Course 02249, DTU Compute
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Computationally Hard Problems – Fall 2020
Assignment 1

Date: 01.09.2020, **Due date**: 07.09.2020, 21:00

The following exercises are **not** mandatory:

Exercise 1.1: Consider the language L_{graphs} from Example 2.3 in the lecture notes. Let $w \in \{0,1\}^k$ be a word of length k over $\{0,1\}$.

a) Describe how one can check that it encodes an undirected graph.

b) If w encodes an undirected graph, describe how one can check whether a certain edge $\{i,j\}$, where $j > i$, is present. Avoid reconstructing the adjacency matrix from w.

End of Exercise 1

Exercise 1.2: Design a language L_{dgraphs} for directed graphs. Do not forget to specify the alphabet Σ_{dgraphs} you use.

End of Exercise 2

Exercise 1.3: Design a language for sequences of fractions. A fraction is the quotient of two integers.

a) Specify the alphabet.

b) Specify how the language is defined.

c) Show how the sequence 1/3, 234/39, −55/16 is coded in your language.

End of Exercise 3

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Exercise 1.4: A *disjunctive form* is a formula over n boolean variables $x_1, \ldots, x_n$ such that

- the formula is a boolean disjunction of k so-called monomials $m_1, \ldots, m_k$,
- where each monomial m_i, $1 \leq i \leq k$, is a boolean conjunction of variables or their negations.

For example, a disjunctive form for $n = 5$ and $k = 4$ is

$$\underbrace{(\overline{x}_1 \wedge x_2 \wedge x_3)}_{m_1} \vee \underbrace{(\overline{x}_1 \wedge x_4)}_{m_2} \vee \underbrace{(x_1 \wedge \overline{x}_3 \wedge x_4 \wedge x_5)}_{m_3} \vee \underbrace{(x_2 \wedge \overline{x}_3 \wedge \overline{x}_5)}_{m_4}.$$

Design a language $L_{\text{disj-form}}$ for the set of disjunctive forms. Do not forget to specify the alphabet $\Sigma_{\text{disj-form}}$ you use.

End of Exercise 4

The following exercise is **mandatory**:

Exercise 1.5: A *hypergraph* $G = (V, E)$ is given by its vertex set $V = \{1, \ldots, n\}$ and its edge set $E \subseteq \{e \mid e \subseteq V\}$, i. e., all $e \in E$ connect a subset of the vertex set. For example, $V = \{1, \ldots, 10\}$ and $E = \{\{1, 2, 5\}, \{4\}, \{3, 5, 6, 9\}, \emptyset\}$ (where $\emptyset$ denotes the empty set). The aim is to design a formal language L_{hg} for hypergraphs.

a) Specify the alphabet Σ_{hg} you use.

b) Specify how a hypergraph is encoded in the language L_{hg}.

c) Describe how one can check whether a given word $w \in \Sigma_{\text{hg}}^*$ is in L_{hg}, and, if so, how the hypergraph can be reconstructed.

d) How would the example from above be encoded in your language?

End of Exercise 5