Week 2

Tuesday, February 9, 2021 9:32 PM

Week2

- Schedule
- Textbooks → Bring to OH

Chapter 4

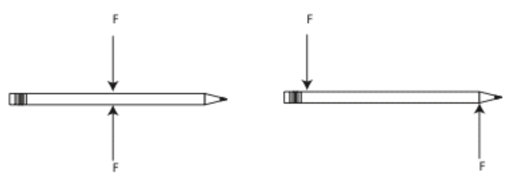
Week 2a: Torque and More Free Body Diagrams

Schedule

4.1 Introduction to Torque [25 minutes]

Last week we worked on vectors, forces, and the idea of free body diagrams - and in creating free body diagrams, we encouraged you to draw a picture to represent the system, and to draw arrows at different locations to represent interactions (forces) between the system and the rest of the world.

We encouraged you to draw the picture – as opposed to just a "dot", which is often how FBD's are drawn in introductory physics – because the location at which forces are applied matters. Consider the two conditions shown below. In both cases the net *force* is zero, but it seems pretty clear that the physical behavior will not be the same!



This qualitative example illustrate the idea of *torque* (denoted by the vector $\boldsymbol{\tau}$ or $\vec{\tau}$). Just as a *force* is colloquially thought of as "the thing that causes acceleration", a *torque* is "the thing that causes rotation". In the first example above, the net torque acting on the pencil was zero - so no rotation occurs, while in the second case, there is a net torque on the pencil – so the pencil "wants" to rotate.

A note about terminology: The terms *torque* and *moment* are often used interchangeably in engineering. In free body diagrams, moments are denoted using a curved arrow, like the example in Figure 4.1.

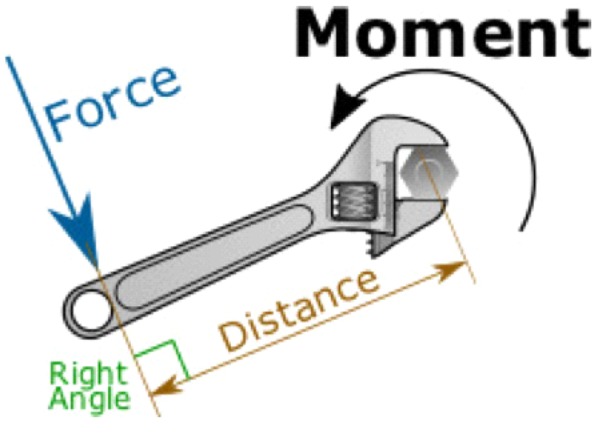


Figure 4.1: The perpendicular force acting on the wrench at some distance from the pivot point creates a torque. This "turning force" can also be represented as a moment at the pivot point.

Torque: The Picture The most common way to think about a torque is a "force applied at a distance from the point of rotation." We can illustrate this concept using the picture in Figure 4.2.

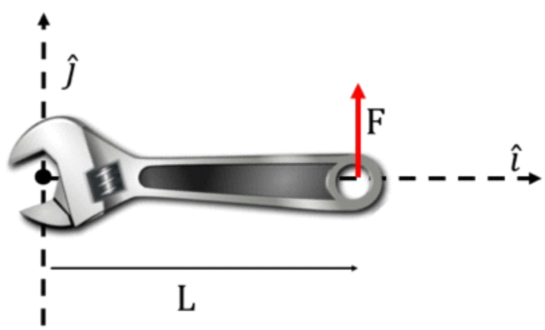


Figure 4.2: Force being applied to a wrench.

In Figure 4.2, a force $\vec{F}$ is being applied to the wrench at a distance $\vec{L}$ from the rotation point at the origin O (note that $\vec{F}$ and $\vec{L}$ are both vectors!). Because of the separation between the application point of the force and the origin, we expect this force would cause the wrench to rotate, increasing its *angular momentum*. While qualitatively this scenario is easy to understand, it is important to add mathematical definition to the scenario, then look at some additional cases.

Torque: Mathematical Definition The mathematical definition for a torque is given as:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

which relies on the *cross product* that you encountered earlier. Looking at the equation for a torque, there are several important characteristics we notice immediately:

1. A torque, $\vec{\tau}$ is a vector, meaning that it has both a magnitude and a direction.
2. Both $\vec{r}$ and $\vec{F}$ are vectors, so both magnitude and direction matter.
3. Torque is dependent on the *cross product* between $\vec{r}$ and $\vec{F}$, so it is maximized when $\vec{r}$ and $\vec{F}$ are perpendicular.

Torque: Understanding the Vector Since torque is a cross product, it is (by definition) a vector quantity – it has both magnitude and direction. Often people want to talk about the "direction" of the torque as being "clockwise" or "counterclockwise" – and as a consequence, they will draw torque as a circular arrow. Indeed, moments are often represented on FBDs with circular arrows indicating direction of rotation that the moment "wants" to cause.

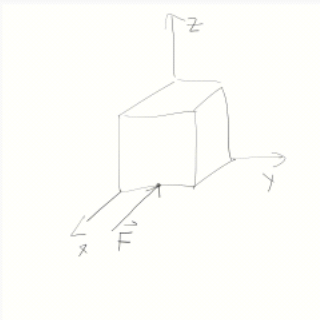
Strictly speaking, though, the direction of the torque vector points along the axis of rotation, and the sign tells you whether the rotation is clockwise (negative) or counterclockwise (positive). You can again use a right hand rule to help you remember the signs; if you curl your fingers in the the direction of the rotation, your thumb will point in the direction of the torque vector.

Now that we have the definition of torque, let's work through a few examples to illustrate the characteristics above.

The ideas we will be exploring in these exercises are also in the videos about the definition of torque and choice of origin if you would like to review after class.

Exercise 4.1

1. Consider a box, suspended in space (e.g. maybe you are actually in a spaceship in zero gravity with the box). You designate the following coordinate system for the box and apply a force at the bottom of the box in the $-x$ direction. How does the box "want" to move? What is the direction of resulting torque about the box's center of mass (in vector form)?



2. Now imagine that a *torque* is applied to the box about the box's COM; the direction of the torque is in the $-x$ direction. How does the box "want" to move?
3. Was our choice of coordinate system and origin a good one? What might have been a better choice?

Exercise 4.2

Consider the wrench shown in Figure 4.3 that rotates about the origin O.

1. For each force vector in Figure 4.3, find the associated torque about the origin O. Make sure to specify both the direction and magnitude of the torque.
2. What change would need to be made to F_B in order for $\vec{\tau}_B = \vec{\tau}_A$ (where $\vec{\tau}_B$ is the torque due to F_B and $\vec{\tau}_A$ is the torque due to F_A)?
3. If point of rotation for the wrench was changed from the origin to the point $(L, 0)$, how would that change the torque due to each force?

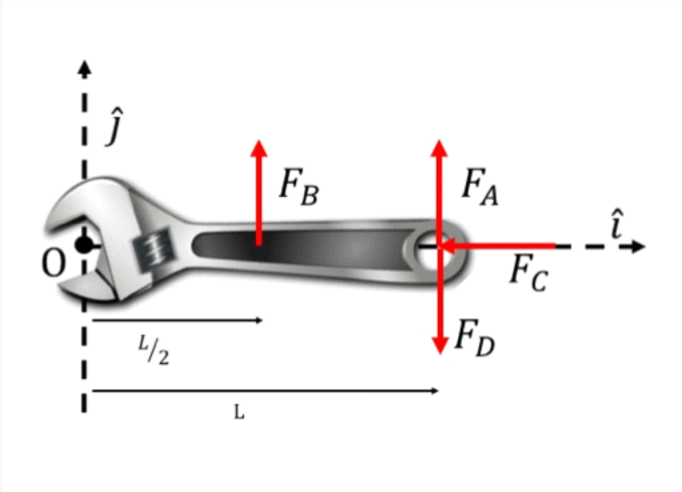


Figure 4.3: Wrench under different force conditions.

Exercise 4.3

Consider the wrench again, but with the force $F = 2\hat{\imath} + 1\hat{\jmath}$ as shown in Figure 4.4.

