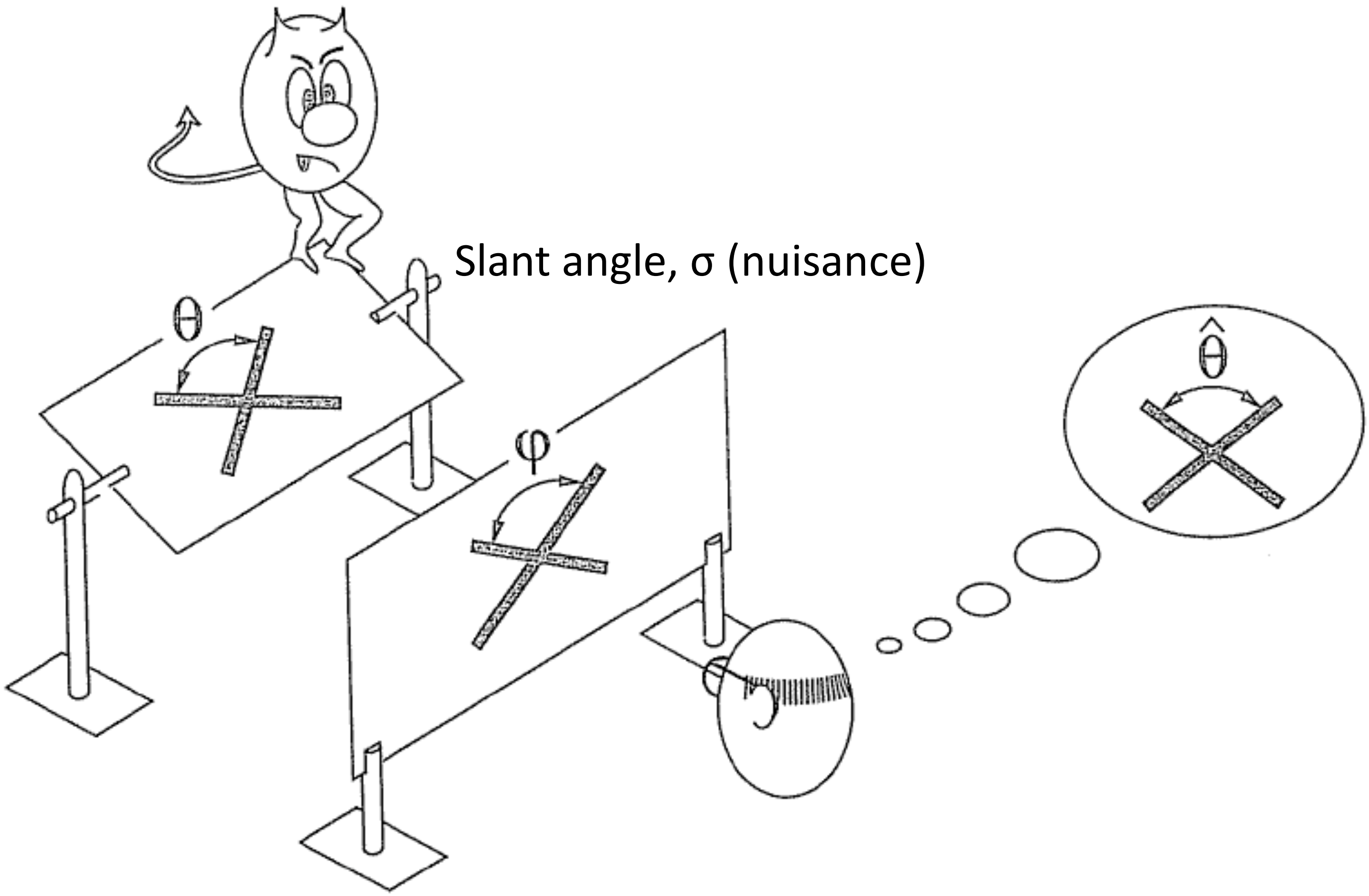
# 02458 Cognitive Modeling

Lecture 4

Bayesian Modeling of Visual Perception

Mamassian, Landy and Maloney

# Example: Spinning demon



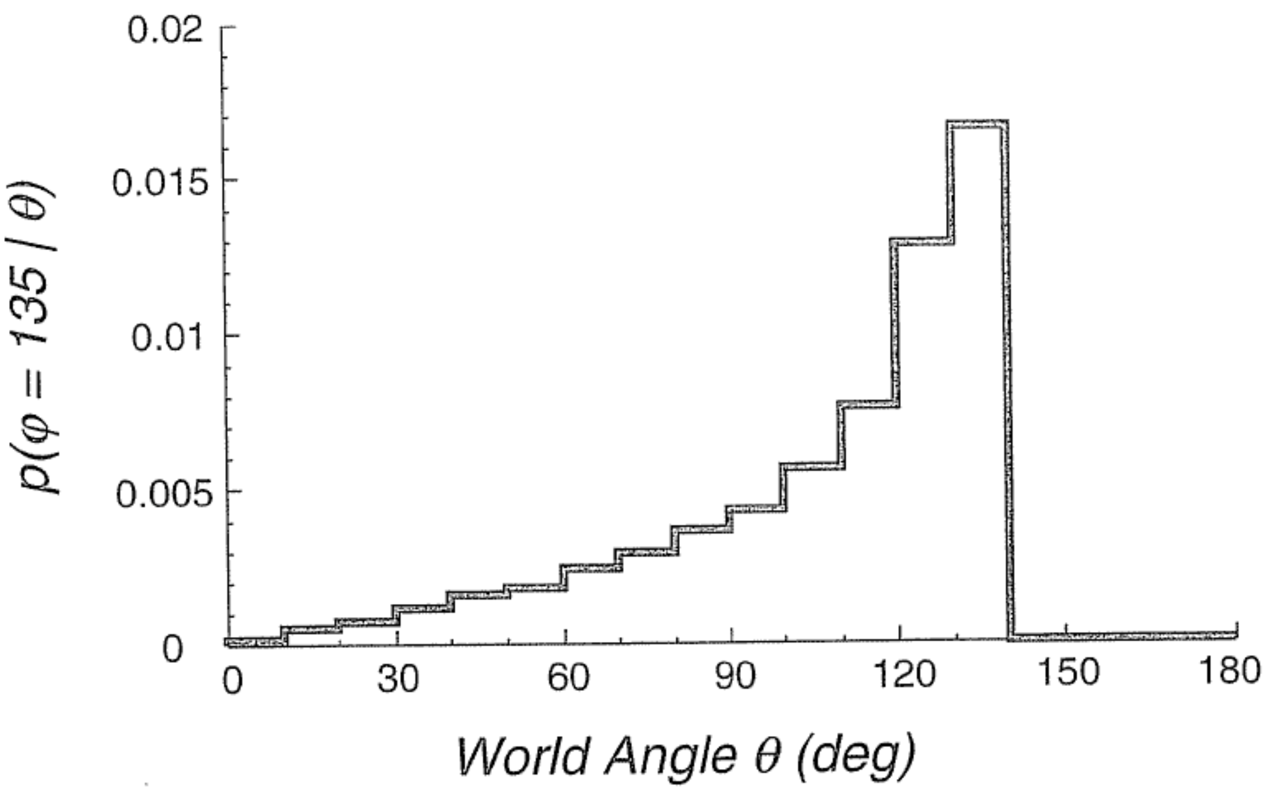

Slant angle, σ (nuisance)

# Example: Spinning demon

- Say, that $\phi=135$ and we want to know $\theta$
- We need the likelihood function *P(I|S)*
- We could use $\phi(\theta,\sigma)$ + some noise
  - Will require some formal geometry, but
    - $\phi(\theta, 90)=\theta$
    - $\phi(\theta, 0)=180$
    - So, $\theta<\phi<180$ ($P(\varphi|S)=0$ for $\phi > \theta$)
- But we just spin the wheel a lot varying $\theta$ and count when $\phi=135$ +/- 5 for each $\theta$

