Week 4

Monday, February 22, 2021 10:31 AM

Safari

Chapter 9

Week 4a: Ducky Tipping Point, Stability

Schedule

This week we're going to use the ducky to understand fundamental ideas about *stability*. The ducky will give us an easy case to introduce stable and unstable equilibria. Ultimately we'll use these ideas to analyze and design stable boats.

9.1 Ducky Tipping Point

As a warmup, let's think about the ducky (Fig. 9.1) once more. Answer the questions below.

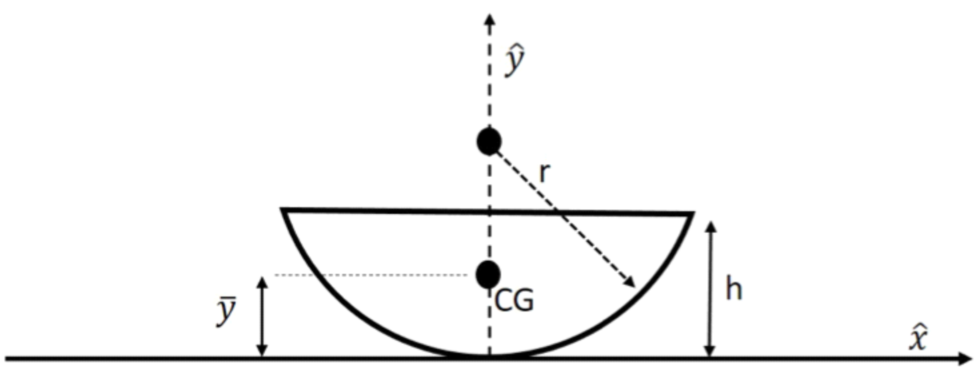
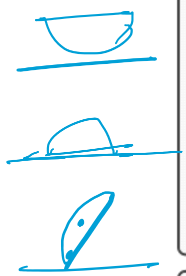


Figure 9.1: Remember the ducky!



Exercise 9.1

1. Remind me briefly: For the ducky to be in static equilibrium, what relationship needs to exist between the contact point and the center of mass? Contact pt & COM must be vertically aligned
2. There are at least three unique angles where the ducky can lay in static equilibrium. What are they? Sketch each case.
3. One of the angles you identified is the "tipping point" of the ducky; the angle beyond which the ducky will tend to fall over. What is happening here—physically—that creates this tipping point? COM goes to other side of contact pt

9.2 Stable vs Unstable Equilibria

In the previous section, we found equilibrium conditions. When these conditions are satisfied, the system has no net forces or torques, and is considered to be in equilibrium. However, equilibria can be of very different natures depending on how they respond to *perturbations* (small disturbances of the system). As an example, consider a ball sitting on a landscape as in Figure 9.2. In both the condition of the ball sitting at the very top of the hill, as well as the case of the ball sitting at the very bottom of a valley, the ball is at static equilibrium: the net force and torque on the ball is zero in both cases. However, the two situations respond very differently to perturbations: the ball at the bottom of the valley is stable to perturbations. The ball at the top of the hill is unstable to perturbations. Naturally for most engineering applications, we would prefer to be at a stable equilibrium than an unstable equilibrium! (Note, the concepts of neutral and metastable equilibria also exist.... feel free to read about these if you are interested).

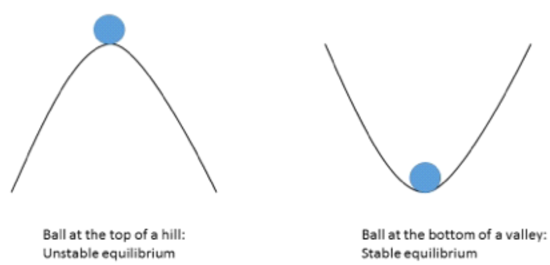


Figure 9.2: Caption

For the following scenarios, determine if the system is stable or unstable.

Exercise 9.2

1. A mass sitting at position x_0 and attached to a spring with force model $F_{\text{spring}} = -k(x - x_0)$. Stable or unstable? stable
2. A person standing still on a completely round floating log. Stable or unstable? unstable
3. A pendulum in a downward position (angle $\theta = 0$) where the torque due to gravity can be written $\tau_g = -mgL\sin(\theta)$ (against the direction of rotation θ). Stable or unstable? stable
4. A pendulum in an upward position (angle $\theta = 180^\circ$) where the torque due to gravity can be written $\tau_g = -mgL\sin(\theta)$ (against the direction of rotation θ). Stable or unstable?
unstable

9.3 Interpreting Graphs for Stability

Above, you probability answered the exercise questions using *intuition*. Next we're going to see how to determine whether an equilibrium is stable or unstable based on analysis. First, for a worked example, see Figure 9.3.

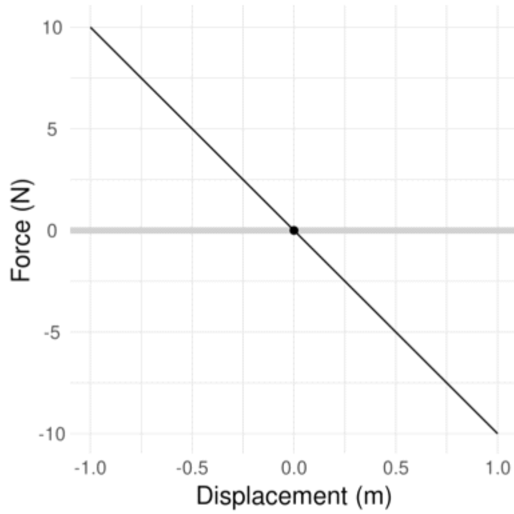


Figure 9.3: Spring force against displacement. Note that as the displacement increases $\delta x = x - x_0 \nearrow$ the force becomes increasingly negative, pushing $\delta x \searrow$. Correspondingly when $\delta x \searrow$ the force increases, pushing $\delta x \nearrow$. When $\delta x = 0$ the force is $F = 0$. It is this effect of the spring force to *restore* the spring to $\delta x = 0$ that makes the equilibrium stable; this system has a *restoring force*.

Next, you'll answer questions about the following torque vs angle graph (Fig. 9.4).

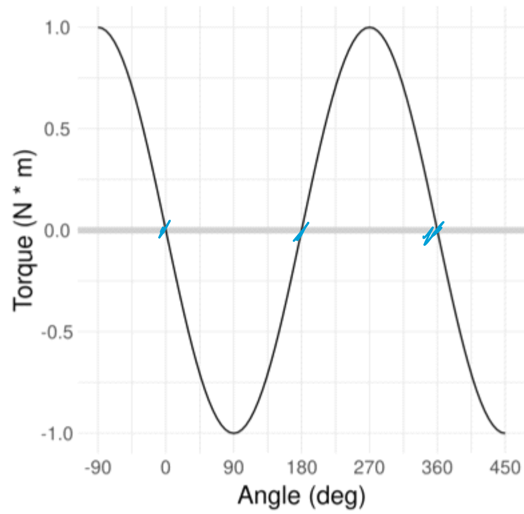


Figure 9.4: Pendulum torque due to gravity against angle. Use this figure to answer the questions below.

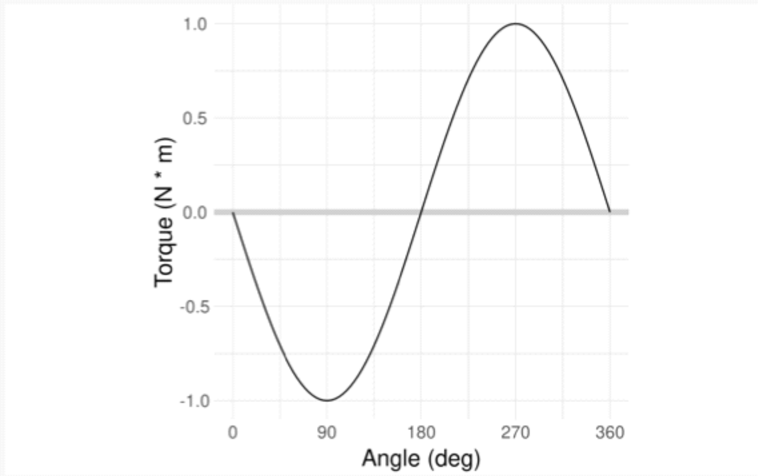
Exercise 9.3

1. Based on Figure 9.4, what angles are equilibria of this system? 0, 180
2. Of the equilibria you identified, which are stable? How do you know this? 0
3. Of the equilibria you identified, which are unstable? How do you know this? 180
4. Look at the following image; how is this graph different from Figure 9.4?

