Homework 1

Monday, February 1, 2021 2:05 PM

QEA2Spring
2021

Chapter 3

Homework 1: Forces and Vector Operations

Contents

Please do the Video Review section (about 1 hour of work) before class on Wednesday—the rest of the homework is due as usual

3.1 Video Review: Force Ideas and Models - Please complete for Wednesday!

In previous exposures to physics content you may have been introduced to various types of forces and force models. Two of the fundamental big-picture takeaways about forces are:

- Force is always a vector, and in order to fully specify the force, you need to give both magnitude and direction.
- Forces must be added vectorially.

Exercise 3.1

Video Review: Hooray, more videos! Like Vectors and Vector Operations, these may or may not be new ideas for you, but these concepts will be important throughout the course. Visit this link, and watch the videos relating to Models for Forces. This should take you about an hour if you watch it at regular speed and watch everything, or maybe 30 minutes if you watch at 2x speed. Which ever you choose to do, please complete the following table by listing the important ideas in the videos.

Ideas in the videos I already knew	Ideas in the videos I had not seen before

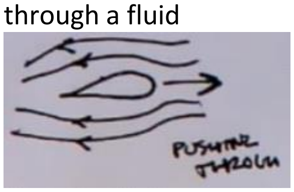
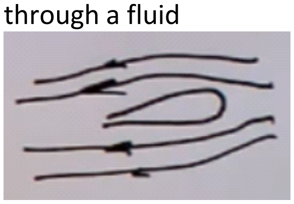
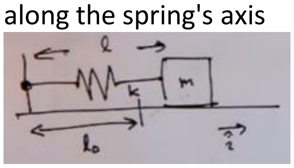
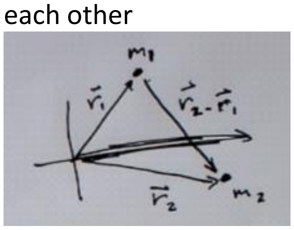
Ideas I already knew	Ideas I didn't already know
What the various types of forces are	Fundamental vs. phenom force distinction
Magnitudes, directions, and use-cases for some of the phenomenological forces	Distributed vs. concentrated forces
	Lots of different information about specific phenomenological forces (magnitudes, use-cases, etc.)

<table>
<tr><td>What is a Force?</td><td>An interaction between two objects that causes a change in momentum (Δp) of a system</td></tr>
<tr><td>What types of forces are there?</td><td>• Fundamental Interaction: the building blocks of the universe (Strong Force, Electromagnetic Force, Weak Force, and Gravity)
• Phenomenological: higher-level interactions built on top of fundamental interactions but modeled separately for simplicity and convenience.</td></tr>
<tr><td>How do the different fundamental forces compare?</td><td><table>
<tr><td>Force</td><td>Strength (Relative)</td><td>Distance (big-O style)</td></tr>
<tr><td>Strong</td><td>1</td><td>Short ($\frac{e^{-r}}{r}$)</td></tr>
<tr><td>Electromagnetic</td><td>10^{-2}</td><td>Long ($\frac{1}{r^2}$)</td></tr>
<tr><td>Weak</td><td>10^{-10}</td><td>Short ($\frac{e^{-r}}{r}$)</td></tr>
<tr><td>Gravity</td><td>10^{-40}</td><td>Long ($\frac{1}{r^2}$)</td></tr>
</table>
The Strong and Weak Forces really only make a difference within the nucleus of an atom. The Electromagnetic Force is responsible for most of the things in our daily lives, and the Gravitational Force is responsible for very large things (planets, etc.).</td></tr>
<tr><td>What is a phenomenological force?</td><td>An interaction caused by a fundamental force(s) but modeled at a higher level for simplicity.

Example: Friction is due to transient bonding (electromagnetic force) between the atoms in an object and in the table. Friction is heating is caused because the atoms shake back and forth when the bonds are stretched and break. The simplified phenom. model we use ($F_{friction} = \mu N$) is actually wrong in many situations, but we use it because it's easy.</td></tr>
<tr><td>What are common phenom forces?</td><td>• Friction
• Drag, Lift (EM interactions with the fluid the object is moving through)
• Spring Force
• Damping Force (similar to drag)
• "Constraint" Forces (forces that prevent objects from doing things they're not allowed to do
    • Normal Force
    • Tension/Compression</td></tr>
<tr><td>What is distributed force?</td><td>For example, if you have a box on a table, every infinitesimally small bit of the box is being pulled down by gravity and every point of contact with the table is being pushed up by the table.</td></tr>
<tr><td>When can we simplify distributed forces into concentrated forces?</td><td>• Not for internal forces
• Not for things that are flimsy or where it doesn't make sense. (For example, if you're modeling a foot standing on the ground, having only one point of contact with the foot doesn't work.)</td></tr>
<tr><td>How can we</td><td>• For gravity, you can replace it with a single force acting on the center of mass.</td></tr>
</table>

simplify distributed forces into concentrated forces?	• Finding the CoM can be hard for asymmetric or non-uniform-density shapes. • For contact forces that are uniform (like normal force on a flat box) and symmetrical, you can have replace it with a single force at the middle. • For something that isn't solid or is asymmetrical, that can be harder.

