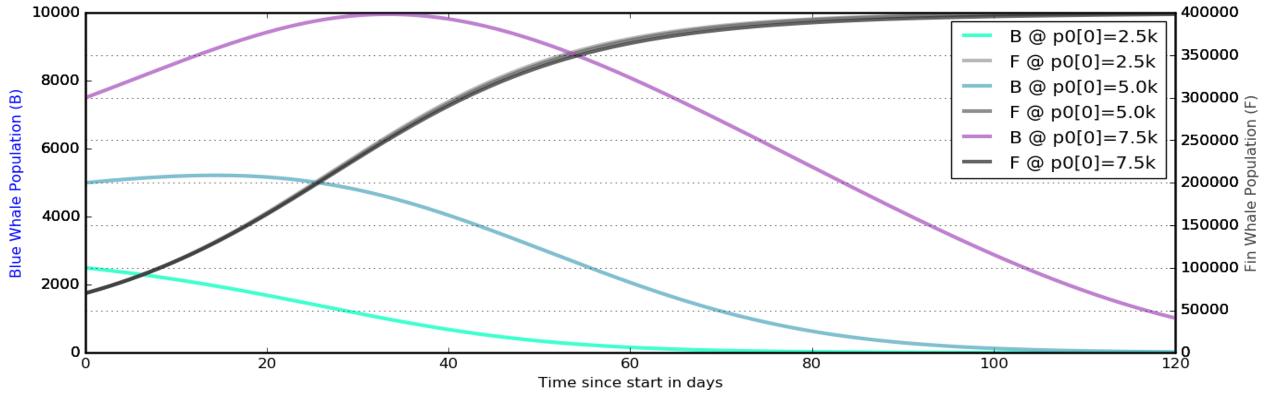
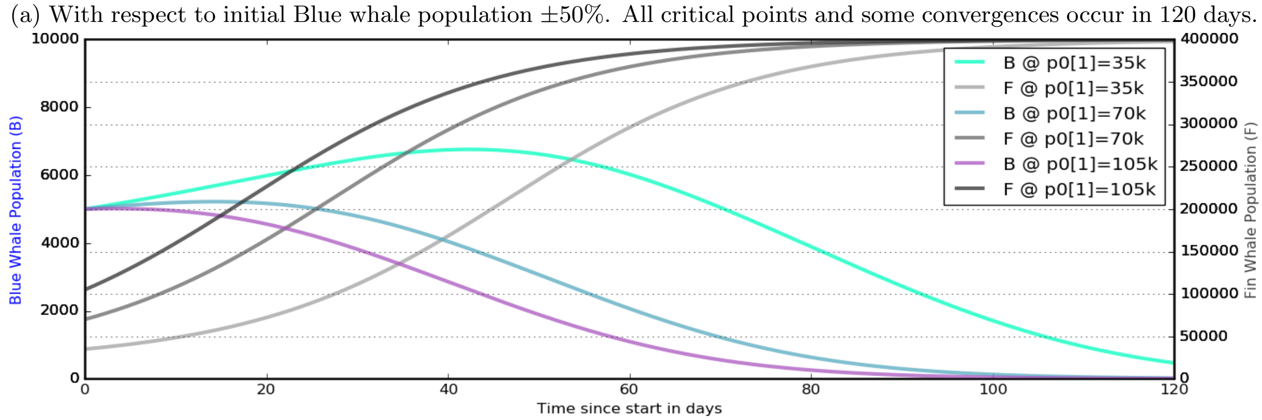
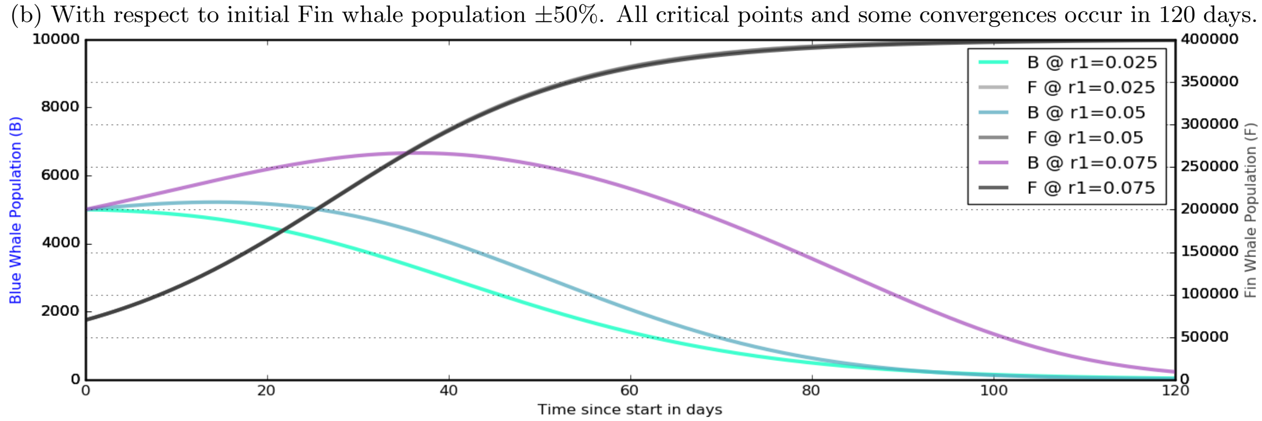