1. Find the torque about the origin O due to the force $\vec{F}$.
2. Find the torque about the point $(L, 0)$.

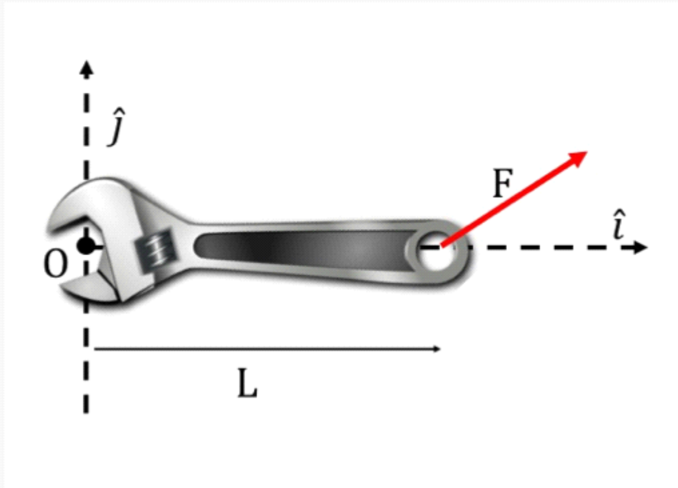


Figure 4.4: Wrench with a non-perpendicular applied force.

4.2 Net Force and Net Torque [15 minutes]

For these next exercises, we want to consider the concepts of *net force* and *net torque* acting on a body. By net force and net torque, we simply mean the vector sum of those quantities:

$$\vec{F}_{net} = \sum \vec{F}$$

and

$$\vec{\tau}_{net} = \sum \vec{\tau}$$

Note: Generally when we calculate net torque for the purposes of dynamics, it's helpful to use the object's center of mass as an origin, whereas when we calculate the net torque for static problems, it is useful to be "clever" about what you choose as an origin. Be careful that you use a single origin for calculating the torques though!

Since we are not getting into the dynamics of objects right now, we're not going to get too formal interpreting the net torque and net force. Qualitatively, the net force tells you how the object's center of mass will accelerate, and the net torque about the COM tells you how about the object's angular acceleration – how much it wants to "spin" about the center of mass, along the axis direction defined by the torque vector. There's a fair bit of detail to getting the dynamics right (which is why we are not going there right now), but if you're curious you can talk to an instructor, and you can also (as always) watch some videos here if you would like to review after class.

Exercise 4.4

Consider the wrench in Figure 4.5. This figure is a snapshot of a moment in time (t=0), right when we apply the force, F.

1. What is the net force acting on the wrench?
2. What is the net torque acting on the wrench?
3. What motion would you expect to see in this scenario at t>0 seconds (assuming a friction-less surface)?

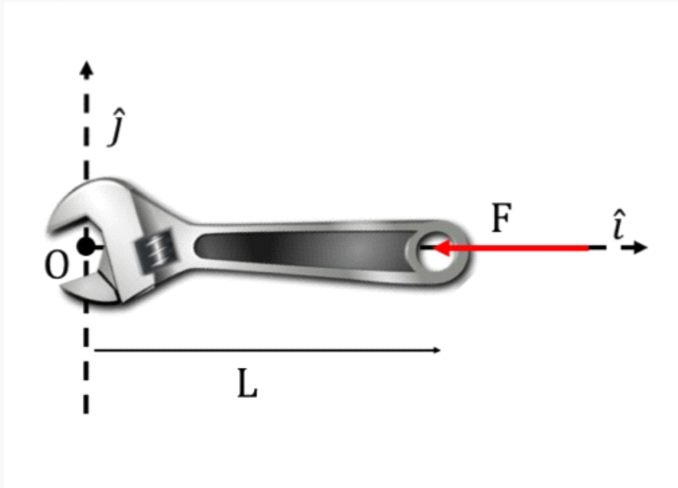


Figure 4.5: Yep, the wrench again.

Exercise 4.5

Consider the pencil in Figure 4.6. Let's say we are looking down on this pencil as it lies on a friction-less surface, at the initial moment (t=0) of applying the forces shown.

1. What is the net force acting on the pencil?
2. What is the net torque acting on the pencil?
3. What motion would you expect to see in this scenario (assuming a friction-less surface) at time >0 seconds?

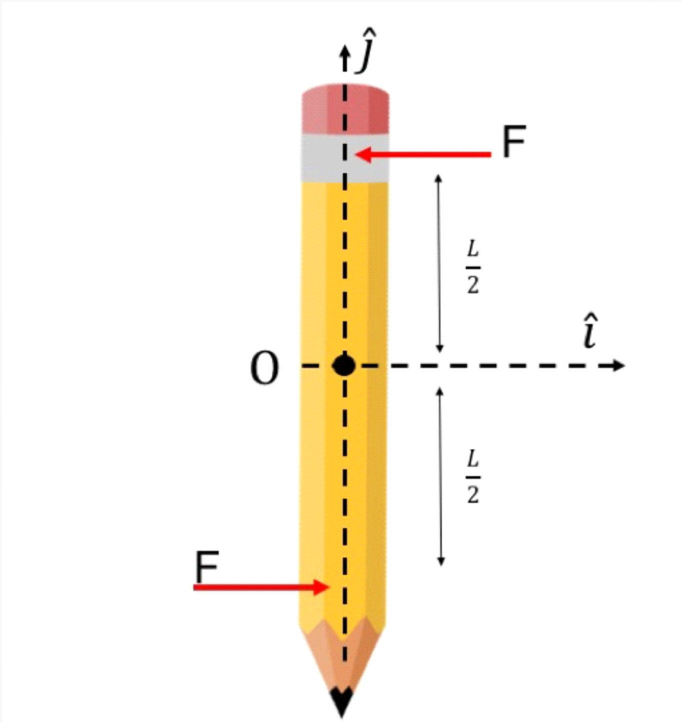
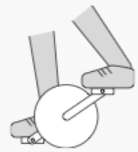


Figure 4.6: Oh hey, a pencil!

4.3 Torque Practice [20 minutes]

Exercise 4.6

Imagine that you are riding a bicycle (without cleats or toe clips so you are only able to push down on the pedals). Assuming you push down with constant force as you pedal, at what point during a pedaling cycle is the torque applied by your feet around the center of the crank greatest? When is it least? What is the direction of the torque?
Why do cleats and toe clips increase the torque you are able to apply?



Exercise 4.7

Say you need a wrench to help you exert a 100 N-m torque. About how long should the handle be and why?

Exercise 4.8

Let's say I'm trying to loosen a very stuck nut which is in a difficult to reach location. I am using a wrench which is 20 cm long. I am capable of exerting a force of 100 N, but because of the awkward position, the direction of the force I exert is at an angle of 60 degrees to the wrench (or 30 degrees off of the perpendicular to the wrench). If the nut will require a torque of 16 N-m to come loose, will I succeed?

4.4 Intro to Statics

In Wednesday's class, we'll take a deeper look at net torque and net force and what happens when these are zero (i.e. statics). If you finish the class work early, please watch the videos on statics for Wednesday (see first section of the homework).

Solution 4.1

1. Consider a box, suspended in space (e.g. maybe you are actually in a spaceship in zero gravity with the box). You designate the following coordinate system for the box and apply a force at the bottom of the box in the $-x$ direction. How does the box "want" to move? What is the direction of resulting torque about the box's center of mass (in vector form)?

   It will want to move counterclockwise around the COM in the direction of the y axis, so the torque is in the $+y$ direction.

2. Now imagine that a *torque* is applied to the box about the box's COM; the direction of the torque is in the $-x$ direction. How does the box "want" to move?

   The block will want to move clockwise around the COM about the x axis (i.e. if you moved the axis so that it went through the COM)

3. Was our choice of coordinate system and origin a good one? What might have been a better choice? It would make more sense to place the origin at the COM.

Solution 4.2

1. For each force vector in Figure 4.3, find the associated torque about the origin O. Make sure to specify both the direction and magnitude of the torque.

$$\vec{\tau}_A = L\hat{\imath} \times F_A\hat{\jmath} = LF_A\hat{k}$$
$$\vec{\tau}_B = \frac{L}{2}\hat{\imath} \times F_B\hat{\jmath} = \frac{L}{2}F_B\hat{k}$$
$$\vec{\tau}_C = L\hat{\imath} \times -F_C\hat{\imath} = 0$$
$$\vec{\tau}_D = L\hat{\imath} \times -F_D\hat{\jmath} = -LF_D\hat{k}$$

2. What change would need to be made to F_B in order for $\vec{\tau}_B = \vec{\tau}_A$? The magnitude of F_B would need to double. Therefore, when you use a crescent wrench, you will need to apply less force if you grip it near the near the end of the handle. By the way, in real life, if you're exerting the forces F_A or F_B, you will want to flip the wrench 180 degrees about its horizontal axis so that the largest force is on the stationary jaw.

