# Ejercicio 1.a

La fuerza entre dos cargas iguales $q$, que pueden ser del mismo signo o no:

$$\frac{d\vec{p}_{\text{tot}}}{dt} = \vec{F}_0 + \vec{F}_{\text{EM}}$$

$$\vec{F}_{\text{EM}} = \int_{\partial V} \mathrm{d}s\ \mathbb{T}\cdot\hat{n} - \frac{1}{4\pi c}\frac{\mathrm{d}}{\mathrm{d}t}\int_V \mathrm{d}v\ \vec{E}\times\vec{B}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\vec{p}_{\text{mec}} + \frac{1}{4\pi c}\int_V \mathrm{d}v\ \vec{E}\times\vec{B}\right] = \vec{F}_0 + \int_{\partial V}\mathrm{d}s\ \mathbb{T}\cdot\hat{n} = 0 \quad \text{(estacionario)}$$

$$\Rightarrow \vec{F}_0 = -\int_{\partial V}\mathrm{d}s\ \mathbb{T}\cdot\hat{n} = -\left[\int_{S_R}\mathbb{T}\cdot\hat{n}_R\,\mathrm{d}s_R + \int_{S_\alpha}\mathbb{T}\cdot\hat{n}_\alpha\,\mathrm{d}s_\alpha\right]$$

Tomamos una semiesfera:

$$S_{\text{tot}} = S_R \cup S_\alpha$$

$$\begin{aligned}\vec{E}_{\text{total}} &= \frac{q(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} + \frac{q'(\vec{r}-\vec{r}'')}{|\vec{r}-\vec{r}''|^3}\\ &= \frac{q(\vec{r}+d\hat{z})}{(r^2+(z+d)^2)^{3/2}} + \frac{q'(\vec{r}-d\hat{z})}{(r^2+(z-d)^2)^{3/2}}\end{aligned}$$

**Mejor con Gauss:**

$$\oint \vec{E}\cdot\mathrm{d}s = 4\pi Q_{\text{enc}} \Rightarrow 4\pi r^2 E(r) = 4\pi q \Rightarrow E(r) = \frac{q}{r^2}$$

$$E'(r) = \frac{q'}{r^2}$$

**Expansión multipolar:**

$$\vec{E}_{r\gg d} = \frac{Q_{\text{total}}}{r^2}\hat{r} + \frac{3(\vec{P}\cdot\hat{r})\hat{r}-\vec{P}}{r^3} \quad \text{con} \quad \vec{P} = q\cdot 2d\hat{z}$$

- Si $q = q'$:

$$\vec{E}_T = \frac{q}{r^2}\hat{r} - \frac{qd}{r^3}\hat{z} + \frac{q'}{r^2}\hat{r} + \frac{q'd}{r^3}\hat{z}$$

- Si $q = -q'$:

$$\vec{E}_T = \frac{q}{r^2}\hat{r} - \frac{qd}{r^3}\hat{z} - \frac{q}{r^2}\hat{r} + \frac{qd}{r^3}\hat{z} \Rightarrow \vec{E}_T = \frac{2qd}{r^3}\hat{z}$$

$$\int_{S_T}\mathrm{d}s\ \mathbb{T}\cdot\hat{n} = \int_{S_R}\mathrm{d}s\ \mathbb{T}\cdot\hat{n} + \int_{S_\alpha}\mathrm{d}s\ \mathbb{T}\cdot\hat{n}$$

$$\mathbb{T} = \frac{1}{4\pi}\left[\vec{E}(\vec{E}\cdot\hat{n}) + \vec{B}(\vec{B}\cdot\hat{n}) - \frac{1}{2}(E^2+B^2)\hat{n}\right]$$

—

## Cálculo de la fuerza

$$\vec{E}^{\text{tot}}_{z=0} = \frac{q(\vec{r}+d\hat{z})}{(r^2+d^2)^{3/2}} + \frac{q'(\vec{r}-d\hat{z})}{(r^2+d^2)^{3/2}} \Rightarrow \frac{2qr\hat{r}}{(r^2+d^2)^{3/2}} - \frac{2qd\hat{z}}{(r^2+d^2)^{3/2}} \quad \text{si } q=-q'$$

$$\Rightarrow \vec{F}_0 = -\int_{S_R} \mathrm{d}s\ \mathbb{T}\cdot\hat{n}_R = -\int_0^{2\pi}\mathrm{d}\varphi\int_0^\infty \mathrm{d}r\left(-\frac{1}{4\pi}\left[\vec{E}(\vec{E}\cdot\hat{z})-\frac{1}{2}E^2\hat{z}\right]\right)$$

**Caso $q=-q'$, $\vec{E}\cdot\hat{z}=0$:**

$$\vec{F}_0 = -\int_0^{2\pi}\mathrm{d}\varphi\int_0^\infty \mathrm{d}r\left(-\frac{1}{4\pi}\left(-\frac{1}{2}E^2\hat{z}\right)\right) = \int_0^{2\pi}\mathrm{d}\varphi\int_0^\infty \mathrm{d}r\left(\frac{1}{8\pi}E^2\right)$$

$$= 2\pi\int_0^\infty \mathrm{d}r\left(\frac{2q^2}{(r^2+d^2)^3}\cdot r\right) = \frac{q^2}{2}\int_0^\infty \frac{d^2\cdot x^2}{(1+x^2)^3}\,\mathrm{d}x\,, \quad x=\frac{r}{d}$$

$$\Rightarrow \frac{-q^2}{2d^2}\int_0^\infty \frac{x^2\,\mathrm{d}x}{(1+x^2)^3} = \frac{-q^2}{2d^2}\int_0^\infty \frac{u-1}{u^3}\frac{\mathrm{d}u}{2}, \quad u=x^2+1$$

$$= \frac{\alpha}{2}\left[\int_0^\infty \frac{1}{u^2}\,\mathrm{d}u - \int_0^\infty \frac{1}{u^3}\,\mathrm{d}u\right] = \frac{q^2}{2d^2}\cdot\left(-\frac{1}{u}+\frac{1}{2u^2}\right)\Big|_1^\infty \Rightarrow \frac{q^2}{8d^2}$$

$$\boxed{\vec{F}_0 = \frac{q^2}{8d^2}}$$