Opinions and conflict in social networks: models, computational problems and algorithms

Lecture 3: Methods for mitigating polarization

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course overview

- lecture 1: introduction
  - polarization in social media; methods for detecting polarization
- lecture 2: mathematical background
  - submodular maximization; spectral graph theory
- lecture 3: methods for mitigating polarization
  - maximizing diversity, balancing information exposure
- lecture 4: signed networks; theory and applications
- lecture 5: opinion dynamics in social networks

overview of this lecture

- methods for mitigating polarization and echo chambers in social networks
- increasing awareness
- recommendation-based methods for increasing diversity and balancing information exposure
  - ideas inspired by influence-maximization setting
- other recommendation-based methods

improving awareness for alternate viewpoints

[Lahoti et al., 2018]

improve awareness

- develop tools for users to perceive their "news diet"
- visualize/navigate in the underlying ideology space, their position, the accounts they follow, the news they read
- offer functionalities such as

  "find a high-quality article on the same topic from the opposing viewpoint"

learning of ideological leanings

- infer ideological stances of users and content

  e.g., liberal–conservative space

- common latent space for users and content

- e.g., substitute ground-truth polarities in previous study with learned polarities

- joint non-negative matrix-factorization task

intuition

- map users and content in a joint latent ideology space

such that

- similar users are more likely to follow each other
- similar users are more likely to share similar content
- similar content is more likely to be shared by similar users

*similar means close in the latent ideology space

the problem setting

- social network $G = (V, E)$
  - adjacency matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$
- user–content matrix $\mathbf{C} \in \mathbb{R}^{m \times n}$
- latent matrix representing user ideology $\mathbf{U} \in \mathbb{R}^{n \times k}$
- latent matrix representing content ideology $\mathbf{V} \in \mathbb{R}^{m \times k}$
- decompose

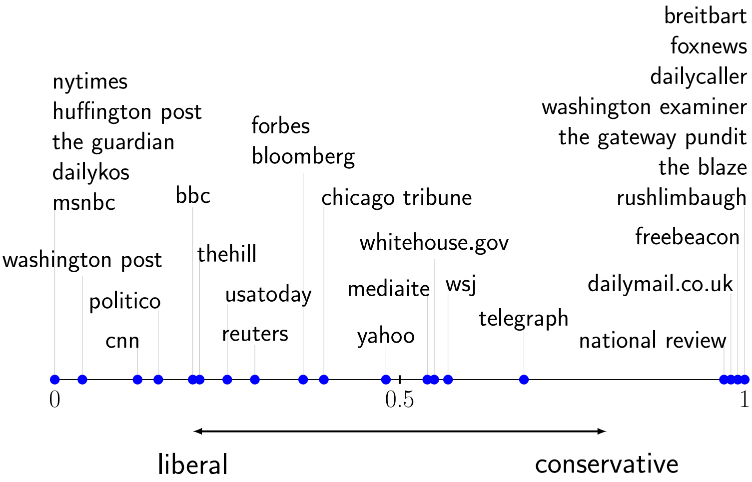
$$\mathbf{A} \approx \mathbf{U}\mathbf{H}_u\mathbf{U}^T \quad \text{and} \quad \mathbf{C} \approx \mathbf{U}\mathbf{H}_v\mathbf{V}^T$$

subject to orthonormal $\mathbf{U}$ and $\mathbf{V}$ and graph-regularization

evaluation

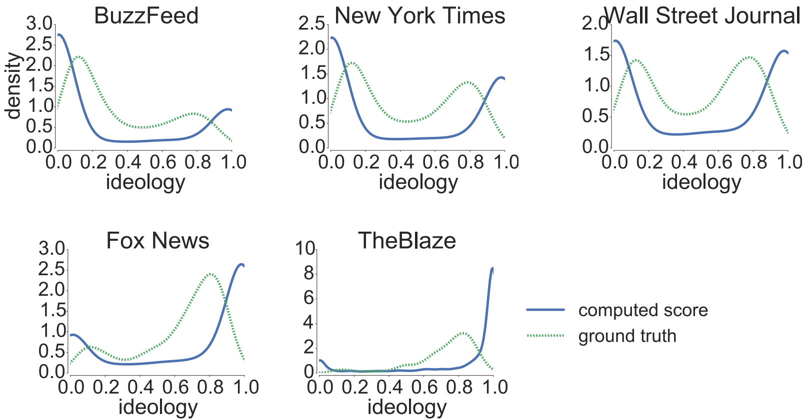
- twitter data from 2011 to 2016, focusing on controversial topics
  - gun control, abortion, obamacare, etc.
- 6 391 users and 19 million tweets
- gather ground-truth polarity scores
  - content polarity [Bakshy et al., 2015]
  - user polarity [Barberá, 2015]

