Z. Phys. B – Condensed Matter 68, 161–167 (1987)

Condensed Zeitschrift für Physik B Matter

Transport as a Consequence of State-Dependent Diffusion

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Received April 27, 1987

Dedicated to Professor Harry Thomas on the occasion of his 60th birthday

Overdamped particles subject to a drift in a force field with sinusoidal space dependence and also a sinusoidally modulated space-dependent diffusion, with the same period as the drift, experience a net driving force. The resulting current depends on the amplitude of the modulation of the diffusion and is a periodic function of the phase difference between the sinusoidal drift and the sinusoidal modulation of the diffusion. For small modulation amplitudes a particle subject to state-dependent noise behaves the same way as a particle subject to thermal noise but with a drift which, in addition to the sinusoidal term, contains a net force term.

I. Introduction

Diffusive motion with a diffusion coefficient which depends on the state of the system plays an important role in a number of physical systems. Some examples are: non-linear self-excited oscillators in the presence of noise [1–4], diodes [5–7], current instabilities in bulk semiconductors [8] and in ballast resistors [9–10]. It was appreciated a long time ago that the Boltzmann factor $\exp(-V/kT)$ which governs systems subject to thermal noise has to be generalized if the system is subject to state-dependent diffusion. In a system with mobility μ subject to drift $v(q)=-\mu \mathrm{d}V/\mathrm{d}q$ and subject to diffusion $D(q)$ the Boltzmann factor has to be replaced by $\exp(-\psi(q))$, where the generalized potential [1–7] is given by

$$\psi(q)=-\int^{q}\mathrm{d}p\,\frac{v(p)}{D(p)}. \tag{1.1}$$

In the case of uniform thermal noise the diffusion constants is determined by the Einstein relation $D=\mu kT$ and (1.1) with $v(q)=-\mu \mathrm{d}V/\mathrm{d}q$ yields $\psi(q)=V(q)/kT$. The innocuous result, (1.1), has a number of interesting and rather deep consequences for the stability and dynamics of non-equilibrium systems. These consequences have been appreciated only over the course of time [11]. The effective potential ψ is not local. Even if the diffusion is thermal near points of local stability of the potential $V(q)$ and deviates from $D=\mu kT$ only in regions which are rarely visited by the particle, the relative stability of the system is nevertheless influenced by the state-dependent diffusion [10]. Indeed a state which is favored in the presence of thermal noise can become a less favored state in the presence of state-dependent diffusion. Therefore, stability criteria which examine only the immediate vicinity of a locally stable state are inadequate to assess the relative stability of states in a non-equilibrium system. Still later, it was appreciated that state-dependent diffusion can generate peaks in the probability distribution $\rho(q)$ at locations where $V(q)$ does not exhibit minima [12, 13]. Such features have received wide attention and have been discussed under the label of "multiplicative noise". In this paper we deal with still another consequence of (1.1). We show that state-dependent diffusion can induce transport in a system which is at equilibrium in the presence of thermal noise only. Van Kampen, in very recent work, has reached similar conclusions [14]. Here we consider a class of systems for which the drift term can be derived from a periodic potential $V(q)=V(q+2\pi)$. In the presence of thermal noise the particle is distributed according to $\exp(-V(q)/kT)$ and the average velocity $\langle \mathrm{d}q/\mathrm{d}t\rangle=0$. However, if the particle is subject to state-dependent diffusion $D(q)$ with the same period as the potential V, a net velocity $\langle \mathrm{d}q/\mathrm{d}t\rangle$ results which depends on the amplitude of the sinusoidal modulation of $D(q)$, and depends on the phase-difference between this modulation and that of $V(q)$. We show that for small modulation amplitudes the motion of a particle subject to state-dependent noise is equivalent to that of a particle subject to thermal noise in a potential $U(q)=V(q)-Fq$, which consists of a driving force potential $-Fq$ superimposed on the periodic potential $V(q)$.

