A Non-autonomous Balanced Chaotic Circuit Based-on A Bipolar Differential-pair

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***Abstract*—A novel cross-coupled bipolar transistor-based non-autonomous chaotic oscillator is proposed. The derivation methodology of this novel chaotic oscillator is based on integrating two of existing chaotic oscillators symmetrically and employing a differential-pair stage. Simulation and experimental results, verifying the feasibility and the correct operation of the circuit are also given.**

***Index Terms*—Chaotic circuit, chaos, non-autonomous, balanced circuit design, differential pair.**

I. Introduction

The chaotic circuits, which include two dictation characteristics, dynamics and non-linearity, shows an extremely unusual and non-periodic behaviors that are very sensitive to the initial conditions. Due to the fact that it has a positive Lyapunov exponent, hypersensitivity to initial conditions makes a chaotic system unpredictable for a long term [2] and hence assures independent trajectories [1].

It is not easy to tell the difference between 2 waveforms coming from a chaotic circuit and from a source of noise [4], both have broadband spectrum and unsteady character which is not predictable [3]. Moreover, it has been shown very successfully in many cases where chaos operates better than noise due to its typical structure in the phase plane [3].

Because of the above mentioned features, chaotic systems can be used for various practical applications and especially for communication and the generation of broadband signals. One of this practical application area is chaotic cryptology.

With regard to the creation of dynamic systems that could be represented in non-linear difference or differential equations, there are 2 kinds of chaotic systems, discrete time and continuous time, respectively. Despite the fact that, discrete-time chaotic maps [7], [5], [6], [10], [9] have been already used in the chaotic cryptology for some time, the usage of continuous-time chaotic oscillators [8], [11], [12] was only recently shown to realize chaotic crypto-systems.

There is another classification for chaotic oscillators other than discrete-time and continuous time, and that is autonomous and non-autonomous. The identification of the characteristic for a non-autonomous chaotic oscillator can be obtained while it produces chaos caused by the excitation of a time varying source. Compared to autonomous chaotic oscillators [15] that has been mentioned in literature (which sustain chaos with no need for an external excitation), there is a relatively very small number of non-autonomous oscillators [13], [14], [17].

It is common to use a sinusoidal waveform as an impulse as seen in [13], [14], [17] where the both frequency and amplitude of the sinusoidal waveform conduce to the chaotic dynamics [20]. Nowadays, chaotic behavior of oscillators has gained interest, where a pulse-train is used to excite the oscillator, [18] along with a motivation primarily based on behavioral models of neural networks [16], [19].

Although it is still not clear what are the proper conditions for obtaining chaos in a continues-time autonomous system [21], the needed conditions are given in the PoincaréBendixson theorem [22] as a minimum of three variables that are represented in energy storage units in implementation and one nonlinearity. Rössler chaotic system [23] is a conventional example of a system like that.

In [24] we have an introduction of the third order passive structure of Chua's circuit that exhibits multiple scroll chaos after its excitation by a periodic pulse-train. It is demonstrated that a second order LC resonator also exhibits chaos at excitation by a bipolar pulse train, which represents a non-autonomous chaotic oscillator introduced in [25].

There isn't a lot of work describing the underlying mechanisms of chaos; however out of all chaotic oscillator designs that are mentioned in literature, there are basically 4 methods for chaos generation, from the point of view of a circuit designer: 1. Implementation of a chaotic mathematical model [26], [27]. 2. Interrupting the current in a coil or short-circuiting the voltage of a capacitor in less than no time [28]. 3. Disturbing the performance of an oscillator by adding some nasty circuit which is not in harmony with the oscillator circuit [29], [30]. 4. Disturbing to charge a capacitor [31].

In this paper, a novel chaotic oscillators is proposed together with its derivation methodology. This methodology do not directly use the aforementioned mechanisms but only based on integrating two of existing chaotic oscillators symmetrically which employ a transistor or a comparator as an ideal switch. Even though there are a lot of chaotic circuits in the literature, only a few of them are designed according to high performance design criteria [32].

To achieve the required nonlinearity, a differential-pair stage is employed by the derived novel chaotic oscillators, and the fact that it has a high IC performance makes it the most universally fundamental analog building block. In addition, an enhance noise immunity and power supply rejection is offered by the proposed chaotic oscillator since it is balanced.

Due to the features indicated above, the proposed chaotic oscillator has more robust chaotic behaviors against parameter variations.