content ideology scores



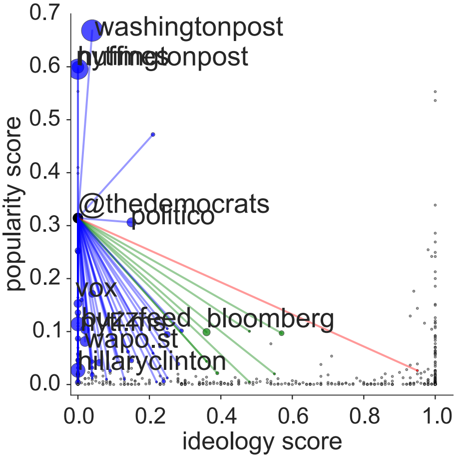
correlation with ground-truth scores 0.82

audience ideology scores

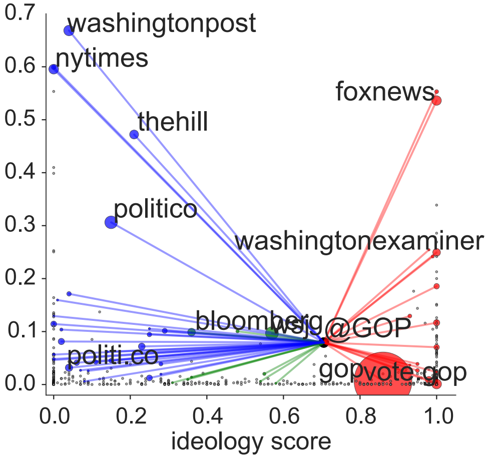


correlation of user ideology scores with ground-truth 0.90

visualizing the information bubble



@thedemocrats



@gop

maximizing diversity in social networks

[Garimella et al., 2017, Matakos et al., 2020b, Matakos et al., 2020a]

maximizing diversity

- goal : make recommendations to maximize diversity
- what is diversity and how to measure it?
- user level : recommend diverse content
- network level : make recommendations so that friends see different content
  - motivation : friends can discuss / debate
- combinations
- another consideration : propagation effects, or not

maximizing diversity in social networks

- one way to achieve this is via recommendations
- important question : what is diversity and how to measure it?
- what kind of recommendations ?
  - friend recommendation : recommend an interesting account to follow
  - content recommendations : recommend interesting content
- considerations :
  - make minimal number of recommendations (k)

    motivation : intervene as little as possible in the “organic” operation of the network
  - make recommendations so that friends see different content

    motivation : friends can discuss / debate
  - consider network-cascade effects

background knowledge

influence maximization in social networks

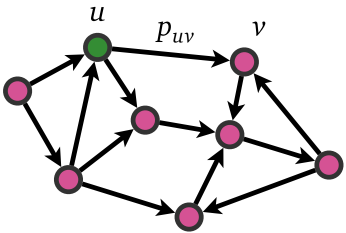
[Kempe et al., 2003]

influence in social networks

- people living in a society are influencing each other
  - behavior they exhibit
  - opinions they form
  - decisions they make
  - products they buy
  - clothes they wear
- viral marketing
  - select few initial adopters in a social network
    to maximize the spread of adoption of a product / action /

modeling influence in a social network

- independent-cascadel: a popular model in the literature
  - introduced by [Kempe et al., 2003] (about 9K citations in Google scholar)
- it assumes a social network, directed graph, $G = (V, E, p)$
  with edge probabilities p_{uv} for each edge $(u, v) \in E$
- basic model considers a single action adopted (spread) in the network
- at time 0: a set of initial nodes (seeds) adopt
- at time t: a node u adopts
- at time $t + 1$: a neighbor v of u adopts with prob p_{uv} -
  - one-time opportunity



the influence-maximization problem

informally

- select k initial adopters to maximize the total number of adopters in the network

a bit more formally

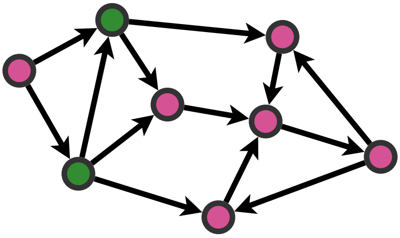
- consider social network $G = (V, E, p)$ with edge probabilities p
- let $\sigma(A)$ be the total number of adopters, for a set of initial adopters $A \subseteq V$

the influence-maximization problem

- select $A \subseteq V$ with $|A| = k$, so as to maximize $\sigma(A)$
- but notice that $\sigma(A)$ is a random variable,
  - so, we are interested to maximize the expectation $\mathbb{E}[\sigma(A)]$

computing $\sigma(A)$

- given a set of seeds A how to compute $\sigma(A)$?
- how to deal with edge probabilities p?

