Week 3

Sunday, February 21, 2021 6:39 PM

All MATLAB code for this assignment can be found at:
https://github.com/ariporad/QEA/tree/main/Boats/hw3

Week3

Chapter 7

Week 3b: Introduction to the Ducky on a Ramp

Schedule

Note (where's 3a?): Due to the Holiday, we only have class once this week. Thus Week 3a has been *obliterated.*

Important setup notes: Jeff made a ton of materials for this problem for the 2020 offering of QEA. Visit this Google Drive folder to download Matlab code for the ducky problem. You can also view a video introduction to the Ducky problem here.

In the overnight assignment you will be solving a timeless problem that has plagued generations of the world's brightest engineers- what angle does a ducky assume when sitting on an inclined surface?

Figure 7.1: It is important not to confuse a duckling on a ramp (left) with a ducky on a ramp (right). Ducklings have fancy balancing systems comprised of brains and muscles and things. Duckies do not.

You may think this problem is ridiculous, and you are absolutely right, especially since our "ducky" is a simple circular segment. Even though the framing of this problem is intentionally silly, the learning objectives are very real and crucially important to engineering practice. The ability to analyze a problem, draw accurate free body diagrams, and interpret and define forces is foundational to the advanced skills you will learn later in QEA, at Olin, and in your professional careers. *Namely*, you'll use a very similar analysis procedure to design a boat at the end of the module!

In the next two overnight assignments you will analyze this problem and ultimately calculate the equilibrium angle for the ducky sitting on an inclined ramp. You will do this through analysis, not through experiments. You will also predict the maximum ramp angle will cause the ducky to either flip over *or* slide down the ramp.

If we were still at Olin, You would then test your predictions live, in class, in real time. You would not be allowed to perform experiments to develop your angle predictions, so you would need to trust your calculations. Unfortunately, in our current remote working environment, we will not be able to do these experiments as a group.

7.1 Upright Ducky

For this exercise, the "ducky" is defined as a circular segment with radius r and segment height h as shown in Figure 7.2. The circular segment is defined in the reference frame of the ramp with unit vectors $[\hat{x}, \hat{y}]$. The circular segment has unit thickness (a thickness of 1 unit), and is constructed from a uniform density material. The center of mass of the circular segment is located at $[\bar{x}, \bar{y}]$, and is denoted by CG in Figure 7.2.

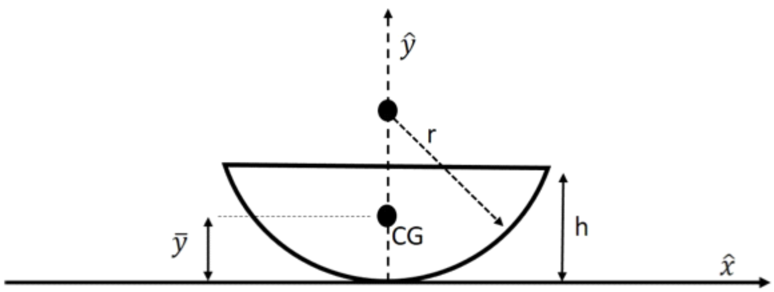


Figure 7.2: The circular segment "ducky."

r

Exercise 7.1

1. How high is the center of the circle above the line of $y = 0$?.
2. Draw a free body diagram for the ducky (circular segment). Label the center of gravity (CG) and point of contact between the segment and the ground. Make sure your forces are applied at one of these two points.

   No, b/c CoG is inline w/ contact pt

   FN CG contact Fg
3. Based on your FBD, will the ducky rotate? Why or why not?

7.2 Rotated Ducky

In order to understand the problem of a ducky on a ramp, we need to draw multiple free body diagrams of relevant scenarios. The goal of these FBDs is to understand the forces and torques acting on the body, and identify if a static equilibrium condition is possible. We'll start by analyzing the case where the ducky is rotated a bit on a flat surface.

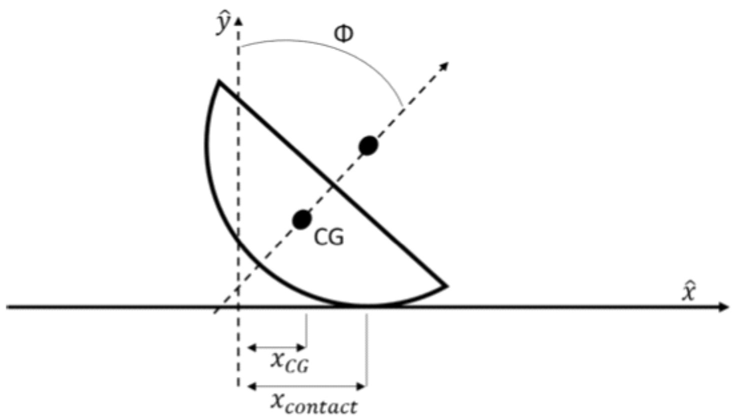


Figure 7.4: The ducky rolled to an angle ϕ relative to the $\hat{y}$ axis.

still r

Exercise 7.2

$(x-c_x)^2+(y-c_y)^2=r^2$

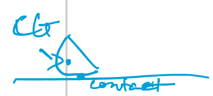
1. What height is the center of the circle (Note: *not* the center of gravity) above the line $y = 0$?
2. Write down (or look up!) the equation for a circle centered about a point c_x, c_y with radius r. You will use this to define the circular segment "ducky".
3. At $\phi = 0$ the center of the circle has $c_x = 0$ and the height you found above. Using those values of c_x, c_y, solve your circle equation for y in terms of x and r. Your equation should have two "branches"; that is, it should have a $\pm$. You will use this expression in the homework assignment.

$(x)^2+(y-r)^2=r^2 \Rightarrow (y-r)^2=r^2-x^2 \Rightarrow$

$y-r=\pm\sqrt{r^2-x^2}$

$y=r\pm\sqrt{r^2-x^2}$

4. Based on the schematic above (Fig. 7.4), which moved to the right *more*: x_{CG} or x_{contact}?
5. Draw a *schematic* of the ducky sitting on flat ground, but rolled to an angle $\phi > 0$ relative to the $\hat{y}$ axis, as shown in Figure 7.4. Your schematic must include both the ducky and the ground. Draw a dot for the CG and the contact point, and label both. (Note that this is *not* a FBD; do not draw any forces on this diagram.)