It's only 1 period of sin

2) small disturbances trigger force towards equilibrium

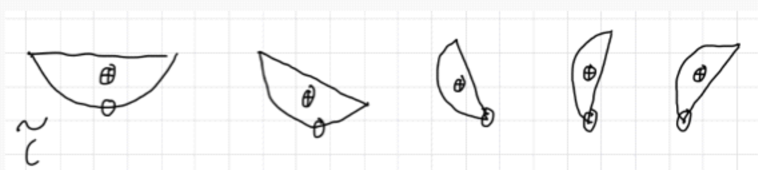
3) small disturbances trigger force away from eq



5. Look at the same image in the previous problem; how would you determine the torque for $\theta < 0$? $0^\circ = 360^\circ$, $-1^\circ = 359^\circ$, etc

Exercise 9.4

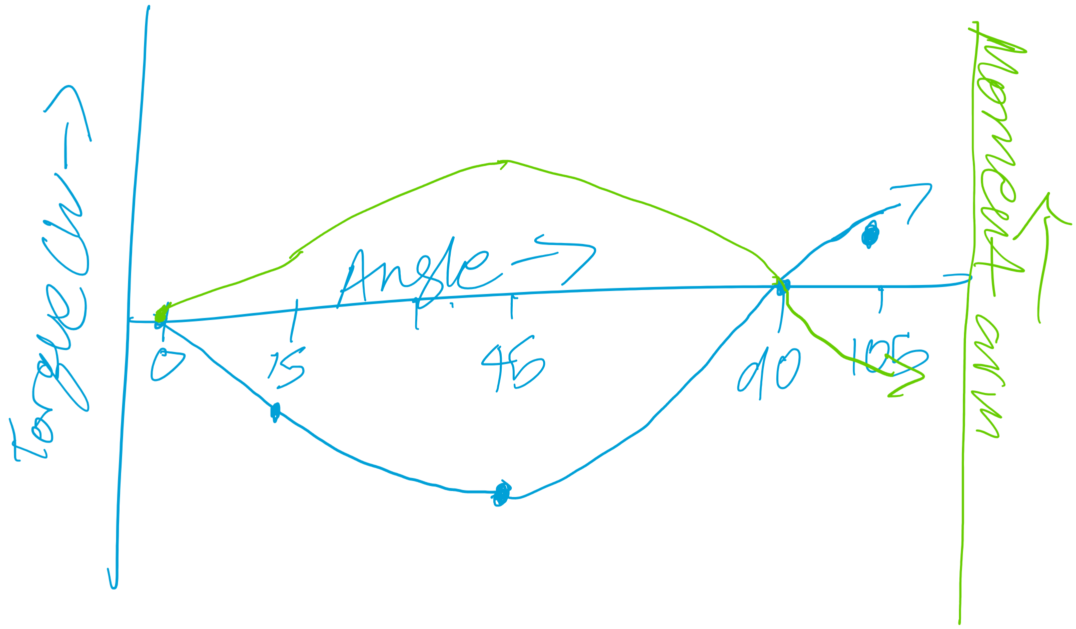
1. Sketch the ducky's net torque vs angle ϕ curve when it is on flat ground $\theta = 0$. (Consider a case where $h < r$, but you need not worry about computing exact torques.) Show a range of angles from $\phi = 0^\circ$ to a bit beyond the ducky tipping point. The following image shows a few suggested "key angles" to consider in your sketch.

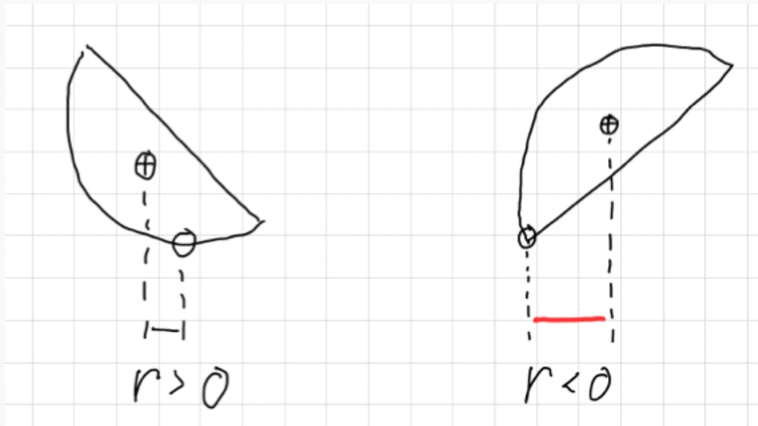


2. Identify the stable and unstable equilibria based on your ducky torque curve. What is the slope of the torque curve at stable points? At unstable points?
3. The moment arm (sometimes called lever arm) is the perpendicular distance between a force applying a torque and its axis of rotation. Draw a moment arm r vs angle curve for the scenario above. Use a sign convention for r based on the diagram below. How do you determine stable vs unstable equilibria with this curve? (How does this differ from using the torque curve?)

It's the same, but mirrored about x axis

2) Stable: 0° Unstable: 90°


Torque cw →
Angle →
0
15
45
90
105
Moment arm

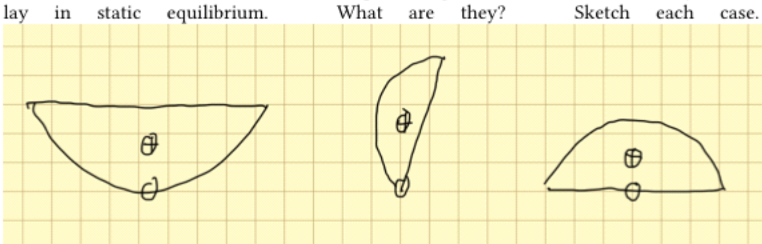

r > 0
r < 0

Solution 9.1

1. Remind me briefly: For the ducky to be in static equilibrium, what relationship needs to exist between the contact point and the center of mass?

   (a) The center of mass (CG) needs to be directly above the contact point, in the global coordinate system.

2. There are at least three unique angles where the ducky can lay in static equilibrium. What are they? Sketch each case.



Unique angles include $\phi = 0°$ (upright), $\phi = 180°$ (overturned), and $\phi > 90°$ (roughly $\phi \approx 100°$ in my sketch) where the ducky is at its tipping point.

3. One of the angles you identified is the "tipping point" of the ducky; the angle beyond which the ducky will tend to fall over. What is happening here—physically—that creates this tipping point?

   (a) The tipping point angle is the angle ϕ where the contact point is the corner of the ducky and the CG is directly above this contact point. Here, further rotation in ϕ will cause the CG to move beyond the contact point, applying a moment that will further rotate the ducky away from equilibrium.

Solution 9.2

1. A mass sitting at position x_0 and attached to a spring with force model $F_{\text{spring}} = -k(x - x_0)$. Stable or unstable?

   (a) Stable.

2. A person standing still on a completely round floating log. Stable or unstable?

   (a) Unstable.

3. A pendulum in a downward position (angle $\theta = 0$) where the torque due to gravity can be written $\tau_g = -mgL\sin(\theta)$ (against the direction of rotation θ). Stable or unstable?

   (a) Stable.

4. A pendulum in an upward position (angle $\theta = 180°$) where the torque due to gravity can be written $\tau_g = -mgL\sin(\theta)$ (against the direction of rotation θ). Stable or unstable?

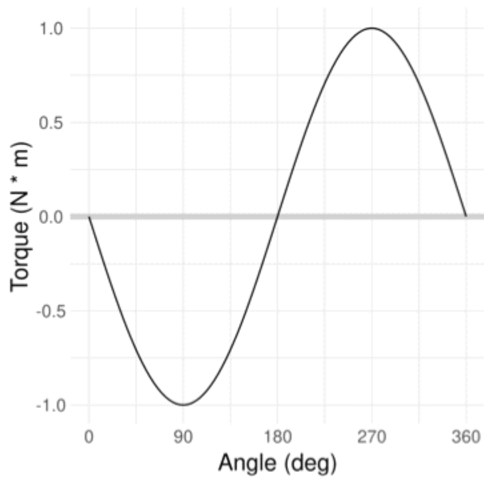
   (a) Unstable.

Solution 9.3

1. Based on Figure 9.4, what angles are equilibria of this system?

   (a) Equilibria include $\theta = 0°, 180°$, plus $360°$ duplicates.

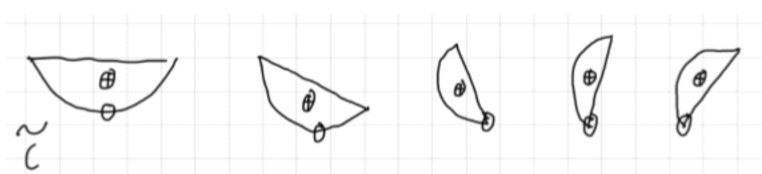
2. Of the equilibria you identified, which are stable? How do you know this?
   (a) The equilibria $\theta = 0^\circ \pm n360^\circ$ are stable. This is because positive perturbations to θ result in a negative torque, and vice versa. Thus these equilibria have a restoring torque.
3. Of the equilibria you identified, which are unstable? How do you know this?
   (a) The equilibria $\theta = 180^\circ \pm n360^\circ$ are unstable. This is because positive perturbations to θ result in a positive torque, and vice versa. Thus these equilibria do not have a restoring torque; the torque instead pushes them "away" from the equilibrium.
4. Look at the following image; how is this graph different from Figure 9.4?