3. If point of rotation for the wrench was changed from the origin to the point $(L, 0)$, how would that change the torque due to each force?

$$\vec{\tau}_A = 0\hat{\imath} \times F_A\hat{\jmath} = 0$$
$$\vec{\tau}_B = -\frac{L}{2}\hat{\imath} \times F_B\hat{\jmath} = -\frac{L}{2}F_B\hat{k}$$
$$\vec{\tau}_C = 0\hat{\imath} \times -F_C\hat{\imath} = 0$$
$$\vec{\tau}_D = 0\hat{\imath} \times -F_D\hat{\jmath} = 0$$

Solution 4.3

Consider the wrench again, but with the force $F = 2\hat{\imath} + 1\hat{\jmath}$ as shown in Figure 4.4.

1. Find the torque about the origin O due to the force $\vec{F}$.

$$\vec{\tau} = L\hat{\imath} \times 2\hat{\imath} + 1\hat{\jmath} = L\hat{k}$$

2. Find the torque about the point $(0, L)$. The torque about $(0, L)$ would be zero because $\vec{r}$=0

Solution 4.4

Consider the wrench in Figure 4.5.

1. What is the net force acting on the wrench? $\vec{F}_{net} = -F\hat{\imath}$
2. What is the net torque acting on the wrench? $\vec{\tau}_{net} = 0\hat{\jmath} \times -F\hat{\imath} = 0$
3. What motion would you expect to see in this scenario (assuming a friction-less surface)? Translation in the $-\hat{\imath}$ direction. Specifically, we would expect a translational acceleration, since $\vec{F}_{net} = -F\hat{\imath} = m\vec{a}$.

Solution 4.5

Consider the pencil in Figure 4.6.

1. What is the net force acting on the pencil? $\vec{F}_{net} = \sum F_x = F\hat{\imath} - F\hat{\imath} = 0$. No forces are acting in the y-direction.
2. What is the net torque acting on the wrench?

$$\vec{\tau}_{net} = \frac{L}{2}\hat{\jmath} \times -F\hat{\imath} + -\frac{L}{2}\hat{\jmath} \times F\hat{\imath} = \frac{L}{2}F\hat{k} + \frac{L}{2}F\hat{k} = FL\hat{k}$$

3. What motion would you expect to see in this scenario (assuming a friction-less surface)? Counter-clockwise (positive) rotation about the center of the pencil. Specifically, we would expect an angular acceleration in the counter-clockwise direction, because $\vec{\tau}_{net} = I\vec{\alpha}$.

Solution 4.6

The maximum torque will be applied when the pedals are horizontal and the force applied by your foot is perpendicular to the crankset. The minimum torque will be applied when the pedals are aligned vertically, and the force from your feet is applied parallel to the crankset. At all points in the pedalling cycle, the torque will be oriented perpendicular to the pedaling plane (i.e. out of the plane in this picture, which is toward your left when you sit on your bike). Cleats and toe clips allow you to push and pull horizontally on the pedals, giving you more torque, especially when the pedals are aligned vertically.

Solution 4.7

Say you need a wrench to help you exert a 100 N-m torque. About how long should the handle be and why?
Most people can lift a 20 lb (10 kg-ish) dumbbell in one hand, so they can probably exert $F = mg \approx 10\text{ kg} \times 10\text{ m}^2/\text{s} = 100\text{ N}$ of force and they could do so perpendicular to the wrench. So, to make a wrench that could exert 100 N-m of torque, you probably want about a 1 m long handle.

Solution 4.8

Let's say I'm trying to loosen a very stuck nut which is in a difficult to reach location. I am using a wrench which is 20 cm long. I am capable of exerting a force of 100 N, but because of the awkward position, the direction of the force I exert is at an angle of 60 degrees to the wrench (or 30 degrees off of the perpendicular to the wrench). If the nut will require a torque of 16 N-m to come loose, will I succeed?
Yup! If you push at the very end of the handle (0.2 m), then your applied torque is $0.2\text{ m} \times 100\text{ N} \times \sin 60°$= 17.3 N-m, which exceeds the required torque.

Chapter 5

Week 2b: Statics

Schedule

5.1 The Idea of Statics and Static Equilibrium - 20 minutes

We discussed the idea of the net force and net torque acting on an object earlier. These ideas of net force and net torque are central to understanding whether the system is *static* – i.e., unchanging in time – or *dynamic* – i.e., changing in time.

A system is said to be in *static equilibrium* when the net force and net torque on every part of the system is zero. The net forces equaling zero is known as the *first condition for equilibrium*, and the net torque equaling zero is known as the *second condition for equilibrium*.

The two conditions for equilibrium lead to the following basic equations (where we use bold to represent a vector):

$$\sum \boldsymbol{F} = 0$$
$$\sum \boldsymbol{\tau} = 0$$

Exercise 5.1

Static Equilibrium Conceptual Questions

1. What do the two conditions for static equilibrium say about the linear and angular acceleration of the system? They're zero
2. For a system to be in static equilibrium, does it need to be at rest? Why or why not?
   No, just not accelerating
3. For a two-dimensional (x,y) system, how many equations are needed to prove static equilibrium? Please write them. How about for a three-dimensional (x,y,z) system?
   2D: $\sum F_x = 0$ $\sum F_y = 0$ $\sum \vec{\tau} = 0$ 3D: Also $\sum F_z = 0$, $\sum \tau_x = 0$, $\sum \tau_y = 0$, $\sum \tau_z = 0$
4. For a two-dimensional (x,y) system in static equilibrium, what is the maximum number of unknown forces/torques that can be solved for? How do you know this?

5. What does it mean for a problem to be "statically determinate" versus "statically indeterminate"? Use the interwebs to investigate this. Then draw a simple example of a statically indeterminate system, and explain what makes it indeterminate. Three

5.2 Free Body Diagrams (again!)

5.2.1 Strategies for FBDs (5 min)

Having a consistent methodology when drawing free body diagrams can be really helpful when analyzing systems. Below are some questions to consider:

Questions to Consider Before the FBD

- How will the system behave? (i.e., Play the movie in your head. Is this a static or dynamic system?)
- What are the aspects of the system you care about?
- Should you divide your system into multiple components or subsystems? Keep in mind that sometimes you need to draw multiple free body diagrams in order to properly analyze a situation and identify interaction forces and torques, etc.
- What interactions do you need to capture in your analysis? (i.e., what are the forces and/or torques acting on the various parts of your system? Are these forces/torques captured in your free-body diagrams?)
- How can you capture those interactions on a diagram or a set of diagrams? (i.e., what does the free-body diagram look like for this system and its components?)
- What are the important directions and locations in the problem – and given this, what are the appropriate frames of references to analyze this system?

How you choose to visualize your system and its components can impact your ability to understand and ultimately complete your analysis. A clear and complete *free body diagram* (or **multiple** *free body diagrams*) can help you clarify your own understanding of the system, derive appropriate equations of motion (more on this later in the course), quickly evaluate the accuracy of your work, and get help from others (because they can see how you are understanding the system!).

Steps for Drawing a FBD As you begin to create free body diagrams, here are some suggested steps to consider following:

- Make a decision as to which system (a body or collection of bodies) you will be analyzing. You may identify the need to break your system into multiple subsystems.
- Choose a body or combination of bodies **isolated** from all surrounding bodies.
- Observe which forces and/or torques are exerted **on** the body or combination of bodies you have chosen.
- Label forces and torques, with appropriate directions. (NOTE: You may need to arbitrarily assign a direction at times, in which case you may subsequently find through analysis that the magnitude of the force in the direction you have chosen is negative, which is fine! Forces do not have to be positive, but it's useful to draw them in the positive direction when possible.)
- Choose and label reference frames.