6. Now draw a FBD of the ducky. Make sure to note the CG and contact point on your FBD, and make sure all forces are applied at one of these two points.

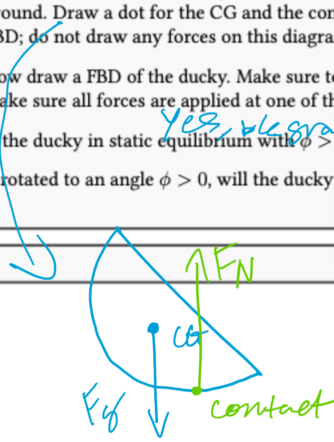
No, FN/Fg apply @ diff points so cause rotation

7. Is the ducky in static equilibrium with $\phi > 0$? Why or why not?

Yes, b/c grav pushes left down, & FN pushes right up

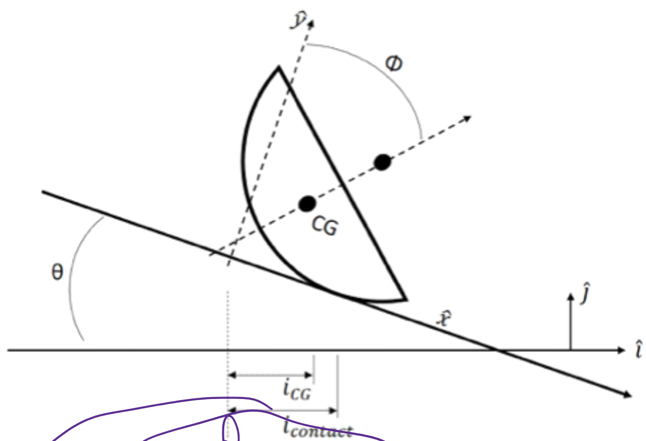
up to a point, then it'd flip over

8. If rotated to an angle $\phi > 0$, will the ducky roll back to $\phi = 0$? How do you know?



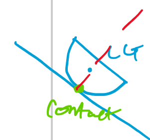
7.3 Ducky on a Ramp

Now, we need to consider the case where the ducky is sitting on a ramp tilted at angle θ. There are two interesting scenarios to consider, the initial condition of the segment before it rolls ($\phi = 0°$), and after the segment has reached an equilibrium condition. A diagram of the ducky on the ramp is shown in Figure 7.7.



Skipped friction

Figure 7.7: The ducky rolled to an angle ϕ on an inclined ramp with angle θ.



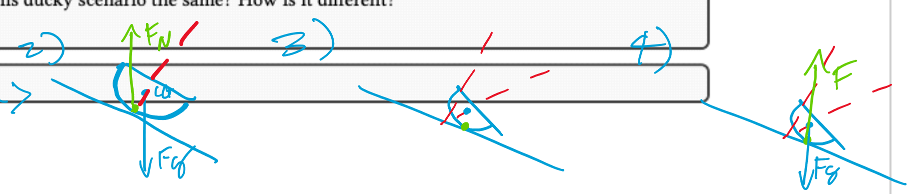
Exercise 7.3

1. Draw a schematic diagram of the ducky on a ramp titled at angle θ and with $\phi = 0$. Your schematic must include both the ducky and the (rotated) ramp. Draw a dot for the CG and the contact point, and label both. (Note that this is *not* a FBD; do not draw any forces on this diagram.)
2. Draw a free body diagram of the ducky on a ramp titled at angle θ and with $\phi = 0$. make sure that all your forces are applied at either the CG or the contact point.
3. Draw a schematic diagram of the ducky on a ramp titled at angle θ and with $\phi > 0$. Your schematic must include both the ducky and the ground. Draw a dot for the CG and the contact point, and label both. (Note that this is *not* a FBD; do not draw any forces on this diagram.)
4. Draw a free body diagram of the ducky on a ramp titled at angle θ and roll angle ϕ. make sure that all your forces are applied at either the CG or the contact point.
5. For the case of $\phi = 0$, is the ducky in static equilibrium? Explain.
6. What conditions would need to exist for the ducky to be in static equilibrium?
7. Compare the scenario where $\theta > 0, \phi > 0$ to the block-on-a-ramp problem from last week. How is this ducky scenario the same? How is it different?

No, F_g & F_N don't ~~add~~ make

The CG & contact pt would have to be vertically aligned, w/o velocity/momentum

Sliding is the same. Block can't rotate around its axis

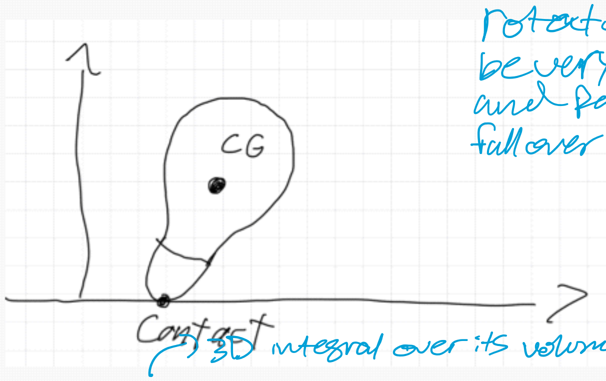


7.4 Key Ingredients

Exercise 7.4

1. Is there anything special about the ducky that causes it to right itself? (aka to not fall over) Compare the ducky against this "top-heavy" body (Fig. 7.12). Does the top-heavy body right itself? Give an explanation in terms of the CG and contact point.

It takes much less rotation for CG to be very ~~off~~-axis and pass the fall over point (AUS)



→ 3D integral over its volume, integrating density

Figure 7.12: A top-heavy body; note how its CG moves compared to its contact point as you rotate the body.

2. The previous question reveals that in order to solve the ducky problem, we'll need to be able to compute the CG and contact point. *Conceptually*, how would you compute the center of mass (CG) of the ducky? (You'll do this numerically in tonight's homework using your boat code.)

3. *Conceptually*, how would you compute the contact point of the ducky? Hint: Above you noted that the center of the circle (not the CG) remains a fixed distance r perpendicular away from the ground (tilted or not).

→ it's always x = Cx, y = 0

Solution 7.1

1. How high is the center of the circle above the line of $y = 0$?.
   (a) The center of the circle is r units above $y = 0$.
2. Draw a free body diagram for the ducky (circular segment). Label the center of gravity (CG) and point of contact between the segment and the ground. Make sure your forces are applied at one of these two points.

