02249 - Computationally Hard Problems, Assignment I

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07 September 2020

Exercise Description

Exercise : A hypergraph $G = (V, E)$ is given by its vertex set $V = \{1, \ldots, n\}$ and its edge set $E \subseteq \{e \mid e \subseteq V\}$, i. e., all $e \in E$ connect a subset of the vertex set. For example, $V = \{1, \ldots, 10\}$ and $E = \{\{1, 2, 5\}, \{4\}, \{3, 5, 6, 9\}, \emptyset\}$ (where $\emptyset$ denotes the empty set). The aim is to design a formal language L_{hg} for hypergraphs.
a) Specify the alphabet Σ_{hg} you use.
b) Specify how a hypergraph is encoded in the language L_{hg}.
c) Describe how one can check whether a given word $w \in \Sigma_{hg}^*$ is in L_{hg}, and, if so, how the hypergraph can be reconstructed.
d) How would the example from above be encoded in your language?

Exercise Resolution

a) $\Sigma_{\text{hg}} = \{0, 1, \ldots, 9, +, -, \#\}$

b) For each edge we list the vertices index numbers with the same polarity, chosen initially as +. Once the list of vertices in that edge is complete we change polarity. Once all edges lists are complete, we include at the end all the vertices that have no edges, preceded by a # symbol without polarity.

c) For a word $w \in \Sigma_{hg}^*$ to be in L_{hg}, each vertex index has to be preceded either by +,- or #.
E.g. 12-1-2#22 is not a word in L_{hg}; +4-1-2#5#3, +1+5-2-5-6+3+4+5, #5#3#7#8#10 are words in L_{hg}
To reconstruct the hypergraph, we need to retrieve the set of vertexes and the edges set from the word. To create the edges set, we move from left to right and register in the same edge the number indexes with the same polarity, until we encounter the first # or we are at the end of the word. To create the vertexes set, we move from left to right and register all the unique numbers, without polarity or #.

d) +1+2+5-4+3+5+6+9#7#8#10