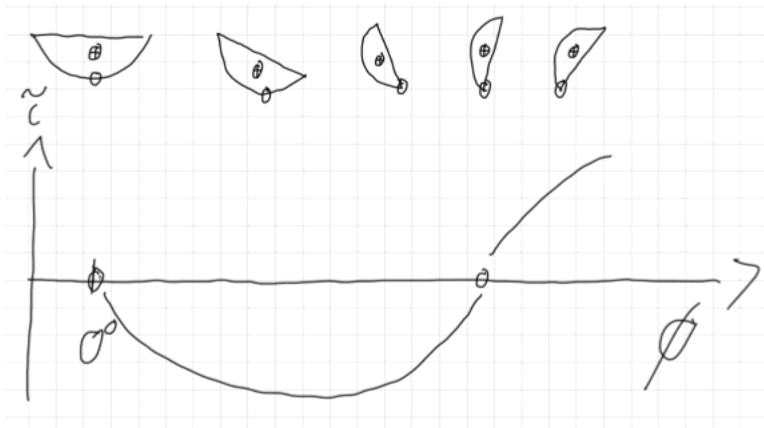
   (a) The graph is the same, except it is restricted to the domain $\theta \in [0, +360^\circ$.
5. Look at the same image in the previous problem; how would you determine the torque for $\theta < 0$?
   (a) The graph is periodic in θ with period 360°; the behavior leading up to $\theta \nearrow 360^\circ$ is identical to the behavior at $\theta \nearrow 0^\circ$.

Solution 9.4

1. Sketch the ducky's net torque vs angle ϕ curve when it is on flat ground $\theta = 0$. (Consider a case where $h < r$, but you need not worry about computing exact torques.) Show a range of angles from $\phi = 0^\circ$ to a bit beyond the ducky tipping point. The following image shows a few suggested "key angles" to consider in your sketch.



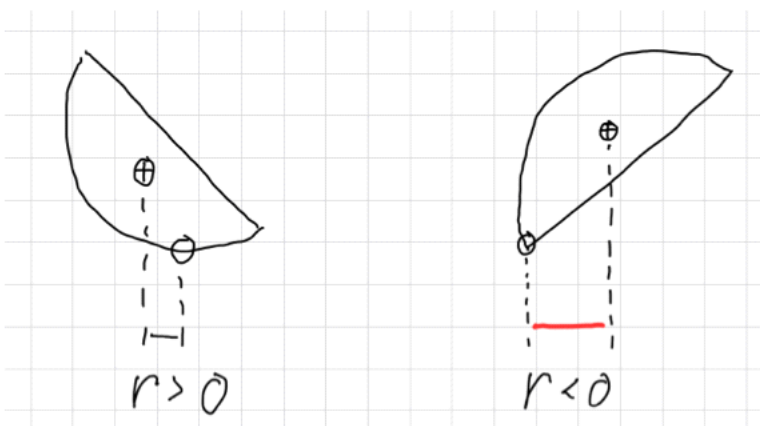
This is my sketch:



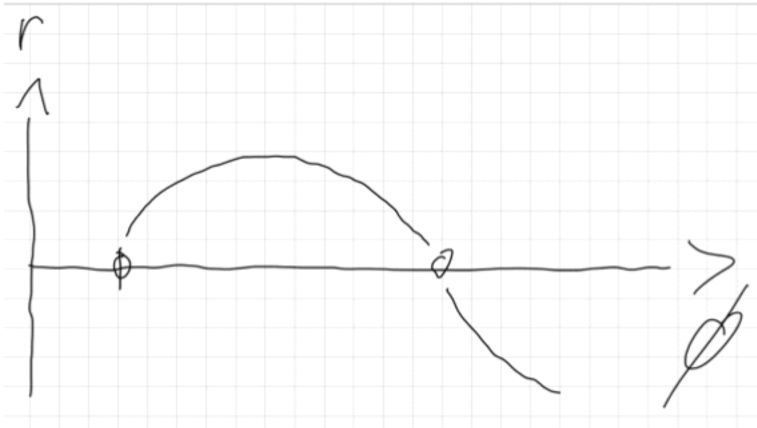
At small $\phi > 0$, the torque τ should be negative; it opposes the direction of motion.

2. Identify the stable and unstable equilibria based on your ducky torque curve. What is the slope of the torque curve at stable points? At unstable points? The angles $\phi = 0^\circ$ and $\phi = 180^\circ$ are stable, these points have a negative slope in the torque curve. The unstable point is a little past $\phi = 90^\circ$; this point has a positive slope on the torque curve.

3. The moment arm (sometimes called lever arm) is the perpendicular distance between a force applying a torque and its axis of rotation. Draw a moment arm r vs angle curve for the scenario above. Use a sign convention for r based on the diagram below. How do you determine stable vs unstable equilibria with this curve? (How does this differ from using the torque curve?)



This is my sketch:



With the moment arm curve, we are looking for a positive slope near the equilibrium, rather than a negative slope as with torque.

Chapter 10

Week 4b: From Ducks to Boats

Schedule

10.1 A prelude to boats

The ducky tipping point introduced us to the concept of stability. Now we'll transition to thinking about the stability of boats. Like with the ducky, there are two "important points" to track for boat stability; the center of mass and the center of buoyancy (COB). For a stable boat, we need the COB to move so as to generate a restoring moment that arrests rolling of the boat. However, this scenario is far more complicated than the duck, as the boat can also *heave*—it can move vertically in the water, which may cause the COB to move. We'll break these complex phenomena down into pieces and understand them bit by bit.

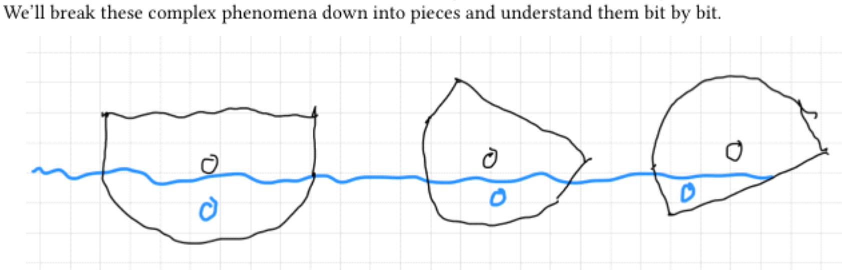


Figure 10.1: An example boat at a few different heel angles. The center of mass is shown as a black circle, and the center of buoyancy is shown as a blue circle.

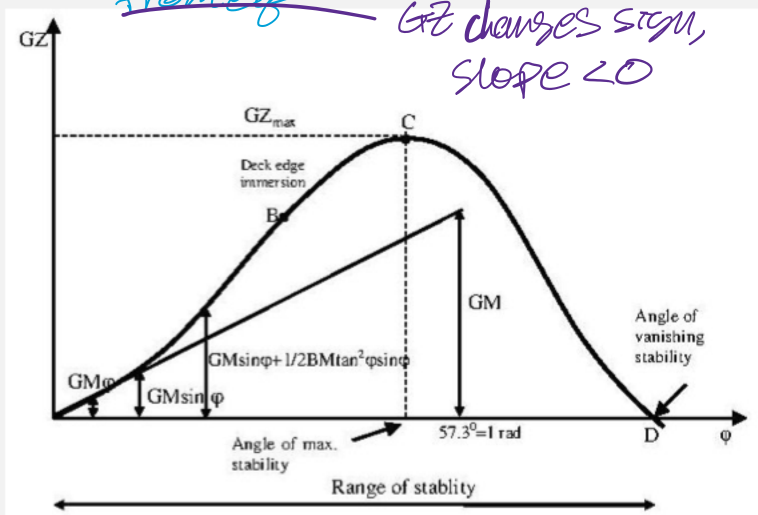
Exercise 10.1

1. As the example boat (Fig. 10.1) rotates away from a zero-heel angle (upright), what tends to happen to the center of buoyancy? It moves (right then left)
2. For the rightmost boat in Figure 10.1, does the torque due to buoyancy tend to restore the boat to a zero-heel angle (upright)? How is this similar to the ducky?