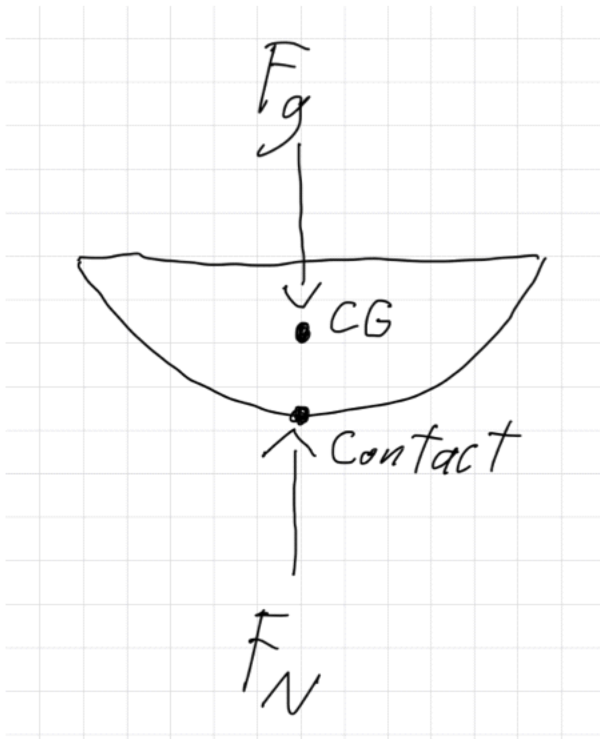


Figure 7.3: Free body diagram for the ducky at $\phi = 0°$.

3. Based on your FBD, will the ducky rotate? Why or why not?
   (a) The ducky will not rotate. Both the force of gravity F_g and normal force F_N act along a line through the center of gravity (aka center of mass). This will not lead to rotation of the body.

Solution 7.2

1. What height is the center of the circle c_y (Note: *not* the center of gravity) above the line $y = 0$?
   (a) The center of the circle is still r units above $y = 0$.

2. Write down (or look up!) the equation for a circle centered about a point c_x, c_y with radius r. You will use this to define the circular segment "ducky".
   (a) The equation for such a circle is $r^2 = (x - c_x)^2 + (y - c_y)^2$.
3. At $\phi = 0$ the center of the circle has $c_x = 0$ and the height you found above. Using those values of c_x, c_y, solve your circle equation for y in terms of x and r. Your equation should have two "branches"; that is, it should have a $\pm$. You will use this expression in the homework assignment.
   (a) The solution is $y = r \pm \sqrt{r^2 - x^2}$.
4. Based on the schematic above (Fig. 7.4), which moved to the right *more*: x_{CG} or x_{contact}?
   (a) The contact point x_{contact} moved further than x_{CG}.
5. Draw a *schematic* of the ducky sitting on flat ground, but rolled to an angle $\phi > 0$ relative to the $\hat{y}$ axis, as shown in Figure 7.4. Your schematic must include both the ducky and the ground. Draw a dot for the CG and the contact point, and label both. (Note that this is *not* a FBD; do not draw any forces on this diagram.)

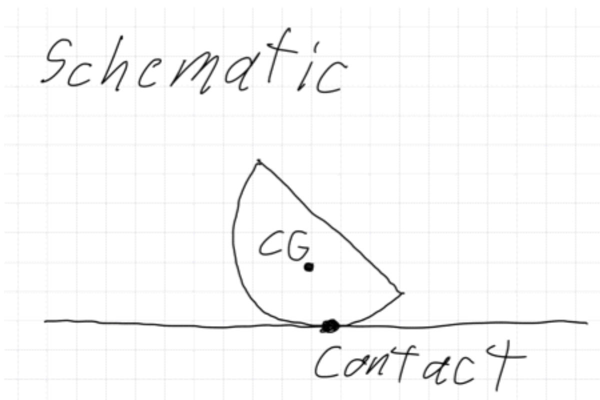


Figure 7.5: Schematic diagram of ducky at an angle of roll ϕ, on flat ground.

6. Now draw a FBD of the ducky. Make sure to note the CG and contact point on your FBD, and make sure all forces are applied at one of these two points.

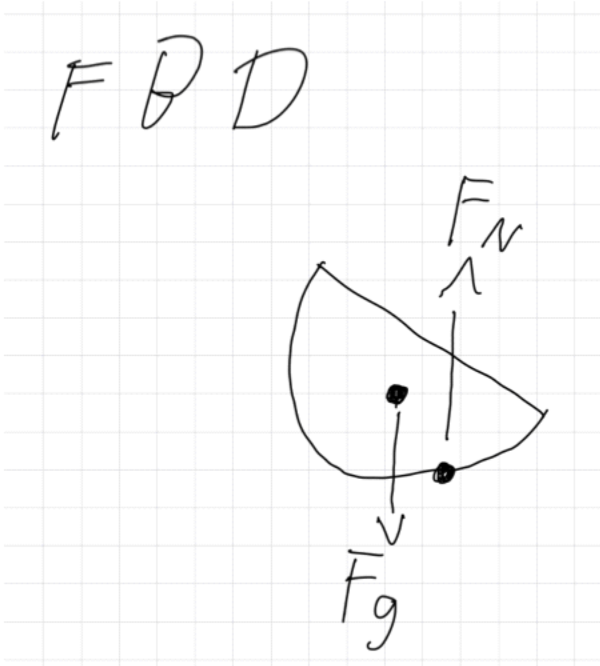


Figure 7.6: Free body diagram for the ducky at an angle of roll ϕ, on flat ground.

7. Is the ducky in static equilibrium with $\phi > 0$? Why or why not?

   (a) The ducky is not in static equilibrium in this condition; the force of gravity and normal force could cancel, but the normal force is no longer applied along a line with the CG. This will apply a torque on the body, causing it to rotate.

8. If rotated to an angle $\phi > 0$, will the ducky roll back to $\phi = 0$? How do you know?

   (a) Trick question—it depends! When $\phi > 0$ but small, a restoring moment is created by the horizontal separation of F_N and F_g. This restoring moment will act to restore ϕ to $\phi = 0$. However if ϕ is too large the CG will move beyond the contact point and cause the ducky to fall over. It will land on its back (and be sad).

Solution 7.3

1. Draw a schematic diagram of the ducky on a ramp titled at angle θ and with $\phi = 0$. Your schematic must include both the ducky and the (rotated) ramp. Draw a dot for the CG and the contact point, and label both. (Note that this is *not* a FBD; do not draw any forces on this diagram.)

