Course 02249, DTU Compute
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Computationally Hard Problems – Fall 2020
Assignment 5

Date: 29.09.2020, **Due date**: 05.10.2020, 21:00

For all problems on this sheet, you may assume that they are in $\mathcal{NP}$.

The following exercises are **not** mandatory:

Exercise 5.1: Prove that the transformation T in the proof of Theorem 4.19 from the lecture notes can be performed in time polynomial in the input size (which is the total length of all clauses plus the number of variables).

End of Exercise 1

Exercise 5.2: We consider the following problem.

Problem: [4-SAT]
Input: A set of clauses $C = \{c_1, \ldots, c_k\}$ over n boolean variables $x_1, \ldots, x_n$, where every clause contains exactly four literals.
Output: YES if there is a truth assignment to the variables $a\colon \{x_1, \ldots, x_n\} \to \{0,1\}$ such that every clause c_j is satisfied. The answer is NO otherwise.

Prove that 4-SAT is $\mathcal{NP}$-complete.
Note: A clause is a set of literals, i. e., it is not allowed to have the same literal multiple times in a clause.

End of Exercise 2

Exercise 5.3: Recall the following problem, which was already defined on Exercise Sheet 2.

Problem: [REFUTATION] Given is a disjunctive form consisting of k monomials $m_1, \ldots, m_k$ over n boolean variables $x_1, \ldots, x_n$. The task is to decide if there is a truth assignment to the variables $a\colon \{x_1, \ldots, x_n\} \to \{0,1\}$ such that the truth value of the disjunctive form is *false.*

Prove that REFUTATION is $\mathcal{NP}$-complete.
Hint: De Morgan's laws from propositional logic can be useful.

End of Exercise 3

Continued on next page.

Exercise 5.4: We consider the following problem.

Problem: [RetiringSalesperson] A salesperson has customers in n different cities. He or she originally lives in city number 1. There is direct road between every pair of different cities and no possibility to change roads outside the cities. The distance going from city i to city j is $d(i, j) \in \mathbb{N}$. The salesperson wants to visit all cities (in any order starting from city 1) exactly once and retires in the city he or she has visited last. Is there such a journey through the cities such that the total distance traveled is at most some given value B?

Prove that RetiringSalesperson is $\mathcal{NP}$-complete. Reduce from an $\mathcal{NP}$-complete problem listed in the lecture notes.
Hint: Reduce from a related graph problem and introduce an extra vertex. Note that we do *not* demand $d(i, j) = d(j, i)$.

End of Exercise 4

The following exercise is **mandatory**:

Exercise 5.5: Consider the following problem.

Problem: [3-Sat-With-Majority]
Input: A set of clauses $C = \{c_1, \ldots, c_k\}$ over n boolean variables $x_1, \ldots, x_n$, where every clause contains exactly three literals.
Output: YES if there is a truth assignment to the variables such that both

- every clause is satisfied under the assignment
- and there are at least $k/2$ clauses in which all 3 literals are set to true under the assignment.

NO otherwise.

Show that this problem is $\mathcal{NP}$-complete. You may use any problem stated as $\mathcal{NP}$-complete in the lecture notes for this course. You may also assume that 3-Sat-With-Majority is in $\mathcal{NP}$.
Hint: Reduce from 3-Sat. Introduce some extra variables and clauses.
Note: A clause is a set of literals, i. e., it is not allowed to have the same literal multiple times in a clause.

End of Exercise 5