5.2.2 Practicing FBDs for static cases (30 minutes)

Exercise 5.2

Consider the *pinned beam* with constant cross section and mass distribution shown in Figure 5.1 below. The beam is supported at one end by a *pinned joint*. This particular type of joint can provide linear support for reaction forces in any direction, but cannot support a turning moment, since it rotates about the pin (If interested, you can learn more about types of load supports here. At the other end of the beam is a rope which can only support tension along its axis, attached to a fixed point. For this exercise, you can assume the beam is stationary ($\boldsymbol{a} = \boldsymbol{\alpha} = 0$, $\boldsymbol{v} = \dot{\boldsymbol{\theta}} = 0$).

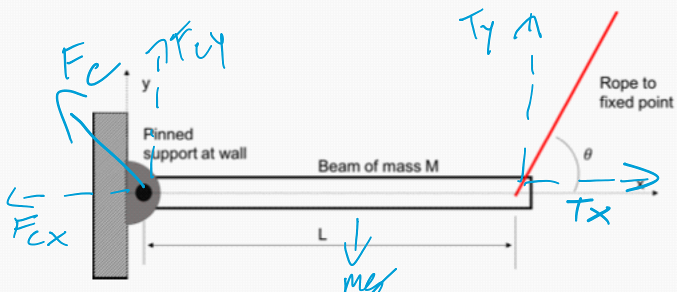


Figure 5.1: A pinned beam supported by a rope.

1. Explain what is meant by "this particular type of joint can provide linear support for reaction forces in any direction, but cannot support a turning moment." The beam can pivot
2. When drawing a free body diagram for the pinned beam in Figure 5.1, which body should you isolate? The beam
3. Draw the FBD for the body you have isolated, paying special attention to the steps outlined above.
4. Is the beam in static equilibrium? How do you know? Yes, b/c neither end can move
5. Write the appropriate equations for find the unknown forces and torques acting on the beam. How many equations are there? $\Sigma F_x = T_x - F_{cx} = 0$ $\Sigma F_y = T_y + F_{cy} - mg = 0$ $\Sigma T = -\frac{MgL}{2} + TMLsin\theta = 0$ $-mg \times \frac{L}{2}$ 6
6. How many unknowns do you have in your system of equations? T_x, T_y, L, T_x, T_y
7. Is the pinned beam system *statically determinant*? Why or why not? Yes, b/c equal unknowns then eqns

5.3 Free Body Diagrams With Multiple Bodies

In the previous exercises you drew simple FBDs for systems with a single body (a beam). Next, we will consider systems with multiple subsystems, building off the work we did last week.

5.3.1 What is a Reference Frame?

A *reference frame* is an explicitly defined framework in which we can make observations and write physical laws. A reference frame is defined by a coordinate system, an observer, and some definition of time. For statics problems, the primary component we are interested in are relationships between different coordinate systems. For dynamic systems, however, time and the observer play a very important role. We will be diving deeper into the concept of reference frames in the Robotics module, and again in QEA 3.

A **coordinate system** consists of an **origin** and a set of **basis vectors**. Recall that a set of vectors form a basis if they are linearly independent—if they are mutually orthogonal then we have an orthogonal coordinate system. The standard basis vectors for 2D are usually labelled $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, but they are sometimes written as $\mathbf{e}_1$ and $\mathbf{e}_2$, or $\mathbf{e}_x$ and $\mathbf{e}_y$, or $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$.

5.3.2 Choosing and Working With Multiple Frames and Multiple Bodies (20 minutes)

Exercise 5.3

For this exercise, refer to the system below.

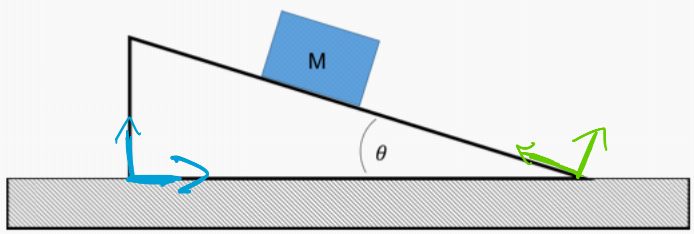
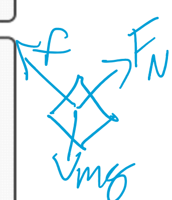


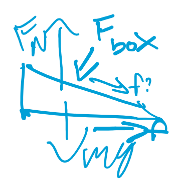
Figure 5.3: Box of mass M on an inclined ramp. The ramp is not fixed to the floor, and the box is not fixed to the ramp.

Suggest two useful reference frames for this system. For each reference frame, draw the two orthogonal unit vectors, as well as what you suggest for an origin.

Exercise 5.4

1. Draw a FBD for the box by isolating it from the ramp. Draw and label appropriate vectors for the forces acting on the box.
2. Draw a FBD for the ramp. Draw and label appropriate vectors for the forces.
3. Compare the two FBDs, and make sure that your interactions make sense – e.g., if the ramp is pushing up on the box, the box better be pushing down on the ramp! friction on ramp?
4. Look carefully at the force vectors you have drawn for the box and ramp FBDs. What reference frame is each force defined in (e.g. for the box, gravity is likely in the global $\hat{j}$ direction, while the normal force is in the ramp $\hat{y}$ direction)? Make a table like the one below explicitly defining each force/frame.





Force	Acting On	Reference Frame	Direction (e.g. $+\hat{\imath}$)

Force	On	Ref Frame	Direction
Fric	Box	Ramp	$-\hat{x}$
Norm	Box	Ramp	$\hat{y}$
Grav	Box	Global	$-\hat{j}$
Fric (Box)	Ramp	Ramp	$\hat{x}$
F_{Box}	Ramp	Ramp	$\hat{y}$
Grav	Ramp	Global	$-\hat{j}$
Norm	Ramp	Global	$+\hat{j}$
Fric	Ramp	Global	$+\hat{j}$

Solution 5.1

Static Equilibrium Conceptual Questions

1. What do the two conditions for static equilibrium say about the linear and angular acceleration of the system?
   Applying Newton's second law, $\sum \boldsymbol{F} = m\boldsymbol{a}$ and $\sum \boldsymbol{\tau} = I\boldsymbol{\alpha}$ (for rotation of a rigid body about the center of mass) we know that the linear acceleration $\boldsymbol{a} = 0$ and the angular acceleration $\ddot{\boldsymbol{\theta}} = \boldsymbol{\alpha} = 0$.

2. For a system to be in static equilibrium, does it need to be at rest? Why or why not?
   No, for a system to be in static equilibrium, the net force and torques acting on the system must be zero, leading to linear acceleration $\boldsymbol{a} = 0$ and the angular acceleration $\ddot{\boldsymbol{\theta}} = \boldsymbol{\alpha} = 0$. This means that the system can be traveling at a constant angular or linear velocity while still satisfying the two conditions for static equilibrium.

3. For a two-dimensional (x,y) system, how many equations are needed to prove static equilibrium? Please write them. How about for a three-dimensional (x,y,z) system?
   For a two-dimensional (x,y) system, three equations are needed:

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum \tau_z = 0$$

   Note that τ_z refers to the torque pointed along the z axis (out of the page or into the page for a 2D system). For a three-dimensional (x,y,z) system, the number of equations increases to six:

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$
$$\sum \tau_x = 0$$
$$\sum \tau_y = 0$$
$$\sum \tau_z = 0$$

4. For a two-dimensional (x,y) system in static equilibrium, what is the maximum number of unknown forces/torques that can be solved for? How do you know this? For a two-dimensional (x,y) system in static equilibrium, we know that there are three equations that must be satisfied as indicated above, all of which represent the net forces/torques in the respective directions,

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum \tau_z = 0$$

   If we have more unknowns than equations, we are not able to determine these unknowns. So, with only these three equations, we can solve a maximum of three unknowns. If the number of unknowns exceeds the number of equations, the system is know as "statically indeterminate," and cannot be solved without additional information. If there are fewer unknowns than the number of equations, the system would be considered "overdetermined," i.e., not physically possible. When the number of equations equals the number of unknowns, our system is "determinate", which means "having exact and discernible limits."

5. What does it mean for a problem to be "statically determinate" versus "statically indeterminate"? Use the interwebs to investigate this. Then draw a simple example of a statically indeterminate system, and explain what makes it indeterminate. See previous problem.