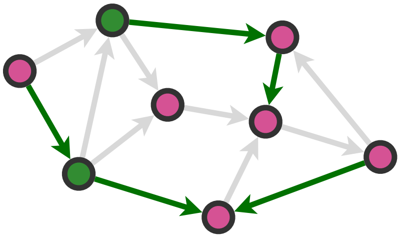


computing $\sigma(A)$

- given a set of seeds A how to compute $\sigma(A)$?
- how to deal with edge probabilities p?

idea:

- assume that edge coin-flips are drawn before the cascade
- sampled edges yield a sample graph X

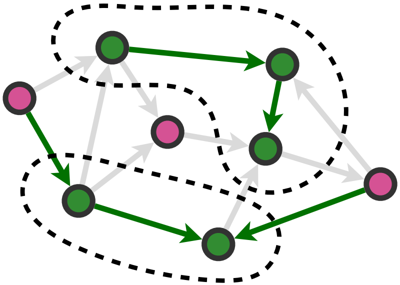


computing $\sigma(A)$

- given a set of seeds A how to compute $\sigma(A)$?
- how to deal with edge probabilities p?

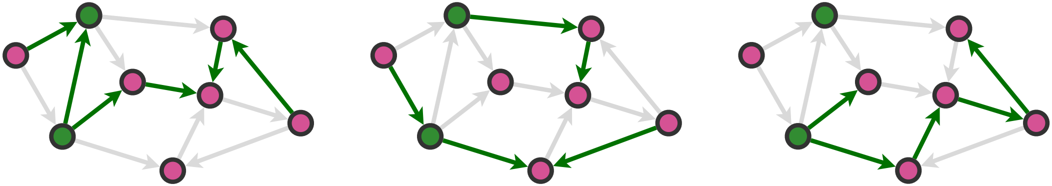
idea:

- assume that edge coin-flips are drawn before the cascade
- sampled edges yield a sample graph X
- $\sigma(A)$ in sample X is the number of nodes reachable from A



computing $\sigma(A)$

- generate a large number of sample graphs X



- compute $\sigma_X(A)$ in each X
  approximate expectation $\mathbb{E}[\sigma(A)]$ via sample mean of all $\sigma_X(A)$
  - Monte-Carlo simulation

- this works OK for approximating $\mathbb{E}[\sigma(A)]$, given A ...
  ... but how to find the optimal A that maximizes $\mathbb{E}[\sigma(A)]$?

the influence-maximization problem

- given $G = (V, E, p)$, find $A \subseteq V$ with $|A| = k$, so as to maximize $\mathbb{E}[\sigma(A)]$

bad news

- the influence-maximization problem is **NP**-hard
  - in fact, it is **NP**-hard to maximize $\sigma_X(A)$ for a sample graph X

good news

- the function $\sigma_X(\cdot)$ is monotonically non-decreasing and submodular

solving the influence-maximization problem

- the function $\sigma_X(\cdot)$ is monotonically non-decreasing and submodular
- it follows that $\mathbb{E}[\sigma(A)]$ is monotonically non-decreasing and submodular
- the Greedy algorithm provides a $\left(1 - \frac{1}{e} - \epsilon\right)$-approximation guarantee
  - (where does ϵ come from?)

