

# Maths

## Objective

The objective of this problem is to test the students' understanding on **Recursion**. Students also need to know the basic algorithm for multiplying and adding 2 matrices.

## Problem Description

Your task is to compute the result of " $\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \mathbf{A}^4 + \dots + \mathbf{A}^N$ ", given a 2x2 matrix  $\mathbf{A}$  and an integer  $N$ ,  $\mathbf{I}$  is an identity matrix,  $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ . If the element  $A[i, j] \geq \mathbf{M}$ , you need to subtract  $\mathbf{M}$  from  $A[i, j]$  until  $0 \leq A[i, j] < \mathbf{M}$ .

## Input

The first two lines of the input contain the information of matrix  $\mathbf{A}$ . The next line contains 2 integers  $N$  ( $1 \leq N \leq 1,000,000,000$ ) and  $\mathbf{M}$  ( $1 \leq \mathbf{M} \leq 1,000,000$ ).

## Output

Output the result of " $\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \mathbf{A}^4 + \dots + \mathbf{A}^N$ ".

## Sample Input

```
1 2
3 4
2 10
```

## Sample Output

```
9 2
8 7
```

## Explanation

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix} = \begin{pmatrix} 9 & 12 \\ 18 & 27 \end{pmatrix}$$

Some elements in the final result are larger than or equal to  $\mathbf{M}$ . Hence we need to subtract  $\mathbf{M}$  from those elements until they are smaller than  $\mathbf{M}$ . The final result is  $\begin{pmatrix} 9 & 2 \\ 8 & 7 \end{pmatrix}$ .

## Note

The main Java class must be called **Maths**, and be in the source file **Maths.java**.