Solution 5.2

1. Explain what is meant by "this particular type of joint can provide linear support for reaction forces in any direction, but cannot support a turning moment." Physically, this means you can "pull" or "push" against the pin in any direction, and it will resist that force, but since it's a pin, the object is free to rotate about it. Therefore the pin can apply an arbitrary force to the object that it is attached to, but cannot apply a torque about the pin.

2. When drawing a free body diagram for the pinned beam in Figure 5.1, which body should you isolate? For the pinned beam in Figure 5.1, you want to isolate just the beam portion of the drawing as shown below.

3. Draw the FBD for the body you have isolated, paying special attention to the steps outlined above. The FBD for the pinned beam is shown in 5.2. Notice that we've chosen to represent all of the weight as a point force, F_G, acting at the center of mass. This ends up being a simplifying shortcut that gives us an accurate result when the mass is equally distributed along the beam (i.e., the mass/length does not change). At the pin, we expect F_R as the reaction force, acting in some unknown direction. For our computational convenience, we define the x-component of F_R, acting along the x-axis, F_{Rx}, and F_{Ry} acting along the y-axis (these vectors are not necessarily drawn to scale in the figure). We are doing this in anticipation of summing the forces in the x and y directions. Incidentally, we are calling them 'reaction' forces as a nod to Newton's law that states, "For every action, there is an equal and opposite 're-' action." These forces are re-acting to the applied force labeled $\vec{T}$. (What happens to these re-acting forces when we remove $\vec{T}$)?

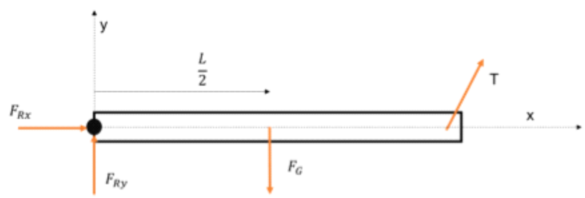


Figure 5.2: FBD for the pinned beam.

4. Is the beam in static equilibrium? How do you know? Yes, in the problem statement we were told for this exercise, you can assume the beam is stationary $\boldsymbol{a} = \boldsymbol{\alpha} = 0$, which satisfies the conditions for static equilibrium.

5. Write the appropriate equations for finding the unknown forces and torques acting on the beam. How many equations are there? There are three equations:

$$\sum F_x = 0 = F_{Rx} + T\cos\theta$$

$$\sum F_y = 0 = F_{Ry} - F_G + T\sin\theta$$

$$\sum \tau_z = 0 = -F_G\frac{L}{2} + T\sin\theta L$$

6. How many unknowns do you have in your system of equations? Three unknowns: T, F_{Rx}, and F_{Ry}.

7. Is the pinned beam system *statically determinant*? Why or why not? Yes, the system is statically determinant because the number of equations equals the number of unknowns.

Solution 5.3

The most logical answer here is to create a "global frame" and a "ramp frame". The global frame is defined by the unit vectors $[\hat{\imath}, \hat{\jmath}]$. The $\hat{\imath}$ vector should be parallel to the floor, with the $\hat{\jmath}$ vector perpendicular to the floor. The "ramp frame" is defined by the unit vectors $[\hat{x}, \hat{y}]$. The $\hat{x}$ vector should be parallel to the ramp, with the $\hat{y}$ vector perpendicular to the ramp. See below. Note that when focusing on forces, the position of the origin of each frame is not particularly important, but if we were concerned with the motion of the ramp and the box, we would want to think carefully about the position of each coordinate system (e.g. we may have three frames, with a global fixed coordinate system, and a coordinate systems with an origin fixed to the ramp and box respectively).

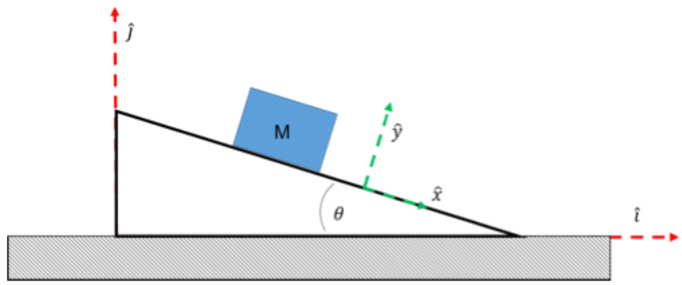


Figure 5.4: Global and ramp reference frames defined for the box-on-a-ramp.

Solution 5.4

1. Draw a FBD for the box by isolating it from the ramp. Draw and label appropriate vectors for the forces acting on the box.

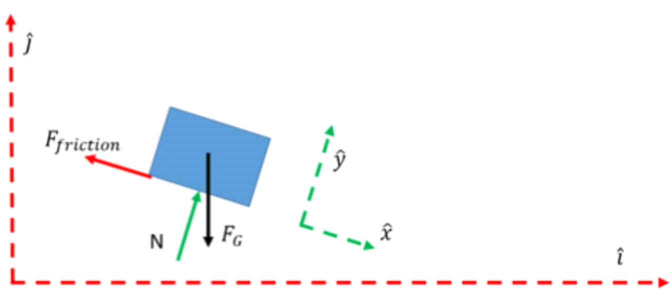


Figure 5.5: FBD for the isolated box.

2. Draw a FBD for the ramp (ramp mass=M_R). Draw and label appropriate vectors for the forces.

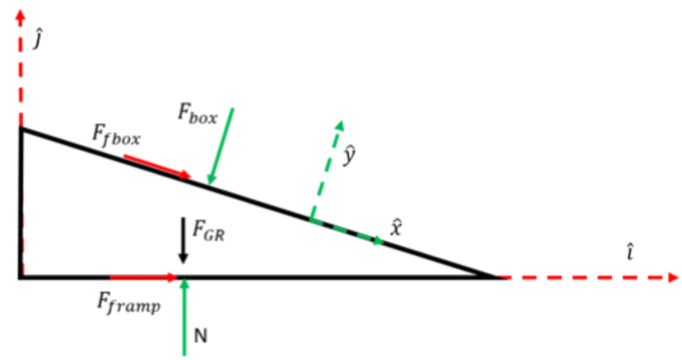


Figure 5.6: FBD for the isolated ramp.

3. Look carefully at the force vectors you have drawn for the box and ramp FBDs. What reference frame is each force defined in (e.g. for the box, gravity is likely in the global $\hat{j}$ direction, while the normal force is in the ramp $\hat{y}$ direction)? Make a table like the one below explicitly defining each force/frame.

Force	Acting On	Reference Frame	Direction
Gravitational force (weight of ramp)	Ramp	Global Frame	$-\hat{j}$
Floor normal force	Ramp	Global Frame	$+\hat{j}$
Floor frictional force	Ramp	Global Frame	$+\hat{\imath}$
Normal force from box to ramp	Ramp	Ramp Frame	$-\hat{y}$
Friction force from box to ramp	Ramp	Ramp Frame	$+\hat{x}$
Friction force from ramp to box	Box	Ramp Frame	$-\hat{x}$
Normal force on box from ramp	Box	Ramp Frame	$+\hat{y}$
Gravitational force (weight of box)	Box	Global Frame	$-\hat{j}$

Chapter 6

Homework 2: Torque and Static Equilibrium

Contents

6.1 Video Review of Statics

Please do the Video Review section (about 20 minutes of work) before class on Wednesday—the rest of the homework is due as usual

Exercise 6.1

Video Review:

1. Visit this link and watch the video about equilibrium titled "A Statics Example". After watching the video, make a table of key concepts. Include simple sketches where appropriate to illustrate these concepts. For extra credit (whatever that means) find the mistake in the video (and yes, there is a REAL mistake in the video, not some minor pedantic thing).
2. Also watch this video, which provides another example of drawing a FBD and thinking about the torques in the problem. This particular example is not a static example, but it talks qualitatively about what the effect of torques will be.

6.2 Statics Continued!

Exercise 6.2

Consider the *cantilevered beam* with constant cross section and mass distribution shown in Figure 6.1 below. The beam has mass M, and is supported at one end by a *fixed joint*, which can provide linear support (or reaction) forces, and moments. For this exercise, you can assume the beam is stationary ($\boldsymbol{a} = \boldsymbol{\alpha} = 0, \boldsymbol{v} = \dot{\boldsymbol{\theta}} = 0$).