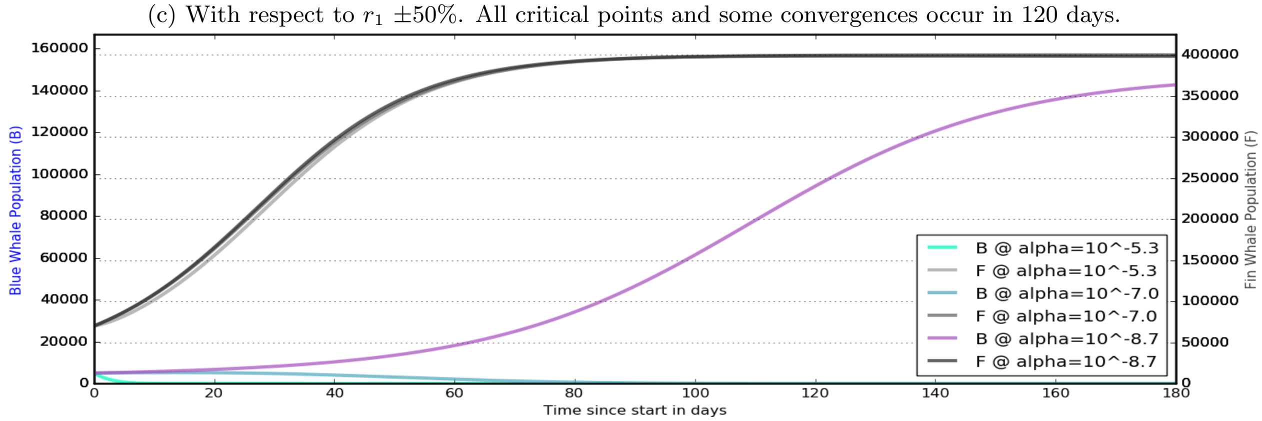

(a) With respect to initial Blue whale population ±50%. All critical points and some convergences occur in 120 days.




(b) With respect to initial Fin whale population ±50%. All critical points and some convergences occur in 120 days.




(c) With respect to $r_1$ ±50%. All critical points and some convergences occur in 120 days.




(d) With respect to $\alpha$ ±25% in $\log_{10}$ scale. All critical points and some convergences occur in 180 days.