II. Transistor-based Non-autonomous Chaotic Oscillator

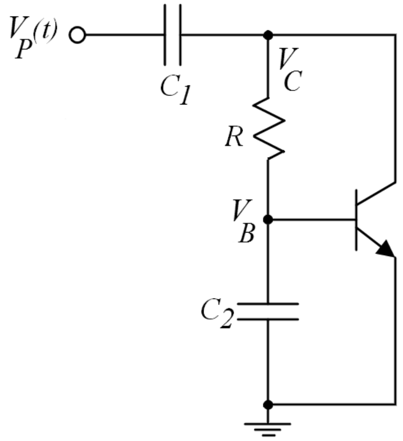

Fig. 1. Transistor-based non-autonomous chaotic oscillator.

In this work, the smallest non-autonomous chaotic oscillator [31] is used as the core of the proposed oscillator. The core circuit is given in Fig. 1 and consists of 1 transistor, 2 capacitors, and 1 resistor. The underlying mechanism of chaos is disturbance of integration where two sources fight with each other to charge a capacitor. Here two sources are the reverse part and the forward part of the BJT. Routine analysis of the circuit yields the following state equations:

$$\begin{array}{l} \dot{V_B} = \frac{V_C - V_B}{RC_2} - \frac{I_B}{C_2} \\ \dot{V_C} = \dot{V_P}(t) + \frac{V_B - V_C}{RC_1} - \frac{I_C}{C_1} \end{array} \tag{1}$$

where $V_P(t)$ is the external sinusoidal voltage source defined as $V_P(t) = V_P sin\Omega t$, the base and the collector currents of the transistor are

$$\begin{array}{ll} I_B = \frac{I_C}{\beta} = \frac{I_S}{\beta} e^{V_B/V_T} & when\ V_B > 0 \\ I_B = I_C \approx 0 & when\ V_B \leq 0 \end{array} \tag{2}$$

where the thermal voltage is V_T and at room temperature it is approximately $25.8mV$ ($V_T = kT/q$).

Using the normalized quantities: $x = V_B/V_S$, $y = V_C/V_S$, $a = C_1/C_2$, $b = \frac{RI_S}{V_S}$, $c = V_P/V_S$, $k = \beta$, $\omega \equiv \Omega\tau$, $\tau = RC$ and taking $0.5V_S = V_T$ in addition to $t_n = t/\tau$, where V_S is a random scaling voltage, Eqn. 1 (the system's equations) can be transformed into:

$$\begin{array}{l} \dot{x} = y - x - \left\{ \begin{array}{ll} \frac{b}{k} e^{2x} & if\ x > 0 \\ 0 & if\ x \leq 0 \end{array} \right. \\ \dot{y} = \frac{x}{a} - \frac{y}{a} + c\omega cos\omega t - \left\{ \begin{array}{ll} \frac{b}{a} e^{2x} & if\ x > 0 \\ 0 & if\ x \leq 0 \end{array} \right. \end{array} \tag{3}$$

PSpice simulations have been made to verify the correct behavior of the chaotic oscillator. In Fig. 1, the bipolar transistor is implemented with 2N2222A. The external signal $V_P(t)$ is generated by a sinusoidal oscillator. The amplitude and frequency of $V_P(t)$ are $10V$ and $10kHz$, respectively.

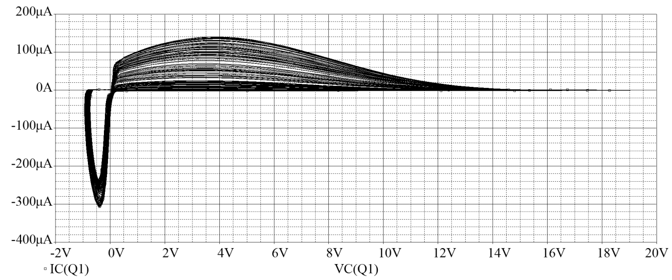

Fig. 2. Chaotic attractor obtained from the PSpice simulation $(Xaxis : V_C, Yaxis : I_C)$.

The passive component values are: $C_1 = 4.7nF$, $C_2 = 1.1nF$, and $R = 994k\Omega$. The collector current and the base voltage of the transistor are obtained as functions of time by PSpice simulation. Fig. 2 and Fig. 3 show the same waveforms as functions of the collector voltage.

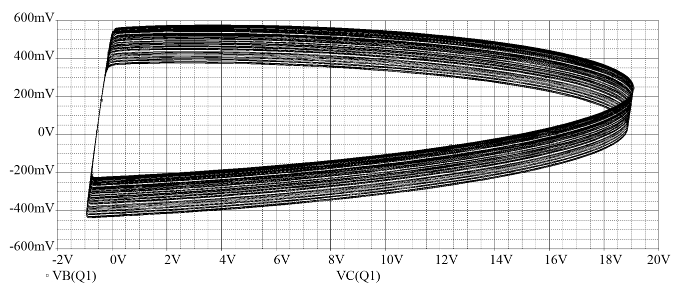

Fig. 3. Chaotic attractor obtained from the PSpice simulation $(Xaxis : V_C, Yaxis : V_B)$.