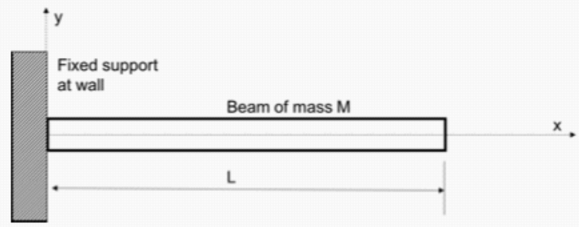
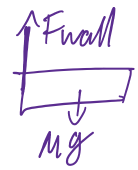


Figure 6.1: A cantilevered beam of mass M.

1. "A fixed joint can provide linear support (or reaction) forces, and moments". Explain why this is so. This question makes no sense (self-definitional)
2. When drawing a free body diagram for the cantilevered beam in Figure 6.1, which body should you isolate? Beam
3. Draw the FBD for the body you have isolated, paying special attention to the steps outlined above.
4. Is the beam in static equilibrium? How do you know? NO. Unbalanced torques
5. Write the appropriate equations for find the unknown forces and torques acting on the beam. How many equations are there? 2 $F_{wall} = ?$ $F_g = mg$
6. How many unknowns do you have in your system of equations? 1
7. Is cantilevered beam system *statically determinant*? Why or why not? Yes, b/c unknowns = eqns



Exercise 6.3

You may have seen this coming, but let's consider the cantilevered beam, of constant cross section and mass distribution, with a rope, shown in Figure 6.3.

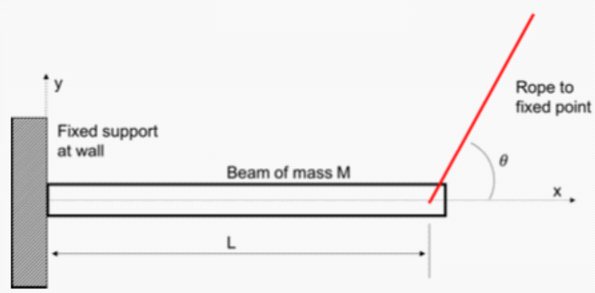
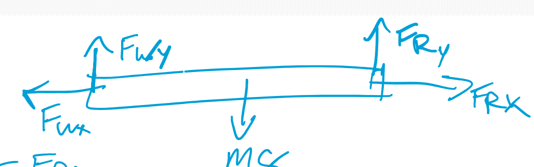


Figure 6.3: A cantilevered beam of mass M with a rope attached to a fixed point.

$0 = F_{wy} + F_{Ry} + mg$

$0 = F_{wx} + F_{Rx}$

$F_{wx} = -F_{Rx}$ $F_{wy} = F_{Ry}$



Didn't understand solutions

1. Draw the FBD for the system in Figure 6.3, paying special attention to the steps outlined above.
2. Write the appropriate equations for find the unknown forces and torques acting on the beam. How many equations are there? 4
3. How many unknowns do you have in your system of equations? 4
4. Is the system *statically determinant*? Why or why not? Yes, # eqns = #?

Exercise 6.4

For this exercise, refer to the system below that we started talking about in class. You should use the reference frames that we developed in class for this problem (i.e., the global frame, and the ramp frame). Assume, for the sake of this problem, that the system is static, that the box has mass M, that the ramp has mass m, and the angle of the ramp is θ.

Box:
$\Sigma F = \sin(\theta) Mg\hat{i} - \cos(\theta) Mg\hat{j}$ } grav
$-\sin(\theta) Mg\hat{i} + \cos(\theta) Mg\hat{j}$ } Norm

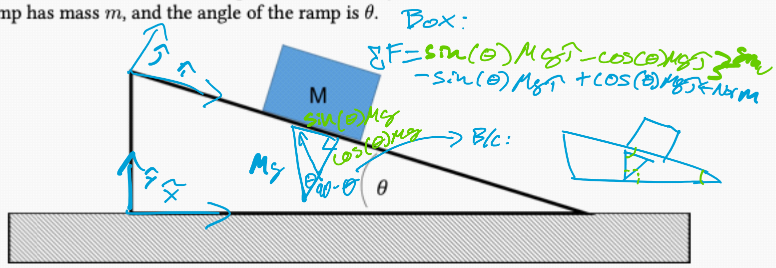
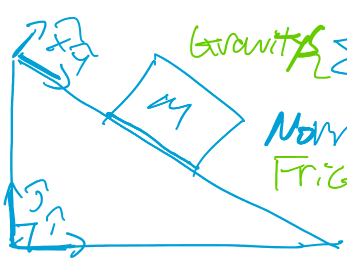


Figure 6.5: Box of mass M on an inclined ramp. The ramp is not fixed to the floor, and the box is not fixed to the ramp.

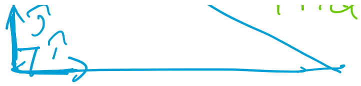
1. Write down an equation for the summation of all of the forces acting on the box, and an equation for all forces acting on the ramp. You should include the unit vector associated with each force (i.e. $\sum F = -F_g\hat{j} - F_{friction}\hat{x} + ...$). How many unknown quantities do you have here? How many equations? Is this problem determinate or indeterminate?
2. Typically, for a 2D problem, we would split the force equation into two equations, $\sum F_{\hat{i}}$ and $\sum F_{\hat{j}}$ (or commonly $\sum F_x$ and $\sum F_y$). Can we do that here? What about the forces that are defined in the $[\hat{x}, \hat{y}]$ frame?
3. We need to represent all of our forces in a common reference frame in order to perform analysis. In the Faces Modules, you encountered the concept of *rotation matrices*, and we can apply that idea here to relate the global frame and the ramp frame.
   (a) Draw the unit vectors that define the global and ramp frame co-located at the same origin. Specify the angle θ that defines the rotation between the two coordinate systems.
   (b) Write equations for $\hat{x}$ and $\hat{y}$ in terms of $\hat{i}$ and $\hat{j}$.
   (c) Rewrite the equations above in the form of a *rotation matrix*.
   (d) Rewrite your force equations so all of the forces are expressed in the global reference frame $[\hat{i}, \hat{j}]$, then split each force equation into the $\hat{i}$ and $\hat{j}$ components (i.e. you should end up with four equations for $\sum F_{box\hat{i}}$, $\sum F_{box\hat{j}}$, $\sum F_{ramp\hat{i}}$, $\sum F_{ramp\hat{j}}$).

Box Forces



Gravity { $\Sigma F_{Box} = \sin(\theta)Mg\hat{i} - \cos(\theta)Mg\hat{j}$
Normal { $+\cos(\theta)Mg\hat{j}$
Friction { $-\sin(\theta)Mg\hat{i}$

Ramp Forces



Friction $\{ -\sin(\theta)\mu g\hat{\imath}$

Ramp Forces

Gravity $\{ \sum F_{ramp} = -mg\hat{y}$

Box$_{fric}$ $\{ +\sin(\theta)\mu g\hat{\imath}$

Box Norm $\{ -\cos(\theta)\mu g\hat{\jmath}$

Normal $\{ +mg\hat{y}$

Friction$_{gnd}$ $\{ -\sin(\theta)\mu g\hat{\imath}$

Take 2

$$\sum F_{Box} = F_{grav}\hat{\jmath} + F_N\hat{y} - F_f\hat{x}$$

$$\sum F_{ramp} = -F_{grav}\hat{\jmath} - F_{fgnd}\hat{\imath} - F_{fbox}\hat{x} - F_N\hat{\jmath} + F_{Ngnd}\hat{\jmath}$$

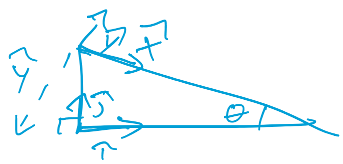
1) 2 unknowns and 2 eqns, so determinate

