Course 02249, DTU Compute
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Computationally Hard Problems – Fall 2020
Assignment 6

Date: 03.11.2020 (only non-mandatory exercises)

The following exercises are **not** mandatory:

Exercise 6.1: Prove that the algorithm from Theorem 5.2 in the lecture notes has an expected approximation ratio of 4/3 (or better) if all clauses have at least 2 literals.

End of Exercise 1

Exercise 6.2: Show that there is a **deterministic** algorithm for MAXSAT with approximation ratio no worse than 2.
Hint: It is enough to consider two assignments.

End of Exercise 2

Exercise 6.3: Consider the following eight clauses over the boolean variables $\{x_1, x_2, x_3\}$:

$$\begin{aligned}
c_1 &= x_1 \vee x_2 \vee x_3 \\
c_2 &= \overline{x_1} \vee \overline{x_3} \\
c_3 &= \overline{x_2} \vee x_3 \\
c_4 &= x_2 \vee x_3 \\
c_5 &= x_1 \vee \overline{x_3} \\
c_6 &= \overline{x_1} \\
c_7 &= \overline{x_3} \\
c_8 &= x_1 \vee \overline{x_2} \vee x_3
\end{aligned}$$

a) Determine whether there is a satisfying assignment.

b) Construct the integer program for the set of clauses.

c) Relax the integer program to a linear program as described in the notes.

d) Solve the linear program.

e) Use randomized rounding to find an assignment and determine how many clauses your rounding satisfies.

Hint: Use an off-the-shelf solver for linear programs. You can find them in lp_solve, Matlab, Maple and many other places.

End of Exercise 3

Exercise 6.4: Consider Algorithm 5.9 from the lecture notes. Suppose it is run on the following 3-SAT instance with variable set $\{x_1, \ldots, x_6\}$ and clause set

$$\begin{gathered}
\overline{x_1} \vee \overline{x_2} \vee \overline{x_3} \\
\overline{x_1} \vee x_2 \vee x_3 \\
x_1 \vee \overline{x_2} \vee x_3 \\
x_1 \vee x_2 \vee \overline{x_3} \\
\overline{x_1} \vee \overline{x_2} \vee x_3 \\
\overline{x_1} \vee x_2 \vee \overline{x_3} \\
x_1 \vee \overline{x_2} \vee \overline{x_3} \\
x_4 \vee x_5 \vee x_6 \\
\overline{x_4} \vee x_5 \vee x_6 \\
x_4 \vee \overline{x_5} \vee x_6 \\
\overline{x_4} \vee \overline{x_5} \vee x_6.
\end{gathered}$$

Suppose furthermore that $S = 1$ and that the random choice of an initial assignment results in $x_i = 0$ for $i \in \{1, \ldots, 6\}$. Find a choice of T such that the algorithm terminates with a satisfying assignment with probability at least $1/2$. Justify your choice.

End of Exercise 4

Exercise 6.5: Suppose that Algorithm 5.9 from the lecture notes is run on a satisfiable 3-SAT instance with parameters $T = n/2$ and $S = \infty$. Show that it outputs a satisfying assignment in an expected number of at most $2(\sqrt{3})^n = O(1.7321^n)$ iterations of the outer loop (over s).
Hint: The random variable d^* used in the proof of Theorem 5.13 is binomially distributed with parameters n and $1/2$. In particular, this means that $d^* \leq n/2$ occurs with probability at least $1/2$.

End of Exercise 5
