Math 471 – Mathematical Modeling

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Location	Thompson 374
Time	MTWF 2-2:50
Textbook	*Mathematical Modeling*, by Mark Meerschaert
Office Hours	MTW 3:00-4:00, or by appointment, or by dropping by if my office door is open
Website	`http://math.pugetsound.edu/~ctoews/teaching/spring11/modeling/`

Welcome to Mathematical Modeling! This broad subject lies at the intersection of pure mathematics, computational methods, and applied science. Although the phrase "mathematical modeling" means different things to different people, for our purposes it will refer to the process of using mathematical language to describe physical phenomena in a way that enhances understanding or control of these phenomena. Methods in mathematical modeling range from the very simple (à la "consider a spherical cow") to the dazzlingly complex (e.g. numerical methods for modeling fluid flows), but good modeling is sensitive to both the practical demands of a given problem, and the formal constraints of mathematical elegance.

The purpose of this course is threefold. First, it is to supply you with a rich mathematical vocabulary with which to describe physical phenomena. Implicit in this goal is the development of a certain "taste", i.e. the capacity to distinguish between processes that should be explicitly modeled and those that are best left implicit. Second, the course will expose you to a broad range of real-world applied problems, thus helping to develop a sense of how this subject unifies the worlds of theory and practice. Lastly, the course will focus on how to actually solve the problems on a digital computer, thus fusing programming and modeling skills.

This is a math course. As such, you will learn some mathematical techniques. But it is important to remember that the purpose of the mathematical results is to solve real world problems, and that real world processes often proceed without theorems (admittedly a scandalous notion.) While an abstract analysis of a mathematical model can shed considerable light on what the model "says", in most cases such analysis benefits from some sort of software simulation. Moreover, in the real world, both solutions and simulations are only useful to the extent that they can be communicated to an appropriate "end user", who in many cases is neither a mathematician nor a computer scientist. So although this is a math course, its focus will be split even between solving mathematical problems, writing software, and communicating your results in both written and oral form.

Pre-requisites: The prerequisites for this course are solid coursework in calculus (through multivariate calculus) and linear algebra. Courses in ordinary differential equations, probability, and statistics will also help, though this material can probably be picked up on the fly. (Though as the mathematician Paul Halmos says, "*The beginner should not be discouraged if he finds he does not have the prerequisites for the prerequisites*"–be bold, take the plunge.)

Course Catalog Description

A study of the process of mathematical modeling as well as specific deterministic (both discrete and continuous) and stochastic models. Certain mathematical topics such as graph theory are developed as needed.

Specific Learning Goals

After successfully completing this course, you should be able to:

1. List examples of environments in which mathematics can play an important role. These examples should include a wide range of subjects: the humanities, the behavior and social sciences, the physical sciences, and engineering.

2. Use a simple description of a real-life problem as the basis for formulating a corresponding mathematical problem, and choose appropriate techniques for its solution.

3. Incorporate multiple multiples, hierarchical or otherwise, into your analysis of a given problem.

4. Justify the importance of sensitivity analysis, and perform such an analysis either analytically or numerically.

5. Communicate your solutions clearly and succinctly to a variety of audiences, including non-mathematical ones.

Provisional List of Topics

This course can be configured in many ways, depending on the level and interests of the class. Although to a certain extent I intend to custom tailor it to your tastes, the following represents a broad outline of expected course content:

Week	**Topic**
1-2	Continuous single variable optimization
3-4	Agent based models
5-6	Continuous multi-variable optimization
7-8	Discrete dynamical systems
9-10	Continuous dynamical systems
11-12	Discrete stochastic models
13-14	Continuous stochastic models

A more detailed course calendar showing due dates, reading assignments, etc. is maintained on the class webpage.

A Note on Expectations

I try to structure a classroom that revolves around the twin pillars of discovery and discussion. I believe that learning is an active process, and that class works best when it functions as an opportunity for guided inquiry, where the "guides" are both the professor and the peer group. In the wider educational community, my approach is labeled "Inquiry Based Learning", or IBL.