NO, b/c it's passed the AVS (COM is right of COB)

3. For the example boat, somewhere between its upright orientation and its "on-its-back" orientation, there needs to be another point where the boat is in static equilibrium. What needs to be the case in this orientation? How is this "special" orientation different from the upright and "on-its-back" orientations? COM & COB vertically aligned, unstable eq

4. The following is an example righting arm curve for a specific boat hull; it visualizes the moment arm (GZ) against boat heel angle (ϕ). The angle of vanishing stability (AVS) is denoted by the point D. What is the slope of the moment arm curve at the AVS, and why does this make physical sense? ~~slope is max, bc it's the farthest from eq~~ GZ changes sign, slope < 0



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There's a subtle difficulty in computing a righting arm curve, which we'll get to soon. But first we need to learn the *boatwright's first rule*....

10.2 Boatwright's First Rule

There's this classic Olin story about a design review involving Dave Barrett. A group of students had designed a boat and planned to load it with all kinds of fancy equipment. The students were excitedly describing all the fancy stuff they would do with their fancy equipment, when someone—a first-year—asked "You designed a boat—how do you know it will float?"

The students replied "It's a boat. Of course it will float." Sensing a problem, Dave dug in, "No, that's a good question. How do you know it will float? Have you done the math?"

Sheepishly, the students admitted that no, they had not done the simple boat math to see if it would float. To their horror, when they did the math they realized their boat could not possibly carry all the fancy equipment they planned to load it with. Their boat would *not* float.

This leads us to the first rule of boats: You have to answer the question:

Will it float?

In the next exercise, you'll remind yourself of the conditions necessary for a boat to float, and some considerations about boat materials.

density of water: ~ 1000 kg/m3

Exercise 10.2

1. Both the ducky and a boat are subject to gravity. For both cases, what *physical* mechanism supplies the vertical force to counter gravity? Bouyancy
2. What must be true about an object in order for it to float? Bouyancy ≥ Gravity
3. The *displacement ratio* is the ratio of the mass of a boat over the maximum mass of water it can displace. What must be true about the displacement ratio of a boat in order for it to float? $\frac{M_{boat}}{P_{water}V_{boat}} < 1$
4. Suppose you make a "boat" out of solid steel (with no gaps inside). Can this boat *possibly* float? Why or why not? NO, steel is denser than water
5. Suppose you make a boat out of steel, but allow for air gaps inside the boat (e.g. like Figure 10.2, but with a hollow hull under the water). What would you need to do to ensure a boat made from this material will float? $\frac{M_{boat}}{P_{water}V_{boat}} < 1$
6. What is the density of wood? Why is this advantageous for building boats? 0.6 - 0.9 $\frac{kg}{m^3}$ It's very low, so displacement ratio can be low
7. For the boat project, you will design your boat to be printed with a plastic filament with density $\approx 1250\text{kg}/m^3$. What ramifications does this have for your boat design? We'll need air gaps to make it float

Making our boat float is *important*. But we want to go beyond just building a boat that floats: We want to build a boat that is *stable*. In order to do this, we'll need to answer a related question:

What is the *waterline* of the hull in static equilibrium?

This question introduces some new concepts: *Heave* is vertical motion of a boat. In static equilibrium a boat will tend not to heave: In equilibrium the distance between the waterline and the bottom of the hull is called *draft*, as seen in Figure 10.2. We call the line the water makes across a section of the hull a *waterline*.

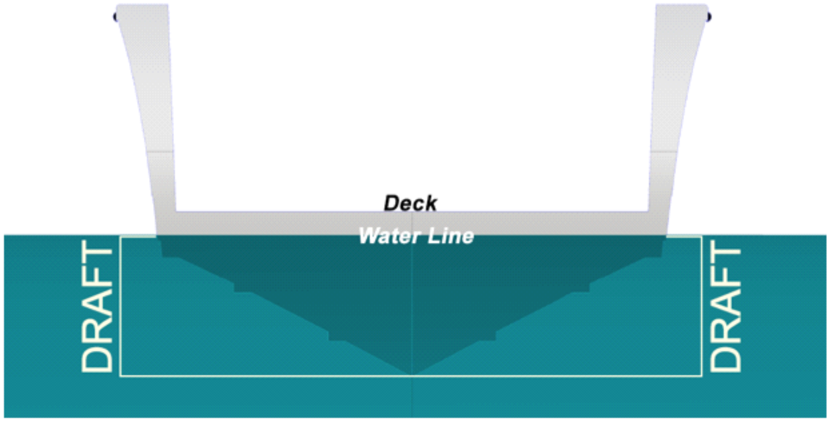


Figure 10.2: Draft of a vessel. From seabornboats.com

Why are these concepts important? As we'll see, in order to remain in (vertical) static equilibrium, the boat must heave as it rotates. This complicates the computation of the righting arm curve. We'll need to understand this complexity if we want to compute the AVS of a boat hull.

Consider the following shapes as the cross-section of a boat; that is, the boat is a prism extruded out-of-the-page with the given shape. Answer the following questions.

CoM

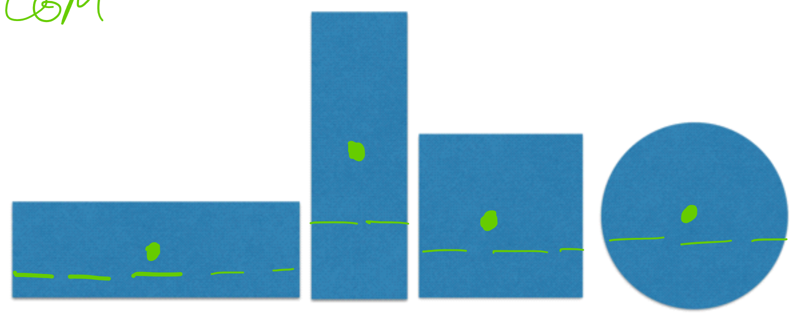


Figure 10.3: Simple extruded (out-of-the-page) hull shapes made out of solid wood with density $\rho_{\text{boat}} = \frac{1}{4}\rho_{\text{water}}$.

Exercise 10.3

1. Sketch in the center of mass (CG) for each boat cross-section.
2. Sketch each boat at its upright heel and at equilibrium waterline. Keep in mind that $\rho_{\text{boat}} = \frac{1}{4}\rho_{\text{water}}$, and each boat is a simple extrusion. How do you know the waterline is at that level?
3. Sketch the wide boat (first from left) and tall boat (second from left) at an angle of about $45°$. Draw a waterline at the approximate equilibrium level, and depict the center of mass and center of buoyancy. How did you determine the equilibrium waterline? Which of the boats is stable about its upright heel? Which is unstable? 1 is stable, 2 3 n 4
4. Next, assume each boat has enough ballast so that its center of mass is at its bottom surface. Sketch each boat cross-section again with its new center of mass.
5. For the new ballast boats, sketch each at its upright heel and at equilibrium waterline. Assume that each boat's total weight is unchanged by the addition of ballast (i.e. we've changed the distribution of material to keep the total mass the same).
6. For the new ballast boats, sketch the wide boat (first from left) and tall boat (second from left) at an angle of about $45°$. Draw a waterline at the approximate equilibrium waterline, and depict the center of mass and center of buoyancy. Which of the boats are stable in their upright position? Which are unstable? Neither are are stable

b/c $P_b = \frac{1}{4} P_w$, only 1/4th the water needs to be displaced

10.3 Righting Arm Curves

We now have the language and concepts to discuss the righting arm curve *properly*: For a given boat, the righting arm curve is a plot of the moment arm due to the center of buoyancy, at a given heel angle ϕ, *assuming the ship is at its equilibrium waterline.* Thus, to construct the righting arm curve, we have to follow a multi-step process:

1. Rotate our boat to a desired heel angle ϕ.
2. Determine the waterline necessary for vertical static equilibrium (where weight equals buoyant force) at the given heel ϕ.
3. Compute the righting arm for the given heel ϕ.
4. Repeat for different heel values ϕ until satisfied.

As we've seen above, the righting moment curve allows us to determine the AVS of a given boat design. In the homework you will build the computational tools necessary to compute a righting moment curve

Exercise 10.4

1. Sketch a righting arm curve for the 'wide' boat. What is the AVS for this shape?
2. Sketch a righting arm curve for the 'tall' boat. Compare it with your curve for the 'wide' boat.