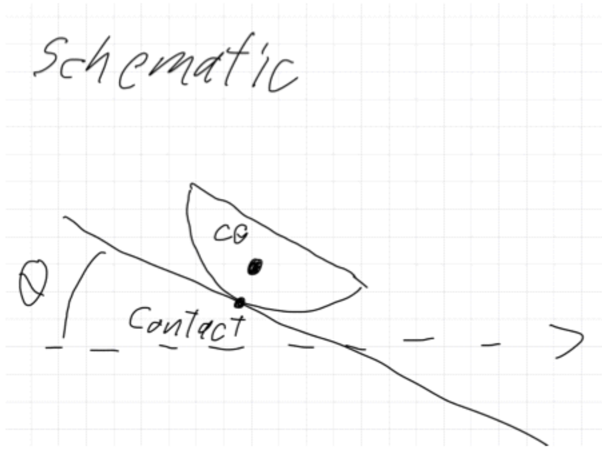


Figure 7.8: Schematic diagram for the ducky on a ramp at angle θ and with $\phi = 0°$.

2. Draw a free body diagram of the ducky on a ramp titled at angle θ and with $\phi = 0$. make sure that all your forces are applied at either the CG or the contact point.

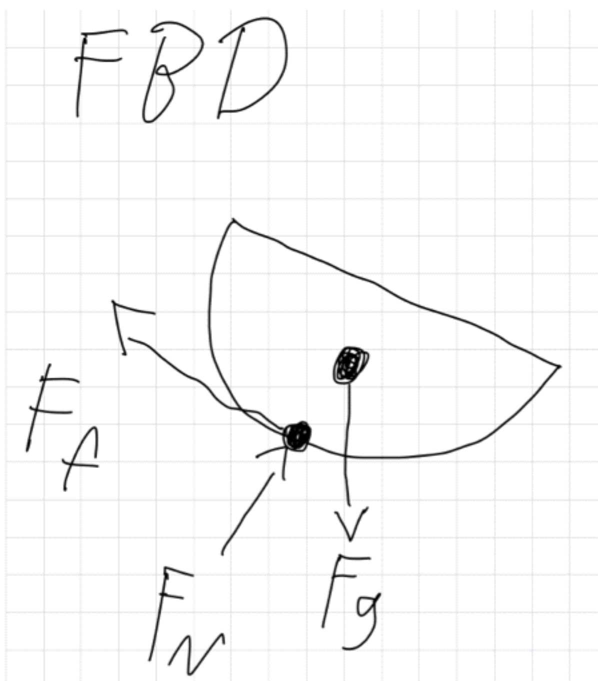


Figure 7.9: Schematic diagram for the ducky on a ramp at angle θ and with $\phi = 0°$.

3. Draw a schematic diagram of the ducky on a ramp titled at angle θ and with $\phi > 0$. Your schematic must include both the ducky and the ground. Draw a dot for the CG and the contact point, and label both. (Note that this is *not* a FBD; do not draw any forces on this diagram.)

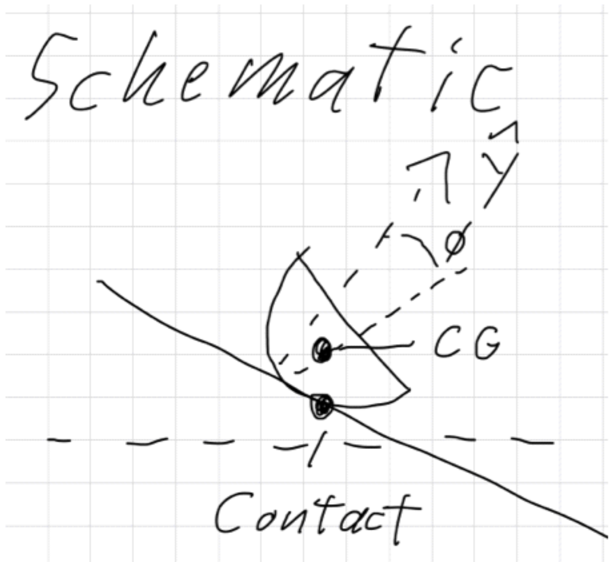


Figure 7.10: Schematic diagram for the ducky on a ramp at angle θ and with $\phi > 0$.

4. Draw a free body diagram of the ducky on a ramp titled at angle θ and roll angle $\phi > 0$. make sure that all your forces are applied at either the CG or the contact point.

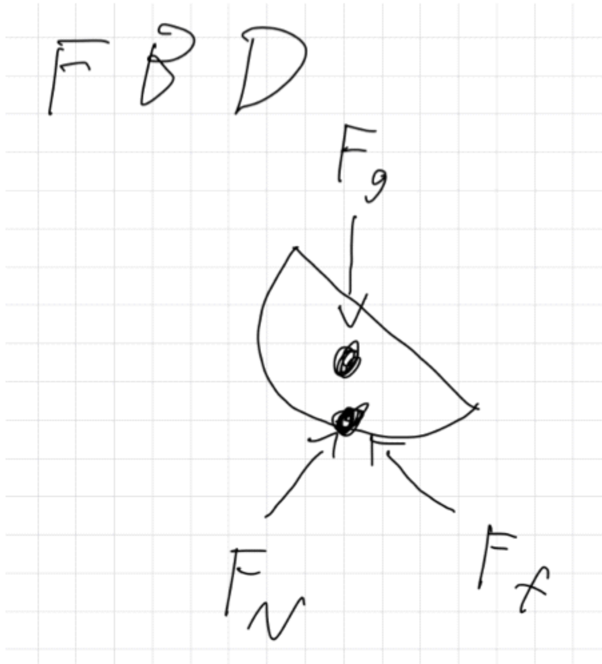


Figure 7.11: Free body diagram for the ducky on a ramp at angle θ and with $\phi > 0$.

5. For the case of $\phi = 0°$, is the ducky in static equilibrium? Explain.

    (a) No, when the ducky is at $\phi = 0°$ and the ramp is at an incline θ, the center of gravity, CG, is no longer directly above the contact point between the ducky and the ramp. Because the gravitational force, F_G acts at CG, and the normal force N acts at the contact point, a clockwise moment is present. The segment might also be out of equilibrium if the frictional force between the ramp and the segment is not sufficient and the segment slips down the ramp.