II. Noise-Induced Current

In the presence of state-dependent diffusion the drift and diffusion coefficient are not unambiguous quantities. The total probability current, however, is well defined and can be expressed in either of two forms,

$$j=v(q)\,\rho(q)-D(q)\,\mathrm{d}\rho(q)/\mathrm{d}q, \tag{2.1}$$

or

$$j=w(q)\,\rho(q)-\mathrm{d}(D(q)\,\rho(q))/\mathrm{d}q. \tag{2.2}$$

Since the probability current is the same for both (2.1) and (2.2) we must have $v(q)=w(q)-(\mathrm{d}D(q)/\mathrm{d}q)\,\rho(q)$. A physical interpretation of these tow equations was proposed in [15]. Equation (2.1) is more convenient and leads to simpler analytical expressions. Reference 15 attributes recognition of this point to Thomas. Thus we assume that the drift and the diffusion have been specified to yield the probability current in the form of (2.1). To be definite we now go on to consider the following class of systems: the drift term is the derivative of a periodic potential $V(q)=V(q+2\pi)$ with period 2π and is given by $v(q)=-\mu\,\mathrm{d}V(q)/\mathrm{d}q$. The mobility μ is independent of the coordinate q. The diffusion is also a periodic function $D(q)=D(q+2\pi)$ with period 2π. (Our results can easily be generalized to the case where $V(q)$ and $D(q)$ have different but commensurable periods.) However to be even more specific we will, later, specialize to the particular cases where the potential and diffusion are simply given by,

$$V(q)=V_0(1-\cos(q)) \tag{2.3}$$

and

$$D^{-1}(q)=D_0^{-1}(1-\alpha\cos(q-\phi)). \tag{2.4}$$

Here α is the amplitude of the modulation and since $D(q)>0$ the amplitude is restricted to the range $0\leq\alpha<1$. The phase ϕ plays an important role. If the phase is a multiple of π the state-dependent noise only causes a redistribution of particles which is periodic with period 2π. If the phase is not a multiple of π then the noise-intensity is asymmetric with regard to the local maxima of the potential. Each local potential hill has a slope with high intensity ("hot slope") and a slope with low intensity ("cold slope"). Transport arises because particles starting in a potential valley can climb the hot slope more easily than they can climb the cold slope. In Sect. 3 we show that the transition rate from valley n to valley $n+1$ differs from the transition rate from valley $n+1$ to valley n. In a double well potential the differing transition rates lead to differing populations in the two wells such that the net flux across the intervening potential hill vanishes. The ratio of the population densities in the two wells is determined by the ratio of the transition rates for the two directions. For the periodic potentials considered here, a steady state without transport is also possible, if the populations in adjacent wells are adjusted to the ratio of the transition rates. This, however, gives a nonuniform distribution with a population per period which increases (or decreases) in a geometrical fashion as we move away from an initial valley. Such a nonuniform polarized distribution is physical only if we terminate the specimen with walls at each end. Alternatively, we can consider a distribution function which is periodic and describes transport and gives rise to a noise-induced velocity $\langle \mathrm{d}q/\mathrm{d}t\rangle \neq 0$.

It is interesting to compare this noise-induced velocity with the velocity resulting from an external driving force potential $-Fq$ in a system subject to uniform thermal noise. Thus for comparison we also consider particles subject to thermal noise $D=D_0=\mu kT$ and subject to a potential

$$U(q)=V(q)-Fq. \tag{2.5}$$

Below we show that for a limited range of modulation amplitudes α there exists an external driving force $F(\alpha)$ such that the two problems can be mapped onto one another.

Let us now give a quantitative analysis of the qualitative discussion given above. In the absence of current, flow $j=0$, (2.1) exhibits a steady state solution

$$\rho_0=C\exp(-\psi(q)) \tag{2.6}$$

with $\psi(q)$ given by (1.1). The key point now is the following: The potential, (1.1), evaluated with a drift and diffusion as specified above is in general not periodic but becomes a tilted sinusoidal function as shown in Fig. 1. Let us introduce Δ to measure the departure of ψ from periodicity

$$2\pi\Delta=\psi(q)-\psi(q+2\pi). \tag{2.7}$$

Δ measures the slope of the potential ψ (see Fig. 1). For our particular example, as specified by (2.3) and

