14 Lenses, Diffraction, and Interference

14-1 Lenses, Telescopes, and Magnifying Glasses

When light shines through a lens, it is **refracted** or bent due to the shape and material of the lens. Parallel rays of light passed through some lenses will eventually converge at the **focal point**. The terminology used for lenses is much the same as that used for mirrors in Chapter 13.

Vocabulary **Object distance:** The distance from the center of the lens to the object.

Vocabulary **Image distance:** The distance from the center of the lens to the image. An image can be **real** (able to be projected on a screen), or **virtual** (not able to be projected on a screen).

Vocabulary **Focal point:** The point where parallel rays meet (or appear to meet) after passing through a lens. The distance from this focal point to the center of the lens is called the **focal length**.

$$\textbf{Thin Lens Equation: } \frac{1}{\textbf{focal length}} = \frac{1}{\textbf{object distance}} + \frac{1}{\textbf{image distance}}$$

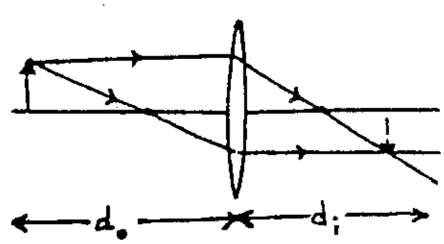
or $$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

NOTE: Many situations involving lenses can also be solved using ray diagrams.

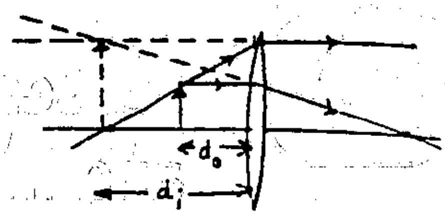
The Converging (Positive) Lens

The focal length of a converging lens is always a positive number.

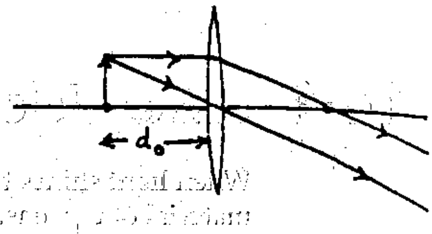
If an object is located outside the focal point of a converging lens, the image it forms is real, inverted, and on the opposite side of the lens. Both d_o and d_i are positive numbers.



If an object is located inside the focal point of a converging lens, the image it forms is virtual, upright, enlarged, and on the same side as the object. In this instance, d_o is positive and d_i is negative.



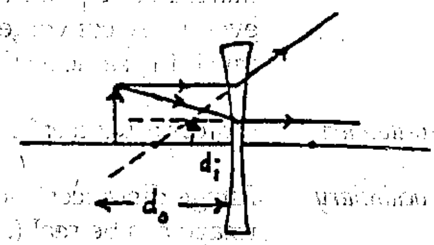
If the object is at the focal point, the rays do not converge and therefore no image is formed.



The Diverging (Negative) Lens

The focal length of a diverging lens is always a negative number.

The image formed by a diverging lens is always virtual, upright, smaller, and on the same side of the lens as the object. In this instance, d_o is positive and d_i is negative.



If an object appears taller when seen through a lens, the object is magnified. The **linear magnification** of an object can be found by comparing the image distance to the object distance, or by comparing the image height, h_i, to the object height, h_o.

$$\textbf{linear magnification} = \frac{\textbf{image distance}}{\textbf{object distance}} = \frac{\textbf{image height}}{\textbf{object height}}$$

$$\text{or} \qquad m = \frac{d_i}{d_o} = \frac{h_i}{h_o}$$

Note that a negative magnification implies a virtual image.

Linear magnification has no units. It is simply a ratio of image to object distance or a ratio of image to object height.