6. What conditions would need to exist for the ducky to be in static equilibrium?

    (a) From looking at the FBD, we can identify two important criteria. The frictional force most be sufficient to prevent slipping, and the gravitational force must act through the contact point. Put differently, the CG and the contact point must be aligned vertically in the global reference frame, so that there is no moment.

7. Compare the scenario where $\theta > 0, \phi > 0$ to the block-on-a-ramp problem from last week. How is this ducky scenario the same? How is it different?

    (a) Like the box-on-a-ramp problem the ducky experiences gravity, a sloped normal force, and a sloped friction force. Unlike the box, the ducky is free to rotate through an additional angle ϕ.

Solution 7.4

1. Is there anything special about the ducky that causes it to right itself? (aka to not fall over) Compare the ducky against this "top-heavy" body (Fig. 7.12). Does the top-heavy body right itself? Give an explanation in terms of the CG and contact point.

   (a) The geometry of the ducky is such that its contact point moves faster than its CG as it rotates through angles ϕ. This means the contact point automatically moves to apply a "righting moment"—a moment (torque) that "corrects" the rotation of the ducky and moves it back towards $\phi = 0$. The top-heavy body has its contact point move slower than its CG as ϕ increases; this leads to a destabilizing moment that tends to topple the object.

2. The previous question reveals that in order to solve the ducky problem, we'll need to be able to compute the CG and contact point. *Conceptually*, how would you compute the center of mass (CG) of the ducky? (You'll do this numerically in tonight's homework using your boat code from last semester.)

   (a) This would be a double-integral over the area of the ducky; the integral would be a weighted average of position within the ducky, with a weight equal to the material density. Note that there is a separate double-integral for each component x_{CG}, y_{CG}.

3. *Conceptually*, how would you compute the contact point of the ducky? Hint: Above you noted that the center of the circle (not the CG) remains a fixed distance r perpendicular away from the ground (tilted or not).

   (a) At $\theta = 0$ the height of the circle center is fixed at a distance r above the ground and at an angle ϕ from the vertical $\hat{y}$. We can use these two quantities in a triangle construction to arrive at x_{contact}. For cases where $\theta \neq 0$ we could apply a coordinate transform to rotate this point by the ramp angle θ.

Chapter 8

Homework 3: The Ducky Angle

Contents

You're going to make use of your boat center of mass code from last semester in this homework. In case you had trouble with Homework 4 from the Fall, here's a reminder that the solutions are available in Matlab Drive.

8.1 Overview

vertically aligned, so they don't exert a torque

Exercise 8.1

1. In order for the ducky to be in static equilibrium, the center of mass and contact point need to have *some* specified relationship. What is that relationship, and why is it necessary?
   they move in the direction of the roll, cg faster
2. How do the center of mass and contact point tend to change as you increase the roll of the ducky ϕ? Do they change at the *same* rate, or does one move faster than the other?

8.2 Geometric Construction

$r\tan(\Phi) = x$

$y = r$

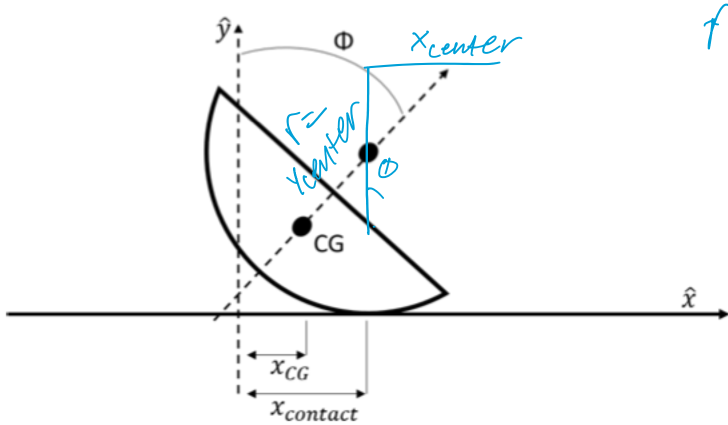


Figure 8.1: The ducky rolled to an angle ϕ relative to the $\hat{y}$ axis.

Exercise 8.2

1. Compute the circle center c_x, c_y as function of ϕ within the x, y coordinate system. *Hint*: Remember from the day assignment that you have *already* determined the value of c_y.

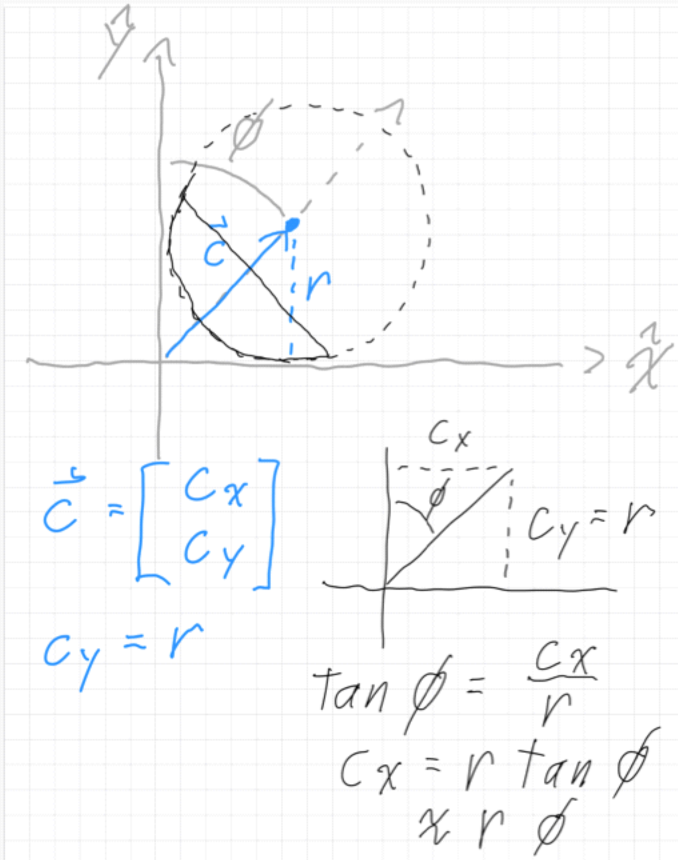


Figure 8.2: Triangle construction to find x-position of circle center.

< Always same

< y=0

2. Study the schematic Figure 8.1 closely: What is the relationship between the contact horizontal position x_{contact} and the center of the circle c_x? What is the y_{contact} value of the contact point?