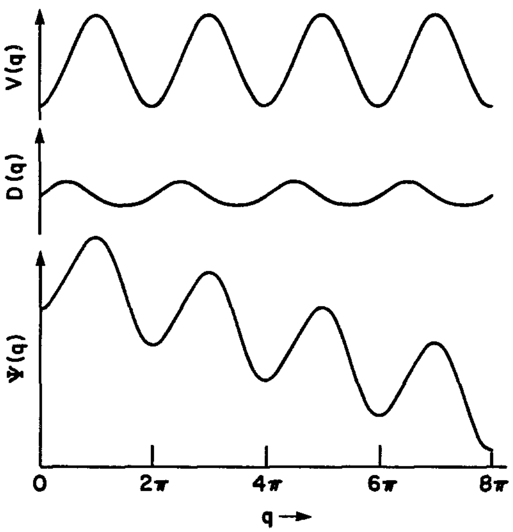

Fig. 1. Potential V, diffusion coefficient D, and generalized potential ψ as a function of q for the example specified by (2.3) and (2.4). The phase difference between the potential V and the diffusion coefficient D is $\pi/2$. Arbitrary units

(2.4), the effective potential, (1.1) is,

$$\psi(q)=\frac{\mu V_0}{D_0}\left[1-\cos(q)-\frac{\alpha}{4}(\cos(\phi)-\cos(2q-\phi)) -\frac{\alpha}{2}\sin(\phi)q\right]. \tag{2.8}$$

The last term is linear in q and gives rise to a slope

$$\Delta=\frac{\mu V_0}{D_0}\frac{\alpha}{2}\sin(\phi). \tag{2.9}$$

Therefore, the steady state solution (2.6) is, in general, not bounded. To find a solution of (2.1) which is periodic we must allow for a non-vanishing current j.

To find the transport solution of (2.1) we proceed as in the case of a particle subject to a tilted sinusoidal potential [3, 4] expect that now the role of the potential $U(q)$ is taken by $\psi(q)$. We search for a solution of the form

$$\rho(q)=h(q)\,\rho_0(q) \tag{2.10}$$

with $\rho_0(q)$ given by (2.6). Periodicity of the solution $\rho(q)$ requires

$$h(q+2\pi)=\exp(-2\pi\Delta)\,h(q). \tag{2.11}$$

Inserting (2.10) into (2.1) yields

$$j=-D(q)\,\rho_0(q)\,\mathrm{d}h(q)/\mathrm{d}q. \tag{2.12}$$

Integration of (2.12) gives,

$$h(q)=-j\int_{\infty}^{q}\mathrm{d}x D^{-1}(x)\,\rho_0^{-1}(x) =(j/C)\int_{q}^{\infty}\mathrm{d}x D^{-1}(x)\exp(\psi(x)). \tag{2.13}$$

Here we have taken the integration limit to infinity. We show that with this choice of the integration constant (2.13) satisfies the periodicity condition (2.11). Consider (2.13) for $q'=q+2\pi$,

$$h(q+2\pi)=(j/C)\int_{q+2\pi}^{\infty}\mathrm{d}x D^{-1}(x)\exp(\psi(x)). \tag{2.14}$$

With the new variable $y=x-2\pi$ we find from (2.14)

$$h(q+2\pi)=(j/C)\int_{q}^{\infty}\mathrm{d}y D^{-1}(y)\exp(\psi(y+2\pi)) \tag{2.15}$$

and using (2.7) yields (2.11). Furthermore, $h(q)$, given by (2.13), is the sum of two integrals,

$$h(q)=(j/C)\int_{q}^{q+2\pi}\mathrm{d}x D^{-1}(x)\exp(\psi(x)) +(j/C)\int_{q+2\pi}^{\infty}\mathrm{d}x D^{-1}(x)\exp(\psi(x)). \tag{2.16}$$

The second term is just the right hand side of (2.15) and hence,

$$h(q)=(j/C)\int_{q}^{q+2\pi}\mathrm{d}x D^{-1}(x)\exp(\psi(x))+h(q+2\pi). \tag{2.17}$$

Using (2.11) to eliminate $h(q+2\pi)$ and solving for $h(q)$ yields

$$h(q)=(j/C)\frac{1}{1-\exp(-2\pi\Delta)}\int_{q}^{q+2\pi}\mathrm{d}x D^{-1}(x)\exp(\psi(x)). \tag{2.18}$$