The Refracting Telescope

A refracting telescope is a device that uses one lens to produce a real image, and a second lens to produce the virtual image that is seen by your eye. The amount of linear magnification you see when you look at an object through a telescope depends upon the focal length of each of the lenses. The lens that points toward the object is the objective lens and the lens you look through is the eyepiece. The focal lengths of each of these lenses are labeled f_o and f_e, respectively.

$$\textbf{linear magnification} = \frac{\textbf{focal length of objective lens}}{\textbf{focal length of eyepiece}} \quad \text{or} \quad m = \frac{f_o}{f_e}$$

The Magnifying Glass

When using a magnifying glass, the amount of **angular magnification** of an object depends upon how close you hold the magnifying glass to the object. It also depends upon the near point of your own eye, which is the closest point at which an unaided eye can focus on an object. A person's near point increases with age and the eyes lose some of their adaptable, elastic properties. However, for the ease of calculations, assume the near point of the eye is 25 cm unless otherwise noted.

$$\textbf{angular magnification} = \frac{\textbf{near point}}{\textbf{focal length}} \quad \text{or} \quad M = \frac{\text{near point}}{f}$$

Solved Examples

Example 1: Mukluk, an Inuit, makes a converging lens out of ice that will enable him to concentrate light from the sun to start a fire. When he holds the ice lens 1.00 m from a snow-covered wall, an image of his 5.00-m-distant igloo is projected onto the snow. a) What is the focal length of the ice lens? b) Draw a ray diagram of the situation.

a. *Given:* $d_o = 5.00$ m
$d_i = 1.00$ m

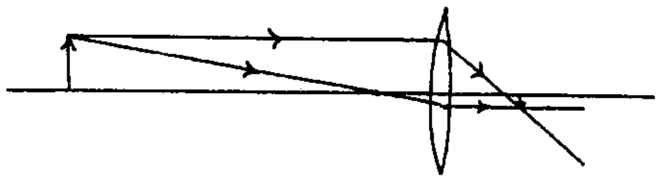
Unknown: $f = ?$
Original equation: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

Solve: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{5.00\text{ m}} + \frac{1}{1.00\text{ m}} = 1.20\text{ m}^{-1}$

Taking the reciprocal gives $f = \frac{1}{1.20\text{ m}^{-1}} = \mathbf{0.833\ m}$

The focal length of 0.833 m is close to the image distance of 1.00 m.

b.



Example 2: A diverging lens is placed 5.0 cm in front of a laser beam to spread the light for the production of a hologram. a) What is the focal length of the lens if the beam of laser light seems to come from a point 2.0 cm behind the lens? b) Draw a ray diagram of the situation.

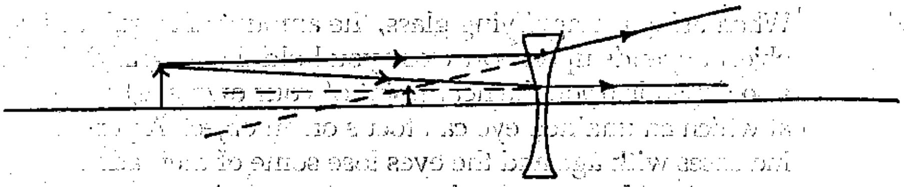
a. *Given:* $d_o = 5.0$ cm
$d_i = -2.0$ cm

Unknown: $f = ?$
Original equation: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

$$\textit{Solve: } \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{5.0 \text{ cm}} + \frac{1}{-2.0 \text{ cm}} = \frac{2}{10. \text{ cm}} - \frac{5}{10.0 \text{ cm}} = -\frac{3}{10.} \text{ cm}^{-1}$$

$$f = -\frac{10.}{3} \text{ cm} = \mathbf{-3.3 \text{ cm}}$$

b.