Mathematical models for phenom forces:

Type	Example	Magnitude	Direction
Static Friction	Pushing an object that isn't moving	Equal to other applied tangential forces, up to a limit ($\mu_s N$). $\lvert F_T \rvert \leq \mu_s N$	Tangent to surface, opposes other tangential forces.
Kinetic Friction	Pushing an object that is in motion	$F = \mu_k N$	Opposite motion ($-\hat{v}$)
Drag	An airfoil moving through a fluid 	Very complicated, heavily dependent on exact shapes. A common formula for moving through air ("inertial drag model"): $\vec{F}_D = -\frac{1}{2} C_D A \rho v^2 \hat{v}$ C_D: Drag coefficient (shape of object, roughly 0.1 - 1, unitless) A: Presented/cross-sectional area (m^2) ρ: Density of the fluid (???) For more viscous fluids: $\vec{F}_D = -\beta \vec{v}$	Opposite motion ($-\hat{v}$)
Lift	An airfoil with different top and bottom geometries moving through a fluid 	Very complicated, no simple expression. Often proportional v^2, often has an area component.	Perpendicular to motion ($\perp \hat{v}$)
Restoring (Spring)	A deformed spring, along the spring's axis 	Has to do with the position/state of the system, not the motion. $\vec{F} = -k(l - l_0)\hat{\imath}$ (NB: this format of formula shows up a lot because it's the first term of a Taylor expansion)	Towards equilibrium
Gravity	Two objects attract each other 	$\vec{F}_{2\ on\ 1} = \frac{G m_1 m_2}{\lvert \vec{r}_2 - \vec{r}_1 \rvert^2} * \frac{\vec{r}_2 - \vec{r}_1}{\lvert \vec{r}_2 - \vec{r}_1 \rvert}$ $= \frac{G m_1 m_2 (\vec{r}_2 - \vec{r}_1)}{\lvert \vec{r}_2 - \vec{r}_1 \rvert^3}$ You may have seen: $\vec{F}_g = -mg\hat{\jmath}$	Towards the other object ($\vec{r}_2 - \vec{r}_1$)

		This only works near the surface of the earth, and assumes that the change in distance between you and the earth is negligible and the earth is always straight down.	
Normal	One object sitting on top of another (or being pushed into another upwards, etc.)	As much as it needs to be to counteract other forces to prevent illegal motion (ie. effectively, the table will push as hard as it needs to to prevent the block from going through it.)	Normal (perpendicular) to the surface.
Tension (String)	An object attached to a center point with a string (ex. Kid on a swing)	As much force as is necessary to ensure the length of the string doesn't exceed the original length (but it can be shorter).	Must be in the radial direction, and can only pull in (you can't push on a string).

3.2 Vector Operations

Exercise 3.2

Make a sketch of the system consisting of the inner solar system. Set the origin on the sun and assume the sun is stationary. Include Mercury, Venus, Earth, and the Earth's moon (no need to be to scale here), and assume that everything is in the plane.
Sketch the following position vectors:

- The displacement vectors of all three planets from the sun (call these $\vec{M}$, $\vec{V}$, and $\vec{E}$)
- The displacement vector of the moon from the earth (label this $\vec{m}$).

With these definitions in place, sketch and find expressions (just in terms of $\vec{M}$, $\vec{V}$, and so forth) for:

- The displacement vector pointing from the Earth to Venus

  $\vec{V}-\vec{E}$ ✓

- A unit vector pointing from Mercury to the sun

  $-\vec{M}/|\vec{M}|$

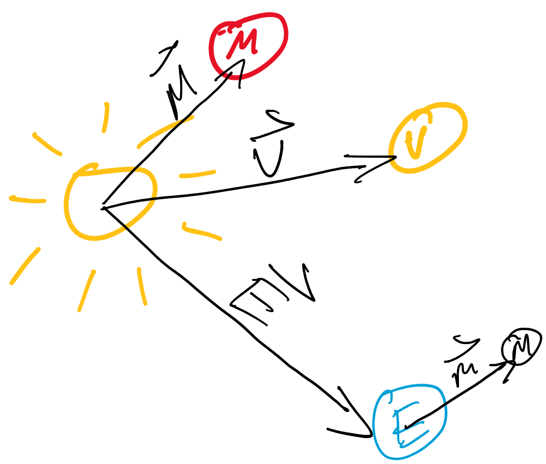
- A unit vector pointing from Venus to the moon

