Midterm for STAT31120

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1 Notice

- Please send your answers with the email address you registered on Canvas to
  zhongjianwang25@gmail.com
- The title should be
  MidTerm-(Your Full Name)
- There is no format requirements, but you have to include *hand-written/typed illustration, program code and a snapshot of the whole screen when program running* in ONE zip file.
- The last version received before 17:30 CDT Apr 29 will be taken as the final submission.
- You can use all existed material during lecture or in the two textbooks, and any computing resources.to answer the questions, unless specifically required by the question.
- You cannot discuss with each other until 17:31 CDT Apr 29.

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2 Problems

1. Derive the solution (not the approximation) to
$$dX_t = (aX_t + b)dt + (cX_t + d)dW_t, \tag{2.1}$$
where a, b, c, d are constant real numbers.

2. (a) Plot one path (within [0,5] with 500 equally distributed observation of W_t, $X_0 = 1$) of the analytic solution to
$$dX_t = 1dt + 2\sqrt{X_t}dW_t \tag{2.2}$$
   (b) Plot the path of its Euler approximation based on the Brownian path same with a).
   (c) Design a weak scheme to calculate EX_5^2 and explain why the scheme is not strong consistent.

3. Consider
$$dX_t = -5X_t dt + dW_t \tag{2.3}$$
$$X_0 = 1 \tag{2.4}$$
with Euler method. The time is discretized into equally distributed sub-interval with length $\delta > 0$. Find the range of δ such that Euler method is asymptotically stable.

4. Consider $u(t, x)$ satisfies
$$u_t = u_{xx} - xu_x + \sin(t + x)u \tag{2.5}$$
$$u(0, x) = \cos(x) \tag{2.6}$$
   (a) Design a numerical scheme to calculate $u(1, 1)$. Monte-Carlo and Euler approximation has to be included in your scheme if you want to answer the next question:)
   (b) Let the largest length of partition interval (δ) in Euler approximation fixed and no bigger than 0.01. Please increase the total particle path number (N) in Monte-Carlo simulation and plot change of values.
   (c) Select a reasonable number of path basing on result in b) and verify the convergence rate against δ. (To do this you should find the reference solution with δ small enough.)

5. Find expressions for the multiple Ito integrals I_α with multi-indices $\alpha = (0, 1), (1, 1), (1, 1, 1)$ and $(1, 1, 1, 1)$ in terms of those with multi-indices $(0), (1)$ and $(1, 0)$.

3 Mid Term Evaluation

Please list at least two aspects that you think should be improved by the instructor.