The distribution function has to be normalized, $\int_{q}^{q+2\pi}\mathrm{d}q\,\rho(q)=1$. Inserting $\rho(q)=h(q)\,\rho_0(q)$ into the normalization integral and solving for j yields a probability current

$$j=\frac{1-\exp(-2\pi\Delta)}{\int_{0}^{2\pi}\mathrm{d}y\exp(-\psi(y))\int_{y}^{y+2\pi}\mathrm{d}x D^{-1}(x)\exp(\psi(x))}. \tag{2.19}$$

Equation (2.19) is the key result of this section and is now discussed in more detail. For small modulation

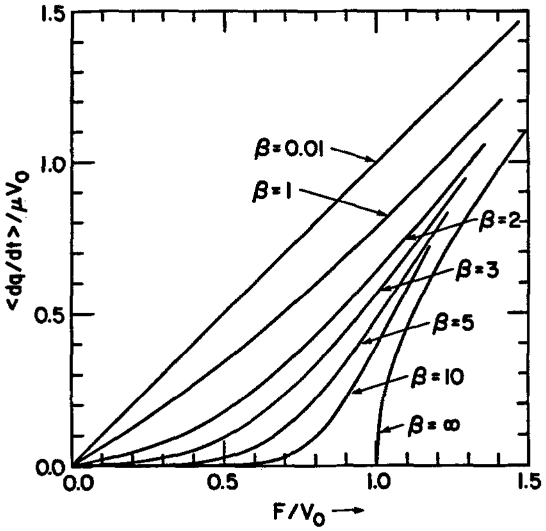

Fig. 2. Average velocity of a particle subject to *uniform* thermal noise and subject to the potential $U(q) = V(q) - Fq$ with V given by (2.3) as a function of the applied force F. The parameter $\beta = V_0/kT$ is a measure of the noise intensity. After [3 and 4]

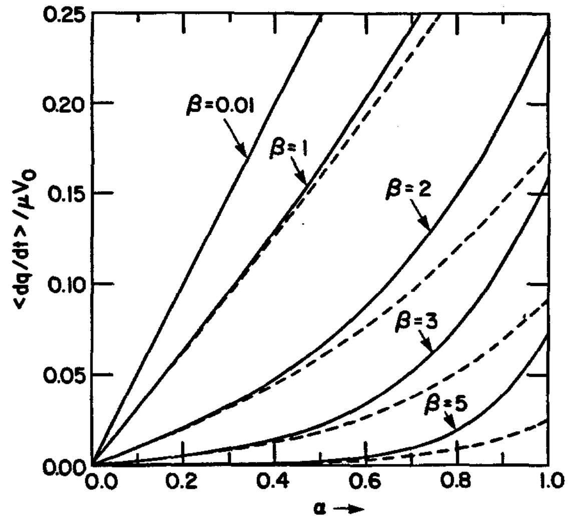

Fig. 3. Average noise-induced velocity (full lines) of a particle subject to state-dependent noise $D(q)$ given by (2.4) and subject to the potential $V(q)$ given by (2.3) as a function of the modulation amplitude α of the diffusion coefficient. The noise-induced velocity (full lines) is compared with the velocity (broken lines) acquired in presence of thermal uniform thermal noise and an external driving force potential (see Fig. 2) with F given by (2.23)

amplitude (2.19) becomes

$$j = 2\pi D_0 \Delta \left(\int_0^{2\pi} \mathrm{d}y \exp(-\mu V(y)/D_0) \cdot \int_y^{y+2\pi} \mathrm{d}x D^{-1}(x) \exp(\mu V(x)/D_0) \right)^{-1}. \tag{2.20}$$

Let us compare (2.19) and (2.20) with the current exhibited by a particle subject to uniform thermal noise in the tilted sinusoidal potential U, (2.5). References 3 and 4 find

$$j = \mu k T \frac{1 - \exp(-2\pi F/kT)}{\int_0^{2\pi} \mathrm{d}y \exp(-U(y)/kT) \int_y^{y+2\pi} \mathrm{d}x \exp(U(x)/kT)}. \tag{2.21}$$

For a very small driving force F, (2.21) becomes

$$j = 2\pi\mu F \left(\int_0^{2\pi} \mathrm{d}y \exp(-V(y)/kT) \cdot \int_y^{y+2\pi} \mathrm{d}x \exp(V(x)/kT) \right)^{-1}. \tag{2.22}$$