There are a number of ways to structure an IBL class, but salient features of this class will include the following:

- Lectures infused with with hands-on activities
- An emphasis on group work
- Lots of dialogue and discussion
- Student presentations
- A supportive environment in which to take "risks"
- An emphasis on communication, both oral and written
- A need for trust and confidence, both student-student and student-professor

Productive failure is an idea that lies at the root of our approach. When you're trying to learn something, never making a mistake is generally a sign that you're not pushing yourself hard enough. This class should be a safe and supportive space in which to get things wrong. When talking or presenting, you are challenged to work slightly outside of your comfort zone, to volunteer answers when you have a pretty good idea but aren't 100% certain, to risk a conjecture that might turn out to be off the mark. And when you are listening to fellow students talk, you are challenged to pay strict attention, to flag small errors of language or comprehension, and to politely and respectfully help guide one another towards a clearer and truer picture of the matter at hand. Failure is part of the design spec for this class, and it can be hard, but you will not be struggling alone.

Although the spirit of what I'm shooting for with this IBL style class is probably clear, here is a minimalist list of concrete expectations:

- attend class daily
- do all assigned work
- do all assigned reading, in a timely fashion
- participate actively in class discussions and class group work activities
- volunteer to present solutions on the board
- volunteer answers to questions I pose, and to ask your own questions when you are confused, uncertain, or simply thinking outside the box
- be courteous and supportive of your fellow learners
- help create a classroom that is a supportive, energetic, respectful place to learn.

More broadly, my basic hope and expectation is that you will engage the spirit of inquiry-oriented learning with enthusiasm, openness, and joy (it is fun), and help make this class a fun and supportive place in which to learn.

Lastly, a word about goals and outcomes: the goal in IBL is to produce critical thinkers who have a strong, creative command of the subject material. There is ample research evidence to support the IBL model, and I'm happy to share it with you if you'd care to see it. For me, one of the strongest element of IBL is the host of secondary skills you develop almost "for free", including skills in abstract thinking, working from first principles, working with other people, and communicating your ideas. As my colleague Dana Ernst has noted, "All of the secondary skills you will develop in this course are highly valued by society. Whether you become a teacher, a lawyer, an engineer, or an artist, what differentiates you from your competition is your ability to think critically at a high level, collaborate professionally, and communicate effectively."

Assessment

Your grade in this class will be based on the following elements:

Participation	5%
Homework	10%
Reports	30%
Presentations	10%
Exams	25%
Final project	20%

Here is a breakdown of what these things mean, and how they are calculated:

- **Participation** In an inquiry-oriented classroom, the active participation of each student is the thing that makes the class work (or not work.) This is the reason I assign a participation grade: I hope that by doing so, I make clear to what extent your contributions to the classroom help shape the learning environment for yourself and for others.

  Ultimately, the participation grade is subjective, and I will mix my own impression with some self-evaluation on your part when I assign these scores. Your participation grade will be an integer between 0 and 10, roughly in accordance with the following standards:

10	An exceptionally strong contributor to classroom environment; asks and answers questions daily; pursues ideas to their logical conclusion; strong contributor to homework solutions.
9	A good contributor to the classroom discussion; asks and answers questions relatively often; solid but not exceptional contributor to homework solutions.
8	Shows up and pays attention, but doesn't speak up very often, either during homework solution day or in conversation. Asks and aswers questions once in a while.
7	Shows up, but sleeps through class.
≤ 6	almost impossible to achieve these scores (except through particular acts of dereliction, as detailed below)

Repeated absences are the one thing that can really bring your participation grade down. You are allowed 3 absences, no questions asked, but each absence after the 3rd reduces your participation grade by one point. Please try to show up.

- **Homework** Homework will be assigned roughly every two weeks. Each assignment will consist of a mixture of written and computer work. Assignments that are neat, complete, correct will receive full points; others will receive fewer, in accordance with the gravity of their omissions. Each homework will be graded on a 3 point scale, as follows:

3	All problems attempted; correct answers, strong effort to articulate thought process
2	Most problems attempted; correct solutions, but not a great articulation of thought process
1	Half or fewer of the problems attempted; incorrect and ill-articulated answers
0	Nothing submitted

Each score gets mapped to a percentage, as follows:

$$3 \to 100\%$$
$$2 \to 80\%$$
$$1 \to 60\%$$
$$0 \to 0\%$$

- **Reports** A major part of your work in this class will be writing formal reports for each modeling project. The reports are to be researched and written in small groups, typeset in Latex, and submitted via Dropbox.

  Each report grade will have several components:

  - My grade for your final report (70%)
  - My grade for your draft (15%)
  - My grade for your peer reviews (15%)

  Since groups will turn in common drafts and final reports, everyone in the group will get the same grade for these components. The peer reviews will be done individually, and your peer review grade will be yours alone.

- **Presentations** Each time you submit a report, your or someone in your group will also give a presentation on the work you've done. We'll discuss what's involved in giving good presentations. You are expected to present at least twice during the term.