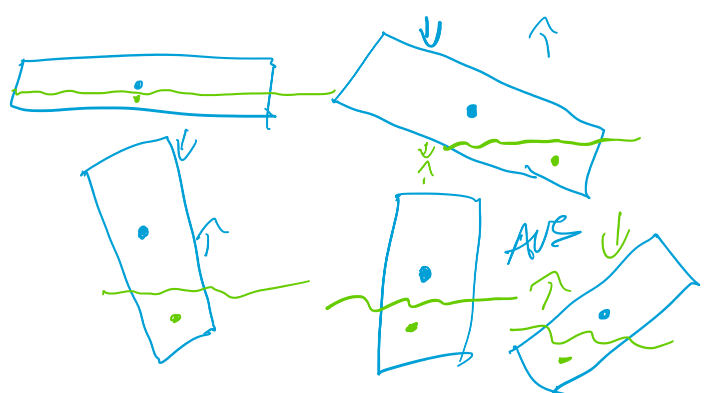
10.4 (Optional) Challenge Problem

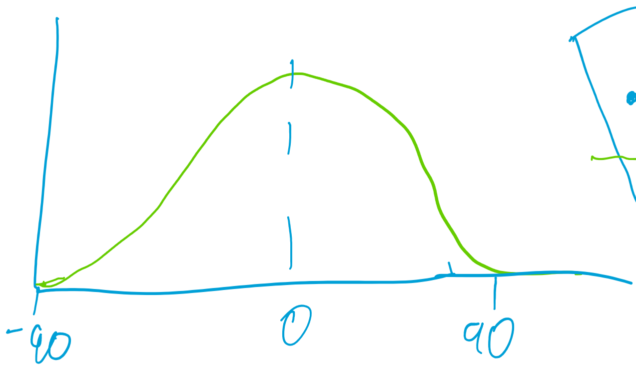
If you're looking for a challenge, try the following exercise!

Exercise 10.5

1. (Optional) Sketch a righting arm curve for the box-shaped boat with displacement ratio 0.25. *Note:* This case is very tricky!
2. (Optional) Draw the box-shaped boat at its stable heel angle.

10.4.1) wide





Tall is the same boat rotated 90°

Solution 10.1

1. As the example boat (Fig. 10.1) rotates away from a zero-heel angle (upright), what tends to happen to the center of buoyancy?

   (a) The center of buoyancy tends to move horizontally, so as to apply a torque to the boat.

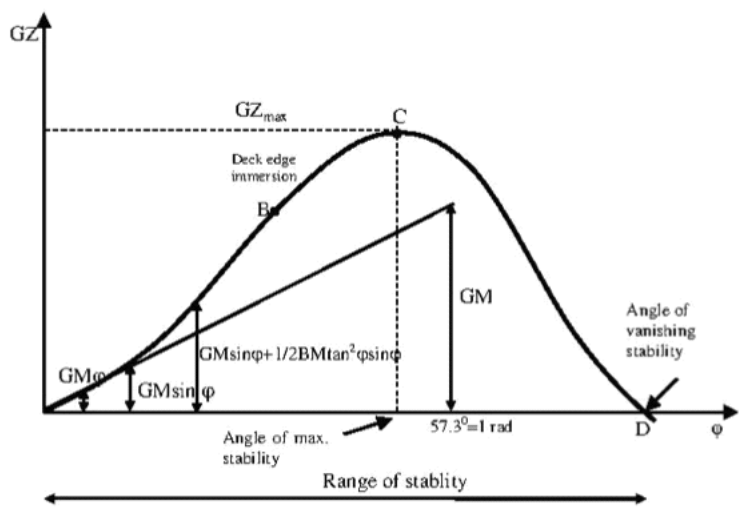
2. For the rightmost boat in Figure 10.1, does the torque due to buoyancy tend to restore the boat to a zero-heel angle (upright)? How is this similar to the ducky?

   (a) No, for the rightmost case the buoyant force tends to apply a torque that restores the boat to an upside-down orientation. This is similar to the duck in that the boat can tip over and settle on its back.

3. For the example boat, somewhere between its upright orientation and its "on-its-back" orientation, there needs to be another point where the boat is in static equilibrium. What needs to be the case in this orientation? How is this "special" orientation different from the upright and "on-its-back" orientations?

   (a) The "tipping-point" angle for the boat is in static equilibrium; therefore the boat center of mass is directly above the center of buoyancy (in the global coordinate system). This orientation differs from the upright and overturned angles because it is an unstable equilibrium; beyond this angle (increasing the heel further) will lead to a non-restoring torque. This "special" angle is called the angle of vanishing stability (AVS).

4. The following is an example righting arm curve for a specific boat hull; it visualizes the moment arm (GZ) against boat heel angle (ϕ). The angle of vanishing stability (AVS) is denoted by the point D. What is the slope of the moment arm curve at the AVS, and why does this make physical sense?



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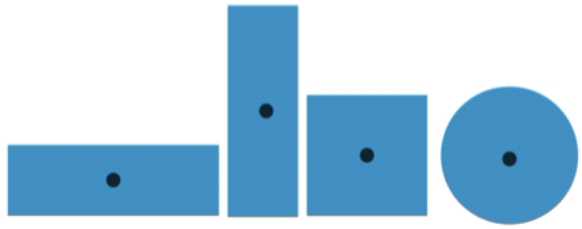
   (a) The slope is negative at the AVS. This makes physical sense because under our sign convention, a positive moment arm corresponds to a restoring torque towards $\phi = 0^\circ$. Beyond the AVS the buoyancy-applied torque pushes the boat away from $\phi = 0^\circ$ and towards $\phi = 180^\circ$.

Solution 10.2

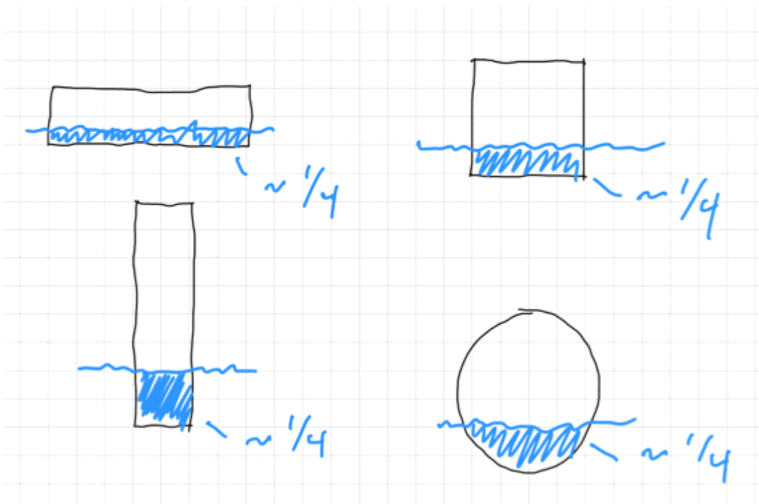
1. Both the ducky and a boat are subject to gravity. For both cases, what *physical* mechanism supplies the vertical force to counter gravity?
   (a) The ducky experiences a normal force that keeps it from falling (this is due to contact with the ground), while a boat experiences buoyancy (this is due to pressure supplied by the surrounding fluid).
2. What must be true about an object in order for it to float?
   (a) In order to float, an object must displace a volume $\forall$ such that the weight of the displaced water equals its own weight $F_b = \rho_{\text{water}} g \forall = W$.
3. The *displacement ratio* is the ratio of the mass of a boat over the maximum mass of water it can displace. What must be true about the displacement ratio of a boat in order for it to float?
   (a) In order for a boat to float, its displacement ratio must be less than one.
4. Suppose you make a "boat" out of solid steel (with no gaps inside). Can this boat *possibly* float? Why or why not?
   (a) A boat made out of *solid* steel could not possibly float. Steel has a density of $\approx 8000\text{kg}/m^3$, which is more than eight times that of water $1000\text{kg}/m^3$. Regardless of the volume that *solid* steel displaces, it will sink.
5. Suppose you make a boat out of steel, but allow for air gaps inside the boat (e.g. like Figure 10.2, but with a hollow hull under the water). What would you need to do to ensure a boat made from this material will float?
   (a) You would need to ensure the volume of displaced water is sufficient to support the total mass of the boat; i.e. displacement ratio less than one. You could do this by enclosing a volume of air with thin steel, which would reduce the "effective" density of the total boat volume. Note that modern battleships are built of steel; this is a proven boat design.
6. What is the density of wood? Why is this advantageous for building boats?
   (a) Density of wood varies quite a bit by species! The Engineering ToolBox lists densities for numerous wood species. For instance, oaks have a density in the range of $[600, 900]\text{kg}/m^3$. Any of these oak varieties would be advantageous for a boat; their density is less than water, so we could potentially build a solid hull and still have positive buoyancy.
7. For the boat project, you will design your boat to be printed with a plastic filament with density $\approx 1250\text{kg}/m^3$. What ramifications does this have for your boat design?
   (a) This material is more dense than water, so you can't make your boat design solid. You will either need to include an air gap inside the boat, or vary the print infill to reduce the density of the material.