Comparing (2.20) and (2.22) we see that they can be mapped onto one another if the driving force F is replaced by an effective noise-induced driving force $F_{\text{eff}} = kT\Delta$, and if the temperature is determined by $kT = D_0/\mu$. For weak modulation of the diffusion constant we thus obtain the same current as for a particle subject to thermal noise and subject to an external driving force. For our particular example specified by (2.3) and (2.4) the effective driving force is

$$F_{\text{eff}} = \frac{\alpha}{2} V_0 \sin(\phi). \tag{2.23}$$

For comparison with the noise-induced velocities we first show in Fig. 2 the velocity-field characteristic, as determined in [3 and 4], for a particle subject to uniform thermal noise and subject to the potential U, (2.5), with $V(q)$ given by (2.3). Figure 2 has been generated by integrating (2.21) and using $\langle \mathrm{d}q/\mathrm{d}t \rangle = 2\pi j$. The strength of the noise is determined by $\beta = V_0/kT$. Figure 3 shows the noise induced velocity calculated for a particle subject to the same potential $V(q)$, (2.3), and state-dependent noise $D(q)$ given by (2.4) as a function of α for $\phi = \pi/2$. Figure 3 has been obtained by using (2.19). The parameter $\beta = \mu V_0/D_0$ characterizes the intensity of the noise. The broken lines in Fig. 3 give the velocity of particles in the potential $U(q)$ subject to thermal noise with $\beta = \mu V_0/D_0 = V_0/kT$ but with F replaced by F_{eff} given by (2.23). Thus the broken lines in Fig. 3 give the same velocities as shown in Fig. 2 but as a function of α. The local minima in the tilted sinusoidal potential U disappear as F increases past V_0. As shown in Fig. 2, for low noise (large β) a sharp increase in velocity occurs as the local minima disappear. In the case of state dependent diffusion, for the example discussed here, the effective driving force always remains smaller than $V_0/2$. Thus in our example the effective driving force is not strong enough to cause an effective potential ψ without local minima. We also point out that the broken curves (i.e. the particle subject to thermal noise and subject to the potential U) give a good approximation to the noise-induced velocities, well beyond the linear range. Note that for our example the difference $\psi(q)-(U(q)/kT)$ between the two potentials, calculated with the help of (2.5), (2.8) and (2.23), is proportional to $(\alpha/4)$ $(\cos\phi-\cos(2q-\phi))$. The q independent term is irrelevant. For small α the second harmonic gives rise to a redistribution of carriers within the wells but has little effect on the net current.

III. Transition Rates

Additional insight why state-dependent diffusion can cause transport can be gained by considering the transition rates between states of local stability of the potential $\psi(q)$. These rates are well defined if the noise-intensity is sufficiently small and the particles are with high probability in the wells of the potential ψ. In this limit it is not necessary to invoke the exact expression, (2.19), the average velocity of a particle can be calculated with the help of such transition rates.

The calculation of transition rates in the presence of state-dependent noise differs from the calculation of escape rates for particles subject to thermally uniform noise [16] and subject to a potential $U(q)$ only in that we have to substitute the generalized potential $\psi(q)$ for the energy $U(q)$. A simple way to determine transition rates is to consider a steady state problem [16, 17]. We feed a current j into the bottom of the well at $q_A\equiv 2\pi n$ and assume a current sink at a coordinate q_1 past the intervening potential maximum at $q_B\equiv\pi(2n+1)$. Thus q_1 is larger than q_B but still far away form the minimum at $q_A+2\pi$. Note that for the class of systems considered here the location of the maxima and minima of V and ψ are the same.