- **Exams** To solidify some of the mathematical content of the course, we'll take two exams. These will be graded on a percentage basis.

- **Final Project** You will be responsible for writing an extended research paper on the subject of your choice. This paper will be scaffolded throughout the semester–you'll submit a proposal, a draft, etc. according to a timeline that I give you. The final written report should display all of the modeling sophistication you've learned over the term. These projects will be written individually, and turned in on the day of the scheduled final. (Thursday, May 12, 4pm.)

Letter grades

Final grades will be weighted as per above, with letter grades roughly as follows:

A	90-100
B	80-89
C	70-79
D	60-69
F	< 60

Getting Help

Mathematical modeling is an advanced class, and there is no official tutoring for this subject. Your first line of attack should be your peers: we?ll work on building peer groups in class, and get in the habit of trying to work with classmates outside of class. I'm around: I have regular office hours, and if you can?t make those, then try just dropping by or setting up an appointment (I am generally in my office if I?m not teaching, eating, or working out.) Make a habit of coming to see me to discuss any questions you might have. I imagine that one of the reasons you chose to go to a liberal arts college was to gain access to your professors, and I am very much on board with that vision. You should leverage me as a resource.

Policies

Attendance

Built into the philosophy of inquiry-based learning is the idea that we help one another to learn. As a consequence, your daily attendance in class is very important, not just for your own benefit, but for that of your peers. I will take role every day and verify attendance. You are allowed to miss up to three days with no penalty. After three missed classes, two things happen: 1) every additional absence will reduce your grade by one level, 2) I reserve the right to drop you from the class.

Late work

In this class, I want you to turn in your work on time. However, I understand that sometimes stuff comes up, and you need to get things in late. Here are my policies on late work:

- **Problem sets:** Late problem sets will lose one point (a 3 becomes a 2, a 2 becomes a 1, and a 1 becomes a 0).
- **Drafts:** Late drafts of articles will lose one point (a 3 becomes a 2, a 2 becomes a 1, and a 1 becomes a 0).
- **Peer reviews:** Late peer reviews will lose one point for each *calendar day* that it is late (a 3 becomes a 2, a 2 becomes a 1, and a 1 becomes a 0).
- **Articles:** Late articles will lose three percentage points for each *class day* it is late.

in the case of extreme emergency, talk to me, and can probably make an exception.

Planned absences

If you need to be absent for some planned family or medical reason, you should contact me in advance. If an emergency arises, contact me as soon as possible after the emergency has passed. Student athletes who need to miss class for games should let me know of this as early as possible.

Classroom policies

You are welcome to bring a cup of coffee or a bottle of water to class, but please eat your meals outside of class. Please turn off your phones and keep your laptops closed, unless we happen to be doing a computer exercise. You can take a bathroom break if necessary, but please make this the exception, not the rule?in general, I don?t want people entering and leaving the room during class.

Academic integrity

It is your responsibility to understand the academic integrity policy of the university. You can find this policy in the Academic Handbook, and it is also available online at:

`http://www.pugetsound.edu/student-life/student-resources/student-handbook/academic-handbook/academic-integrity/`.

Not citing other people?s work, turning in the same work to satisfy two different classes, citing false information, or plagiarism are all violations of the academic integrity policy. Such violations are taken very seriously, and will be reported if discovered.

Disabilities

If you have a physical, psychological, medical or learning disability that may impact your course work, please contact Peggy Perno, Director of Student Accessibility and Accommodation, 105 Howarth Hall, 253-879-3395. She will determine with you what accommodations are necessary and appropriate. All information and documentation is confidential.

Classroom Emergency Response Guidance

Please review university emergency preparedness and response procedures posted at www.pugetsound.edu/emergency/. There is a link on the university home page. Familiarize yourself with hall exit doors and the designated gathering area for your class and laboratory buildings. If building evacuation becomes necessary (e.g. earthquake), meet your instructor at the designated gathering area so she/he can account for your presence. Then wait for further instructions. Do not return to the building or classroom until advised by a university emergency response representative.

If confronted by an act of violence, be prepared to make quick decisions to protect your safety. Flee the area by running away from the source of danger if you can safely do so. If this is not possible, shelter in place by securing classroom or lab doors and windows, closing blinds, and turning off room lights. Lie on the floor out of sight and away from windows and doors. Place cell phones or pagers on vibrate so that you can receive messages quietly. Wait for further instructions.

"The search for truth should be the object of our activity; it's the only goal worthy of it." –**Henri Poincaré**