stove, the electrons and protons in the heating element begin to shake around more as the temperature of the element rises, and consequently the heating element begins to glow (emit visible light).

Much of the light that is emitted due to thermal radiation is infrared – too long a wavelength to see – but you can still very much feel thermal radiation (e.g., even before the heating element begins to glow, you can still hold your hand above it and feel the radiated heat).

So long as an object's temperature is above absolute zero (i.e., any object in the universe), it will radiate some amount of energy. The power emitted (P_{emitted}) – and hence the rate of loss of internal energy – is strongly dependent on the temperature of the object, and also depends on the object itself:

$$P_{\text{emitted}} = -\frac{dU}{dt} = e\sigma AT^4$$

where A is area of the object (which we have to think carefully about), T is temperature of the object (in Kelvin), e is the emissivity of the surface of the object, and σ is the Stefan-Boltzmann constant, which has a value of $5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$. Emissivity is a material property, which ranges between near 0 for shiny stuff (e.g., for aluminum foil, $e = 0.04$) and near 1 for dark, dull stuff. Human skin has an emissivity near 1.

Now if you think about it, this sounds a bit weird – everything is giving off energy all the time?? Wouldn't we all be getting colder as a result? The tradeoff, of course, is that things are constantly absorbing radiation from the environment as well. In fact, it turns out (and can be fairly easily proven) that the absorptivity of a surface is equal to its emissivity: if you shine a flashlight with power P on a surface, the time rate of change of the body's internal energy will be given by

$$\frac{dU}{dt} = eP$$

and when a body is in an environment of temperature T_0 , it is absorbing energy at a rate (P_{absorbed}) of

$$P_{\text{absorbed}} = +\frac{dU}{dt} = e\sigma AT_0^4$$

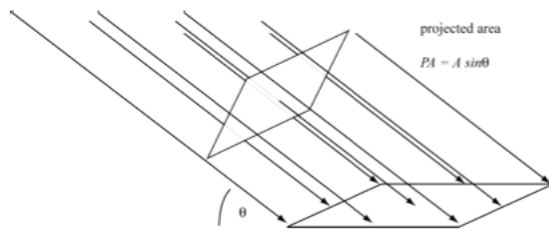
Because of the T^4 dependence, radiation becomes very important in cases where there are high temperatures involved. It also is very important in cases where other mechanisms (e.g., conduction, convection) are not present. For example, in a vacuum, convection and

conduction are zero, since there is no matter to carry the heat, so radiation is the only relevant mechanism.

Solar Radiation. When you are lying in the sun, you are absorbing solar radiation at a pretty constant rate: insolation (incident solar radiation) has a particular value for a given location, weather condition, and time of day – for example, at noon at sea level, insolation is about 1000 W/m^2 . Your body reflects some of the radiation, so that your rate of change in internal energy will be given by:

$$\frac{dU}{dt} = eIA$$

where I is insolation in W/m^2 , A your effective surface area in m^2 and e is the efficiency of absorption. Note in this example that you have to be a bit careful when you define A : it is the projected exposed surface area:



Volumetric Transfer

Consider the case of going outside through a revolving door on a cold day. When you do this, you exchange a volume V of warm interior air for the same volume of cold, nasty exterior air. Thus, this will lead to a change in internal energy of

$$\Delta U = -V\rho c(T_{in} - T_{out})$$

where V is the exchanged volume of air, ρ is the density of the air, c is the specific heat of the air, T_{in} is the temperature of the interior air, and T_{out} is the temperature of the exterior air.

Similarly, most houses lose air continually through poorly sealed windows, vents, etc. If we assume that the rate of exchange is $\frac{dV}{dt}$ m^3/s , the rate of change in the internal energy would be:

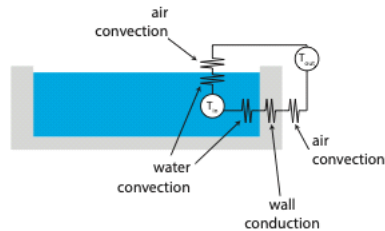
$$\frac{d(\Delta U)}{dt} = -\frac{dV}{dt}\rho c(T_{in} - T_{out})$$

This works as long as the flow is slow enough that the exiting air is at internal temperature.

Multiple Mechanisms

Now if you think about it a bit, it seems like there's something a little fishy with the discussion above about convection versus conduction. Convection carries heat away from a hot surface due to motion of the fluid (gas or liquid) that is in contact with the surface. Conduction carries heat through a solid material. So in fact, if you have a pan full of boiling water, there's *convection* between the water and the inner wall of the pan, *conduction* through the wall of the pan, and then *convection* again between the outer wall of the pan and the atmosphere. At the same time, there's *convection* between the exposed surface of the water and the atmosphere. So how do we combine all of these mechanisms?

It's useful to think about this using a resistance analogy: just as electrical resistors can be in parallel and in series, thermal resistances can also be put in series or in parallel. In this case, the convection between the water and the inner wall of the pan, the conduction through the wall of the pan, and then convection between the outer wall of the pan and the atmosphere are all in series: the heat needs to first go into the wall, then through the wall, and finally into the atmosphere. This series resistance is in parallel with a thermal resistance due to convection between the water and the surface and a thermal resistance due to convection between the surface and the air. In other words, the heat could go through the wall *or* leave from the surface:



To think in this resistance way, we need to define what we mean by a thermal resistance. To do this, we make an analogy between current and heat flow, and between voltage and temperature drop:

$$R\dot{Q} = -\Delta T$$

For conduction, this gives us

$$R_{cond} = \frac{d}{Ak}$$

Similarly, for convection, we get

$$R_{conv} = \frac{1}{hA}$$

Note that this makes physical sense: increasing the area results in more heat flow (less resistance); increasing the heat transfer coefficient results in less resistance; increasing the thickness of a wall results in greater resistance. Similarly, blowing across the water or on the pot would increase the convection, leading to a lower value of R_{conv} .

Once we've defined the thermal resistance for each flow, we can add them the same way we add electrical resistance. When they're in series, like the conductive and convective flows to, through, and away from the wall of the pot, we simply add them up. When they're in parallel, like the flow through the wall of the pot and the flow from the surface of the liquid, we add their reciprocals and take the reciprocal of that:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Remember that resistance is multiplied by \dot{Q} in our equation, so that's where we plug in our total resistance:

$$R_{total}\dot{Q} = -\Delta T$$

We can then algebraically rearrange the expression to solve for \dot{Q} .