III. Proposed Chaotic Oscillator

As shown in Fig. 4, a novel cross-coupled transistor-based non-autonomous chaotic oscillator circuit is also proposed as a design example.

Proposed chaotic oscillator is derived from the smallest non-autonomous chaotic oscillator given in Fig. 1 by the symmetric integration of two of them in addition to the employment of a differential-pair stage so we can achieve the required nonlinearity. The state equations below are obtained from the routine analysis of the circuit:

$$\begin{array}{l} \dot{V_1} = \frac{V_2}{RC_2} - \frac{V_1}{RC_2} \\ \dot{V_2} = 2V_P\Omega cos\Omega t - \frac{V_2}{RC_1} + \frac{V_1}{RC_1} - \frac{I_0}{C_1} tanh(\frac{V_1}{2V_T}) \end{array} \tag{4}$$

where I_0 is the tail current of the differential pair composed of the emitter currents, the external sinusoidal voltage is $V_P(t) = V_P sin\Omega t$, and the thermal voltage is $V_T = kT/q$ and at room temperature it is approximately $25.8mV$.

Using the normalized quantities: $x = V_1/V_S$, $y = V_2/V_S$, $a = C_1/C_2$, $b = \frac{RI_0}{V_S}$, $c = V_P/V_S$, $\omega \equiv \Omega\tau$, $\tau = RC$ and taking $0.5V_S = V_T$ in addition to $t_n = t/\tau$, where V_S is a

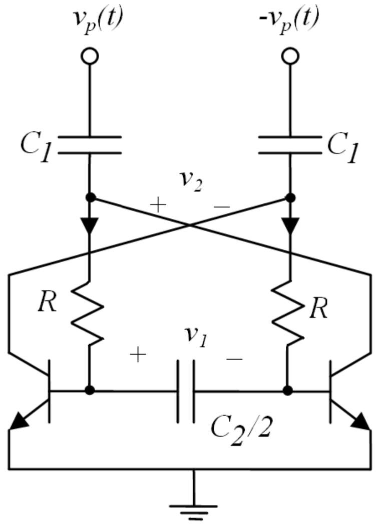

Fig. 4. Proposed Chaotic Oscillator.

random scaling voltage, Eqn. 4 (the system's equations) can be transformed into:

$$\begin{aligned}\dot{x} &= y - x \\ \dot{y} &= \frac{x}{a} - \frac{y}{a} + 2c\omega cos\omega t - \frac{b}{a}tanh(x)\end{aligned} \tag{5}$$

IV. Simulation Results

A simulation of the circuit given in Fig. 4 is done via PSpice to verify the expected operation and feasibility of the proposed chaotic oscillator. The external signals $V_P(t)$ and $-V_P(t)$ shown in Fig. 4 are generated by introducing a sinusoidal oscillator through a non-inverting and an inverter stages that nearly generate the exactly same delays. Amplitudes of $V_P(t)$ and $-V_P(t)$ are $\pm10V$ while theirs frequencies are adjusted to $10kHz$. The chosen values of the passive components for the realization are: $C_1 = 4.7nF$, $C_2/2 = 0.55nF$, and $R = 994k\Omega$. Differential-pair stage is implemented with 2N2222A transistors.

The differential collector currents and the chaotic waveform V_1 are obtained by PSpice simulation as functions of time, respectively. Fig. 5 and Fig. 6 show the PSpice simulation results and corresponding attractors that verify the feasibility of the proposed circuit.

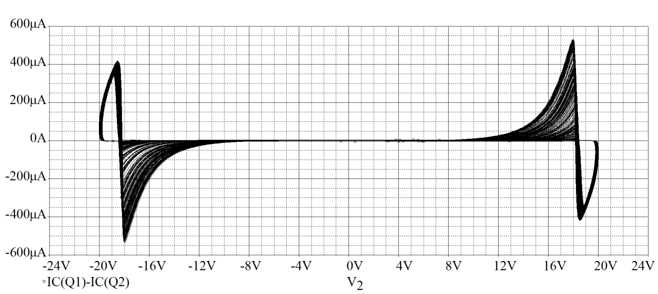

Fig. 5. A PSpice simulation showing the chaotic attractor in X-Y axis $(Xaxis : V_2, Yaxis : I_{C1} - I_{C2})$.

