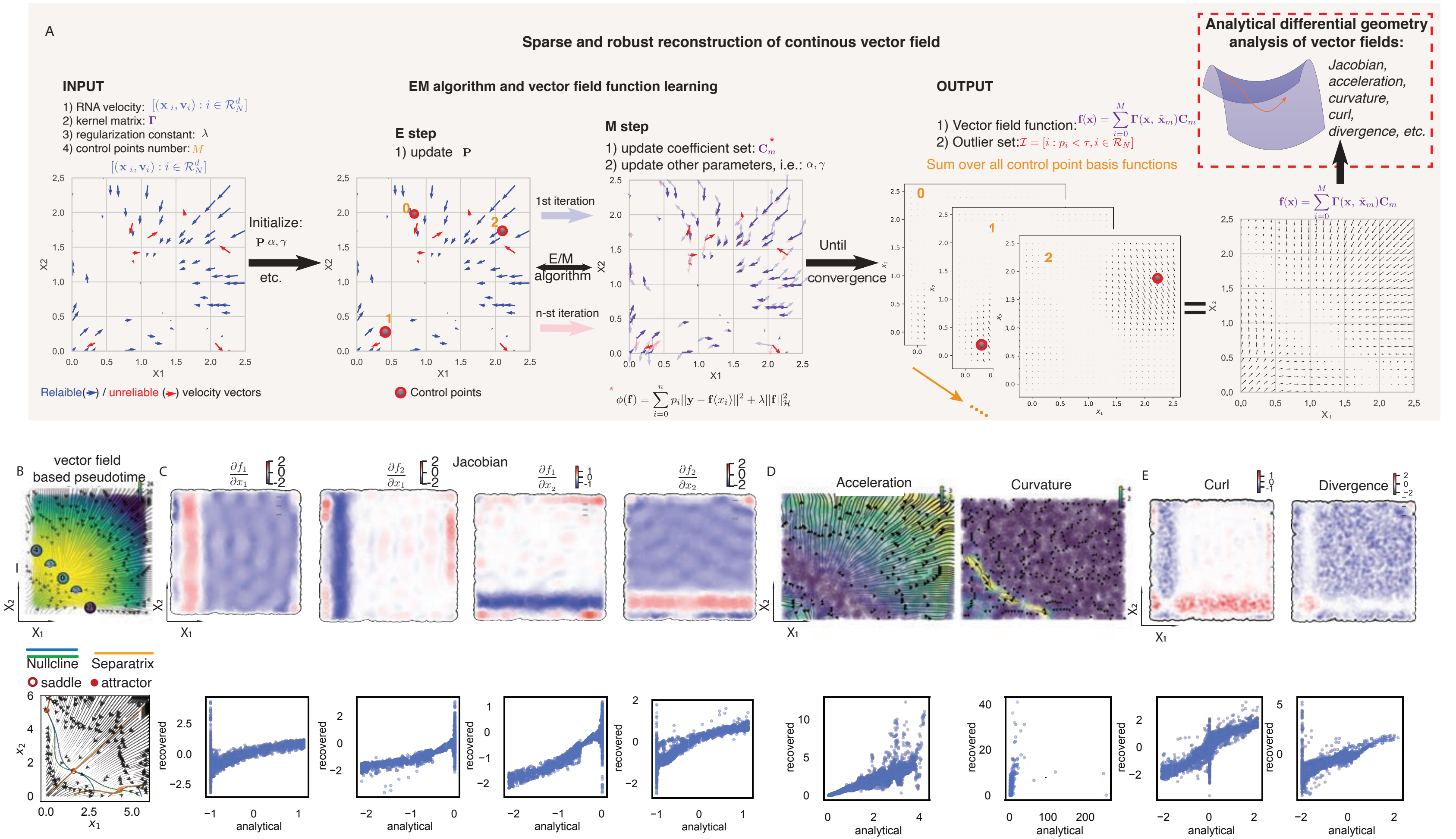

A
Sparse and robust reconstruction of continous vector field
INPUT
1) RNA velocity: $[(\mathbf{x}_i, \mathbf{v}_i) : i \in \mathcal{R}_N^d]$
2) kernel matrix: Γ
3) regularization constant: λ
4) control points number: M
Relaible / unreliable velocity vectors
Initialize: $\mathbf{P}\ \alpha, \gamma$ etc.
EM algorithm and vector field function learning
E step
1) update $\mathbf{P}$
Control points
M step
1) update coefficient set: $\mathbf{C}_m^*$
2) update other parameters, i.e.: α, γ
1st iteration
E/M algorithm
n-st iteration
$\phi(\mathbf{f}) = \sum_{i=0}^{n} p_i \|\mathbf{y} - \mathbf{f}(x_i)\|^2 + \lambda \|\mathbf{f}\|_{\mathcal{H}}^2$
Until convergence
OUTPUT
1) Vector field function: $\mathbf{f}(\mathbf{x}) = \sum_{i=0}^{M} \Gamma(\mathbf{x}, \tilde{\mathbf{x}}_m)\mathbf{C}_m$
2) Outlier set: $\mathcal{I} = [i : p_i < \tau, i \in \mathcal{R}_N]$
Sum over all control point basis functions
Analytical differential geometry analysis of vector fields:
Jacobian, acceleration, curvature, curl, divergence, etc.
B
vector field based pseudotime
Nullcline
Separatrix
saddle
attractor
C
Jacobian
$\frac{\partial f_1}{\partial x_1}$
$\frac{\partial f_2}{\partial x_1}$
$\frac{\partial f_1}{\partial x_2}$
$\frac{\partial f_2}{\partial x_2}$
D
Acceleration
Curvature
E
Curl
Divergence
recovered
analytical