  X Backward

  $(\vec{V}-(\vec{E}+\vec{m}))/|\vec{V}-(\vec{E}+\vec{m})|$

- A unit vector that is perpendicular to the orbital plane of the earth

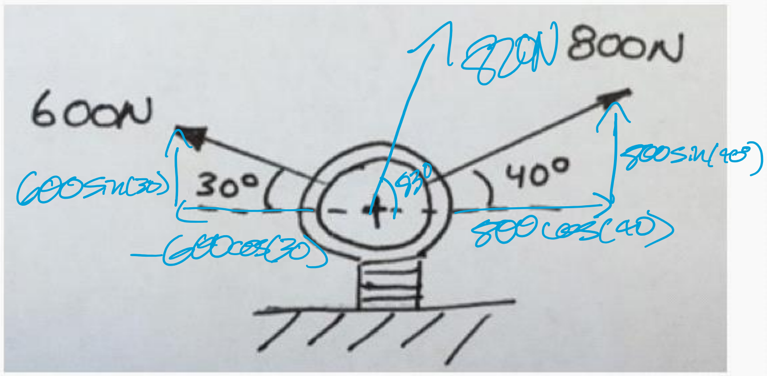
  $\frac{\vec{V}\times\vec{E}}{|\vec{V}\times\vec{E}|}$



3.3 Modeling and Manipulating Forces

Exercise 3.3

1. Two forces are applied to an eye hook. They lie in the plane of the page with the orientation and magnitudes as shown. Determine the orientation and magnitude of the sum of these two forces acting on the hook.



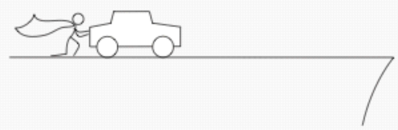
$\langle 800\cos(40°) - 600\cos(30°),\ 600\sin(30°) + 800\sin(40°) \rangle$

$\langle 93.22,\ 814.23 \rangle$

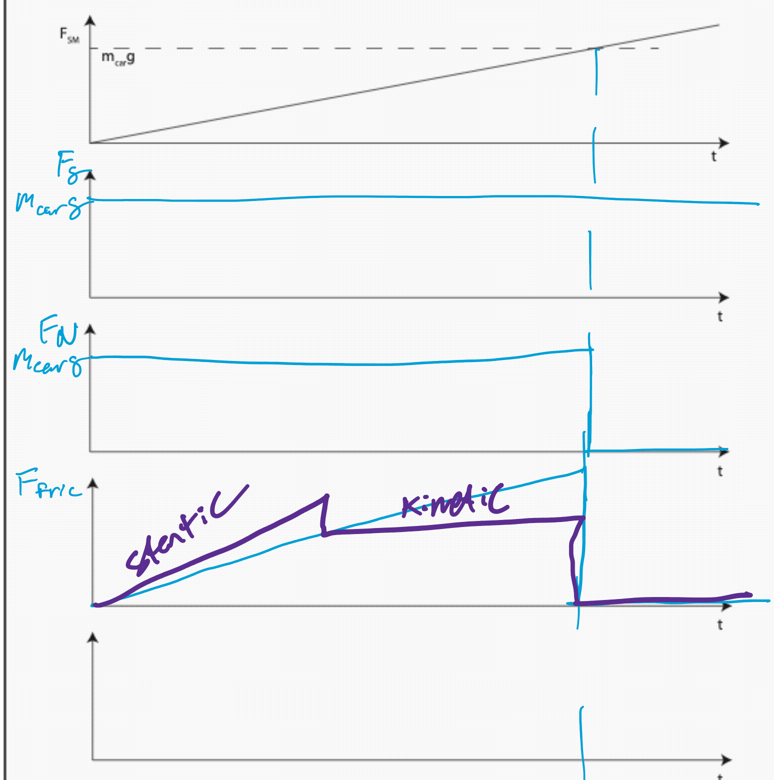
$\langle 83.47°,\ 819.55 \rangle$

Exercise 3.4

In order to save us all from a horrible fate involving radioactivity, genetically-engineered super bugs, and bad script writing, Superman must push a car off a cliff.



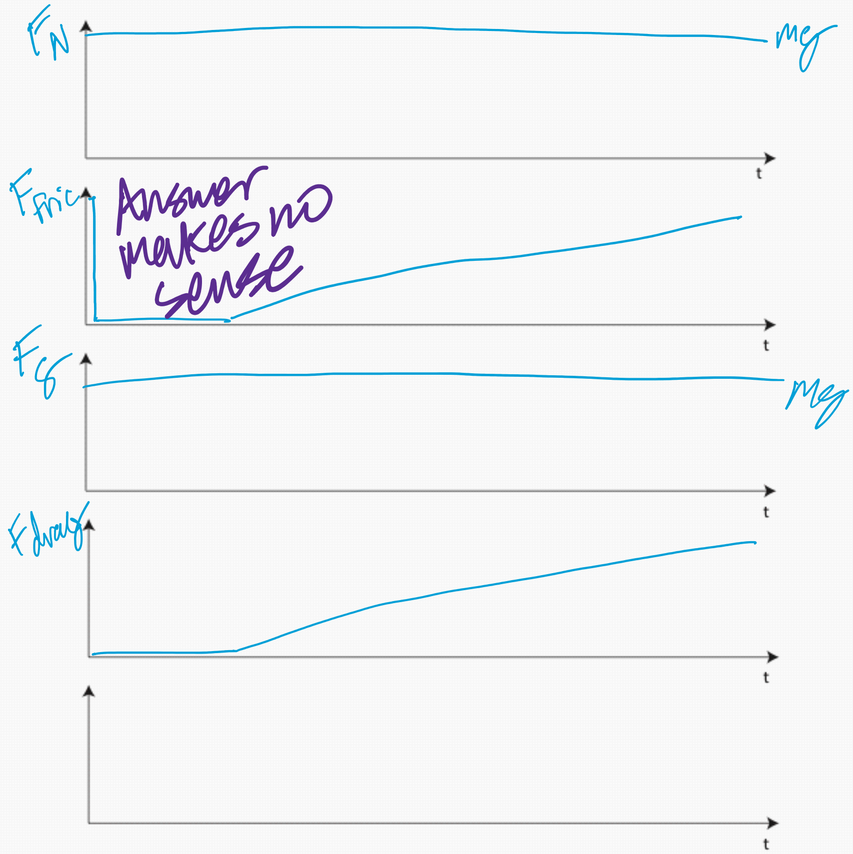
The car's emergency brake is engaged, which might make things harder. But since he's super, he's able to push against the back of the car as hard as he wants to. So he chooses to apply a force that increases linearly in time. As he reaches the cliff, the force he is applying happens to be $m_{car}g$. Let's call the interaction that the car experiences with Superman's hands F_{SM}. What other interactions does the car experience during this time? Identify as many as you can. Then, using the axes below, sketch how the magnitude of each of these interactions changes in time.