The chaotic system that starts at the exact same initial conditions $x(0)$ and $x(0) + \Delta(0)$, have an entirely different result or output $x(t)$ and $x(t) + \Delta(t)$ at a time t where $\Delta(t) \propto e^{\lambda t}$ for $\lambda > 0$. λ in the above expression is known as the system's maximum Lyapunov exponent [33] which is a real positive number for chaotic systems. The maximum Lyapunov exponent has been calculated from the PSpice times series using the software given in [34] as 0.203 which confirms the chaotic motion of the system.

Discrete components were used to implement the proposed chaotic oscillator because of the inability to reach a proper fabrication facility. The impact of possible time delays between the external sinusoidal signals $V_P(t)$ and $-V_P(t)$ (Fig. 4) on the performance of the chaotic oscillator is brought to a minimum level by generating them through the introduction of a sinusoidal oscillator through a non-inverting and an inverting stages that nearly generate the exactly same delays.

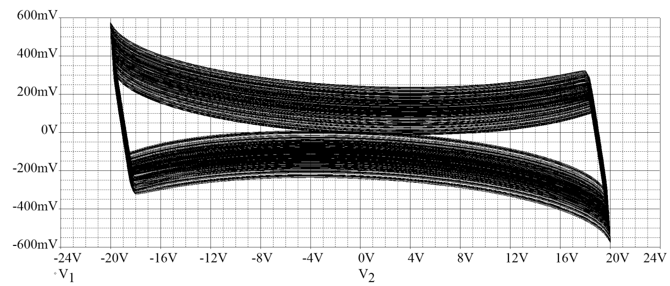

Fig. 6. A PSpice simulation showing the chaotic attractor $(Xaxis : V_2, Yaxis : V_1)$.

Amplitudes of $V_P(t)$ and $-V_P(t)$ are $\pm6.8V$ while their frequencies are adjusted to $f = 7.6kHz$. The passive component values used for the realization are: $C_1 = 4.7nF$, $C_2/2 = 0.6nF$, and $R = 910k\Omega$. Differential-pair stage shown in Fig. 4 is implemented with 2N2222A transistors. Verification of the feasibility and correct operation of the proposed chaotic oscillator has been obtained via an experimental tests and observed attractor shown in Fig. 7 is similar to the one obtained from the PSpice simulation.

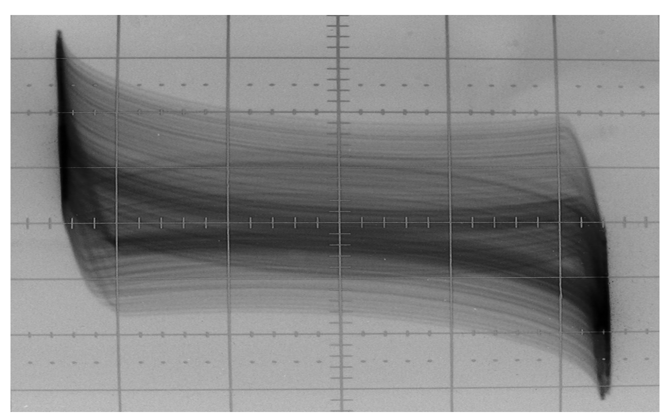

Fig. 7. Experimental results of the proposed chaotic oscillator $(Xaxis : V_2, Yaxis : V_1, f = 7.6kHz)$.

To achieve the required nonlinearity, a differential-pair stage is employed by the proposed chaotic oscillators, and the fact that it has a high IC performance makes it the most universally fundamental analog building block. Additionally, an enhanced noise immunity and power supply rejection is offered by the proposed chaotic oscillator since it is balanced. Also, a clock signal that produces a periodic pulse train can be found on the chip by default and can be used as the external source for driving the circuit. All of the above mentioned features are advantages offered by the proposed chaotic oscillator over the existing one [31].

Additionally, since the circuit of the proposed chaotic oscillator has an RC design with no inductors, it is safer when it comes to external interference and side channel attack. Finally, proposed circuit is simple due to the absence of large blocks and can be effectively realized on IC. Due to the features indicated above, proposed chaotic oscillator has more robust chaotic behavior; hence the usage of it is preferable as the core of a practical application. The chaotic oscillator can be considered as a periodic to aperiodic waveform transformer and can be used as an entropy source for cryptographic applications.

V. Conclusions

Many chaotic oscillator circuits can be designed by using the methodology introduced in this paper. According to the procedure described here, balanced oscillators can be derived from existing chaotic oscillators, which employ a transistor or a comparator as an ideal switch, by the symmetric integration of two of them in addition to the employment of a differential-pair stage. Note that, undertaking a real design example will be the most effective way to develop a hierarchical design methodology. Simulation results presented in this paper not only verify the feasibility and the correct operation of the proposed circuit, but also encourage its use as the core of a high-performance entropy source as well.

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