2) No for all forces, yes for $[\hat{x}\ \hat{y}]$ only

3) a)

b) $\hat{x} = \cos(\theta)\hat{\imath} - \sin(\theta)\hat{\jmath}$

$\hat{y} = \cos(\theta)\hat{\jmath} + \sin(\theta)\hat{\imath}$



c)
$$\begin{array}{cc} \hat{x} & \hat{y} \\ \end{array}$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \cos(0) & -\sin\theta \\ \sin(0) & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos a + \sin b & \sin a + \cos a \\ & \end{bmatrix}$$

d)
$$\begin{array}{c|c} & \begin{array}{cc} \hat{x} & \hat{y} \end{array} \\ \begin{array}{c} \text{box} \\ \text{ramp} \end{array} & \begin{bmatrix} F_{fbox} & F_{Nbox} \\ -F_{fbox} & -F_{Nbox} \end{bmatrix} \end{array} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} + \begin{array}{c} \begin{array}{cc} \hat{\imath} & \hat{\jmath} \end{array} \\ \begin{bmatrix} & F_g \\ F_{fgnd} & F_g + F_{Ng} \end{bmatrix} \end{array} = \begin{bmatrix} F_{b\hat{\imath}} & F_{b\hat{\jmath}} \\ F_{r\hat{\imath}} & F_{r\hat{\jmath}} \end{bmatrix}$$

$$\begin{bmatrix} F_{fbox} & F_{Nbox} \\ -F_{fbox} & -F_{Nbox} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} 0 & Mg \\ F_{fgnd} & mg + F_{Ng} \end{bmatrix} = \begin{bmatrix} F_{b\hat{\imath}} & F_{b\hat{\jmath}} \\ F_{r\hat{\imath}} & F_{r\hat{\jmath}} \end{bmatrix}$$

✓ 4 eqns, 4 unknowns

4. Assume now that the box has mass $M = 1$ kg, that the ramp has mass $m = 2$ kg, and the angle of the ramp is $\theta = \pi/6$. Using the magic of matrices, solve the resulting system of equations for the unknown forces.

5. In solving for this system of equations, you should have obtained two normal forces as well as two frictional forces. You found these by *assuming* that the system was at equilibrium. Now you should check that assumption: are the resulting normal and frictional forces possible? In particular, what coefficients of static friction are needed between the block and the ramp and between the ramp and the ground in order to meet the static condition?

6.3 Revisiting Torque

Exercise 6.5

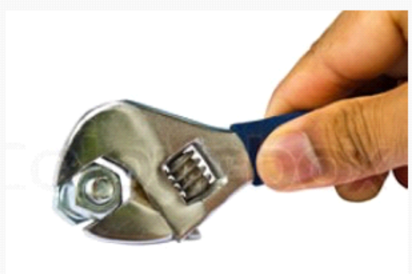
A force with vector representation $\vec{F} = 3\hat{\imath} + 2\hat{\jmath}$ is acting at a location $\vec{r} = 1\hat{\jmath} + 5\hat{k}$. What is the torque due to this force about the origin? What is the torque due to this force about the point $\vec{r}_0 = 7\hat{i} + 3\hat{k}$? You might need to review the process of computing the cross-product of two vectors to complete this exercise. Consider the Rule of Sarrus for the procedure, or the physical interpretation.

Exercise 6.6

The picture below shows a person trying to tighten a nut with a wrench. The point of view is that of standing, facing the end of the bolt as shown and turning clockwise. Draw a free body diagram for

1. The wrench only
2. The nut only

In drawing your free body diagram, call out the forces acting between surfaces (i.e., don't just draw a moment on the nut!) Then, using your FBDs, explain how the wrench is used to tighten the nut.



Solution 6.2

1. "A fixed joint can provide linear support (or reaction) forces, and moments". Explain why this is so. Since the end is fixed, you can imagine trying to pull on the joint in any direction – and the joint resisting – but you can also imagine trying to rotate the object about the joint, and the joint reacting.

2. When drawing a free body diagram for the pinned beam in Figure 5.1, which body should you isolate? For the cantilevered beam in Figure 6.1, you want to isolate just the beam portion of the drawing as shown below.

3. Draw the FBD for the body you have isolated, paying special attention to the steps outlined above. The FBD for the cantilevered beam is shown in 6.2. We've used the same treatment for the reaction force at the wall as we did for the pin in the above cantilever; its components, F_{Rx} and F_{Ry} are shown at the joint in the x and y directions, respectively. We have also drawn a circular arrow to represent a reactive turning moment, M_{Rz}, at the wall. We drew this on our FBD because we know by experience that the F_G produces a clockwise turning moment. Since the beam is not moving, this moment must be offset by an equal an opposite one at the wall connection.

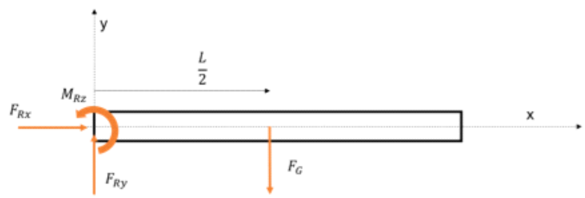


Figure 6.2: FBD for the cantilevered beam. Vectors are not necessarily drawn to scale.

4. Is the beam in static equilibrium? How do you know? Yes, in the problem statement we were told For this exercise, you can assume the beam is stationary $\boldsymbol{a} = \boldsymbol{\alpha} = 0$, which satisfies the conditions for static equilibrium.

5. Write the appropriate equations for find the unknown forces and torques acting on the beam. How many equations are there? There are three equations:

$$\sum F_x = 0 = F_{Rx}$$
$$\sum F_y = 0 = F_{Ry} - F_G$$
$$\sum \tau_z = 0 = -F_G \frac{L}{2} + M_{Rz}$$

6. How many unknowns do you have in your system of equations? Three unknowns: M_{Rz}, F_{Rx}, and F_{Ry}.

7. Is the cantilevered beam system *statically determinant*? Why or why not? Yes, the system is statically determinant because the number of equations equals the number of unknowns.

Solution 6.3

1. Draw the FBD for the system in Figure 6.3, paying special attention to the steps outlined above.

   The FBD for the cantilvered beam with a rope is shown in 6.4. In the diagram, F_{Rx} and F_{Ry} are the reaction forces from the fixed joint in the x and y directions respectively, and M_{Rz} is the reaction moment.

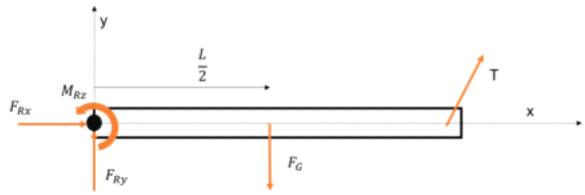


Figure 6.4: FBD for the cantilevered beam with a rope.

2. Write the appropriate equations for find the unknown forces and torques acting on the beam. How many equations are there? There are three equations:

$$\sum F_x = 0 = F_{Rx} + T\cos\theta$$

$$\sum F_y = 0 = F_{Ry} - F_G + T\sin\theta$$

$$\sum \tau_z = 0 = -F_G\frac{L}{2} + T\sin\theta L + M_{Rz}$$

3. How many unknowns do you have in your system of equations? Four unknowns: M_{Rz}, F_{Rx}, F_{Ry}, and T.

4. Is the system *statically determinant*? Why or why not? No, the system is statically indeterminant because the number of equations is less than the number of unknowns.

Solution 6.4

1. Write down an equation each for the summation of all of the forces acting on the box and ramp. You should include the unit vector associated with each force (i.e. $\sum F_{box} = -F_g\hat{j} - F_{friction}\hat{x} + ...$).

$$\sum F_{box} = -F_g\hat{j} - F_{friction}\hat{x} + N\hat{y}$$

$$\sum F_{ramp} = -F_{GR}\hat{j} + N\hat{j} + F_{framp}\hat{i} + F_{fbox}\hat{x} - F_{box}\hat{y}$$

2. Typically, for a 2D problem, we would split the force equation into two equations, $\sum F_{\hat{i}}$ and $\sum F_{\hat{j}}$ (or commonly $\sum F_x$ and $\sum F_y$). Can we do that here? What about the forces that are defined in the $[\hat{x}, \hat{y}]$ frame? Can forces in different framed be added?

   Yes, we can split these equations into two separate equations, but not before we represent all of the forces in the same reference frame. Forces in different reference frames do not add, so we must choose one frame for the whole system.