Again we start with the distribution for $j=0$ given by (2.6) and seek a solution for $j\neq 0$ of the form given by (2.10). Integration of (2.12) from q_A to q_1 gives

$$j=\frac{h(q_A)-h(q_1)}{\int\limits_{q_A}^{q_1} \mathrm{d}q D^{-1}(q)\,\rho_0^{-1}(q)}. \tag{3.1}$$

The correction factor $h(q)$ which multiplies the steady state solution accounts for transport. To maintain a current j the correction factor varies strongly in regions of q-space where the steady state distribution function is small and varies little in regions where the steady state solution provides an appreciable concentration of particles. Deep in the well the actual distribution function $\rho(q)$ is close to the steady state solution, $h(q)$ approaches 1. Thus at the bottom of the well we have $h(q_A)=1$. Further, since at q_1 we have a current sink the distribution function must vanish, $h(q_1)=0$. Taking this into account we find from (3.1)

$$j=\frac{1}{\int\limits_{q_A}^{q_1} \mathrm{d}q D^{-1}(q)\,\rho_0^{-1}(q)}. \tag{3.2}$$

The integral in (3.2) is dominated by the maximum of $\rho_0^{-1}(q)\propto\exp(\psi(q))$ at $q=q_B$. We can, therefore, approximate the potential by

$$\psi(q)=\psi(q_B)-(1/2)\,\lambda_B(q-q_B)^2 \tag{3.3}$$

and the diffusion coefficient by

$$D^{-1}(q)=D^{-1}(q_B)+\mathrm{d}'(q-q_B)+(1/2)\,\mathrm{d}''(q-q_B)^2. \tag{3.4}$$

With these approximations (3.2) gives a current

$$j=C\left(\frac{\lambda_B}{2\pi}\right)^{1/2}\left(\frac{1}{D(q_B)}+\frac{\mathrm{d}''}{2\lambda_B}\right)^{-1}\exp(-\psi(q_B)). \tag{3.5}$$

In (3.5) C is the normalization constant of the steady-state distribution function (2.6). Using

$$\psi(q)=\psi(q_A)+(1/2)\,\lambda_A(q-q_A)^2 \tag{3.6}$$

to evaluate $\int \mathrm{d}q\,\rho_0(q)=N$ yields $C=(\lambda_A/2\pi)^{1/2}N$ $\exp(\psi(q_A))$. With this and (3.5) we find a transition rate from valley n to valley $n+1$,

$$r_{n+1,n}=j/N=\left(\frac{\lambda_A\,\lambda_B}{2\pi}\right)^{1/2}\left(\frac{1}{D(q_B)}+\frac{\mathrm{d}''}{2\lambda_B}\right)^{-1} \cdot\exp(-[\psi(q_B)-\psi(q_A)]). \tag{3.7}$$

Repeating these considerations for the transition rate from valley $n+1$ to valley n yields

$$r_{n,n+1}=\left(\frac{\lambda_A\,\lambda_B}{2\pi}\right)^{1/2}\left(\frac{1}{D(q_B)}+\frac{\mathrm{d}''}{2\lambda_B}\right)^{-1}\exp(-[\psi(q_B) -\psi(q_A+2\pi)]). \tag{3.8}$$

Note that the correction proportional to d'' is small since $\lambda_B\propto\mu V_0/D_0$ is large in the limit of small noise intensity. These rates for transitions in the positive and negative q-direction differ if Δ defined in (2.7) is different from zero. The ratio of these two rates is given by

$$r_{n,n+1}/r_{n+1,n}=\exp(\psi(q_A+2\pi)-\psi(q_A))=\exp(-2\pi\Delta). \tag{3.9}$$

The differing rates give rise to an average particle velocity

$$\langle \mathrm{d}t/\mathrm{d}t\rangle=2\pi(r_{n+1,n}-r_{n,n+1})=2\pi r_{n+1,n}(1-\mathrm{e}^{-2\pi\Delta}). \tag{3.10}$$

For the particular example specified by (2.3) and (2.4) we find

$$r_{n+1,n}=\left(\frac{\mu V_0}{2\pi}\right)\left(\frac{1-\alpha\cos(\phi)}{1+\alpha\cos(\phi)}\right)^{1/2} \cdot\exp\left(-\left[\frac{2\mu V_0}{D_0}-\pi\Delta\right]\right) \tag{3.11}$$

for the transition rate to the right and

$$r_{n,n+1}=\left(\frac{\mu V_0}{2\pi}\right)\left(\frac{1-\alpha\cos(\phi)}{1+\alpha\cos(\phi)}\right)^{1/2} \cdot\exp\left(-\left[\frac{2\mu V_0}{D_0}+\pi\Delta\right]\right) \tag{3.12}$$