3. The center of mass remains at a fixed-distance from the center of the circle. Solve for the location of the center of mass x_{CG}, y_{CG} using Figure 8.5 in terms of this fixed distance d. Your solution should be a function of r, ϕ, involve trigonometric functions, and should include the unknown constant d.

$$x = r\tan(\Phi) - d\sin(\Phi)$$

$$y = r - d\cos(\Phi)$$

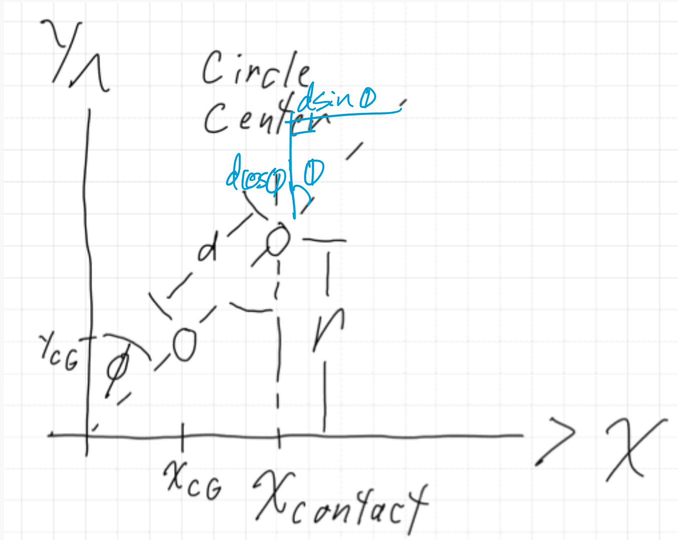


Figure 8.3: Schematic to construct the location of the CG x_{CG}, y_{CG}.

8.3 Center of Gravity

Next, you will use what you learned from the Fall semester to compute the center of mass of the ducky, which will let you solve for the unknown parameter d. We will build on what you did in class and use a discrete approximation to the double-integral to compute x_{CG}, y_{CG}. Remember that solutions to the Fall homework are in Matlab Drive.

As a brief reminder on how we tackled the boat center of mass problem in the fall:

1. We set out to approximate the double-integral $\frac{1}{m}\int\int \vec{r}\rho(\vec{r})dxdy$ as a summation $\frac{1}{m}\sum_{i=1}^{n} \vec{r}_i\rho(\vec{r}_i)\Delta x\Delta y$.
2. We created a grid of points and *filtered* that grid based on two inequalities; a lower curve for the bottom of the boat $y(x)_{\text{boat}} \leq y_i$ and an upper curve for the top of the boat $y_i \leq h$.
3. We computed the summation over *only* those points that met our filter condition (the inequalities above) to approximate the center of mass.

We will apply the same approach to compute the center of mass (CG) in the local $\hat{x}, \hat{y}$ coordinate system, then use that CG to compute the d parameter we introduced above. Once we have the value of d, we'll be well on our way to solving the ducky angle problem!

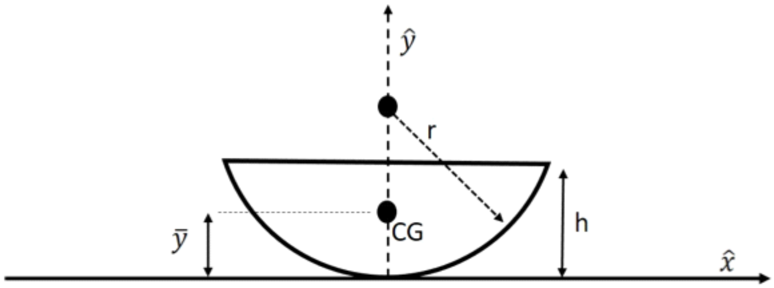


Figure 8.7: The circular segment "ducky." We'll approximate the location of the CG and compute d as the distance between CG and the center of the circle.

Exercise 8.3

1. Remember that the ducky is defined by a circular *segment* with height h (Fig.8.7). What two inequalities in y must points inside the ducky satisfy? (You found the bottom curve in-class for the case $\phi = 0$; use your results from that activity.)

2. Adapt your code from the Fall boat problem (See Matlab Drive for solutions!) to find the CG of the ducky. You will need to adapt the bounds in **`linspace`** and the inequalities for the `insideBoat` vector. Test your code with $r = 6, h = 1$; you should find that $x_{CG} = 0, y_{CG} \approx 0.6$.

   - *Note:* The purpose of this part of the HW is to get you to re-engage with your boat code from last semester. You will use this code again for the boat design project at the end of the module. We recommend making a copy of your boat code (or the solution code) and adapting it for the ducky problem. You should not have to make too many changes to compute the CG for the ducky!

3. Use your CG code to compute d for the case where $r = 6, h = 2$. Remember that d is the distance between the CG and the circle center. What is the value of d?

CG: (0, 1.19)
C: (0, 6)
d: <0, 4.81>

1) $\phi = 0$ $(x)^2 + (y-r)^2 = r^2 \Rightarrow (y-r)^2 = r^2 - x^2 \Rightarrow$

$y - r = \pm\sqrt{r^2-x^2}$
$y = r \pm \sqrt{r^2-x^2}$

$r\tan(\phi) = x_{center}$

$y_{center} = r$

$(x - r\tan(\phi))^2 + (y-r)^2 = r^2 \Rightarrow (y-r)^2 = r^2 - (x - r\tan\phi)^2$

$y - r = \pm\sqrt{r^2-(x-r\tan\phi)^2}$

Desmos shows neg branch is relevant $\Rightarrow$ $y = r \pm \sqrt{r^2-(x-r\tan\phi)^2}$

$y \ge r - \sqrt{r^2-(x-r\tan\phi)^2}$

$y \le h$

8.4 Transformation and Solution

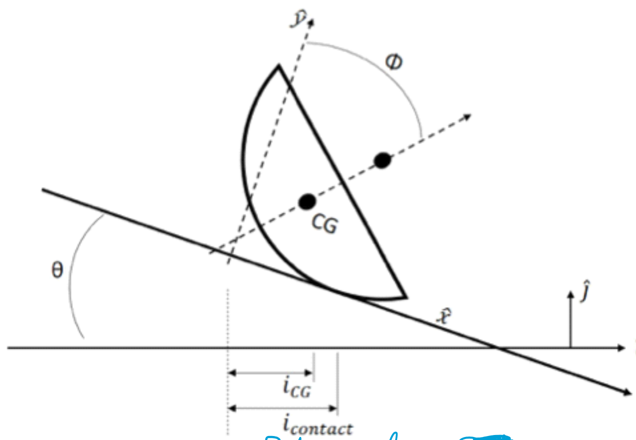


Figure 8.8: The ducky rolled to an angle ϕ on an inclined ramp with angle θ.

