02249 - Computationally Hard Problems, Mandatory 7

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Exercise Description

Consider a TSP instance on $n \in N$ points, where $n = 64k$ for some $k \in N$, in the two-dimensional Euclidean plane such that point $i, 1 \leq i \leq n$, has coordinates (a_i, b_i) where $a_i = \cos(i \cdot 2\pi/n)$ and $b_i = \sin(i \cdot 2\pi/n)$. Hence, the points lie equidistantly on a circle of perimeter 2π. We work with the Euclidean distance. Suppose that the following two tours are given:

$$
\begin{gathered}
x = (1, 2, \ldots, \\
n/8-2, n/8, n/8+2, n/8-1, n/8+1, n/8+3, n/8+4, \ldots, \\
3n/8-3, 3n/8-1, 3n/8+1, 3n/8-2, 3n/8, 3n/8+2, 3n/8+3, \ldots, \\
7n/8-3, 7n/8+3, 7n/8-2, 7n/8, 7n/8+2, 7n/8-1, 7n/8+1, 7n/8+4, 7n/8+5, \ldots, \\
n)
\end{gathered}
$$

and

$$y = (1, 2, \ldots, 5n/8-2, 5n/8, 5n/8+2, 5n/8-1, 5n/8+1, 5n/8+3, 5n/8+4, \ldots, n)$$

Describe the output of Generalized Partition Crossover (GPX) applied to x and y. While doing so, determine the set of common edges of the union graph and the connected components remaining after removal of the common edges. To simplify the comparison of lengths of alternative subtours, you may assume that n is sufficiently large.

Exercise Resolution

Considerations

The problem is related to the approximation of the circumference by a regular polygon with n sides inscribed in it. The circumference C of a circle of radius r is equal to $2\pi r$, ad can be approximated by $2nr\sin\left(\frac{360}{2n}\right) = 2nr\sin\left(\frac{\pi}{n}\right)$.

Approach

I had an hands on approach, I have implemented my own GPX in this notebook (link, but i also attach it at the end of this pdf as it is at the time of delivery) and the following consideration and observations come from there.

GPX Method

Generalized Partition Crossover (GPX) is an evolutionary algorithm developed to address TSP, and it generalize Partition CrossOver (PX) in the sense that it considers more than one cut. GPX steps:

1. Identify common edges between parent x and parent y
2. Remove the common edges and find the connected components
3. find the shortest path through each component
4. Concatenate common edges and shortest paths to get the child

Input description

x and y are a described by two series of nodes depending on a parameter $n = 64 * k$, for simplicity I will consider cases with $k \geq 2$. Each $node_i$ has its own coordinates over a circumference of radius 1. All the n nodes are in each of the two paths, with different order.

Common Edges

The list of common edges between x and y is given by the union of the following sets of edges:

- $\big[(1,2),\dots,\big(\frac{n}{8}-3,\frac{n}{8}-2\big)\big]\cup$
- $[(\frac{n}{8}+3,\frac{n}{8}+4),\dots,(\frac{3n}{8}-4,\frac{3n}{8}-3)]\cup$
- $[(\frac{3n}{8}+2,\frac{3n}{8}+3),\dots,(\frac{5n}{8}-3,\frac{5n}{8}-2)]\cup$
- $[(\frac{5n}{8}+3,\frac{5n}{8}+4),\dots,(\frac{7n}{8}-4,\frac{7n}{8}-3)]\cup$
- $[(\frac{7n}{8}+4,\frac{7n}{8}+5),\dots,(n-1,n)]\cup$

The number of common edges increases linearly with n.

Connected Components

There are always 4 connected components:

1. $[\frac{n}{8}-2,\dots,\frac{n}{8}+3)]$
2. $[\frac{3n}{8}-3,\dots,\frac{3n}{8}+2)]$
3. $[(\frac{5n}{8}-2,\dots,\frac{5n}{8}+3)]$
4. $[(\frac{7n}{8}-3,\dots,\frac{7n}{8}+4)]$

The number of not common edges and the size of each connected component is constant.

Shortest path

In each connected component there are in this case only two "entry nodes", meaning nodes that are part of the connected component and of one common edge. These are the extremes of the lists reported above. The shortest path between these two entry nodes is found in each connected component.

1. $[(\frac{n}{8}-2,\frac{n}{8}-1)\dots,(\frac{n}{8}+2,\frac{n}{8}+3)]$
2. $[(\frac{3n}{8}-3,\frac{3n}{8}-2),\dots,(\frac{3n}{8}+1,\frac{3n}{8}+2)]$
3. $[(\frac{5n}{8}-2,\frac{5n}{8}-1),\dots,(\frac{5n}{8}+2,\frac{5n}{8}+3)]$
4. $[(\frac{7n}{8}-3,\frac{7n}{8}-2),\dots,(\frac{7n}{8}+3,\frac{7n}{8}+4)]$

Offspring

The offspring is given by the concatenation of the common edges and the shortest paths per connected component:

- $\big[(1,2),\dots,\big(\frac{n}{8}-3,\frac{n}{8}-2\big)\big]\cup$
- $[(\frac{n}{8}-2,\frac{n}{8}-1)\dots,(\frac{n}{8}+2,\frac{n}{8}+3)]\cup$
- $[(\frac{n}{8}+3,\frac{n}{8}+4),\dots,(\frac{3n}{8}-4,\frac{3n}{8}-3)]\cup$
- $[(\frac{3n}{8}-3,\frac{3n}{8}-2),\dots,(\frac{3n}{8}+1,\frac{3n}{8}+2)]\cup$
- $[(\frac{3n}{8}+2,\frac{3n}{8}+3),\dots,(\frac{5n}{8}-3,\frac{5n}{8}-2)]\cup$
- $[(\frac{5n}{8}-2,\frac{5n}{8}-1),\dots,(\frac{5n}{8}+2,\frac{5n}{8}+3)]\cup$
- $[(\frac{5n}{8}+3,\frac{5n}{8}+4),\dots,(\frac{7n}{8}-4,\frac{7n}{8}-3)]\cup$
- $[(\frac{7n}{8}-3,\frac{7n}{8}-2),\dots,(\frac{7n}{8}+3,\frac{7n}{8}+4)]\cup$
- $[(\frac{7n}{8}+4,\frac{7n}{8}+5),\dots,(n-1,n)]\cup$