

# Current Estimation Using Thevenin Battery Model

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**Abstract**— Current sensor is an important part of Battery Management System (BMS). Current information of a battery is very important to estimate the value of State of Charge (SOC) as a component of fault detection. Fault detection is very important to implement in BMS because of its function to protect battery from damage caused by over discharge and overcharge. The issue here is expensive current sensors. To overcome this issue, this research aims to design a current estimation algorithm which is based on a sensorless current method where the battery is modeled in a Thevenin equivalent circuit model. The Thevenin model is then formed into autoregressive exogenous (ARX) model and the parameters are extracted by using MATLAB identification toolbox. This research uses lithium polymer battery with a capacity of 2200 mAh and the tests conducted in this research are constant pulse load test and load variation test to see the performance of the algorithm. The results show that the current estimation using Thevenin model results better than the one using RC model as shown in the estimation test with constant pulse load and load variation.

**Keywords**— component; Lithium Polymer Battery, Thevenin Battery Model, Transfer Function, ARX

## I. INTRODUCTION

Electrical vehicles are designed to anticipate the issues of energy crisis. An innovation leading to electrical vehicles becomes a solution as they are low cost in the operation and environment-friendly. On the other hand, a prestige factor also takes part in the development of electrical vehicle technology.

A battery is one among many vital components of an electrical vehicle because of its role as a main power source. It is necessary that the battery has high performances such as saving energy in large amounts, being able to do current expansion, and furthermore rechargeable. Those performances will actually result in expensive cost for its investment. A battery Management System (BMS) is a technology to maintain the battery so that it operates optimally. BMS should fulfill its main ability to monitor a condition of a battery so that information of the battery's ability and proper usage can be well understood.

State of Charge (SOC) is a component of a battery which describes a condition of how much the capacity remained. It is defined as a ratio of remaining capacity of the battery to nominal capacity of the battery[1].

Current information of a battery is used to estimate the value of State of Charge (SOC) in BMS with Coulomb

Counting approach by integrating current with time [1]. SOC is a component of fault detection system. Fault detection is very important to implement in BMS because of its function to protect battery from damage caused by over discharge and overcharge. On the other hand, expensive current sensors become an issue in BMS. Furthermore if in the next development it is necessary to have a system which monitors a battery's condition in every cell. Therefore, a research in current estimation becomes an interesting topic as it is possible to save cost in the building of BMS.

A research in current estimation has been done lately as in [2] and [3]. In [2] RC battery model is used to determine a formula for current estimation. RC battery model has a weakness as it models the impedance in the form of a resistor only [4]–[8]. It is not relevant as the electrochemical process in battery is complex. In this research, a development of current estimation is proposed by fixing the weakness in the previous researches, a better Thevenin battery model is then presented. The Thevenin battery model is used in many cases as it is more accurate in describing the characteristic of a real battery [3], [9], [10].

## II. BATTERY MODELING

Battery modeling acts very important in this research. Equivalent circuit model (ECM) is one of battery models commonly used to describe a battery's characteristic. There are many types of ECM battery model which have been used in past researches. However, they can be just classified into two type namely simple battery model and Thevenin battery model. The Thevenin battery model is widely used in many experiments because of its accuracy. However, simple battery model were used in certain researches as it reduces complexity in the computation. Current estimation proposed in this research also uses a battery model proposed in [2] even though it has a weakness in the selection of battery model as it used simple RC battery model and didn't represent the whole characteristics of a battery.

Thevenin battery model as shown in Fig.1 is selected because the model represents the dynamic response of a battery in the form of components  $R_0$  and  $C_d$  as an equivalent circuit. The results of Thevenin model are later compared to the results of the RC battery model.

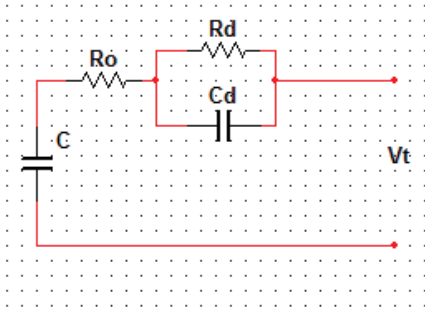


Fig. 1. Battery voltage response toward pulse load

### III. PROPOSED METHOD

The equivalent circuit in Fig.1 consist of capacitor  $C$  and  $C_d$ , ohmic resistance  $R_0$  and  $R_d$  and terminal voltage  $V_t$ .  $R_0$  indicates internal resistance of battery.  $C$  indicates battery capacity. The Equivalent impedance of  $C$  ( $Z_c$ ) can be drawn as

