I. Introduction

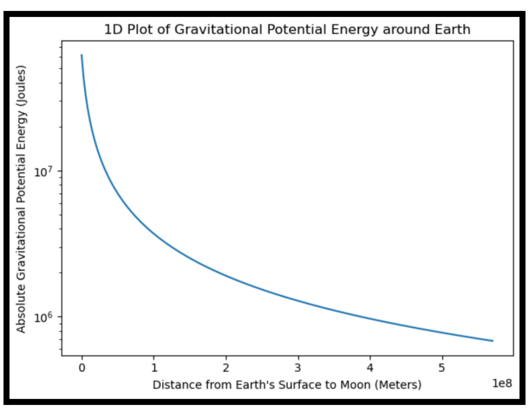
This report will detail the gravitational potential energy of the Earth and Moon, the gravitational force field produced by this system and the effect it has on the Apollo 11 command module, and the efficiency of the Saturn V rocket.

II. The gravitational potential of the Earth-Moon system

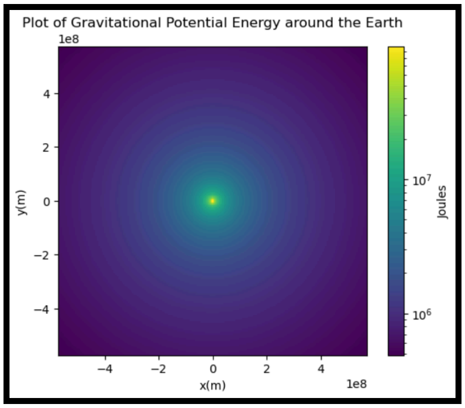
The following calculations were performed for the Earth and Moon using this formula for gravitational potential energy, where G is the gravitational constant, M is the mass of the object (in this case the Earth or Moon), and r is the distance from that object:

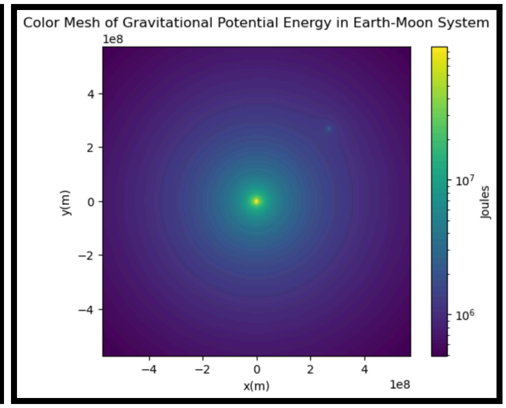
$$\boxed{\Phi(r) = -\frac{GM}{r}}$$

First, observe the speed with which the gravitational potential energy falls as one moves away from the Earth. This indicates that the core challenge is getting off the ground and into orbit. Once there, changing position will require far less energy. The plot below demonstrates the fall off in potential energy logarithmically to help visualize the curve.



Now we turn to a 2D plot of the gravitational potential energy. Note the effect that the moon has on the plots, with the second plot corresponding to the entire Earth-Moon system.





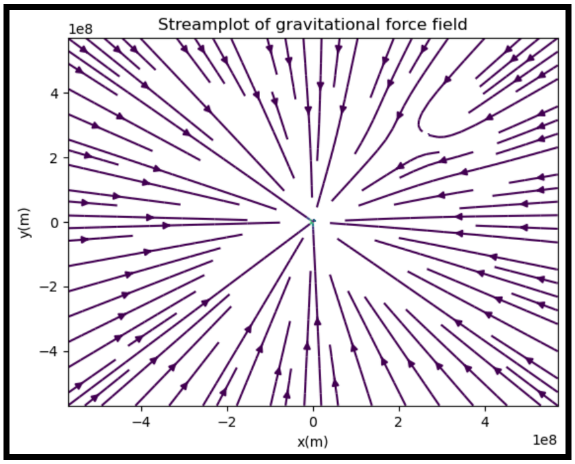
These findings indicate that the Earth is the primary obstacle to overcome in space travel, although there is a noticeable jump in gravitational potential energy around the moon. The plotting is, again, logarithmic. Without it, all the colors outside of the immediate vicinity of the Earth are homogeneous, again demonstrating how quickly the gravitational potential falls off.

III. The gravitational force of the Earth-Moon system

The following calculations were performed using this formula for gravitational force, where G is the gravitational constant, M_1 is the mass of one body (in this case the Earth or Moon), m_2 is the mass of some other body (for this, the Apollo 11 command module), and r_21 is the displacement vector between M_1 and m_2

$$\boxed{\vec{F}_{21} = -G\frac{M_1 m_2}{|\vec{r}_{21}|^2}\hat{r}_{21}}$$

We can see in the following figure the gravitational influence of the Earth vs that of the Moon. Again, this demonstrates just how much more of a gravitational effect the Earth has on anything in the Earth-Moon system.



IV. Projected performance of the Saturn V Stage 1

This report found that the burn time of the Saturn V Stage 1 rocket would be 157.69 seconds, based on the following formula, where m_0 is the mass of the rocket with all of its fuel, m_f is the mass after burning it all off, and m’ is the rate at which fuel is consumed:

$$\boxed{T = \frac{m_0 - m_f}{\dot{m}}}$$

The critical thing to determine with this number is the altitude that the Saturn V could reach within this time. This report will not provide the entire formula for the velocity of the rocket overtime, but the altitude is found by integrating (essentially instantaneous summation at each point in time) the velocity like so:

$$\boxed{h = \int_0^T \Delta v(t)dt}$$

This report calculates that the rocket would reach an altitude of roughly 74.093 kilometers in 157.69 seconds.

V. Discussion and Future Work

While actual test results found that the burn time was 160 seconds and the height was 70 kilometers, this makes sense as the calculations were done without accounting for drag. When drag is accounted for, the rocket moves slower and so does not get as high, but also burns fuel slower since it cannot accelerate quite as fast. nFuture work could be done to determine the burn time for rockets launched off of other bodies, namely the Moon, the compare the effects of different gravitational strengths.