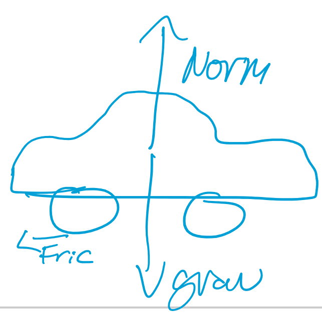


Exercise 3.5

Somehow the villain escaped. She hops her speedy getaway car, which looks like a poor drawing of a sedan but which is actually equipped with all sorts of really cool villain stuff, including a very powerful engine, and a top speed of 200 mph. She floors the accelerator, so initially her wheels spin and smoke (which looks cool, but probably increases her chance of being caught). And she keeps the accelerator pressed to the floor, and so she speeds away at ever-increasing speed (until, of course, she reaches 200 mph).

What interactions does the car experience during the time that she is getting away? identify them, and then sketch how each changes in time using the axes below.

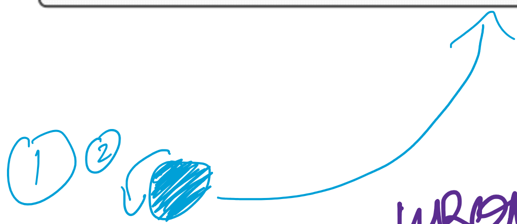




Exercise 3.6

The Magnus force can be important for the motion of rotating bodies. You should be able to find a youtube video that demonstrates the Magnus force using paper cups (search for Curve Ball Demonstrator from the Exploratorium Teacher Institute, or follow this link.
It's a nifty demo, and easy to try out. It works even better if you use styrofoam cups (which are a bit lighter than paper).

1. Write out/sketch the following: (1) a mathematical expression for the Magnus force, (2) a sketch of the trajectory of the cups in the demo, and (3) a set of free body diagrams that explain the motion.
2. In the video as set up, the spin is counterclockwise, and the launch is to the right. How do things change if the spin is clockwise? Sketch and explain.



Includes shape, surface area, liquid, etc

Angular velocity

WRONG: $F = S(\omega \times \vec{V})$

Air Resist Consta

① It's going to include:
circumference: $2\pi r$ (m)
Rotational velocity: V_R (m/s)
Some sort of constant: $C_?$ (unitless?)
So: $(2\pi r)(V_R)(C_?)$ has units of m^2/s
Unless we say that $C_?$ includes some measure of the amount of mass of air that's pushed down
↳ Probably dependent on surface area
so maybe $C_?$ has units of $\frac{\text{mass}}{\text{area}}$ $\left(\frac{kg}{m^2}\right)$
and the formula is:

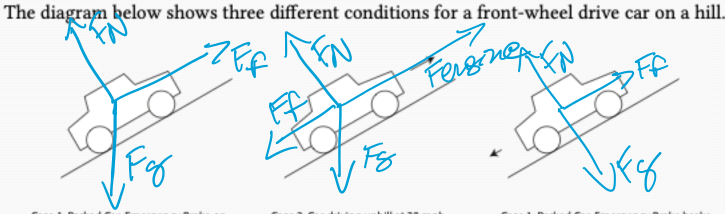
$$F_{magnus} = \underset{m}{(2\pi r)}\ \underset{m/s}{(V_R)}\ \underset{m^2}{(SA)}\ \underset{kg/m^2}{(C_?)}$$

3.4 Free Body Diagrams

In class, with the balloon problem, we started looking at different ways we can define the system boundaries for free body diagrams. For a more in depth look, watch the second video on Free Body Diagrams "System and subsystem boundaries" here.

Exercise 3.7

The diagram below shows three different conditions for a front-wheel drive car on a hill.