Figure 10: Sensitivity analysis with respect to initial whale populations and other parameters on the dynamical system from the second model.

space $\Omega \subset \mathbb{R}^2 = [-1,1] \times [-1,1]$ as

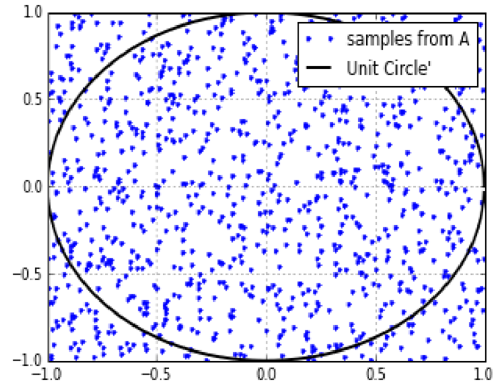
$$A = |\Pi_{i=1}^{n} \arg\max_{x\in\Omega}(x[i]) - \arg\min x \in \Omega(x[i])|$$
$$= (1-(-1)) \times (1-(-1)) = 4$$

Thus, the probabilistic problem we must solve is $4\frac{1}{N}\sum_{i=1}^{N} f(\vec{x}_i)$ for $N$ samples of $\vec{x}$. With a pseudo-random number generator and a given $N$, this is no more than a problem of summing a map of $N$ generated numbers.

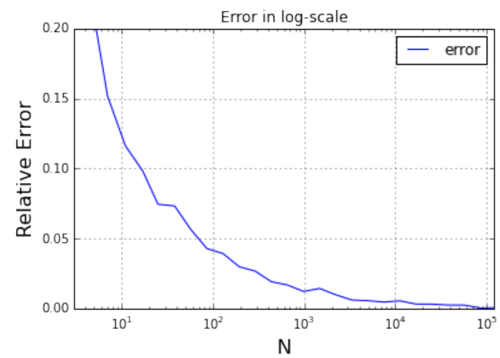
We calculate the error in our approximation using the Central Limit Theorem [4]. Let $I$ denote the true integral $I$ and $S_n = A\sum_{i=1}^{N} f(\vec{x}_i) = \sum_{i=1}^{N} Af(\vec{x}_i)$. Note that $\sigma$, the standard deviation of the integral, is a constant whose value won't vary. With respect to how many standard deviations it is from the true value [3, p.77], $\sigma \times \epsilon, \epsilon \in \mathbb{Q}$ and positive, the error for the approximation $I_n$ can be calculated probabilistically:

$$P(|\frac{S_n - N\mu}{\sqrt{N\sigma^2}}| \leq \epsilon)$$
$$= P(|\frac{(\frac{N}{A} \times I_n) - N(I)}{N}| \leq \frac{\epsilon \times \sigma}{\sqrt{N}})$$
$$= P(|I_n - I| \leq \frac{\epsilon \times \sigma}{\sqrt{N}})$$

We can thus say that, for a certain number of standard deviations from the true integral, the estimate $I_n$ converges within it with a probability of $P(\chi \leq \epsilon)$. Typically, in applications, $\sigma$ is taken to just be 1 [5, p.398], but should still be considered an important factor due to the probablistic nature of the Monte Carlo Method. Similarly, for error-analysis, we say that the Monte Carlo Integration method converges to an error inversely-proportional to the square-root of $N$, such that the error$\approx \frac{1}{\sqrt{N}}$, which lacks the $\sigma$ to signify how the error is assumed to scale with the variability of the problem.