Greedy

1. $A \leftarrow \emptyset$
2. **while** $(|A| < k)$ **do**
3. $\quad v \leftarrow \arg\max_{u \in V} \{\sigma(A \cup \{u\}) - \sigma(A)\}$
4. $\quad A \leftarrow A \cup \{v\}$

the influence-maximization problem — extensions and variants

- the influence-maximization problem is widely-studied problem, many extensions and variants have been proposed
  - inferring the network structure and the influence probabilities
  - scaling up the method and avoiding Monte-Carlo simulation
  - multiple actions and topic-aware influence
  - competitive and game-theoretic formulation
  - time-varying networks
  - reinforcement-learning formulation
  - ...

balancing information exposure

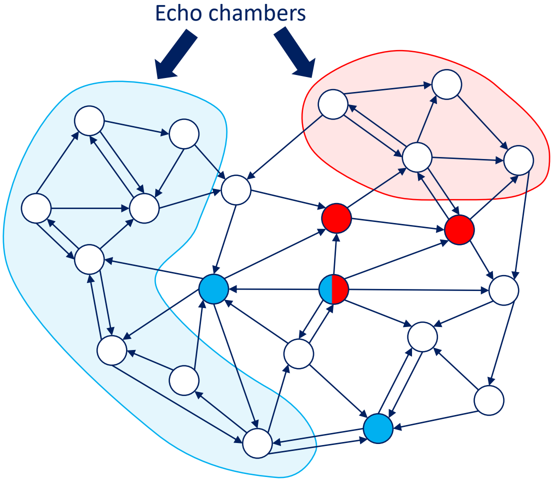
[Garimella et al., 2017]

balancing information exposure

- setting inspired by influence maximization and viral marketing
  - a social network and two campaigns
  - seed nodes $\mathcal{I}_1$ and $\mathcal{I}_2$ for the two campaigns
  - a model of information propagation
  - for each edge e, we assume two probabilities, $p_1(e)$ and $p_2(e)$, one for each campaign
- the problem of balancing information exposure
  - find additional seeds S_1 and S_2, with $|S_1| + |S_2| \leq k$
  - s.t. minimize # of users who see only one campaign or maximize # of users who see both or none

illustration

social discussion on fracking



balancing information exposure — results

- optimization problem is **NP**-hard
- minimization problem is **NP**-hard to approximate
- maximization problem: objective function non monotone and non submodular
  - but there is some structure that we can exploit
- different models of how the two campaigns propagate
- approximation guarantee $\frac{1}{2}(1 - \frac{1}{e})$

adapted greedy algorithm

Algorithm 2: Hedge, greedy algorithm, where each step is as good as adding the best common seed

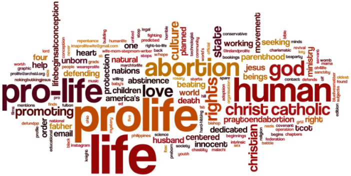
$S_1 \leftarrow S_2 \leftarrow \emptyset$;
while $|S_1| + |S_2| \leq k$ **do**
  $c \leftarrow \arg\max_c \Phi(S_1 \cup \{c\}, S_2 \cup \{c\})$;
  $s_1 \leftarrow \arg\max_s \Phi(S_1, S_2 \cup \{s\})$;
  $s_2 \leftarrow \arg\max_s \Phi(S_1 \cup \{s\}, S_2)$;
  add the best option among $\langle c, c\rangle$, $\langle \emptyset, s_1\rangle$, $\langle s_2, \emptyset\rangle$, $\langle s_2, s_1\rangle$, to $\langle S_1, S_2\rangle$ while respecting the budget.

balancing information exposure — example

Side 1
Pro-Choice

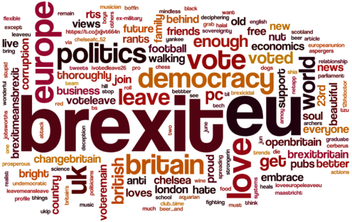
Side 2
Pro-Life

Hedge



Pro-Remain

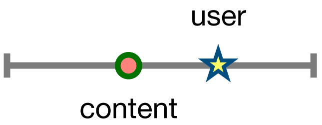
Pro-Leave



maximizing content diversity in social networks

[Matakos et al., 2020a]

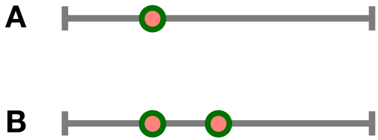
a simple model



a simple model:

represent user and content leanings as values in the $[0, 1]$ interval

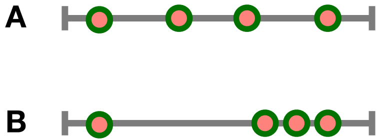
a simple model



for diversity :

B is preferable to **A**

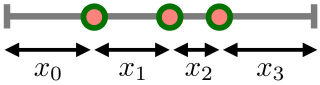
a simple model



for diversity :