Case 1: Parked Car, Emergency Brake on.

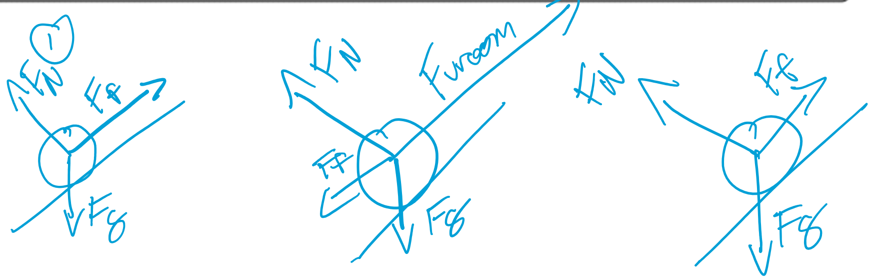
Case 2: Car driving uphill at 30 mph.

Case 1: Parked Car, Emergency Brake broke (starting to roll downhill).

For each condition, draw free-body diagrams for the following systems:

1. System = the entire car.
2. System = the rear wheel of the car.

Not @ CoM:
Distribute Forces



Exercise 3.8

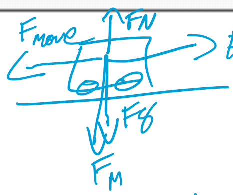
Draw free body diagrams for the following problems.

1. The root educational robot uses magnetic wheels to move around on a whiteboard. Draw a FBD for the root robot while a) Moving on a flat magnetic surface and b) on a vertical whiteboard.
2. Some robots such as the Hull Skater use magnetic wheels as well, but to stick to ships instead of whiteboards. Draw a new FBD for this robot when underwater.

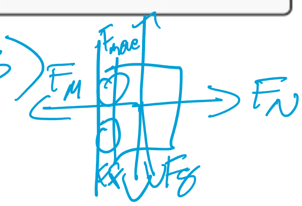
Exercise 3.9

A chain, consisting of N links, each of mass m, hangs from the ceiling. Find an expression for the tension in the nth link in the chain ($n = 1$ for the chain that is attached to the ceiling; $n = N$ for the lowest link in the chain).

3.8.1A)



3.8.1B)



3.8.2)

$$F_T = m_{link}\, g\, (N - n + 1)$$

Exercise 3.10

The diagram below shows different cross-sections of a boat in water. For the purposes of this problem, make the following assumptions:

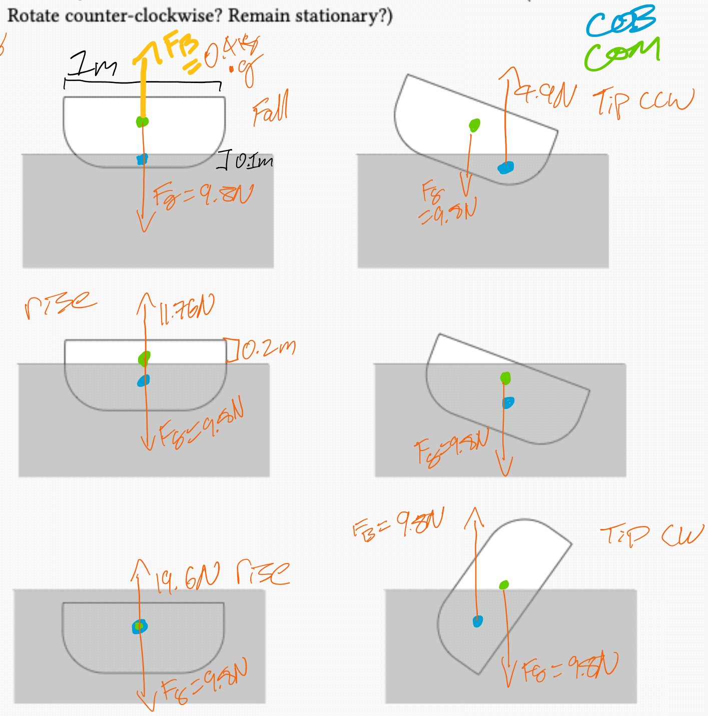
- The boat is of uniform cross-section from stem to stern (i.e., it is a simple extrusion of the shape going into the page)
- The boat has a uniform density of $\rho = 0.5$ g/cm^3.
- The boat is 4 meters in length, has a beam of 1 meter, and is about 0.5 meters from the bottom of the hull to the deck (freeboard plus draft).
- The water has a uniform density of $\rho = 1$ g/cm^3.

With those assumptions in mind, please do the following for each of the conditions drawn.

1. Calculate (approximately) the magnitudes of the gravitational force acting on the boat and the buoyant force acting in the boat. Pretend boat is rectangle
2. Calculate (approximately) the locations of the center of mass and center of buoyancy of the boat.
3. Draw an appropriate free body diagram for each condition.
4. In words, describe how the boat would move for each condition (rise? Fall? Rotate clockwise? Rotate counter-clockwise? Remain stationary?)

