11 Simple Harmonic Motion

11-1 Springs

Vocabulary **Period:** The time it takes for a vibrating object to repeat its motion.

Vocabulary **Frequency:** The number of vibrations made per unit time.

Period and frequency are the reciprocals of each other. In other words,

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{T}$$

Since period is a measure of time, its SI unit is the **second,** while the unit for frequency is the reciprocal of this, or **1/second.** Another way of writing 1/s is with the unit **hertz (Hz).**

You may recognize this as being similar to the explanation of period and frequency in Chapter 6 on circular motion.

Hooke's Law

Whenever a spring is stretched from its equilibrium position and released, it will move back and forth on either side of the equilibrium position. The force that pulls it back and attempts to restore the spring to equilibrium is called the **restoring force.** Its magnitude can be written as

restoring force = (force constant)(displacement from equilibrium)
or $F = kx$

This relationship is known as **Hooke's law.** The force constant is a measure of the stiffness of the spring. The SI unit for the force constant is the **newton per meter (N/m).**

Keep in mind that this is the force required to restore the spring back to its original position. The force that acts to move the spring *away* from the equilibrium position is equal in magnitude to the restoring force, but opposite in direction.

Simple harmonic motion is motion that occurs when the restoring force acting on an object is proportional to the object's displacement from its rest position. Objects at the end of springs move in simple harmonic motion when they are displaced from their rest position and bounce up and down on the spring, or oscillate.

Period of a Mass on a Spring in Simple Harmonic Motion

The only two things that affect the period of an object hanging on a bouncing spring are the object's mass and the force constant of the spring on which the object is oscillating.

$$\textbf{Period} = 2\pi\sqrt{\frac{\textbf{mass}}{\textbf{force constant}}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

To prove that this equation does indeed give the period in seconds, simplify the units for $\sqrt{m/k}$ by writing

$$\sqrt{\frac{\text{kg}}{\text{N/m}}} = \sqrt{\frac{\text{kg}}{\frac{\text{kg}\cdot\text{m/s}^2}{\text{m}}}} = \sqrt{\text{s}^2} = \text{s}$$

Solved Examples

Example 1: A hummingbird beats its wings up and down with a frequency of 80.0 Hz. What is the period of the hummingbird's flaps?

Given: $f = 80.0$ Hz

Unknown: $T = ?$
Original equation: $T = \frac{1}{f}$

Solve: $T = \frac{1}{f} = \frac{1}{80.0 \text{ Hz}} = \mathbf{0.0125\ s}$

Example 2: In anticipation of her first game, Alesia pulls back the handle of a pinball machine a distance of 5.0 cm. The force constant of the spring in the handle is 200 N/m. How much force must Alesia exert?

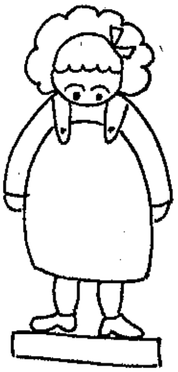
Solution: First, convert cm to m. 5.0 cm = 0.050 m

Given: $k = 200$ N/m
$x = 0.050$ m

Unknown: $F = ?$
Original equation: $F = kx$

Solve: $F = kx = (200 \text{ N/m})(0.050 \text{ m}) = \mathbf{10\ N}$

Example 3: As Bianca stands on a bathroom scale, whose force constant is 220 N/m, the needle on the scale vibrates from side to side. a) If Bianca has a mass of 180 kg, what is the period of vibration of the needle as it comes to rest? b) If Bianca goes on a diet, how will this change the period of vibration?



a. *Given:* $m = 180$ kg
$k = 220$ N/m

Unknown: $T = ?$
Original equation: $T = 2\pi\sqrt{\frac{m}{k}}$

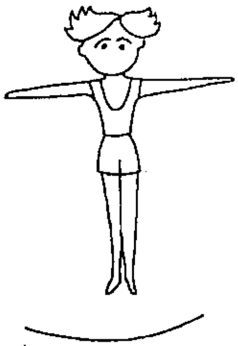
Solve: $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{180\text{ kg}}{220\text{ N/m}}} = \mathbf{5.7\ s}$

In other words, this is the amount of time for one complete oscillation.

b. Because the mass will be smaller, the period of vibration will be smaller. In other words, it will take less time for the needle on the scale to bounce from side to side as it comes to rest.

Practice Exercises

Exercise 1: Terry jumps up and down on a trampoline with a frequency of 1.5 Hz. What is the period of Terry's jumping?



Answer: ______________________

Exercise 2: Gary Stewart of Reading, Ohio set a pogo stick record in 1990 by jumping 177 737 times. a) If the pogo stick he used had a force constant of 6000. N/m and was compressed 0.12 m on each jump, what force must Gary have exerted on the pogo stick upon each jump? b) What force would be exerted back up on Gary each time he went up?

Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 3: At the post office, Cliff, a postal worker, places a 0.60-kg package on a scale, compressing the scale by 0.03 m. a) What is the force constant of the spring in the postal scale? b) What happens to the force constant if Cliff weighs a heavier package?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 4: A jack-in-the-box lid will pop open when a crank is turned on the outside of the box. If Jack pushes against the inside of the box with a force of 3.00 N when the lid is closed, and the spring is compressed 10.0 cm from equilibrium, what is the force constant of the spring?

Answer: ____________________

Exercise 5: Sam, a butcher, puts 3.0 kg of chopped beef on the 1.0-kg pan of his scale, which has a spring whose force constant is 400. N/m. What is the period of vibration of the pan as it comes to rest? b) If Sam adds more beef to the scale, what will this do to the period of vibration?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 6: A toy bobs up and down over Campbell's crib with a period of 1.0 s. The toy hangs from the end of a spring whose force constant is 20.0 N/m. What is the mass of the toy?

Answer: ______________________

11-2 Pendulums

The period of a pendulum depends only upon the pendulum's length (if the angle of swing is not too large). A long pendulum has a longer period than a short pendulum. The relationship between period and length can be shown with the following equation.

$$\textbf{Period} = 2\pi\sqrt{\frac{\textbf{length}}{\textbf{acceleration due to gravity}}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{L}{g}}$$

It should be noted that this equation works only for a pendulum whose mass is considerably larger than the mass of the string from which it swings. To simplify calculations, in the following exercises you will be working with pendulums swinging from strings of negligible mass.

Solved Examples

Example 4: A tall, thin tree sways back and forth in the breeze with a frequency of 2 Hz. What is the period of the tree?

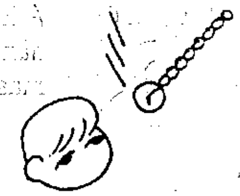
Given: $f = 2$ Hz

Unknown: $T = ?$

Original equation: $T = \frac{1}{f}$

Solve: $T = \frac{1}{f} = \frac{1}{2\text{ Hz}} = \mathbf{0.5\ s}$

Example 5: World-reknowned hypnotist Paulbar the Great swings his watch from a 20.0-cm chain in front of a subject's eyes. What is the period of swing of the watch?



Solution: First, convert cm to m. 20.0 cm = 0.20 m

Given: $L = 0.20\ \text{m}$
$g = 10.0\ \text{m/s}^2$

Unknown: $T = ?$
Original equation: $T = 2\pi\sqrt{\dfrac{L}{g}}$

Solve: $T = 2\pi\sqrt{\dfrac{L}{g}} = 2\pi\sqrt{\dfrac{0.20\ \text{m}}{10.0\ \text{m/s}^2}} = \mathbf{0.89\ s}$

Therefore, it takes 0.89 s for the watch to swing in one direction and back again, through one full cycle.

Example 6: A spider swings in the breeze from a silk thread with a period of 0.6 s. How long is the spider's strand of silk?

Solution: The answer is determined using the pendulum equation, but now it must be set up in terms of the unknown, L. First, square all of the terms to get rid of the radical. The equation becomes

$T^2 = 4\pi^2 \dfrac{L}{g}$. Then rearrange the equation as shown.

Given: $T = 0.60\ \text{s}$
$g = 10.0\ \text{m/s}^2$

Unknown: $L = ?$
Original equation: $T = 2\pi\sqrt{\dfrac{L}{g}}$

Solve: $L = \dfrac{gT^2}{4\pi^2} = \dfrac{(10.0\ \text{m/s}^2)(0.6\ \text{s})^2}{4\pi^2} = \mathbf{0.09\ m}$

Practice Exercises

Exercise 7: A metronome is a device used by many musicians to get the desired rhythm for a musical piece. If a metronome is clicking back and forth with a frequency of 0.5 Hz, what is the period of the metronome?

Answer: ____________________

Exercise 8: Many amusement parks feature a ride in which a giant ship swings back and forth. If the period of the ship is 8.00 s, what is the frequency of the swinging ship?

Answer: ____________________

Exercise 9: Tegan, a trapeze artist, swings from a 2.5-m-long trapeze, high above the three-ring circus. a) What is Tegan's period of swing? b) Would Tegan's period of swing change if she were more massive? If so, how?

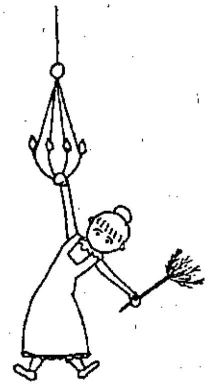
Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 10: Danielle is pushing her twin Daniel on a swing that hangs from a tree branch by 2.0-m-long ropes. With what frequency will Danielle have to push Daniel as he swings?

Answer: ____________________

Exercise 11: Marla, a maid, is standing on the Vanderbilt's dining room table dusting the chandelier. While Marla is reaching up, she slips and grabs hold of the chandelier to catch her balance. When she lets go, the chandelier begins to swing with a period of 1.6 s. How long is the cable connecting the chandelier to the ceiling?