Solution 10.3

1. Sketch in the center of mass (CG) for each boat cross-section.

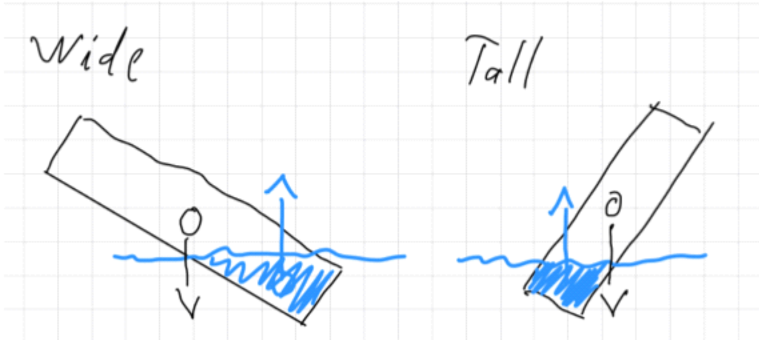


2. Sketch each boat at its upright heel and at equilibrium waterline. Keep in mind that $\rho_{\text{boat}} = \frac{1}{4}\rho_{\text{water}}$, and each boat is a simple extrusion. How do you know the waterline is at that level?



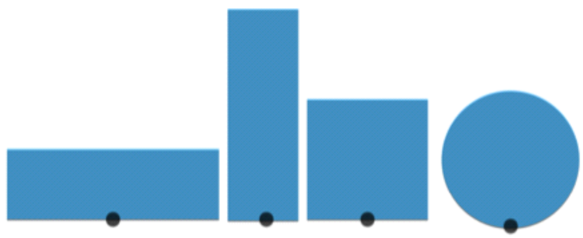
Each boat displaces a volume of water equal to $1/4$ of its total volume.

3. Sketch the wide boat (first from left) and tall boat (second from left) at an angle of about $45°$. Draw a waterline at the approximate equilibrium level, and depict the center of mass and center of buoyancy. How did you determine the equilibrium waterline? Which of the boats is stable about its upright heel? Which is unstable?

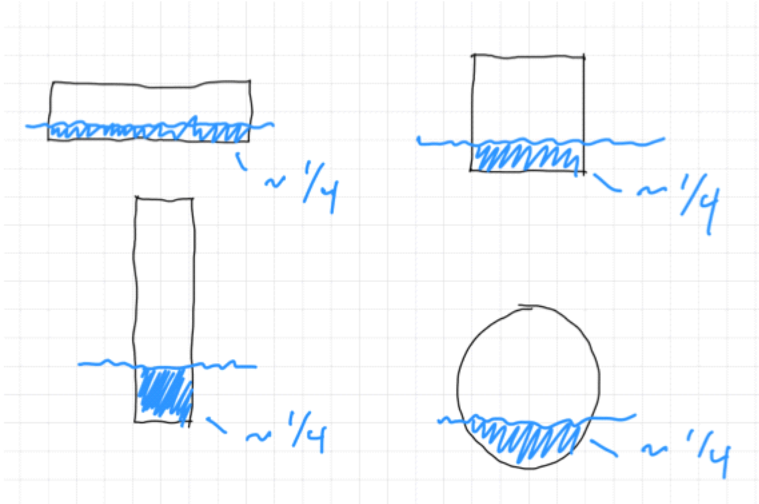


The equilibrium waterline is where a quarter of the boat's volume is underwater (see above). The 'Wide' boat is stable, while the 'Tall' boat is unstable.

4. Next, assume each boat has enough ballast so that its center of mass is at its bottom surface. Sketch each boat cross-section again with its new center of mass.

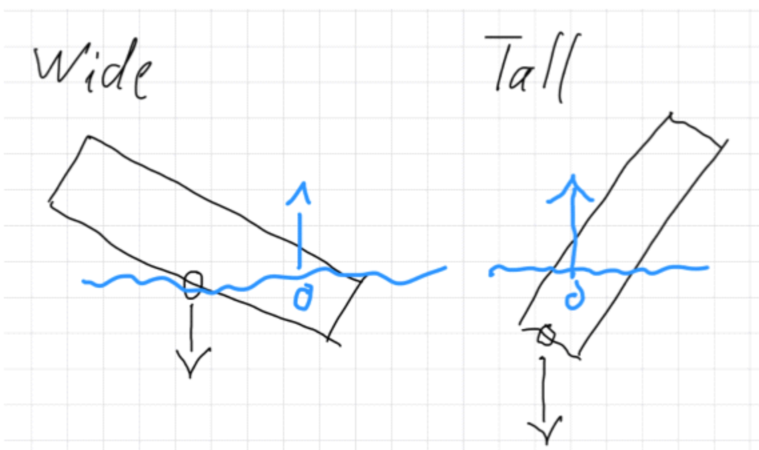


5. For the new ballast boats, sketch each at its upright heel and at equilibrium waterline. Assume that each boat's total weight is unchanged by the addition of ballast (i.e. we've changed the distribution of material to keep the total mass the same).



The waterlines are unchanged: Each boat still displaces a volume of water equal to $1/4$ of its total volume.

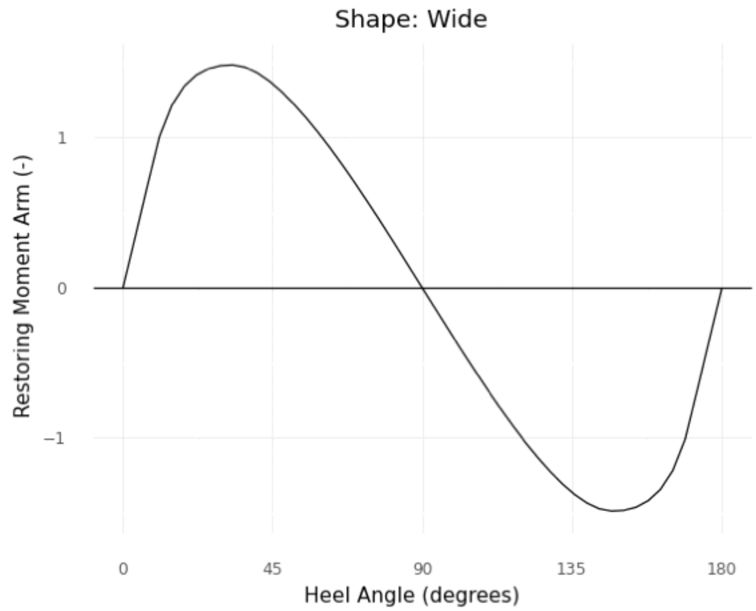
6. For the new ballast boats, sketch the wide boat (first from left) and tall boat (second from left) at an angle of about 45°. Draw a waterline at the approximate equilibrium waterline, and depict the center of mass and center of buoyancy. Which of the boats are stable in their upright position? Which are unstable?



Now both the wide and tall boats are stable in their upright position.

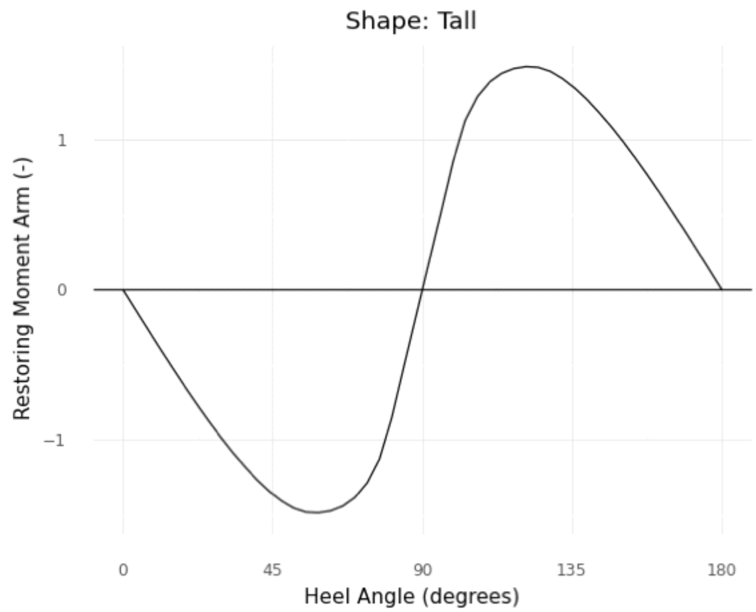
Solution 10.4

1. Sketch a righting arm curve for the 'wide' boat. What is the AVS for this shape?



The shape can be in equilibrium at $\phi = 90°$, but the slope is negative at this point so this is the AVS.