$V_{boat} = 0.5 \times 1 \times 4 = 2m^3$
$M_{boat} = 1kg$

COB
COM



COB = Center of Mass of Displaced water
Force of Buoyancy = - Fg of displaced water

QEA 2 HW
1 Update...

QEA 2 HW 1 Update...

Solution 3.1

Please watch the videos and make your own table based on personal experience with the content.

Solution 3.2

Make a sketch of the system consisting of the inner solar system. Set the origin on the sun and assume the sun is stationary. Include Mercury, Venus, Earth, and the Earth's moon (no need to be to scale here), and assume that everything is in the plane.
Sketch the following position vectors:

- The displacement vectors of all three planets from the sun (call these $\vec{M}$, $\vec{V}$, and $\vec{E}$)
- The displacement vector of the moon from the earth (label this $\vec{m}$).

With these definitions in place, sketch and find expressions (just in terms of $\vec{M}$, $\vec{V}$, and so forth) for:

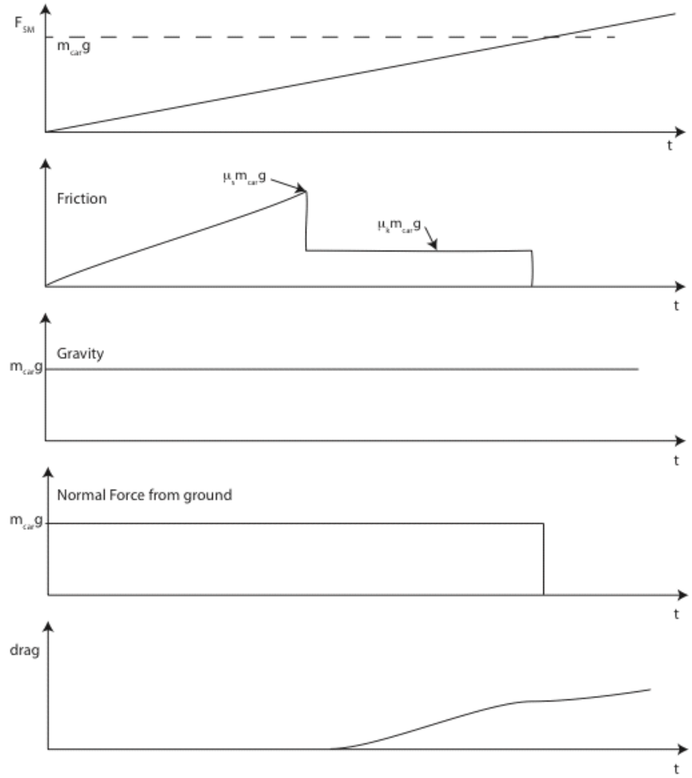
- The displacement vector pointing from the Earth to Venus
  $\vec{V} - \vec{E}$
- A unit vector pointing from Mercury to the sun
  $-\frac{\vec{M}}{|\vec{M}|}$
- A unit vector pointing from Venus to the moon
  $\frac{\vec{E}-\vec{V}+\vec{m}}{|\vec{E}-\vec{V}+\vec{m}|}$

Solution 3.3

The horizontal component of the resultant force would be $800\cos 40 - 600\cos 30$ and the vertical component would be $800\sin 40 + 600\sin 30$. Magnitude can be found from the square root of the sum of the squares of the two component. Angle can be found by taking the arc-tangent of the ratio, but it is also acceptable to just leave it in components.

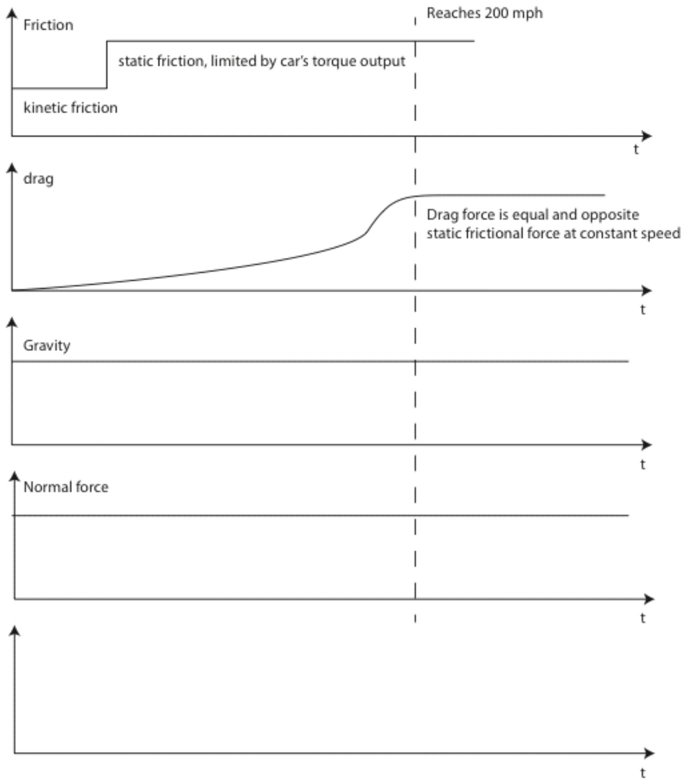
Solution 3.4

Major interactions will be gravity, normal force, friction, and maybe drag. Friction will be static friction (brake is locked) until enough force is applied to break static friction, at which point it becomes kinetic friction. Normal force and friction stop once the car goes over the cliff. Drag will increase as the car's speed increases (so it's 0 until static friction is broken). Note that the solution sketch assumes a coefficient of static friction of around 0.5.