$\vec{v}_{CG} = \begin{bmatrix} r\tan\phi - d\cos\phi \\ r - d\sin\phi \end{bmatrix}$ $\vec{v}_{contact} = \begin{bmatrix} r\tan\phi \\ 0 \end{bmatrix}$

$\begin{bmatrix} \cos(-\theta), & -\sin(-\theta) \\ \sin(-\theta), & \cos(-\theta) \end{bmatrix}$

Exercise 8.4

1. Write the *vector* location for the contact point and CG in the *local* x, y coordinate system. Both vectors $\hat{v}_{\text{contact, local}}, \hat{v}_{CG,\text{local}}$ should be two-dimensional vectors.
2. Write down the rotation matrix $R_{\text{ramp}}(\theta)$ that will rotate the local coordinate system *clockwise* by an angle θ. (*Note*: The ordinary rotation matrix rotates a vector *counter-clockwise*. You will need to adapt this formula to rotate in the direction matching Figure 8.8.)
3. In Matlab, write code to use the local-coordinate vectors $\hat{v}_{\text{contact, local}}, \hat{v}_{CG,\text{local}}$ and matrix $R_{\text{ramp}}(\theta)$ to compute the global-coordinate vectors $\hat{v}_{\text{contact, global}}, \hat{v}_{CG,\text{global}}$.
4. In Matlab, write a function `compute_arm(phi)` that has hard-coded values for r, d, θ, takes in the argument ϕ, and returns the value $i_{\text{contact}} - i_{CG}$—the horizontal distance between the contact point and CG in the global coordinate system.

   As always, fzero is broken

5. Compute the angle of the ducky ϕ when $r = 6, h = 2, \theta = 10°$. Keep in mind the *condition* that we discussed above, and remember that the Matlab function **`fzero`** will find the root of a function; the point where $f(x^*) = 0$.

Solution 8.1

1. In order for the ducky to be in static equilibrium, the center of mass and contact point need to have *some* specified relationship. What is that relationship, and why is it necessary?
   (a) For static equilibrium we must have the center of mass and contact point vertically aligned in the *global* coordinate system (note, *not* the ramp coordinate system). In the global $\hat{\imath}, \hat{\jmath}$ coordinates, this corresponds to $i_{CG} = i_{\text{contact}}$. Under this condition the contact forces can align with the center of mass so as to impose a zero net torque. Zero net torque is necessary for static equilibrium.

2. How do the center of mass and contact point tend to change as you increase the roll of the ducky ϕ? Do they change at the *same* rate, or does one move faster than the other?
   (a) Both x_{CG} and x_{contact} tend to increase as you increase ϕ. However x_{contact} tends to increase faster.

Solution 8.2

1. Compute the circle center c_x, c_y as function of ϕ within the x, y coordinate system. *Hint*: Remember from the day assignment that you have *already* determined the value of c_y.
   (a) From the day assignment we know $c_y = r$ is constant. We can find from a simple triangle argument that $c_x = r\tan(\phi) \approx r\phi$. See Figure 8.4 for details.

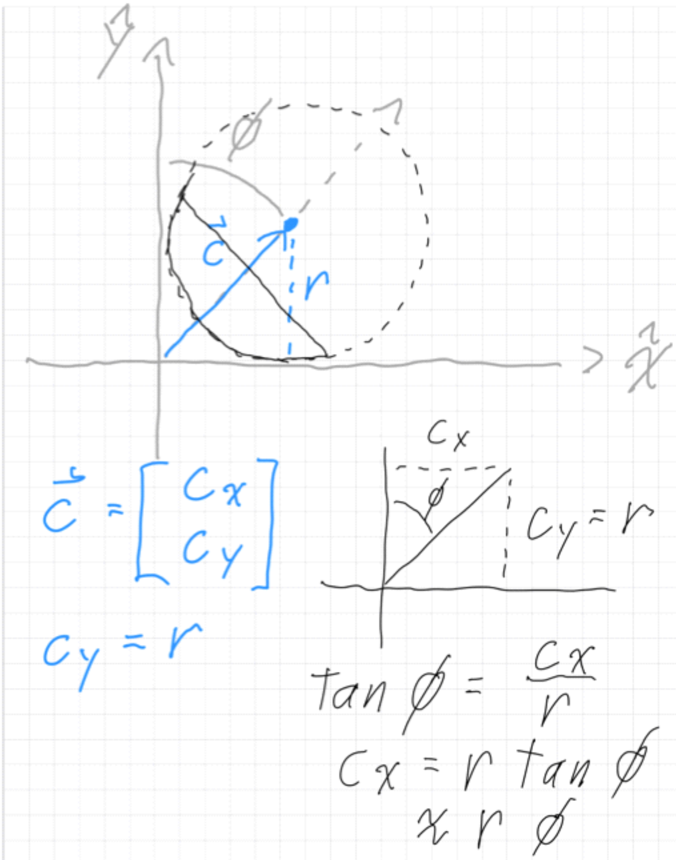


Figure 8.4: Triangle construction to find x-position of circle center.

2. Study the schematic Figure 8.1 closely: What is the relationship between the contact horizontal position x_{contact} and the center of the circle c_x? What is the y_{contact} value of the contact point?

   (a) The contact horizontal is identical to the circle center horizontal position; that is $x_{\text{contact}} = c_x = r\tan(\phi)$. We also have $y_{\text{contact}} = 0$.

3. The center of mass remains at a fixed-distance from the center of the circle. Solve for the location of the center of mass x_{CG}, y_{CG} using Figure 8.5 in terms of this fixed distance d. Your solution should be a function of r, ϕ, involve trigonometric functions, and should include the unknown constant d.