Example 3: Irwin, a coin collector, is looking at a rare coin held behind a magnifying glass whose focal length is 5.0 cm. a) If the eyes' near point is 25 cm, what is the angular magnification? b) If the coin is 2.0 cm in diameter, how large will its diameter appear to be when it is held in this position under the magnifying glass?

a. *Given:* near point = 25 cm, $f = 5.0$ cm

Unknown: $M = ?$

Original equation: $M = \frac{\text{near point}}{f}$

Solve: $M = \frac{\text{near point}}{f} = \frac{25 \text{ cm}}{5.0 \text{ cm}} = \mathbf{5.0}$ The coin is magnified 5.0 times.

b. *Given:* $m = 5.0$, $h_o = 2.0$ cm

Unknown: $h_i = ?$

Original equation: $m = \frac{h_i}{h_o}$

Solve: $h_i = mh_o = (5.0)(2.0 \text{ cm}) = \mathbf{10. \text{ cm}}$

Example 4: The ship *Speedwell* brought many early settlers to this country in the 1600s. Oceanus sits high above the ship's deck in the crow's nest watching through a telescope for the first sign of land. How much does the telescope magnify if the eyepiece has a 2.0-cm focal length and the objective lens has a 80.-cm focal length?

Given: $f_o = 80.$ cm, $f_e = 2.0$ cm

Unknown: $m = ?$

Original equation: $m = \frac{f_o}{f_e}$

Solve: $m = \frac{f_o}{f_e} = \frac{80. \text{ cm}}{2.0 \text{ cm}} = \mathbf{40.}$ The telescope magnifies 40. times.

Practice Exercises

Exercise 1: Harold and Roland find a discarded plastic lens lying on the beach. The boys discuss what they learned in physics last semester and argue whether the lens is a converging or a diverging one. When they look through the lens, they notice that objects are inverted. a) If an object sitting 25.0 cm in front of the lens forms an image 20.0 cm behind the lens, what is the focal length of the lens? b) Is it a converging or a diverging lens?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 2: Sadie looks at her friend's face through a diverging lens. a) Is the image real or virtual? b) If her friend's face is 50.0 cm from the lens that forms an image at a distance of 20.0 cm, what is the focal length of the lens? c) Draw a ray diagram of the situation.

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 3: Giorgio is clicking shots of the fashion model Nadine as she walks toward him across the studio. Giorgio's camera contains a lens with a focal length of 0.0500 m. a) How far back must the film be located when Nadine is 3.00 m from the camera? b) Should the lens be moved in or out as Nadine approaches closer to the photographer? c) Draw a ray diagram of the situation with Nadine at 3.00 m and 1.00 m from the camera.

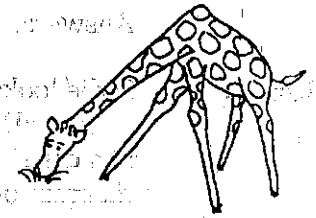
Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 4: Dr. Wasserman is showing slides to his biology class. a) If the slides are positioned 15.5 cm from the projector lens that has a focal length of 15.0 cm, where should the screen be placed to produce the clearest image of the slide? b) Draw a ray diagram of the situation.

Answer: **a.** ____________________

Exercise 5: Marlin is out on a safari. Looking through his telescope, he spots a giraffe in the distance. The telescope has an objective lens of 40-cm focal length and an eyepiece of 2-cm focal length. a) What is the magnification of the giraffe? b) How large is the image formed by the telescope if the giraffe appears to be 1.5 cm high to the naked eye?



Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 6: Emilio, an entomologist, studies a millepede that he holds behind a magnifying glass whose focal length is 2.00 cm. a) Assuming Emilio's near point is 25.0 cm, what is the angular magnification? b) Does Emilio have to bring the magnifying glass closer to, or farther from, the millipede in order to make it appear larger?

Answer: **a.** ______________________

Answer: **b.** ______________________

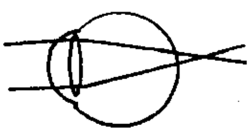
Exercise 7: Mr. Crabtree, a jeweler, looks through his jeweler's loupe (a small magnifying glass attached to his glasses) in order to read the engraving on a pewter bowl. The loupe has a focal length of 3 cm. If Mr. Crabtree's near point is 24 cm, what is the angular magnification of the engraving?