$$Z_c = \frac{1}{Cs}. \quad (1)$$

The pair of  $R_d$  and  $C_d$  describes the dynamic effect of battery. The Equivalent impedance of  $R_d$  and  $C_d$  ( $Z_d$ ) can be drawn as

$$Z_d = \frac{R_d}{1 + R_d C_d s}. \quad (2)$$

Transfer function from terminal voltage to current will be derived by Ohm law as in

$$V_{t(s)} = I_{(s)}(Z_c + R_0 + Z_d). \quad (3)$$

Equation (1) and (2) substitutes in (3) as in

$$V_{t(s)} = I_{(s)} \left( \frac{1 + R_d C_d s + R_0 C s (1 + R_d C_d s) + R_d C s}{C s (1 + R_d C_d s)} \right). \quad (4)$$

transformation. The discrete form is necessary as the method will be implemented with a microcontroller. Formula of Bilinear Transformation can be drawn as

$$G(z) = G(s) \Big|_{s = \frac{2}{T_s} \frac{z-1}{z+1}}, \quad (6)$$

where  $z$  is discretization operator. Equation (6) is implemented in (5) as in

$$\frac{I(z)}{V_{t(z)}} = \frac{R_d C_d C \frac{4}{T_s^2} \frac{(z-1)^2}{(z+1)^2} + C \frac{2}{T_s} \frac{(z-1)}{(z+1)}}{R_0 R_d C_d C \frac{4}{T_s^2} \frac{(z-1)^2}{(z+1)^2} + (R_d C + R_0 C + R_d C_d) \frac{2}{T_s} \frac{(z-1)}{(z+1)} + 1}. \quad (7)$$

Equation (7) is simplified as

$$\frac{I(z)}{V_{t(z)}} = \frac{(m + C.k)z^2 - 2mz + m - C.k}{(R_0.m + nk + 1)z^2 + (2 - 2R_0m)z + R_0m - nk + 1}. \quad (8)$$

We define

$$k = \frac{2}{T_s};$$

$$m = R_d C_d \cdot C \cdot k^2; \quad (9)$$

$$n = R_d C + R_0 C + R_d C_d.$$

Equation (8) is converted to a delay form as in

$$\frac{I(z)}{V_{t(z)}} = \frac{(m + Ck) - 2mz^{-1} + (m - Ck)z^{-2}}{(R_0m + nk + 1) + (2 - 2R_0m)z^{-1} + (R_0m - nk + 1)z^{-2}}. \quad (10)$$

Discrete form of (10) is defined by

### IV. EXPERIMENT SETUP

$$I[z] = \frac{(m + Ck)}{(R_0m + nk + 1)} V_{t[z]} - \frac{2m}{(R_0m + nk + 1)} V_{t[z-1]} + \frac{(m - Ck)}{(R_0m + nk + 1)} V_{t[z-2]} - \frac{(2 - 2R_0m)}{(R_0m + nk + 1)} I[z-1] - \frac{(R_0m - nk + 1)}{(R_0m + nk + 1)} I[z-2]. \quad (11)$$

Transfer function from terminal voltage to current defined by

$$\frac{I(s)}{V_{t(s)}} = \frac{R_d C_d C s^2 + C s}{R_0 R_d C_d C s^2 + (R_d C + R_0 C + R_d C_d) s + 1}. \quad (5)$$

The transfer function in (5) is then converted to a discrete form with sampling interval  $T_s$  by using Bilinear

The experiment setup is shown in fig. 2 consist of battery, sensor for current and voltage information, dummy load for load simulation, switch, microcontroller and computer for processing unit. The battery used in this research is Turnigy lithium polymer battery with a capacity of 2200 mAh and the supporting tools are current sensor ACS712 to obtain the real value of current, DC Electronic load GW Instek PEL2000 as a

dummy load, and microcontroller Chipkit UNO32 as a mechanism controller. The value of current sensed with current sensor is later compared to the result of current estimation to determine the error information produced in this research.

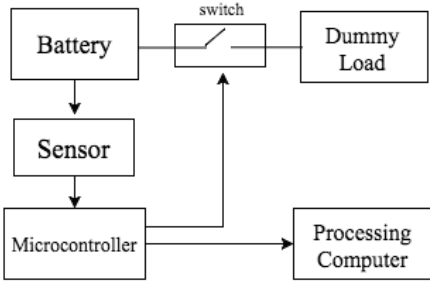


Fig. 2. Configuration of experimental setup

One of the proposed tests in this research is pulse load test which is done by connecting the battery to a DC Electronic load in 30 seconds and then in the next 30 seconds the battery is in a rest condition before it is connected to the load again. The purpose of this test is to determine the characteristic of the battery if constant load is applied. The second test is load variation test to get the information of battery terminal voltage toward dynamic load. A validation is done by comparing the estimation current with the current from current sensor. The difference of the current estimation and the current from current sensor is then considered as estimation error.