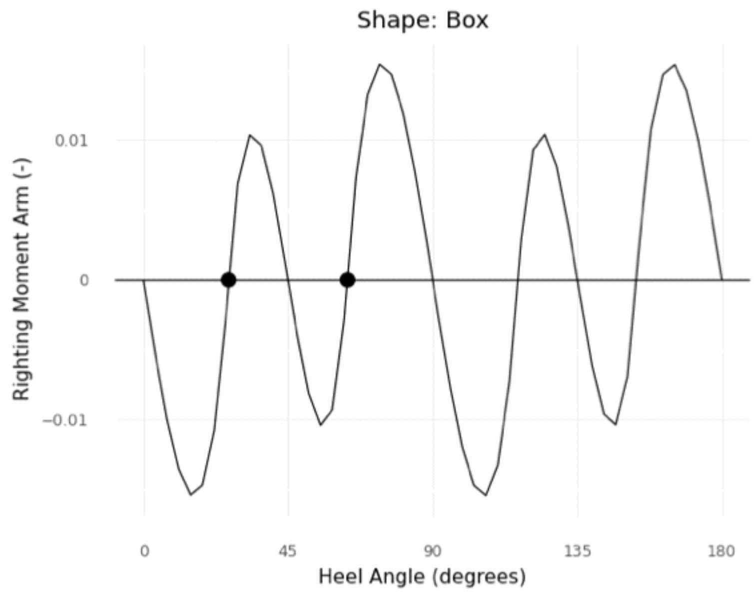
2. Sketch a righting arm curve for the 'tall' boat. Compare it with your curve for the 'wide' boat.



The 'tall' boat's curve should look familiar; it's nothing more than a phase-shifted version of the 'wide' boat! Put differently, the tall boat is just the wide boat on its side.

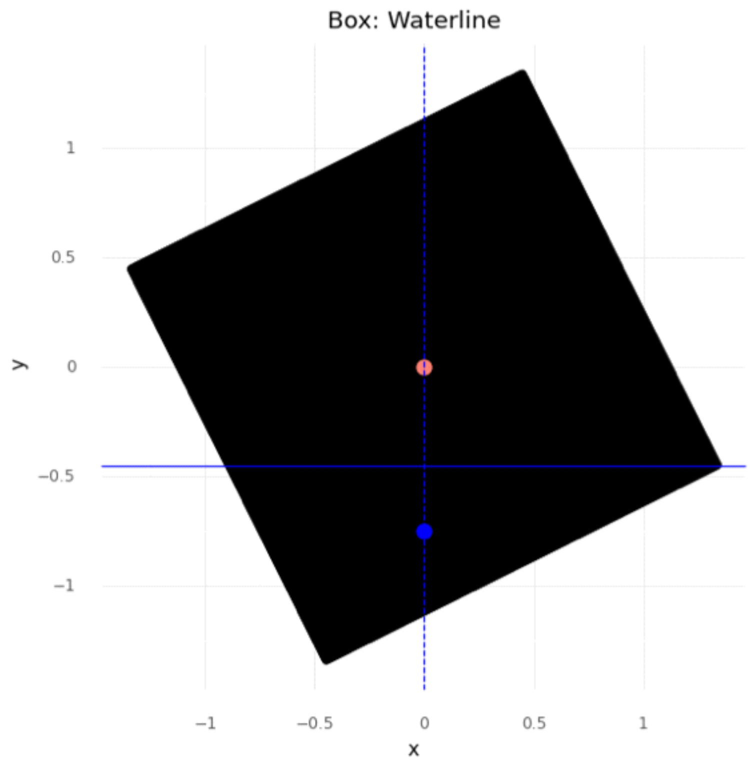
Solution 10.5

1. (Optional) Sketch a righting arm curve for the box-shaped boat with displacement ratio 0.25.
   Note: This case is very tricky!



Note that the box hull is not stable around $\phi = 0°$, nor around $\phi = 45°$. Instead it is stable at $\pm 26.5°$.

2. (Optional) Draw the box-shaped boat at its stable heel angle.


Box: Waterline
y
1
0.5
0
−0.5
−1
x
−1
−0.5
0
0.5
1

Chapter 11

Homework 4: Analyzing Boat Stability

Contents

You're going to make use of your boat center of mass code from last semester in this homework. In case you had trouble with Homework 4 from the Fall, here's a reminder that the solutions are available in Matlab Drive.

11.1 Infill, Ballast, and Waterlines

Through the following exercise you will build up your own analysis code. Remember that you can use the Fall Homework 4 solution in Matlab Drive as a starting point. This exercise will also help you connect 3d print infill to the concept of ballast; you can use these ideas in your own boat design!

Exercise 11.1

1. In your own words, what is *infill* in the context of 3d printing? How could you use the concept of infill to add ballast to your boat?

2. Modify your boat code to allow for a variable density of the hull. Use a base density of `rho100 = 1250` (the density of the 3d print filament, in kg/m^3), a 50% infill density of `rho50 = 0.5 * 1250`, and combine these quantities in a piecewise density function `fun_rho = @(y) rho100 * (y < h) + rho50 * (y >= h)`. Use this function to replace the density in the line `masses = insideBoat .* fun_rho(P(2, :)') * deltaA * L`. Test your code with boat shape 2 with `W = 1.0; D = 0.5; L = 0.6`. At `h = 0.6` you should find `CoM = [0.0; 0.3015]`, and at `h = 0.2` you should find `CoM = [0.0; 0.2652]`. Describe—in your own words—how the parameter h allows you to vary the ballast of the boat.

3. For the boat with piecewise infill, use the sum of the `masses` to compute the `mass_boat`, and compute a vector of `masses_water` based on the density of water (`rho_water`) to compute `mass_water` (the mass of the maximum displaced water). Use these two quantities to compute `displacement_ratio = mass_boat / mass_water`. What is the displacement ratio for the case where `W = 1.0; D = 0.5; L = 0.6; h = 0.2`? Can this boat float?

4. Suppose you wanted to 3d print a boat with a displacement ratio of 0.25 with a *fixed* infill

1) The inside of a 3D print isn't solid plastic, it's a mostly hollow pattern that takes up less material (and therefore less weight and less time), but is reasonably strong. Think honeycomb.

2) All ys less than h are 2x dense as the rest of the boat, pulling the CoM down

3) 0.7853, so it will float!

percentage across the boat. *Approximately* what infill percentage would be necessary? Neglect the fact that the walls are solid and of finite thickness. 20%

5. Modify your boat code to set the waterline at a distance `d` above $z = 0$ and compute the mass of the displaced water. Use the boolean vector `underWaterAndInsideBoat` to create a vector of submerged masses. Compute the difference of the boat mass minus the mass of the displaced water `mass_diff` for distance `d = 0.2`. Is this above, below, or at the equilibrium waterline? Basically there: disp. ratio = .9997
6. Modify your boat code to first rotate the boat, set a waterline offset `d`, then compute the `mass_diff`. What is the mass difference for heel angle $\phi = 90°$? -109.78kg
7. Explain *conceptually* how you would use the Matlab routine **`fzero`** to find the value of the waterline offset `d` where `mass_diff` is zero for a specified heel ϕ. Why is finding this point `d` important? Pass in a func that does the math & returns mass_diff. Finds the eq waterline

Solutions for this exercise are in this Matlab drive folder in the file `solutions_e11p1.mlx`.

11.2 Complete the Righting Arm Code

For the next exercise you will work from an existing righting arm code. From this Matlab drive folder download `boat_analysis_assignment.m` and modify the code to complete the following.

Exercise 11.2

1. Complete the `boat_analysis_assignment.m` analysis code by implementing the buoyancy function at the bottom of the file. Note that there is no solution file for this task: You should be able to do this using techniques you learned in the previous parts. ✓?

This was weird, I feel like I missed a part

11.3 Analyze the Triangle Boat

For the following exercise, use your righting arm code to analyze the case where $W = 4, H = 2, L = 10, n = 1$. This will give a triangular hull, as shown in Figure 11.1.

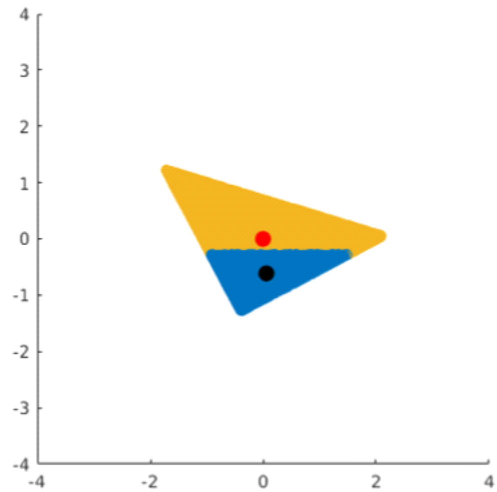


Figure 11.1: Triangular hull at a small heel angle.

Exercise 11.3

1. Set `dispratio = 0.30` and generate a righting arm curve. What is the AVS for this hull? 53°
2. Set `dispratio = 0.20` and generate a righting arm curve. Is this hull stable at heel $\phi = 0°$? Why or why not? Yes, b/c slope < 1 @ 0°
3. Set `dispratio = 0.25` and generate a righting arm curve. Is this hull stable at heel $\phi = 0°$? Why or why not? No, b/c slope > 1 @ 0°

Solution 11.1

1. In your own words, what is *infill* in the context of 3d printing? How could you use the concept of infill to add ballast to your boat?