Answer: ______________________

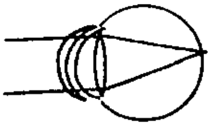
14-2 Eyeglasses

When the eye is unable to focus incoming light directly on the retina (a layer of tissue in the back of the eye that is sensitive to light), eyeglasses or contact lenses are usually prescribed.

If the lens, or cornea, is curved so that light would focus behind the retina, the result is a condition called **farsightedness**, where only objects at a distance can be seen clearly. To correct this problem, glasses for a farsighted person have lenses that are thicker in the middle and thinner near the edges (converging lenses).

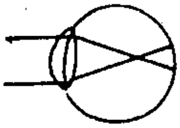


Uncorrected

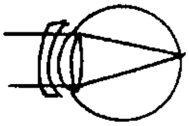


Corrected

If the lens, or cornea, is curved so that light would focus in front of the retina, the result is a condition called **nearsightedness**, where only objects close up can be seen clearly. To correct this problem, glasses for a nearsighted person have lenses that are thinner in the middle and thicker near the edges (diverging lenses).



Uncorrected



Corrected

The power of a pair of prescription glasses is the reciprocal of the focal length, if the focal length is measured in meters.

$$\textbf{Power} = \frac{\textbf{1}}{\textbf{focal length}} \quad \text{or} \quad P = \frac{1}{f}$$

The SI unit for the power of eyeglasses is the **diopter**, which equals the **reciprocal of a meter ($\mathbf{m^{-1}}$)**.

For all the following exercises, assume that the preferred far point of the eye is infinity, ∞, and the preferred near point is 25 cm. To find the power of the lenses in a pair of glasses, take the difference between the reciprocal of how far the eye can see without glasses and how far it can see with glasses.

$$\text{power} = \frac{1}{f_{\text{glasses}}} = \frac{1}{d_{o(\text{glasses})}} - \frac{1}{d_{o(\text{no glasses})}}$$

If you wear glasses or contact lenses, ask your doctor about the power of your prescription. You may find that it can be different for each eye!

Solved Examples

Example 5: Craig is nearsighted, so he must wear glasses to see objects that are far away. If his glasses have a focal length of 0.5 m, what is their power in diopters?

Solution: The focal length must be written as a negative number because a nearsighted person will always wear glasses with diverging lenses. A diverging lens has a negative focal length.

Given: $f_{\text{glasses}} = -0.5\text{ m}$

Unknown: $P = ?$

Original equation: $P = \frac{1}{f}$

Solve: $P = \frac{1}{f} = \frac{1}{-0.5\text{ m}} = \mathbf{-2\text{ diopters}}$

Example 6: In the previous exercise, if Craig can see to infinity with his glasses on, what is the maximum distance he can see clearly with the glasses off?

Given: $f_{\text{glasses}} = -0.5\text{ m}$, $d_{\text{o(glasses)}} = \infty$

Unknown: $d_{\text{o(no glasses)}} = ?$

Original equation:

$$\frac{1}{f_{\text{glasses}}} = \frac{1}{d_{o\text{(glasses)}}} - \frac{1}{d_{o\text{(no glasses)}}}$$

Solve:

$$\frac{1}{d_{o\text{(no glasses)}}} = \frac{1}{d_{o\text{(glasses)}}} - \frac{1}{f_{\text{glasses}}} = \frac{1}{\infty} - \frac{1}{-0.5\text{ m}} = 0 - (-2) = 2\text{ diopters}$$

$$d_{\text{o(no glasses)}} = \frac{1}{2\text{ diopters}} = \mathbf{0.5\text{ m}}$$

The farthest Craig can see clearly without glasses is 0.5 m.