## V. RESULT AND DISCUSSION

The current estimation proposed in [2] uses an RC battery model to estimate the current of the battery. ARX model to estimate the current with RC battery model in [2] as in

$$I_{[k]} = b_0 V_{T[k]} + b_1 V_{T[k-1]} - a_1 I_{[k-1]}, \quad (12)$$

where  $b_0, b_1$  and  $a_1$  are parameters of simple RC battery ARX model. Equation (11) is converted to ARX model as in

$$I_{[k]} = b_0 V_{T[k]} - b_1 V_{T[k-1]} + b_2 V_{T[k-2]} - a_1 I_{[k-1]} - a_2 I_{[k-2]}, \quad (13)$$

where  $b_0, b_1, b_2, a_1$  and  $a_2$  are parameters of Thevenin battery ARX model. We define

$$\begin{aligned} b_0 &= \frac{(m+Ck)}{(R_0 m+nk+1)}; \\ b_1 &= \frac{2m}{(R_0 m+nk+1)}; \\ b_2 &= \frac{(m-Ck)}{(R_0 m+nk+1)}; \\ a_1 &= \frac{(2-2R_0 m)}{(R_0 m+nk+1)}; \\ a_2 &= \frac{(R_0 m-nk+1)}{(R_0 m+nk+1)}. \end{aligned} \quad (14)$$

The parameters in (12) and (13) are identified from the input – output ( $I/O$ ) data of the lithium polymer battery by using MATLAB Identification toolbox. The function is defined as  $arx(model, order)$  with the model is based on the  $I/O$  data and the order is the order of ARX model. Table 1 consists of data parameters which have been extracted with MATLAB Identification toolbox.

TABLE I. RESULT OF EXTRACTED PARAMETERS

Parameters	RC Model	Thevenin Model
$a_1$	-0.9966	-0.5444
$a_2$	0	-0.4521
$b_0$	18.24	18.35
$b_1$	-18.24	9.567
$b_2$	0	-8.784

### A. Constant Load

The first conducted test uses constant pulse load. Fig. 3 is the response of terminal battery voltage to constant pulse load. The value of the load used in this research is constant current of 2.2 A. The dynamic response data of terminal voltage ( $V_T$ ) with constant load are then considered as input data to estimate the current in (12) and (13) using the extracted parameters in Table 1.

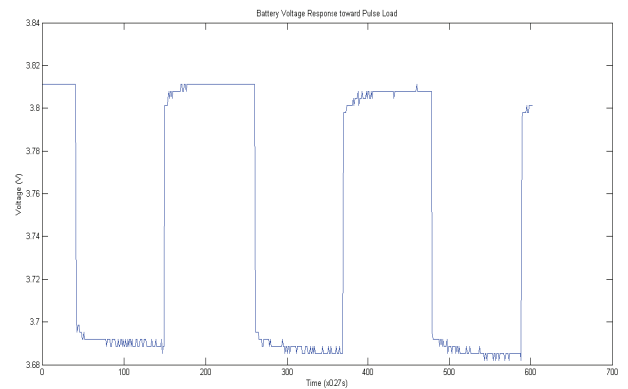


Fig. 3. Battery voltage response toward pulse load

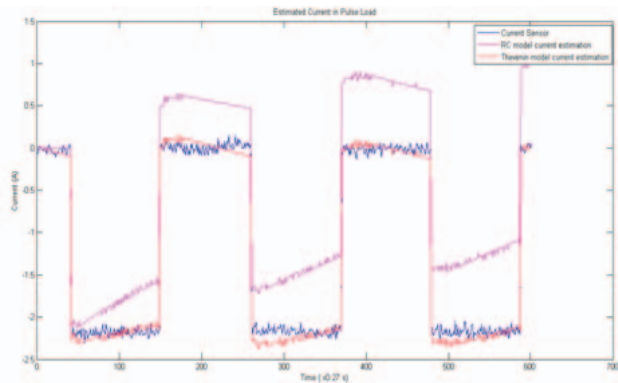


Fig. 4. Estimated current in pulse load

Fig. 4 shows the comparison between the estimated current and the current from current sensor ACS712. It shows that the estimated current with Thevenin battery model (shown in red) follow the change of actual current more accurately than the estimated current of RC battery model (shown in magenta) do. Error estimation in constant load is shown in Table II.