Solution 3.5

Major interactions will be gravity, normal force, friction, and drag. Friction will start as kinetic (tires spinning) and then will become static. Note that the static friction that the car can exert on the ground is limited by the car's ability to turn the tires – which depends on the motor, so I haven't tried to represent that as anything other than a constant output. Normal force and gravity are constant and equal and opposite. Drag will increase as the car's speed increases, and will asymptote as the car approaches its top speed so that the frictional force from the tires and the drag force balance.



Solution 3.6

The Magnus force can be important for the motion of rotating bodies. You should be able to find a youtube video that demonstrates the Magnus force using paper cups (search for Curve Ball Demonstrator from the Exploratorium Teacher Institute, or follow this link.
It's a nifty demo, and easy to try out. It works even better if you use styrofoam cups (which are a bit lighter than paper).

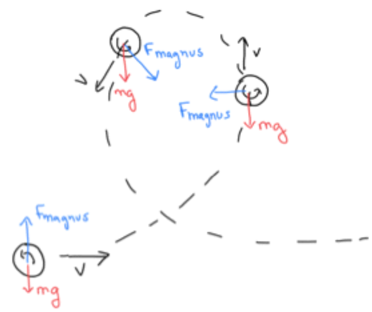
1. Write out/sketch the following: (1) a mathematical expression for the Magnus force, (2) a sketch of the trajectory of the cups in the demo, and (3) a set of free body diagrams that explain the motion.

   (a) $F_m = S(\omega \times v)$ where F_m is the Magnus force, S is air resistance coefficient, v is the velocity and ω is the angular velocity.

   (b) The cup should make a loop-de-loop.

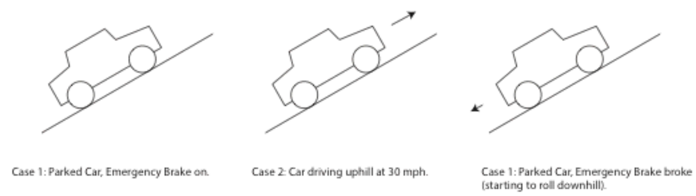
   (c) After the cup is released, spinning and traveling forward, the drag force on the ball will result in an overall Magnus force upward (you can check this with your Right Hand Rule). As the ball accelerates upward, the velocity vector changes; this will change the cross product and thus the Magnus force. This creates the loop-de-loop. See below for an example of a sketch.

2. In the video as set up, the spin is counterclockwise, and the launch is to the right. How do things change if the spin is clockwise? Sketch and explain. The cup will curve backwards. Now the Magnus force is pointing downward. As the cup accelerates downward and the velocity vector changes, the Magnus force will start to point backwards.



Solution 3.7

The diagram below shows three different conditions for a front-wheel drive car on a hill.

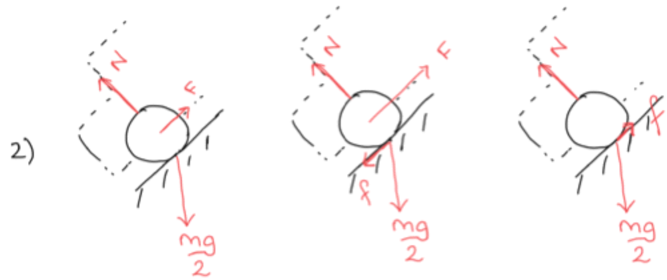


For each condition, draw free-body diagrams for the following systems:

1. System = the entire car.
2. System = the rear wheel of the car.

See example in figure.

1. The stationary car should have mg going downward, normal forces at each of the tires, and a frictional force pointing along the incline acting on the front wheel. Accelerating up is the same, but the friction force on the front wheel should be larger, and there should be a frictional force on the back wheel pointing backwards. Rolling downhill, again is similar but now both frictional forces are pointing up the hill and they should be smaller than the x-component of mg.
2. Rear wheel is similar but with mg/2 and one normal force. In the stationary and accelerating cases, there should be some force representing the force pulling on the axle (rather than friction from the front wheel)



Solution 3.8

Draw free body diagrams for the following problems.

1. The root educational robot uses two magnetic wheels to move around on a whiteboard. Draw a FBD for the root robot while a) Moving on a flat magnetic surface and b) on a vertical whiteboard. For simplicity, assume the robot is moving straight and accelerating a) mg and $F_{magnets}$ going down, normal force is equal to the sum of both, friction is going in the direction of motion. b) mg points down $F_{magnets}$ points into the wall, N is equal and opposite to $F_{magnets}$, f is greater than mg and pointing up.