(A) A graphical representation of the integration process for a sample of 100 points. The ratio of points inside the points and points overall is approximately $\pi$




(B) How the error of the approximation decreases as $N$ increases

Figure 1. Monte Carlo Integration for a pseudo-random number generator, integrating the unit circle

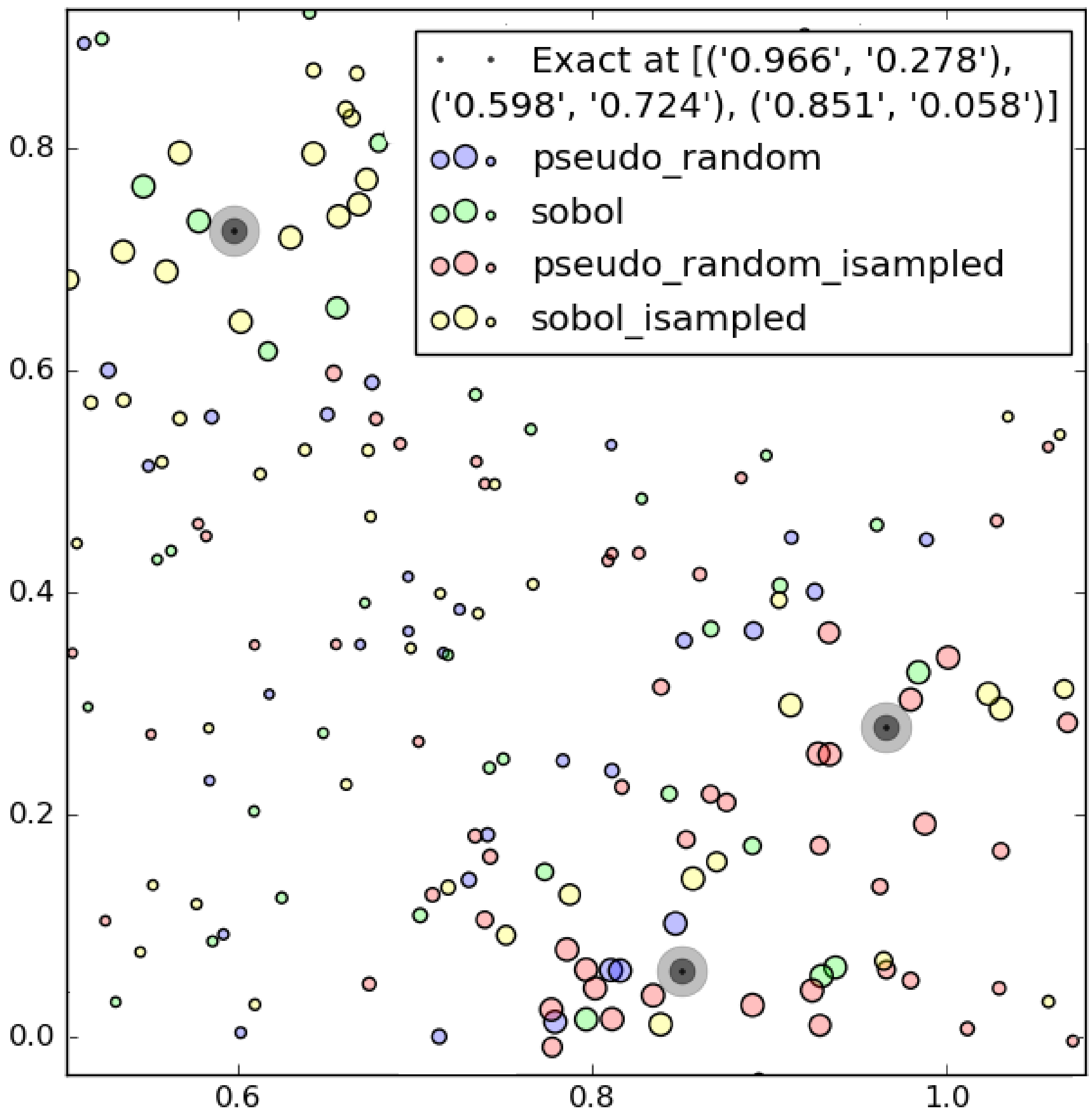



Figure 3: Demonstration of how the four uninformed global optimization algorithms perform for a fixed number of samples. Samples taken from 0 to 1 on both axes for three optima in random points, all equally.