for the transition rate to the left. Here Δ is given by (2.9). Corrections of the prefactor proportional to d'' are neglected in (3.11) and (3.12). For $\alpha=0$, i.e. thermally uniform noise, $r_{n+1,n}=r_{n,n+1}$. For $\alpha \neq 0$ the two rates are the same if ϕ is multiple of π. In this case the diffusion coefficient is symmetric with respect to the local maxima of the potential V. For $\phi=0$ the rate decreases with increasing α since diffusion is reduced near the potential peaks. For $\phi=\pi$ the escape rate increases with increasing α since the noise intensity near the barrier peaks is enhanced. For $0<\phi<\pi$ the diffusion coefficient is larger to the left of the local maxima then to the right of the local maxima. The escape out of the well along the "hot slope" is favored compared to the escape out of the well along the "cold slope". Therefore, for ϕ in this range the transition rates over a local maximum depend on the direction of the transition. Transitions in the direction of positive q are more likely. Finally for $\pi<\phi<2\pi$ the escape in the direction of negative q is favored. In the limit of weak noise the average particle velocity is then given by

$$\langle \mathrm{d}q/\mathrm{d}t\rangle=2\mu V_0\left(\frac{1-\alpha\cos(\phi)}{1+\alpha\cos(\phi)}\right)^{1/2} \cdot\sinh(\pi\Delta)\exp\left(-2\frac{\mu V_0}{D_0}\right). \tag{3.13}$$

Again we compare this result with the average velocity of a particle subject to uniform thermal noise and subject to a potential U (see Fig. 2). For kT much smaller than the potential barrier the velocity is given by

$$\langle \mathrm{d}q/\mathrm{d}t\rangle=2\mu V_0(1-(F/V_0)^2)\sinh(\pi F/kT)$$
$$\exp\left(-\left(\frac{2V_0}{kT}\right)\left[\sqrt{1-\left(\frac{F}{V_0}\right)^2} +\frac{F}{V_0}\arccos\left(\sqrt{1-\left(\frac{F}{V_0}\right)^2}\right)\right]\right). \tag{3.14}$$

For small α (3.13) can be mapped onto (3.14) if F is replaced by F_{eff} (2.23) and $kT=D_0/\mu$. As α, respectively F, increases the two results differ, chiefly because the exponent in (3.14) deviates from $2V_0/kT$ by a term which is proportional to $(F/V_0)^2$.

To summarize: if the periodic potential V and the diffusion coefficient D are not in phase state-dependent noise breaks the symmetry between the rate for transition to the right and transitions to left. Naturally if these rates differ but the populations in adjacent valleys remain the same transport must arise.

IV. Discussion

In this paper we have pointed out that state-dependent noise can induce carrier flow. The effect we have discussed is an interplay between a periodic drift and a periodic noise-intensity. Current flow arises when these two modulations are not in phase. From a broad point of view this effect is not new: transport theory of electrical conduction teaches that thermal gradients can induce current flow. Similarly the Soret effect is an example, where thermal gradients lead to a concentration flux. The effect we have discussed is perhaps more elementary than all of these. Experimental tests of this effect should also be possible. Consider atoms diffusing on a surface consisting of alternating strips of two materials (the surface of a superlattice). The surface can be heated with a laser-induced grating [16]. If the period of the laser-grating is commensurable with the superlattice, the adatoms acquire an average velocity perpendicular to the strips. Much simpler experiments are also possible. Consider a gas, dilute enough to avoid equilibration via molecule-molecule collisions, enclosed in a circular tube and fill a segment of the tube with porous material. If the segment with the porous material is heated at one end and cooled at the other the gas starts to circulate.

We have focused on the case of a *single* overdamped particle subject to state-dependent noise. A number of interesting questions arise if we consider state-dependent noise in systems with many degrees of freedom, such as a sine-Gordon chain or a chain of coupled particles in a double well potential [17]. In the presence of thermally uniform noise the propagation of transition regions (domain walls, kinks, solitons) is a deterministic problem [17, 19]. Thermal noise leads to small corrections of the deterministically determined mobility [20]. Landauer has argued that state-dependent noise only affects the diffusive behavior of kinks but does not give rise to a noise-induced kink velocity [21]. Calculations by Engel indicate that this is indeed correct [22]. Engel finds a noise-induced propagation velocity for domain walls in chains of finite length. With increasing chain length the noise-induced velocity of the domain walls decreases and vanishes in the limit of an infinitely long chain. Even if state-dependent noise does not give rise to kink propagation, it certainly affects the rate of nucleation of kink-antikink pairs. For the sine-Gordon chain with sinusoidal state-dependent noise we can thus expect that a particle of the chain also acquires an average velocity. However, the effect of state-dependent noise is much smaller in such a many particle system and less dramatic then in the single particle problems considered in this paper.