A is preferable to **B**

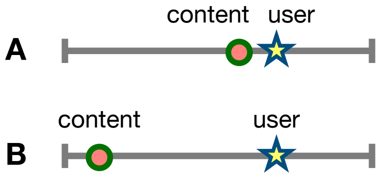
a simple model



user v diversity score for exposure to set of items $\mathcal{I}$

$$d_v(\mathcal{I}) = 1 - \sum_i x_i^2$$

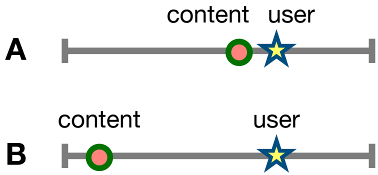
a simple model



for a cascade effect :

A is preferable to **B**

a simple model



for modeling cascades :

- if user u shares item i
- a user v, friend of u, may reshare item i
- reshare probability depends on leanings $\ell(u)$, $\ell(v)$, and $\ell(i)$

maximizing diversity of exposure

- given a set of items with different leanings, and possibly different viralities
- goal : recommendations to maximize user diversity
- consideration : recommended content may be shared among users, creating possible cascades
- make a small number of recommendations
  - why ? intervene as little as possible in the organic operation of the network

[Matakos et al., 2020a]

maximizing diversity of exposure

- problem formulation inspired by influence maximization
- item propagation modeled by independent cascade
  - influence prob. depend on user and item leanings
- recommend a (small) number of items to users so as to maximize the expected diversity score

$$E\left[\sum_{v \in V} d_v(\mathcal{I}(v))\right]$$

$\mathcal{I}(v)$: items that v is exposed, considering also cascades

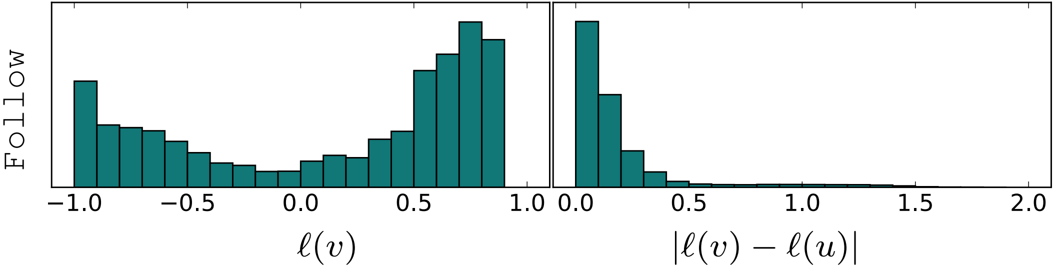
- small number of recommendations
  - at most k recommendations in total
  - at most k_v recommendations to user v

maximizing diversity of exposure — theoretical results

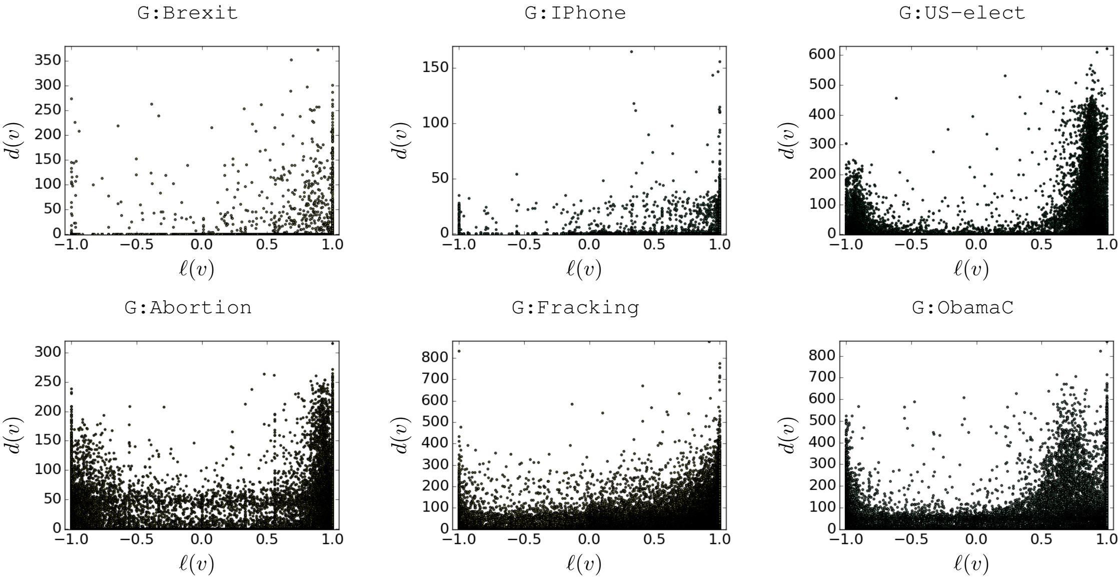
- diversification problem is **NP**-hard
- diversity objective is submodular
- greedy algorithm provides $\frac{1}{2}$ approximation
  - maximizing a submodular function under partition matroid constraints
- but computation prohibitively expensive
  - Monte-Carlo simulations
- adapt recent techniques to obtain highly scalable algorithm
  - generalize the idea of reverse-reachable sets
  - sample-size estimation for early stopping lower bound OPT, which gives bound on sample size

