Cryptology Packet Pt1

ID: 1747

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1. Problem: Find the greatest common divisor for each of the following pairs of integers.

   a. $15, 35$
   Solution: For 15, these are the divisors, $1, 3, 5, 15$ and for 35, the divisors are $1, 5, 7, 35$. The only common divisor between the two is 5 so therefore, it is also the greatest common divisor.

   b. $0, 111$
   Solution: For 0, all integers are common divisors of it and for 111, the greatest divisor will be 111. So, since all common divisors of 0 are integers, the greatest common divisor will be 111.

   c. $-12, 18$
   Solution: For -12, the divisors are $-12, -6, -4, -3, -2, -1, 1, 2, 3, 4, 6, 12$ and for 18, the divisors are $1, 2, 3, 6, 9, 18$. Therefore, 6 is the greatest common divisor between $-12, 18$.

   d. $99, 100$
   Solution: For 99, the divisors are $1, 3, 9, 11, 33, 99$ and for 100, the divisors are $1, 2, 4, 5, 10, 20, 25, 50, 100$. The only common divisor between 99 and 100 is 1 so it is also therefore the greatest common divisor between the two.

   e. $11, 121$
   Solution: For 11, the divisors are $1, 11$ and for 121, the divisors are $1, 11, 121$. Therefore, the greatest common divisor between the two is 11.

   f. $100, 102$
   Solution: For 100, the divisors are $1, 2, 4, 5, 10, 20, 25, 50, 100$ and for 102, the divisors are $1, 2, 3, 6, 17, 34, 51, 102$. Therefore, the greatest common divisor between $100, 102$ is 2.

2. Problem: Let a be a positive integer. What is $\gcd(a, 2a)$?

Solution: Since a is a divisor of $2a$, the greatest common divisor of a and $2a$ is a.

3. Problem: Let a be a positive integer. What is $\gcd(a, a^2)$?

   Solution: Since a is a divisor of a^2, the greatest common divisor of a and a^2 will be a

4. Problem: Let a be a positive integer. What is $\gcd(a, a+1)$?

   Solution: The greatest common divisor between a and $a+1$ is 1. This can be seen with a basic example. If $a = 3$ then $a + 1 = 4$ and the only common divisor is 1. This holds true for all positive integers.

5. Problem: Let a be a positive integer. What is $\gcd(a, a+2)$?

   Solution: If a is even, the greatest common divisor between a and $a+2$ will be 2. This is because $a+2$ will also be even and all even numbers are divisible by 2. If a is odd, the greatest common divisor between a and $a+2$ will be 1. We can use an example to prove this. If $a = 3$, $a + 2 = 5$ and the greatest common divisor between them is 1. This will hold true when a is odd and a positive integer.

6. Problem: Find the greatest common divisor for each of the following sets of integers.

   a. $8, 10, 12$
   The divisors of 8 are $1, 2, 4, 8$, for 10 they are $1, 2, 5, 10$, and for 12, they are $1, 2, 3, 4, 6, 12$. Therefore, the common divisors between the three are $1, 2$, and 2 is the greatest common divisor.

   b. $6, 15, 21$
   The divisors of 6 are $1, 2, 3, 6$, for 15, they are $1, 3, 5, 15$, and for 21, they are $1, 3, 7, 21$. There are two common divisors, $1, 3$, and 3 is the greatest common divisor.

   c. $-7, 28, -35$
   The divisors of -7 are $-7, -1, 1, 7$, for 28, they are $1, 2, 4, 7, 14, 28$, and for -35, they are $-35, -7, -5, -1, 1, 5, 7, 35$. Between the three, 7 is the greatest common divisor.

7. Problem: Find a set of three integers that are mutually relatively prime, but any two of which are not relatively prime.

   Solution: A set of three integers would be $10, 12, 15$ as they are mutually relatively prime. Since the greatest common divisor between the numbers $10, 12, 15$ is 1, the numbers are relatively prime to each other. However, combinations of these numbers in sets of two won't be relatively prime.

Mutually relatively prime integers have a greatest common divisor of 1. For example, $10, 12$ will have a greatest common divisor of 2 and isn't prime. $12, 15$ will have a greatest common divisor of 3 and isn't prime. $10, 15$ will have a greatest common divisor of 5 and isn't prime. Therefore, $10, 12, 15$ satisfies the conditions.

8. Problem: Find four integers that are mutually relatively prime such that any three of these integers are not mutually relatively prime.

   Solution: A set of four integers would be $14, 30, 42, 105$ as they are mutually relatively prime since their greatest common divisor would be 1. However, combinations of any three of these integers wouldn't be mutually relatively prime. Mutually relatively prime integers have a greatest common divisor of 1. For example, $30, 42, 105$ would have a greatest common divisor of 3 and isn't prime. $42, 105, 14$ would have a greatest common divisor of 7 and isn't prime. $14, 30, 42$ would have a greatest common divisor of 2 and isn't prime. Therefore, $14, 30, 42, 105$ satisfies the conditions.