This paper is in honor of Prof. Dr. Harry Thomas on the occasion of his 60th birthday. I am greatly indebted to Thomas who, as my PhD advisor, strongly influenced and shaped my development as a physicist. I have learned to admire him both for the modesty he shows toward his own achievements and for his deep devotion to the physical sciences. Despite the distance which has separated us, over a number of years, we have continued to collaborate.

References

1. Kuznetsov, P.I., Stratonovich, R.L., Tikhonov, V.I.: Dokl. Adad. Nauk. SSSR **97**, 639 (1954)
2. Kuznetsov, P.I., Stratonovich, R.L., Tikhonov, V.I.: Sov. Phys. JETP **1**, 510 (1955)
3. Tikhonov, V.I.: Automatika i Telemekanika **20**, 298 (1959)
4. Stratonovich, R.L.: Topics in the theory of random noise. New York, London, Paris: Gordon and Breach 1967
5. Landauer, R.: J. Appl. Phys. **33**, 2209 (1962)
6. Haken, H.: Rev. Mod. Phys. **47**, 67 (1977)
7. Hänggi, P., Thomas, H.: Phys. Rep. **88**, 207 (1982)
8. Thomas, H.: In: Synergetics. Haken, H. (ed.), p. 87. Stuttgart: Teubner 1973; Gunn, J.B.: IBM J. Res. Dev. **13**B, 591 (1969); Büttiker, M., Thomas, H.: Z. Phys. B – Condensed Matter **34**, 301 (1979)
9. Busch, H.: Ann. Phys. **64**, 401 (1921)
10. Landauer, R.: Physics Today **31**, 23 (November 1978); Phys. Rev. A**15**, 2117 (1977)
11. For an extended historical account see, Landauer, R.: In: Noise in nonlinear dynamical systems: theory, experiment, simulation. Moss, F., McClintock, P.V.E. (eds.). Cambridge: Cambridge University Press (in press)
12. Horsthemke, W., Lefever, R.: Phys. Lett. **64**A, 19 (1977)
13. Landauer, R.: Phys. Lett. **68**A, 15 (1978); Ryter, D.: Z. Phys. B – Condensed Matter **41**, 39 (1981)
14. van Kampen: IBM J. Res. Develop. (in press)
15. Landauer, R.: In: Synergetics. Haken, H. (ed.), p. 97. Stuttgart: Teubner 1973
16. Kramers, H.A.: Physica **7**, 284 (1940)
17. Büttiker, M., Landauer, R.: In: Nonlinear phenonema at phase transitions and instabilities. Riste, T. (ed.), p. 111. New York, London: Plenum Press 1982
18. Eichler, H.J.: In: Advances in Solid State Physics, Festkörperprobleme XVIII, p. 241. Treusch, J. (ed.). Braunschweig: Vieweg 1978; Hoffman, C.A., Jarasiunas, K., Gerritsen, H.J., Nurmikko, A.V.: App. Phys. Lett. **33**, 536 (1978); Salcedo, J.R., Siegman, A.E.: IEEE J. Quantum Electron. QE**15**, 250 (1979); Nelson, K.A., Lutz, D.R., Fayer, M.D., Madison, L.: Phys. Rev. B**24**, 3261 (1981)
19. Büttiker, M., Thomas, H.: Phys. Lett. **77**, 372 (1980); unpublished (1987)
20. Kaup, D.J.: Phys. Rev. B**27**, 6787 (1983)
21. Landauer, R.: In: The maximum entropy formalism. Levine, R.D., Tribus, M., (eds.), p. 321. Cambridge, Mass: M.I.T. Press 1978
22. Engel, A.: Phys. Lett. **113**A, 139 (1985)

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