   (a) Infill is the interior print pattern that fills the structure. It allows the interior of a 3d printed part to be nearly hollow while having solid walls. You can add ballast to your boat by making the infill pattern more dense near the bottom of the boat, and more sparse towards the deck.

2. Modify your boat code to allow for a variable density of the hull. Use a base density of `rho100 = 1250` (the density of the 3d print filament, in kg/m^3), a 50% infill density of `rho50 = 0.5 * 1250`, and combine these quantities in a piecewise density function `fun_rho = @(y) rho100 * (y < h) + rho50 * (y >= h)`. Use this function to replace the density in the line `masses = insideBoat .* fun_rho(P(2, :)') * deltaA * L`. Test your code with boat shape 2 with `W = 1.0; D = 0.5; L = 0.6`. At `h = 0.6` you should find `CoM = [0.0; 0.3015]`, and at `h = 0.2` you should find `CoM = [0.0; 0.2652]`. Describe—in your own words—how the parameter `h` allows you to vary the ballast of the boat.

   (a) Increasing `h` makes a larger fraction of the bottom of the boat heavier; this increases the ballast. See the solution for an implementation of this work.

3. For the boat with piecewise infill, use the sum of the `masses` to compute the `mass_boat`, and compute a vector of `masses_water` based on the density of water (`rho_water`) to compute `mass_water` (the mass of the maximum displaced water). Use these two quantities to compute `displacement_ratio = mass_boat / mass_water`. What is the displacement ratio for the case where `W = 1.0; D = 0.5; L = 0.6; h = 0.2`? Can this boat float?

   (a) For the case where `W = 1.0; D = 0.5; L = 0.6; h = 0.2` we have `displacement_ratio = 0.7813`. Since the displacement ratio is less than one, this boat will float. See the solution for an implementation of this work.

4. Suppose you wanted to 3d print a boat with a displacement ratio of 0.25 with a *fixed* infill percentage across the boat. *Approximately* what infill percentage would be necessary? Neglect the fact that the walls are solid and of finite thickness.

   (a) The displacement ratio is $W_{\text{boat}}/W_{\text{water}} = \rho_{\text{boat}}/\rho_{\text{water}}$. Therefore we need the effective density of the boat to be 25% that of water. Setting this yields $\rho_{\text{boat}} = 0.25\rho_{\text{water}} = 250\text{kg}/m^3 \approx 0.2\rho_{\text{filament}}$.

5. Modify your boat code to set the waterline at a distance `d` above $z = 0$ and compute the mass of the displaced water. Use the boolean vector `underWaterAndInsideBoat` to create a vector of submerged masses. Compute the difference of the boat mass minus the mass of the displaced water `mass_diff` for distance `d = 0.2`. Is this above, below, or at the equilibrium waterline?

   (a) With distance `d = 0.2` the draft is far less than the equilibrium draft; the equilibrium draft is around `0.425`. See the solution for an implementation of this work.

6. Modify your boat code to first rotate the boat, set a waterline offset `d`, then compute the `mass_diff`. What is the mass difference for heel angle $\phi = 90°$?

   (a) With distance `d = 0.2` at heel angle $\phi = 90^\circ$, `mass_diff = -0.9336`; this is near the equilibrium condition. See the solution for an implementation of this work.

7. Explain *conceptually* how you would use the Matlab routine **`fzero`** to find the value of the waterline offset `d` where `mass_diff` is zero for a specified heel ϕ. Why is finding this point `d` important?

(a) You could write a function that assumes a heel angle ϕ and takes d as an argument, computes the mass of the displaced water based on the proposed waterline, and returns the difference in displaced water mass and total boat mass. Passing this function to **`fzero`** will search for the waterline d that corresponds to vertical static equilibrium. This process is important because the righting arm curve is based on the moment arm of buoyancy at equilibrium draft; finding the correct d gives the correct center of buoyancy for this calculation.

Solution 11.2

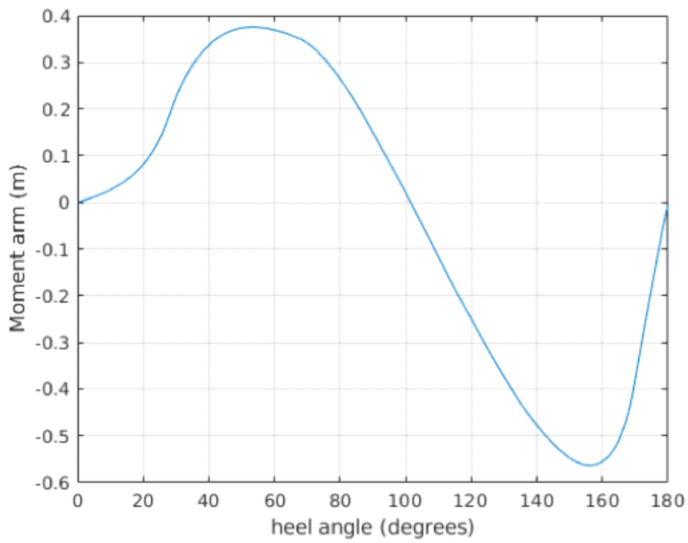
1. Complete the `boat_analysis_assignment.m` analysis code by implementing the buoyancy function at the bottom of the file. Note that there is no solution file for this task: You should be able to do this using techniques you learned in the previous parts.

   (a) You can do it!

Solution 11.3

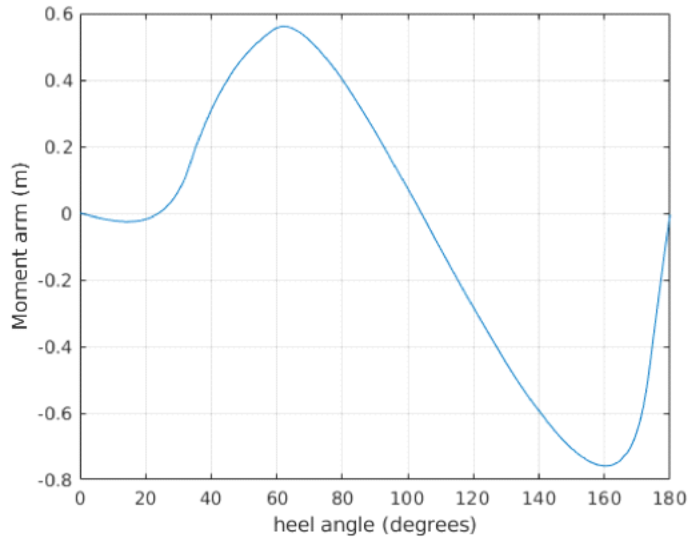
1. Set `dispratio = 0.30` and generate a righting arm curve. What is the AVS for this hull?

   (a) The AVS is just above $100°$.



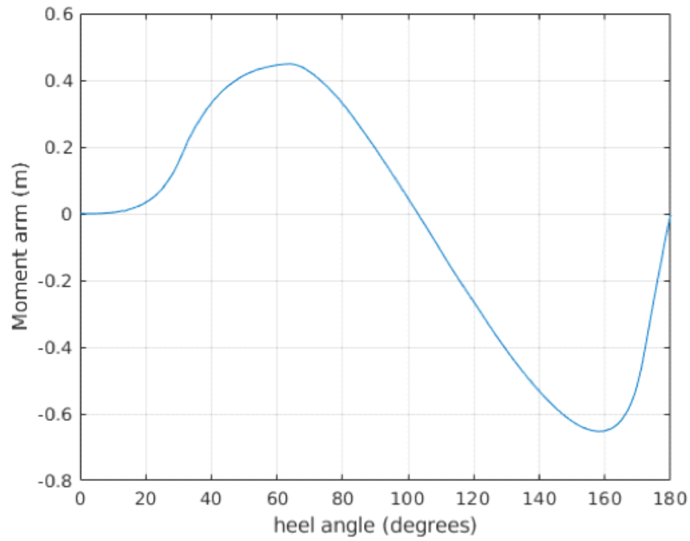
2. Set `dispratio = 0.20` and generate a righting arm curve. Is this hull stable at heel $\phi = 0°$? Why or why not?

   (a) This hull is not stable at $\phi = 0°$; note that the slope of the moment arm curve is negative at $\phi =°$. This boat has a finite angle of loll.



3. Set `dispratio = 0.25` and generate a righting arm curve. Is this hull stable at heel $\phi = 0°$? Why or why not?

   (a) This hull is not technically stable at $\phi = 0°$ as it does not provide a restoring torque.


0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
Moment arm (m)
0
20
40
60
80
100
120
140
160
180
heel angle (degrees)