experiments — exploration of datasets

Histograms of node leanings (left) and leaning differences across the edges (right) of `Twitt` network.



experiments — exploration of datasets



comparison against baselines

Dataset (k, k_u)	diversity score: Myopic	diversity score: Max-Var	diversity score: Min-Var	diversity score: TDEM	memory (mb)	run time (s)
`DBLP:BSch` (5, 1)	91	38	112	132	457	2
`DBLP:CPap` (5, 1)	177	44	160	254	276	1
`DBLP:PYu` (5, 1)	535	100	706	912	285	2
`TPair:X` (5, 1)	198	58	160	289	279	1
`TPair:Y` (5, 1)	946	246	841	1 051	174	2
`TPair:Z` (5, 1)	4 147	3 020	3 243	3 997	1 658	42
`Tweet:S5` (50, 5)	816	492	1 006	1 298	5 943	24
`Tweet:S2` (50, 5)	6 127	1 500	10 673	12 386	656	9
`Tweet:M5` (50, 5)	15 509	4 273	21 189	27 659	3 100	77
`Twitt:Follow` (50, 5)	17 635	17 396	8 138	28 164	373	44
`G:Brexit` (50, 5)	542	340	477	1072	23 725	72
`G:IPhone` (50, 5)	14 758	4 129	9 567	26 633	1 803	15
`G:US-elect` (50, 5)	427	311	380	844	45 828	525
`G:Abortion` (50, 5)	640	278	520	3 608	154 588	1 275
`G:Fracking` (50, 5)	561	339	514	4 575	400 565	4 785
`G:ObamaC` (50, 5)	523	311	536	4 420	360 449	3 936
`Twitt:XL` (50, 5)	394 203	360 079	261 281	939 215	3 438	806

diversity-aware friend recommendation

[Matakos et al., 2020b]

problem setting and modeling assumptions

- a social graph $G = (V, E, w)$, with adjacency matrix A
- w_{ij} denotes the connection strength between people i and j
- s_i denotes person's i content exposure on some topic, say, $s_i \in [-1, 1]$
- an echo chamber is when people around you are exposed to similar content
- diversity index : $\eta = \sum_{(i,j)\in E} w_{ij}(s_i - s_j)^2 = \mathbf{s}^\top \mathbf{L}\,\mathbf{s}$
  - high diversity : many individuals have different exposure than social connections
- a recommendation to person i may lead to a change in their exposure value s_i
- probability of accepting the recommendation
- problem : make k recommendations to maximize the diversity index η

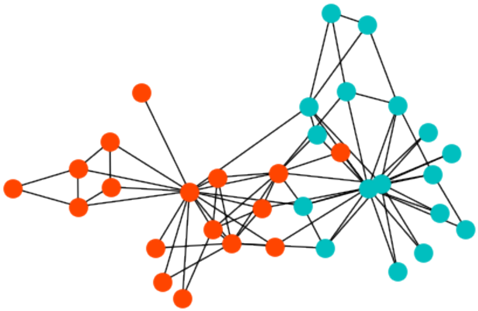
[Matakos et al., 2020b]