Example 7: Dorcas must hold the phone book 0.5 m from her eyes in order to find the eye doctor's phone number. a) If Dorcas would like to read the phone book at a more comfortable distance of 0.25 m, what power glasses does she need? b) What type of lenses would these glasses contain?

a. *Given:* $d_{\text{o(no glasses)}} = 0.5\text{ m}$, $d_{\text{o(glasses)}} = 0.25\text{ m}$

Unknown: $P = ?$

Original equation:

$$\frac{1}{f_{\text{glasses}}} = \frac{1}{d_{o\text{(glasses)}}} - \frac{1}{d_{o\text{(no glasses)}}}$$

Solve:

$$\frac{1}{f_{\text{glasses}}} = \frac{1}{d_{o\text{(glasses)}}} - \frac{1}{d_{o\text{(no glasses)}}} = \frac{1}{0.25\text{ m}} - \frac{1}{0.5\text{ m}} = 4 - 2 = \mathbf{2\text{ diopters}}$$

b. Because the power of the glasses in this example is a positive number, the lenses must be converging lenses. This is supported by the fact that farsightedness must be corrected with converging lenses.

Practice Exercises

Exercise 8: Beth is farsighted, so she must wear glasses to see objects close by. If her glasses have a focal length of 0.30 m, what is their power in diopters?

Answer: ____________________

Exercise 9: Herman is able to read the newspaper at a distance of 0.75 m, but no closer. a) Is he farsighted or nearsighted? b) What power lens should he use to allow him to read the paper at 0.25 m? c) What type of lens does he need?

Answer: **a.** ____________________

Answer: **b.** ____________________

Answer: **c.** ____________________

Exercise 10: At the beach, Maria can see Sandy, a surfer, clearly only when he is standing closer than 2.0 m. a) What power prescription sunglasses would Maria need in order to see Sandy when he is out on the ocean riding a wave? b) What type of lenses will her glasses contain?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 11: Matt is driving his "18-wheeler" while wearing his new pair of glasses whose focal length is -0.40 m. If the glasses allow Matt to see clearly at an infinite distance for normal driving, how far could Matt see clearly before he bought the glasses?

Answer: ____________________

Exercise 12: Moshe has gone to Bermuda for spring vacation and when he is on the beach realizes that he has picked up his father's pair of prescription sunglasses by mistake. The glasses have a power of $+3.0$ diopters. a) What type of eye problem does Moshe's father have, and how do you know? b) What is the closest that Moshe's father can see clearly without his glasses? c) Will these glasses produce an image in front of, or behind, the image formed by Moshe's normal eye?

Answer: **a.** ____________________

Answer: **b.** ____________________

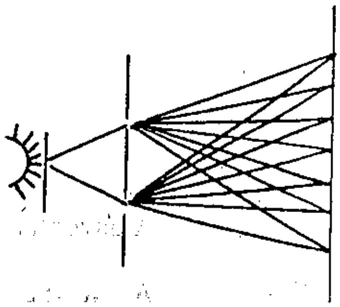
Answer: **c.** ____________________

14-3 Diffraction and Interference

Vocabulary **Diffraction:** The spreading of a wave as it passes around an obstacle or through an opening.

Vocabulary **Interference:** When two waves overlap to produce one new wave.

In 1801, Thomas Young attempted to prove that light was a wave by showing that it has the ability to diffract and interfere. Young passed white light through two closely-spaced slits and noticed that the light spread out as it passed through the openings (diffracted), and overlapped on a screen a few meters away (interfered), to produce alternating bands of light and dark.



Whether light is passed through two slits or through the multiple, closely-spaced slits of a diffraction grating, the grating equation can be written as

$$\textbf{wavelength} = \frac{\textbf{(slit separation)(space between bright bands)}}{\textbf{(distance from slits to screen)}} \quad \text{or} \quad \lambda = \frac{dx}{L}$$

This equation is a good approximation when the angular separation between the bright bands is very small. When used with a diffraction grating, however, it could produce an answer with as much as 10% error. Nevertheless, to simplify calculations and avoid the use of trigonometry, the equation will be used in this form in all exercises.

The common unit for the wavelength of light is the **nanometer (nm)**, which equals 10^{-9} m.