We need to represent all of our forces in a common reference frame in order to perform analysis. In the Faces Modules, you encountered the concept of *rotation matrices*, and we can apply that idea here to relate the global frame and the ramp frame.

1. Draw the unit vectors that define the global and ramp frame co-located at the same origin. Specify the angle θ that defines the rotation between the two coordinate systems.

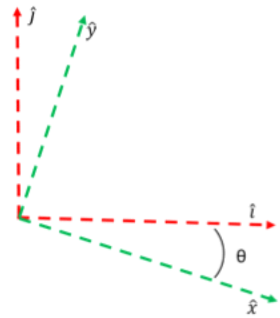


Figure 6.6: Unit vectors that define the two coordinate systems in the box-on-a-ramp problem.

2. Write equations for $\hat{x}$ and $\hat{y}$ in terms of $\hat{\imath}$ and $\hat{\jmath}$.

$$\hat{x} = \cos(\theta)\hat{\imath} - \sin(\theta)\hat{\jmath}$$
$$\hat{y} = \sin(\theta)\hat{\imath} + \cos(\theta)\hat{\jmath}$$

3. Rewrite the equations above in the form of a *rotation matrix.*

$$\begin{bmatrix}\hat{x}\\ \hat{y}\end{bmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\hat{\imath}\\ \hat{\jmath}\end{bmatrix}$$

4. Rewrite your force equations so all of the forces are expressed in the global reference frame $[\hat{\imath}, \hat{\jmath}]$, then split each force equation into the $\hat{\imath}$ and $\hat{\jmath}$ components (i.e. you should end up with four equations for $\sum F_{box\hat{\imath}}$, $\sum F_{box\hat{\jmath}}$, $\sum F_{ramp\hat{\imath}}$, $\sum F_{ramp\hat{\jmath}}$).

Note: $F_{friction} = F_f$ for simplicity

$$\sum F_{box\hat{i}} = -F_f \cos\theta \hat{i} + N \sin\theta \hat{i} = 0$$

$$\sum F_{box\hat{j}} = -F_G \hat{j} + F_f \sin\theta \hat{j} + N \cos\theta \hat{j} = 0$$

$$\sum F_{ramp\hat{i}} = F_{framp} \hat{i} + F_{fbox} \cos\theta \hat{i} - F_{box} \sin\theta \hat{i} = 0$$

$$\sum F_{ramp\hat{j}} = -F_{GR} \hat{j} + N \hat{j} - F_{fbox} \sin\theta \hat{j} - F_{box} \cos\theta \hat{j} = 0$$

Solution 6.5

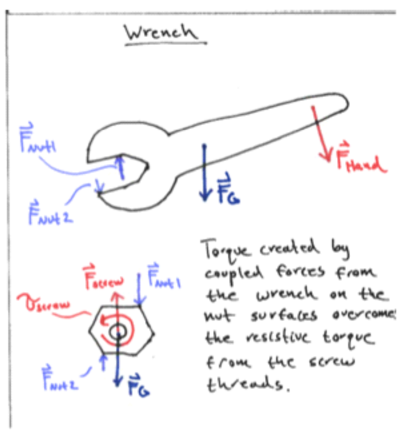
A force with vector representation $\vec{F} = 3\hat{i} + 2\hat{j}$ is acting at a location $\vec{r} = 1\hat{j} + 5\hat{k}$. What is the torque due to this force about the origin? What is the torque due to this force about the point $\vec{r}_0 = 7\hat{i} + 3\hat{k}$?
$\vec{r} \times \vec{F}$ gives you $-10\hat{i} + 15\hat{j} - 3\hat{k}$, $(\vec{r} - \vec{r}_0) \times \vec{F}$ gives you $-4\hat{i} + 6\hat{j} - 18\hat{k}$

Solution 6.6

The picture below shows a person trying to tighten a nut with a wrench. The point of view is that of standing, facing the end of the bolt as shown and turning clockwise. Draw a free body diagram for

1. The wrench only
2. The nut only

In drawing your free body diagram, call out the forces acting between surfaces (i.e., don't just draw a moment on the nut!) Then, using your FBDs, explain how the wrench is used to tighten the nut.



Boats2

```
syms F_bi F_bj F_ri F_rj;
syms M m;
syms F_f_box F_f_gnd;
syms F_n_box F_n_gnd;
syms g;
syms theta;

F_XY = [F_f_box, F_n_box; -F_f_box, -F_n_box]
```

F_XY =

$$\begin{pmatrix} F_{f,\text{box}} & F_{n,\text{box}} \\ -F_{f,\text{box}} & -F_{n,\text{box}} \end{pmatrix}$$

```
F_IJ = [0, M * g; F_f_gnd, (m * g) + (-m * g)]
```

F_IJ =

$$\begin{pmatrix} 0 & M\,g \\ F_{f,\text{gnd}} & 0 \end{pmatrix}$$

```
trans = [cos(theta), -sin(theta); sin(theta), cos(theta)]
```

trans =

$$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$$

```
XYIJ = [1 1; 1 1] * trans
```

XYIJ =

$$\begin{pmatrix} \cos(\theta)+\sin(\theta) & \cos(\theta)-\sin(\theta) \\ \cos(\theta)+\sin(\theta) & \cos(\theta)-\sin(\theta) \end{pmatrix}$$

```
XIJ = XYIJ(:, 1)
```

XIJ =

$$\begin{pmatrix} \cos(\theta)+\sin(\theta) \\ \cos(\theta)+\sin(\theta) \end{pmatrix}$$

```
YIJ = XYIJ(:, 2)
```

YIJ =

$$\begin{pmatrix} \cos(\theta)-\sin(\theta) \\ \cos(\theta)-\sin(\theta) \end{pmatrix}$$

```
M * g
```

ans = $M\,g$

```
F_net = (F_XY * trans) + F_IJ
```

```
F_net =
```

$$\begin{pmatrix} F_{f,\text{box}} \cos(\theta) + F_{n,\text{box}} \sin(\theta) & M\ g + F_{n,\text{box}} \cos(\theta) - F_{f,\text{box}} \sin(\theta) \\ F_{f,\text{gnd}} - F_{f,\text{box}} \cos(\theta) - F_{n,\text{box}} \sin(\theta) & F_{f,\text{box}} \sin(\theta) - F_{n,\text{box}} \cos(\theta) \end{pmatrix}$$

```
F_g_box_in_xy = [1; 1] * (M * g)
```

```
F_g_box_in_xy =
```

$$\begin{pmatrix} M\ g \\ M\ g \end{pmatrix}$$

```
F_n_box = - dot(F_g_box_in_xy, YIJ)
```

`F_n_box =` $-2\,\overline{M}\,\overline{g}\,(\cos(\theta) - \sin(\theta))$

```
M = 1; m = 2; theta = (pi / 6); g = 9.8;
F_n_gnd = -(m * g);

F_net
```

```
F_net =
```

$$\begin{pmatrix} F_{f,\text{box}} \cos(\theta) + F_{n,\text{box}} \sin(\theta) & M\ g + F_{n,\text{box}} \cos(\theta) - F_{f,\text{box}} \sin(\theta) \\ F_{f,\text{gnd}} - F_{f,\text{box}} \cos(\theta) - F_{n,\text{box}} \sin(\theta) & F_{f,\text{box}} \sin(\theta) - F_{n,\text{box}} \cos(\theta) \end{pmatrix}$$

```
subs(F_net)
```

```
ans =
```

$$\begin{pmatrix} \frac{\sqrt{3}\ F_{f,\text{box}}}{2} - \sigma_2 & \frac{49}{5} - \sigma_1 - \frac{F_{f,\text{box}}}{2} \\ F_{f,\text{gnd}} - \frac{\sqrt{3}\ F_{f,\text{box}}}{2} + \sigma_2 & \frac{F_{f,\text{box}}}{2} + \sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = \sqrt{3}\ \overline{M}\,\overline{g}\,(\cos(\theta) - \sin(\theta))$$

$$\sigma_2 = \overline{M}\,\overline{g}\,(\cos(\theta) - \sin(\theta))$$

```
result = solve(F_net)
```

```
result = struct with fields:
    F_f_box: [0×1 sym]
          M: [0×1 sym]
          g: [0×1 sym]
      theta: [0×1 sym]
```