Answer: ________________

Exercise 12: You have been commissioned by NASA to travel to Jupiter's innermost Galilean satellite, Io, to learn more about this volcanic moon. As you board the spacecraft, you are handed a rock tied to a 10.0-cm string, and a stopwatch, and are asked to derive an experiment that would allow you to determine the acceleration due to gravity on Io. You must use both pieces of equipment and nothing more. a) Describe how you would calculate Io's gravitational acceleration. b) If the pendulum swings with a period of 1.48 s, what is the gravitational acceleration on Io?

Answer: **a.** ________________

Answer: **b.** ________________

Additional Exercises

A-1: Mr. Knote, a piano tuner, taps his 440-Hz tuning fork with a mallet. What is the period of the vibrating tuning fork?

A-2: Denny jumps up and down on his bed, taking 0.75 s for each jump. What is the frequency of Denny's jumping?

A-3: Inside most ball-point pens is a small spring that compresses as the pen is pressed against the paper. If a force of 0.1 N compresses the pen's spring a distance of 0.005 m, what is the force constant of the tiny spring?

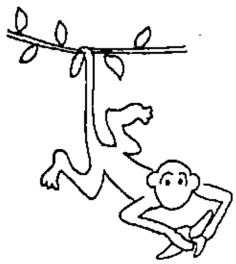
A-4: Maureen is trying to predict the period of a mass hung on a spring. She has a spring of negligible mass and four weights to hang on the end. Maureen collects the following data as she observes the stretch of the spring:

force (N)	displacement (m)
2.5	0.050
5.0	0.102
7.5	0.149
10.0	0.199

a) Plot a graph of force (on the y-axis) vs. displacement (on the x-axis). b) Find the slope of the graph. What does this slope represent? c) Use the information you have obtained to find the period of the spring when a 3.0 kg mass is hung on the end.

A-5: Kim drives her empty dump truck over a berm (also called a speed bump) at the construction site. The truck has a mass of 3000. kg and the force constant for one of the truck's springs is 100 000. N/m. (Remember, the truck has 4 wheels.) a) What is the resulting period of the bouncing truck as it goes over the bump? b) If Kim leaves the construction site with a load of dirt in her truck, what will this do to the period of her dump truck as it crosses the berm?

A-6: A monkey swings from a jungle vine by his 0.30-m-long tail. a) What is the period of swing of the monkey? b) With what frequency does the monkey swing?



A-7: A wrecking ball used to demolish buildings swings from a 10.0-m-long cable. What is the period of the wrecking ball as it swings?

A-8: A crow attempts to land on a small bird feeder, causing it to swing back and forth with a frequency of 0.350 Hz. How long is the wire from which the feeder hangs?

A-9: The acceleration due to gravity on the moon is 1/6 that on Earth. a) If you wanted a pendulum clock to tell time on the moon the same as it does on Earth (i.e., have the same period), would you need to lengthen or shorten the pendulum? b) If the pendulum was originally 24.0 cm long on Earth, how long should it be on the moon?

Challenge Exercises for Further Study

B-1: Ezra, a 60.0-kg high school student, is sleeping on his waterbed when his 2.0-kg cat, Muffin, jumps onto his back, causing Ezra to sink 2.0 cm deeper into the waterbed. a) If Muffin then jumps off Ezra from this new equilibrium position, what will be the period of Ezra's bobbing motion on the waterbed? b) Will this period slow down, speed up, or remain the same as the amplitude of the bounces gets smaller and smaller? Explain your answer.

B-2: Andy (mass 80.0 kg), Randy (mass 60.0 kg), and twins Candy and Mandy (each with a mass of 70.0 kg) climb into a 1000.-kg car, causing each of the four springs to compress 4.00 cm. Find the period of vibration of the car as it comes to rest after the four get in.

B-3: Tanja talks long distance with her boyfriend every night from her dormitory pay phone, and her phone bills are getting rather high. She has decided that she must limit each of her calls to 10 minutes. Since Tanja doesn't have a watch, she devises a unique way to time her calls. Tanja notices that the pay phones each have a cord that is 0.800 m long. Therefore, as she talks on one phone, she can swing the receiver of the adjacent phone to time her call. How many complete swings will the nearby phone receiver make before Tanja must hang up?

B-4: On a 0°C-winter day, a 10.000-m-long brass Foucault pendulum hanging in the covered entrance to the science museum swings back and forth with the rotation of Earth. The outdoor temperature variations range from 0°C in the winter to 30.0°C in the summer. How does the period of the pendulum change throughout the year? ($\alpha_{brass} = 19 \times 10^{-6}°C^{-1}$)

B-5: Gillian buys a pendulum clock at a discount store and discovers when she gets it home that it loses 6.00 minutes each day. a) Should she lengthen or shorten the pendulum in order for it to keep accurate time? b) If the pendulum has a period of 2.00 s, by how much must the length be changed so that the clock keeps accurate time?