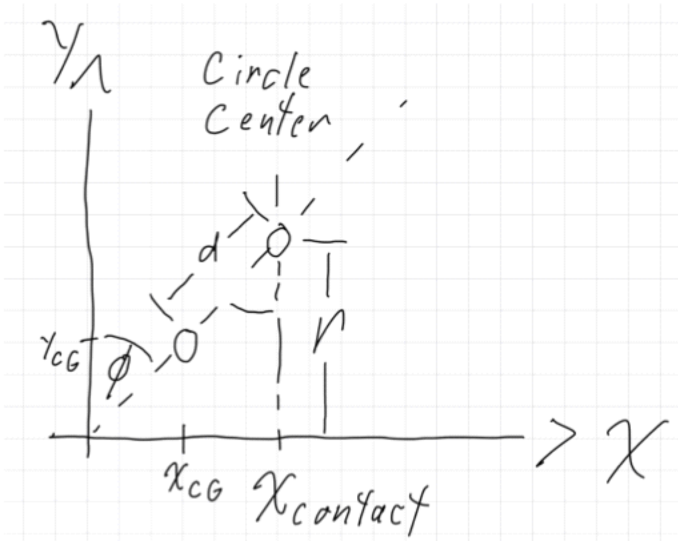


Figure 8.5: Schematic to construct the location of the CG x_{CG}, y_{CG}.

(a) We have $x_{CG} = x_{\text{contact}} - d\sin(\phi) = r\tan(\phi) - d\sin(\phi) \approx (r-d)\phi$ and $y_{CG} = r - d\cos(\phi)$.

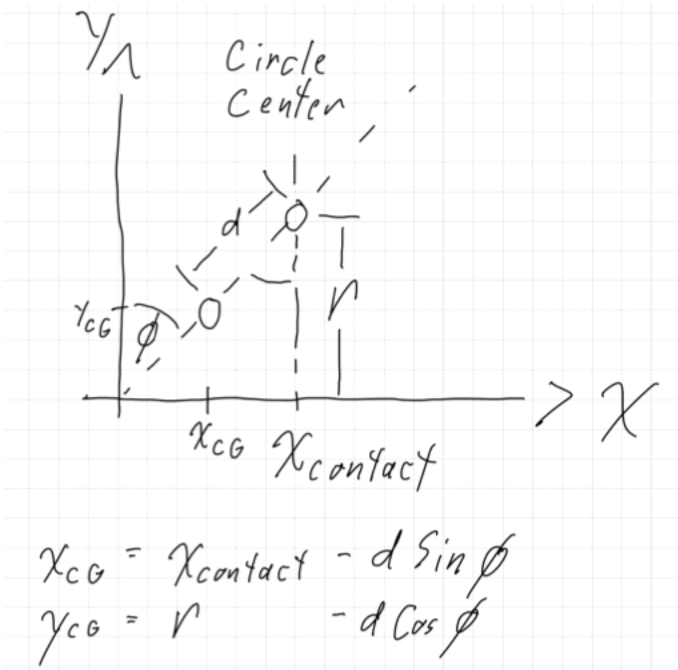


Figure 8.6: Triangle construction to find x and y position of the CG. Note that this is a function of ϕ, and depends on a yet unknown quantity d. You'll solve for d in the next problem.

Solution 8.3

1. Remember that the ducky is defined by a circular *segment* with height h (Fig.8.7). What two inequalities in y must points inside the ducky satisfy? (You found the bottom curve in-class for the case $\phi = 0$; use your results from that activity.)

   (a) Points inside the ducky satisfy $r - \sqrt{r^2 - x^2} \leq y$ and $y \leq h$.

2. Adapt your code from the Fall boat problem (See Matlab Drive for solutions!) to find the CG of the ducky. You will need to adapt the bounds in **`linspace`** and the inequalities for the `insideBoat` vector. Test your code with $r = 6, h = 1$; you should find that $x_{CG} = 0, y_{CG} \approx 0.6$.

   - *Note:* The purpose of this part of the HW is to get you to re-engage with your boat code from last semester. You will use this code again for the boat design project at the end of the module. We recommend making a copy of your boat code (or the solution code) and adapting it for the ducky problem. You should not have to make too many changes to compute the CG for the ducky!

   (a) An implementation of this solution is available in this Matlab Drive folder; see `duck_angle.mlx`.

3. Use your CG code to compute d for the case where $r = 6, h = 2$. Remember that d is the distance between the CG and the circle center. What is the value of d?

(a) For $r = 6, h = 2$, the value is $d \approx 4.8$.

Solution 8.4

1. Write the *vector* location for the contact point and CG in the *local* x, y coordinate system. Both vectors $\hat{v}_{\text{contact, local}}, \hat{v}_{CG,\text{local}}$ should be two-dimensional vectors.

   (a) The solution is

$$\begin{aligned} \hat{v}_{\text{contact, local}} &= \begin{bmatrix} r\tan(\phi) \\ 0 \end{bmatrix} \\ \hat{v}_{CG,\text{local}} &= \begin{bmatrix} r\tan(\phi) - d\sin(\phi) \\ r - d\cos(\phi) \end{bmatrix} \end{aligned} \tag{8.1}$$

2. Write down the rotation matrix $R_{\text{ramp}}(\theta)$ that will rotate the local coordinate system *clockwise* by an angle θ. (*Note*: The ordinary rotation matrix rotates a vector *counter-clockwise*. You will need to adapt this formula to rotate in the direction matching Figure 8.8.)

   (a) The solution is

$$\begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} \tag{8.2}$$

3. In Matlab, write code to use the local-coordinate vectors $\hat{v}_{\text{contact, local}}, \hat{v}_{CG,\text{local}}$ and matrix $R_{\text{ramp}}(\theta)$ to compute the global-coordinate vectors $\hat{v}_{\text{contact, global}}, \hat{v}_{CG,\text{global}}$.

   (a) An implementation of this solution is available in this Matlab Drive folder; see `duck_angle.mlx`.

4. In Matlab, write a function `compute_arm(phi)` that has hard-coded values for r, d, θ, takes in the argument ϕ, and returns the value $i_{\text{contact}} - i_{CG}$—the horizontal distance between the contact point and CG in the global coordinate system.

   (a) An implementation of this solution is available in this Matlab Drive folder; see `duck_angle.mlx`.

5. Compute the angle of the ducky ϕ when $r = 6, h = 2, \theta = 10°$. Keep in mind the *condition* that we discussed above, and remember that the Matlab function **`fzero`** will find the root of a function; the point where $f(x^*) = 0$.

   (a) An implementation of this solution is available in this Matlab Drive folder; see `duck_angle.mlx`. The angle is $\phi \approx 2.5°$.