Solved Examples

Example 8: Miss McGillivray loses the specifications for her diffraction grating and must recalibrate it in order to determine the grating spacing. She shines a red helium-neon laser, whose wavelength is 633 nm, through the grating. Two bright spots that are each 1.40 m from the central maximum fall on the wall 4.00 m away. What is the space between the grooves on the diffraction grating?

Solution: First, convert nm to m. $633 \text{ nm} = 6.33 \times 10^{-7} \text{ m}$

Given: $\lambda = 6.33 \times 10^{-7}$ m
$L = 4.00$ m
$x = 1.40$ m

Unknown: $d = ?$
Original equation: $\lambda = \dfrac{dx}{L}$

Solve: $d = \dfrac{\lambda L}{x} = \dfrac{(6.33 \times 10^{-7} \text{ m})(4.00 \text{ m})}{1.40 \text{ m}} = \mathbf{1.81 \times 10^{-6}\ m}$

Example 9: In the previous exercise, Miss McGillivray uses her newly calibrated grating to determine the wavelength of a green helium-neon laser. Keeping the laser at the same distance from the wall as before, the distance from the central maximum to the first bright fringe is 1.20 m. What is the wavelength of the green HeNe laser?

Given: $d = 1.81 \times 10^{-6}$ m
$L = 4.00$ m
$x = 1.20$ m

Unknown: $\lambda = ?$
Original equation: $\lambda = \dfrac{dx}{L}$

Solve: $\lambda = \dfrac{dx}{L} = \dfrac{(1.81 \times 10^{-6} \text{ m})(1.20 \text{ m})}{4.00 \text{ m}} = 5.43 \times 10^{-7} \text{ m} = \mathbf{543\ nm}$

Practice Exercises

Exercise 13: Judy and Earl are sitting under the boardwalk one warm summer evening while the light of a low-pressure sodium vapor lamp whose wavelength is 589 nm passes through two small cracks in a board, producing fringes of light 0.0020 m apart on the ground. a) If the boardwalk is 3.0 m above the sand, what is the distance between the two cracks in the board? b) If the distance between the cracks were smaller, would the fringes of light on the ground be closer together or farther apart?

Answer: a. ______________________

Answer: b. ______________________

Exercise 14: Two large speakers broadcast the sound of a band tuning up before an outdoor concert. While the band plays an A whose wavelength is 0.773 m, Brenda walks to the refreshment stand along a line parallel to the speakers. If the speakers are separated by 12.0 m and Brenda is 24.0 m away, how far must she walk between the "loudspots"?

Answer: ______________________

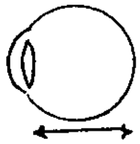
Exercise 15: In an attempt to test the particle nature of matter, Claus Jönsson performed an experiment in 1961 that was very similar to Young's Double Slit experiment for light done in 1801. Jönsson sent a beam of electrons through two slits separated by 2.00×10^{-6} m onto a fluorescent screen 0.200 m away. Due to their high speed, the electrons behaved like waves with a wavelength of 2.40×10^{-11} m. How far apart were the bright lines formed on the screen?

Answer: ______________________

Additional Exercises

A-1: A photocopy machine is set to reduce the size of printed material by 50%. When the print is regular size, both the image and object distance are 16.0 cm. If the lens is then moved 24.0 cm from the object, how large is the new image distance?

A-2: The average normal human eye forms an image on the retina at a distance of about 0.0240 m from the lens, as shown. How much must the focal length of the lens change in order to accommodate an object moved from 10.0 m to 0.250 m? (This change in focal length is accomplished by small muscles in the eye called *cilliary muscles*. These muscles actually stretch and relax the lens.)



0.024m

A-3: Lisa is posing for her senior class picture and sits 2.00 m from the camera lens whose focal length is 17.0 cm. The camera lens is positioned 21.0 cm in front of the film. Will the photographer obtain a clear image of Lisa? If not, by how much must the camera lens be moved in our out?