a simple problem abstraction

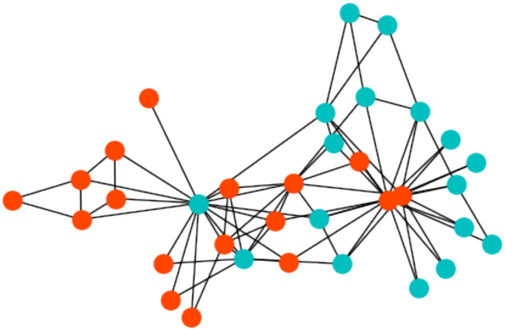
- simplify the problem to understand its complexity
- assume "extreme" exposure values $s_i \in \{-1, 1\}$
- assume that a recommendation reverses a user's exposure value $-1 \longleftrightarrow 1$
- our problem becomes: given a graph whose vertices are painted in two colors (say, `red` or `blue`), select k vertices and swap their color, so as to maximize the number of bi-chromatic edges
- in math form

$$\begin{array}{ll} \max & \mathbf{x}^\mathsf{T} \mathbf{P}\, \mathbf{x} \\ \text{subject to} & \mathbf{b}^\mathsf{T} \mathbf{x} \leq k \\ & \mathbf{x} \in \{0,1\}^n \end{array}$$

toy example



(a) Echo-chamber graph



(b) Graph with diversified exposure

optimal solution for $k = 4$

problem complexity

- problem **NP**-hard (generalization of MAX-CUT)
  - also **NP**-hard to approximate
- problem is non-convex
  - matrix $\mathbf{P}$ is not positive-semidefinite
  - an instance of quadratic knapsack (QK)

solution I — SDP relaxation

- inspired by MAX-CUT and QK solutions
- "lift" the program to the space of square matrices
  - set $\mathbf{X} = \mathbf{x}\,\mathbf{x}^\intercal$ (not convex)
  - relax $\mathbf{X} - \mathbf{x}\,\mathbf{x}^\intercal \succeq 0$ (it implies $\mathbf{X} \succeq 0$)
- objective and constraints are relaxed to corresponding convex ones
- solve the resulting SDP relaxation for $Opt = (\mathbf{X}^*, \mathbf{x}^*)$
- sample fractional $\mathbf{z}$ from a certain distribution specified by $(\mathbf{X}^*, \mathbf{x}^*)$
- round $\mathbf{z}$ to binary $\bar{\mathbf{x}}$
- repeat until constraints are satisfied

solution II — greedy

- SDP solvers use expensive interior-point methods
- efficient methods are needed for large-scale data
- consider simple greedy method that select the most cost-effective item
  - linear-time complexity

experimental results

Table 2. Solution quality and bounds from the relaxations

Dataset	k	IQP	SDP-Relax	Glover	I-Greedy	S-Greedy
Karate	$0.1\,n$	184	184 (185.72)	184 (209.12)	184	184
	$0.2\,n$	224	224 (236.52)	216 (276.2)	224	204
	n	244	244 (253.92)	208 (312.00)	228	204
Karate-D	$0.1\,n$	200	200 (215.84)	196 (263.4)	200	200
	$0.2\,n$	220	220 (242.96)	200 (319.36)	212	208
	n	244	244 (253.92)	200 (372.00)	220	192
Books	$0.1\,n$	828	828 (831.24)	828 (943.6)	828	828
	$0.2\,n$	1056	1048 (1089.04)	996 (1320.04)	1056	992
	n	1236	1224 (1273.72)	1068 (1788.00)	1192	1012
Books-D	$0.1\,n$	1060	1048 (1091.8)	996 (1313.28)	1052	1008
	$0.2\,n$	1140	1124 (1193.92)	1120 (1555.28)	1136	1016
	n	1236	1228 (1273.76)	1128 (1990.0)	1144	972
Twitter100	$0.1\,n$	1700	1700 (1700.76)	1696 (1918.08)	1700	1696
	$0.2\,n$	2396	2396 (2405.52)	2356 (3166.2)	2396	2368
	n	–	3172 (3216.84)	2932 (5 624.00)	3160	2588
Twitter100-D	$0.1\,n$	2972	2968 (3011.76)	2888 (3671.96)	2968	2860
	$0.2\,n$	–	3.028 (3102.12)	2916 (4 285.28)	3044	2860
	n	–	3172 (3216.84)	3100 (5 984.00)	3064	2948
Blogs	$0.1\,n$	–	–	39 512 (50 636.24)	39 516	35 556
Elections	$0.01n$	–	–	–	471 800	469 928
Twitter	$0.001n$	–	–	–	–	6 715 012

summary and discussion

- different ideas to mitigate polarization and information bottlenecks in social networks
  - improve awareness for opposing viewpoints
  - balance information exposure, maximize diversity of content
  - friend recommendations
- an important aspect that was not discussed:
  - how to give control to users over these choices

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