2. Some robots such as the Hull Skater use magnetic wheels as well, but to stick to ships instead of whiteboards. Draw a new FBD for this robot when underwater. You can pick its orientation (maybe it's upside down), but similarly assume it's accelerating forward and wheels are direct drive. f is acting on both wheels in the direction of motion. mg goes down, F_{mag} goes into the boat surface at each wheel. There should be some distributed buoyant force from the hydrostatic pressure. N is perpendicular to the boat surface and at each wheel.

3. How could do these FBDs impact your design choices for each robot? For root and the Hull Skater, the friction force on the wall (or boat) must be larger than the weight of the robot and will determine the strength of the magnets and material of the wheels.

Solution 3.9

A chain, consisting of N links, each of mass m, hangs from the ceiling. Find an expression for the tension in the nth link in the chain ($n = 1$ for the chain that is attached to the ceiling; $n = N$ for the lowest link in the chain). Hint: Consider plugging in some real numbers to check your answer. Since the chain is not moving, the sum of the forces is zero and tension in the chain must be equal to the force due to gravity from both the nth link and all the links below it. Therefore $T = (N-n+1)mg$

Solution 3.10

The diagram below shows different cross-sections of a boat in water. For the purposes of this problem, make the following assumptions:

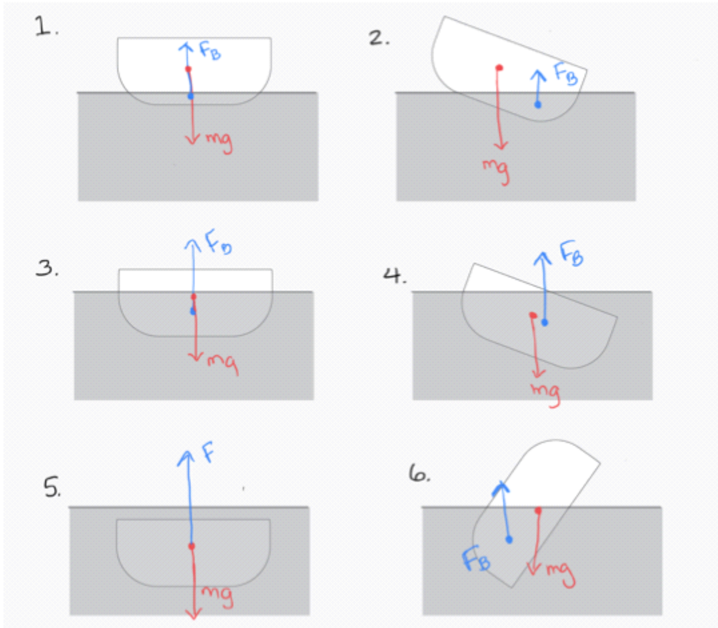
- The boat is of uniform cross-section from stem to stern (i.e., it is a simple extrusion of the shape going into the page)
- The boat has a uniform density of $\rho_b = 0.5$ g/cm^3.
- The boat is 4 meters in length, has a beam of 1 meter, and is about 0.5 meters from the bottom of the hull to the deck (freeboard plus draft).
- The water has a uniform density of $\rho_w = 1$ g/cm^3.

With those assumptions in mind, please do the following for each of the conditions drawn.

1. Calculate (approximately) the magnitudes of the gravitational force acting on the boat and the buoyant force acting in the boat. The magnitude of the gravitational force is always the same. $F_g = \rho_b V g = \rho_b A l g$. The cross sectional area can be approximated a few ways, most simply (though least accurately) as a rectangle with dimensions .5 m x 1 m. Thus $v = 2m^3$ and $F_g = \rho_b A l g = .5g/cm^2 * \frac{1kg}{1000g} * \frac{100^3 cm^3}{1m^3} * 2m^3 * 9.8m/s^2 = 9.8kN$. The buoyant force F_B is the density of water multiplied by the submerged volume V_s. Here we approximate V_s as some fraction of the whole boat V_B
2. Sketch (approximately) the locations of the center of mass and center of buoyancy of the boat.
3. Draw an appropriate free body diagram for each condition.
4. In words, describe how the boat would move for each condition (rise? Fall? Rotate clockwise? Rotate counter-clockwise? Remain stationary?)

See diagram for numbering and FBDs.

1. F_B is approximately $\frac{1}{6} V_B \rho_w g = 3.3kN$. $F_B < mg$ so the boat will fall.
2. The submerged volume is about 1/4 of the boat. Following a similar line of calculation, $F_B = 4.9kN$. $F_B < mg$ and is not aligned so the boat will rotate counterclockwise (CCW) and sink slightly.
3. Similar to(1), 2/3 of the boat is submerged, F_B is approximately 13.1 kN and is greater than mg, so the boat will rise.
4. $F_B = 14.7kN$, the boat will rise and turn CCW.
5. $F_B = 19.6kN$ The boat will rise (and might even pop out of the water before settling at equilibrium).
6. $F_B = 9.8kN$ and equals mg. The boat will turn CW.


1.
F_B
mg
2.
F_B
mg
3.
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mg