A-4: Cindy is lying on the beach focusing her camera on a friend standing 5.00 m away. Her camera has a focal length of 5.00 cm. a) Where must Cindy position the camera lens relative to the film for the sharpest focus? b) What type of lens must her camera have, and why?

A-5: Sherlock Holmes discovers some telltale hairs at the scene of a crime. He views the hairs with his magnifying glass from a distance of 6.0 cm. If the hairs are magnified 4.0 times, how far is the magnified image from the lens?

A-6: Jacob attaches a solar filter to his telescope and projects an image of the sun through the objective lens that has a focal length of 2.00 m. Jacob can't decide whether to use a 40.0-mm eyepiece or a 16.0-mm eyepiece to study the solar features. a) What amount of magnification will each eyepiece provide? b) Someone may look through a telescope and ask, "What is the magnification of this instrument?" Why is it impossible to give one standard answer to the question? c) If the sun appears to be 1.00 cm across to the naked eye, how large will it appear when viewed with the 16.0-mm eyepiece?

A-7: To the naked eye, Jupiter appears to be about 0.10 cm in diameter. In a telescope whose objective lens has a focal length of 2.0 m, Jupiter appears to be 1.2 cm in diameter. What is the focal length of the eyepiece used to produce this image?

A-8: Ms. Chang is standing by the slide projector in the back of the room when she realizes that the screen is in the wrong location to get a clear image. a) If the projector has a lens with a focal length of 20.0 cm, and the slides sit 20.6 cm behind the lens, in which direction should one of the students move the screen that sits 7.00 m from the lens? b) How far away should the screen be from the projector lens?

A-9: Beverly wears bifocals. She can read close up when she looks through the bottom portion and can read far away when she looks through the top portion. a) The top of her glasses has a focal length of -0.25 m. What is the power, in diopters, of this part of the glasses? b) The bottom portion has a power of 3.5 diopters. What is the focal length of this part of the glasses?

A-10: In exercise A-9, if Beverly can see to infinity with her glasses on, a) what is the maximum distance she can see clearly with the glasses off? b) If Beverly can see an object at 25 cm with her glasses on, what is the minimum distance she can see clearly with the glasses off?

A-11: Rachel brings a note home from school. The note advises her mother that "Rachel is having a difficult time reading the words on the board and can only see the words if she is sitting closer than 2.0 m." If Rachel wants to be able to read the words from 3.0 m away, what power glasses does she need?

A-12: Joon puts on a pair of diffraction grating glasses that he bought in a novelty shop and looks at a mercury vapor street lamp that is 5.00 m away. He sees a yellow spectral line 1.16 m on either side of the light source. If the diffraction grating glasses have a slit separation of 2.49×10^{-6} m, what is the wavelength of the light Joon is observing?

A-13: Radio station WLLH has two transmitters that sit atop nearby hillsides broadcasting a wave that is 214 m long. As Kiesha drives down the interstate parallel to the two transmitters at a distance of 1000. m, she hears an increase in signal from the station every 30.0 m. How far apart are the two transmitters?

Challenge Exercises for Further Study

B-1: The Hale telescope at the Yerkes Observatory in Wisconsin has an objective lens with a focal length of 19 m. (For an object at infinity, the image distance equals the focal length.) If the telescope is used to observe Saturn that is 1275×10^9 m from Earth, what will be the apparent diameter of the rings if their actual diameter is 27×10^7 m?

B-2: Dr. Kirwan is preparing a slide show that he will present to the executive board at tonight's committee meeting. He places a 3.50-cm slide behind a lens of 20.0 cm focal length in the slide projector. a) How far from the lens should the slide be placed in order to shine on a screen 6.00 m away? b) How wide must the screen be to accommodate the projected image?

B-3: Madeline is working for the Eye-Spy Detective Agency and her assignment is to secretly photograph the pages of a journal. Madeline's tiny camera has the film located 2.10 cm behind the lens, and she must fill the entire piece of 1.00-cm film with the picture of the 25.0-cm-tall document. How close must Madeline be to the journal pages to get a clear image on the film?