Lab 2 Report

I. Introduction

This report will consider how a 1kg mass falls. Initially, we will look at the effect of variable gravity and drag on the fall of a 1kg mass down a 4km shaft. Accounting for the Coriolis force, we will determine the feasibility of this approach. We will continue stripping assumptions and account for the nonhomogeneity of the Earth's density, and see how this affects the crossing time of an object dropped through the entire Earth. Finally, these results will be extended to the lunar case for a discussion of how density affects the crossing time.

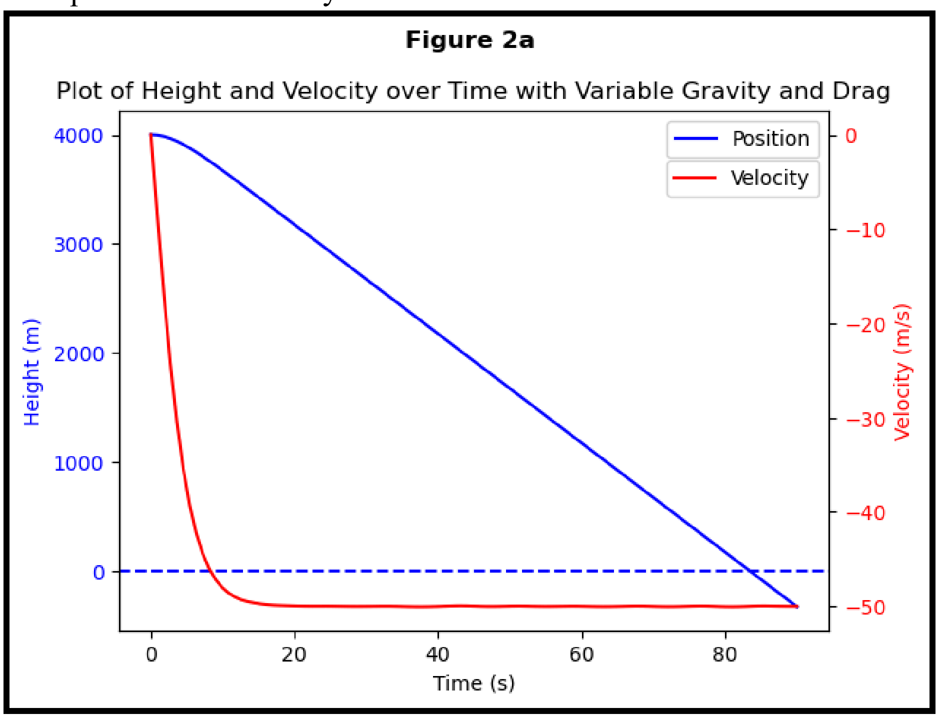
II. Calculation of Fall Time

The calculations for fall time are based on the following system of differential equations where t = time, y = height, v = velocity, g_0= gravitational acceleration on Earth's surface, α = drag coefficient, γ = speed dependence on drag, and R_E is the radius of the Earth:

(1) $\frac{dy}{dt} = v$

(2) $\frac{dv}{dt} = -g_0(\frac{R_E - 4000 + y}{R_E}) + \alpha|v|^{\gamma}$

The g_0 term is multiplied by $(\frac{R_E - 4000 + y}{R_E})$ to find the radius at some position y over R_E. The drag coefficient α is calculated $\alpha = \frac{g_0}{v_t^2}$ where v_t = 50 m/s is the terminal velocity. With an initial height of 4000 meters and a velocity of 0 m/s since it falls from rest, the above system gives the following plot of position and velocity over time:



The object rapidly approaches its terminal velocity of -50m/s and then flattens out due to drag. The variable gravity in the system does slightly increase the time it takes to fall as well, since as y decreases, the numerator, which corresponds to the radius at some position y, of

$\frac{R_E - 4000 + y}{R_E}$ decreases, so g_0 is scaled by smaller and smaller values over time. It takes 83.5 seconds for the object to reach the bottom of the shaft. This makes sense intuitively, since the terminal speed is 50 m/s, and $\frac{4000m}{50\ m/s}$ = 80 seconds, which gives a lower bound for how long it could take the object to fall, since 50 m/s is its maximum speed.

III. Feasibility of Depth Measurement

Unfortunately for this approach of depth measurement, the coriolis force prevents it. Including the Coriolis force in the x (transverse) and y (height) directions means that our system of differential equations must be updated where Ω = the Earth's rotation at the equator:

(1) $\frac{dx}{dt} = v_x$

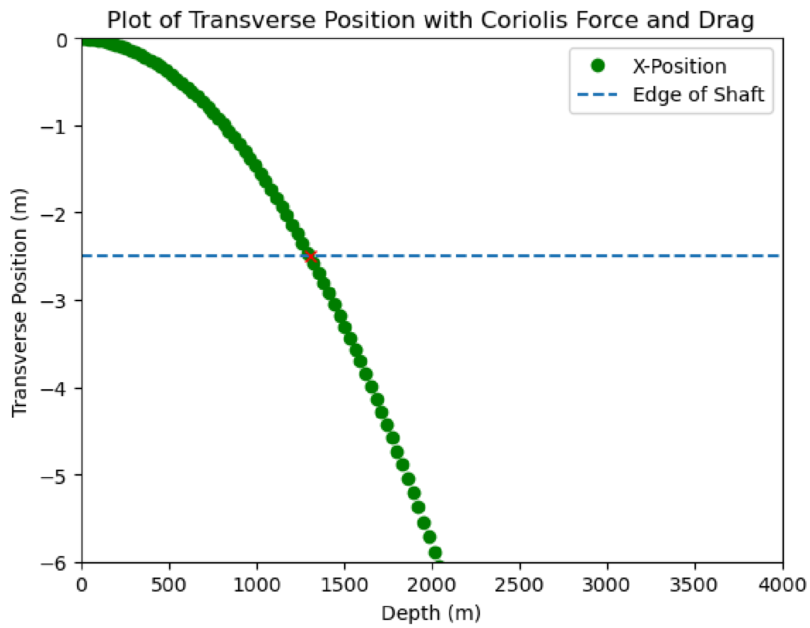
(2) $\frac{dy}{dt} = v_y$

(3) $\frac{dv_x}{dt} = -\alpha|v_x|^{\gamma} + 2\Omega v_y$

(4) $\frac{dv_y}{dt} = -g_0(\frac{R_E - 4000 + y}{R_E}) + \alpha|v_y|^{\gamma} - 2\Omega v_x$

This updated model means that the object now moves in the transverse direction due to the rotation of the Earth. Since the shaft is only 5 meters wide, the object will hit the side of the shaft at a depth of 1300.4 meters after only 29.5 seconds. However, drag is not responsible for this effect. Even without accounting for drag, the object still hits the side of the shaft at a depth of 2353.9 meters after 21.9 seconds. Plotting, we can see how dramatically the Coriolis force affects the transverse position of the object:

Figure 3a



IV. Crossing times

In the previous models, we have assumed that the density of the Earth is constant. However, this is not true, and removing this assumption has dramatic effects on the movement of the object. In actuality, the gravitational force for the Earth must be calculated:

$$g(r) = \frac{4\pi G}{r^2}\int_0^r p_n\left(1 - \frac{r^2}{R_E^{\ 2}}\right)^n r^2\, dr$$

Here, p_n is a normalizing constant for our density function $(1 - \frac{r^2}{R_E^{\ 2}})^n$, which ensures that the gravitational acceleration at the surface of the Earth is always 9.81 m/s^2. In the plots below, we see the position and velocity over time for our object with different density functions. For n = 0, we have a homogeneous Earth, whereas for n = 9, we have a nonhomogeneous Earth with widely varying densities depending on the radius from the center of the earth.

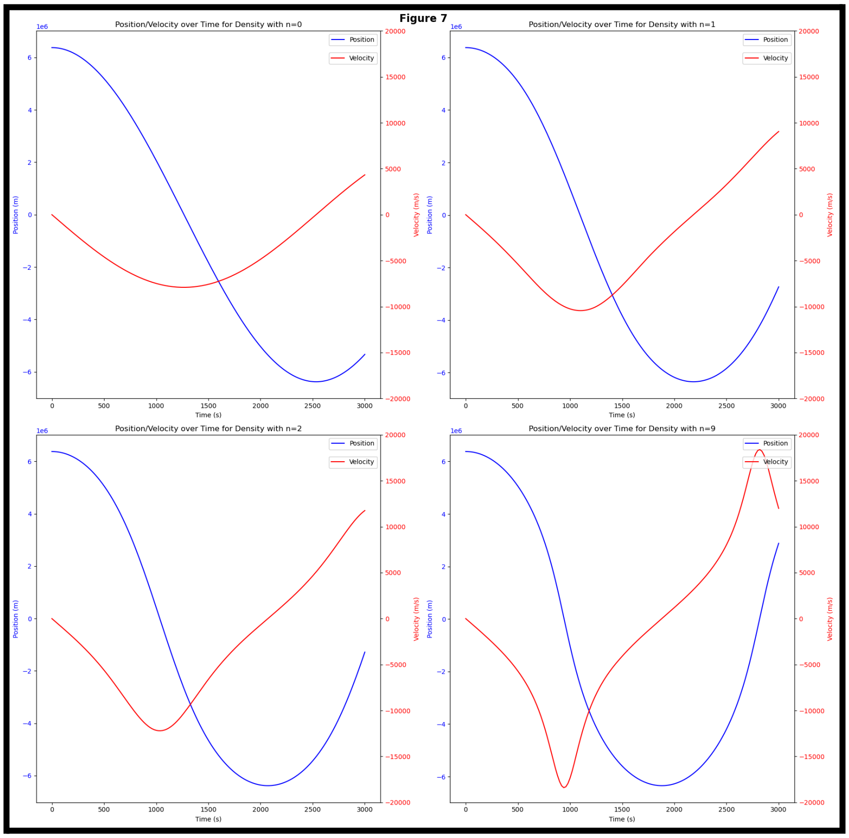


Figure 7

For the n = 0 case, the object crosses the center of the Earth after 1266.5 seconds, but for the n = 9 case, the object crosses the center of the Earth after 943.3 seconds. The higher density in the n = 9 case causes the object to accelerate much faster due to the increased gravitational force. Extending this further, in the lunar case with a homogeneous density, the crossing time of a 1kg object dropped from rest on the moon's surface to the center of the moon is 1626.8 seconds. Varying the density constant p_n, we find that the crossing time in these calculations is inversely proportional to the square of p_n. More details are available in the code for this plot as I am exceeding the page limit for this report.

V. Discussion and Future Work

One of the most important assumptions in this model is that the density is constant over radius. That is, for at radii +r and -r, the density will be the same. However, this is not necessarily the case, and should be accounted for in future calculations. Moreover, the Earth itself is not spherical. Removing this assumption would be difficult though as it greatly increases the difficulty of calculation the mass as a function of radius.