TABLE II. ERROR ESTIMATION IN CONSTANT LOAD

Battery Model	MSE	RMSE
THEVENIN	0.0085 A <sup>2</sup>	0.091989 A
SIMPLE RC	0.4465 A <sup>2</sup>	0.668244 A

Table II shows the comparison of estimation error of the current in constant load. It is easily understood that the estimation error in Thevenin battery model is smaller than simple RC battery model. Current estimation with Thevenin battery model has 0.0085 A<sup>2</sup> of MSE and 0.091989 A of RMSE.

### B. Load Variation

The second conducted test uses load variation. The purpose of this test is to obtain the response of terminal battery voltage toward dynamic load as shown in Fig. 5. The load used in it is structured variant current starting from 0A to 3A which is repeated periodically.

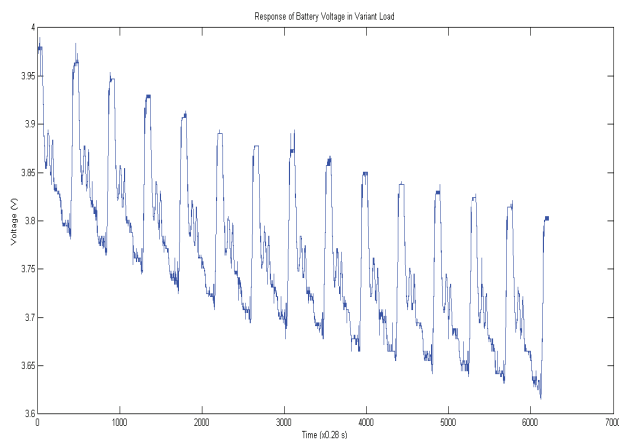


Fig. 5. Response of battery voltage in load variation

The dynamic response data of terminal voltage with load variation are then considered as the input to estimate the current in (12) and (13) using the extracted parameters in Table 1. The purpose of this estimation is to know the performance if it is implemented in real applications as the load is actually not constant but variant. Fig. 6 shows the comparison of estimated current with load variation.

The curves in Fig. 6 show that the estimated current with Thevenin battery model (shown in red) is better than the one with RC battery model (shown in magenta) with the current from current sensor (shown in blue) as a reference. Even though both of the estimated currents have estimation error, it is noted that the one with Thevenin battery model has the smallest amount of error. Error estimation in load variation is shown in Table III.

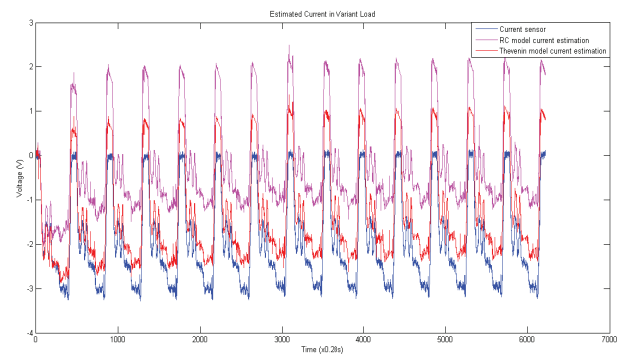


Fig. 6. Estimated current in load variation

TABLE III. ERROR ESTIMATION IN LOAD VARIATION

Battery Model	MSE	RMSE
THEVENIN	0.4209 A <sup>2</sup>	0.6488 A
SIMPLE RC	3.0719 A <sup>2</sup>	1.7527 A

Table III shows the comparison of error estimation in RC battery model with one in Thevenin battery model. It is easily understood that the estimation error in Thevenin battery model smaller than RC battery. Current estimation with Thevenin battery model has 0.4209 A<sup>2</sup> of MSE and 0.4209 A<sup>2</sup> of RMSE.

Considering the result, it is noted that the current estimation method with Thevenin battery model is better than the one with RC battery model. Even though the current estimation with Thevenin battery model results better, it still has a problem in its estimation error which is still considered large especially in dynamic load. Therefore, a future work is necessary to enhance the current estimation.

## VI. CONCLUSION

Based on the experiment result, it can be concluded that the current estimation with Thevenin battery model results better than the one with RC battery model. Current estimation with Thevenin battery model has 0.0085 A<sup>2</sup> of MSE and 0.091989 A of RMSE in constant load. Even though the current estimation with Thevenin battery model results better, it still has error estimation 0.4209 A<sup>2</sup> MSE and 0.4209 A of RMSE in load variation. Adaptive estimation method will be proposed in the

next work to increase the performance of the current estimation algorithm.

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