# Example: Spinning demon

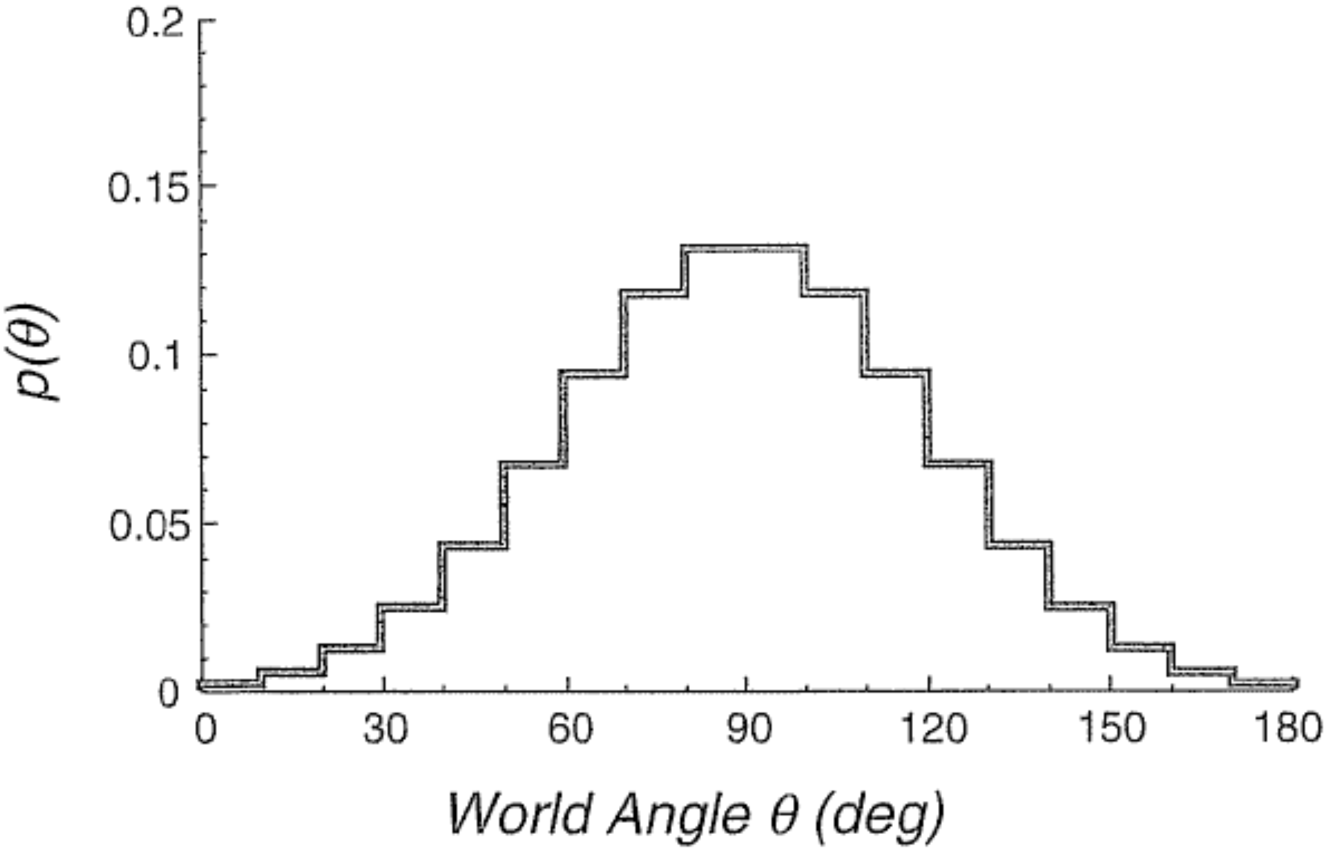
- Solution: ML estimate of ϕ = 135



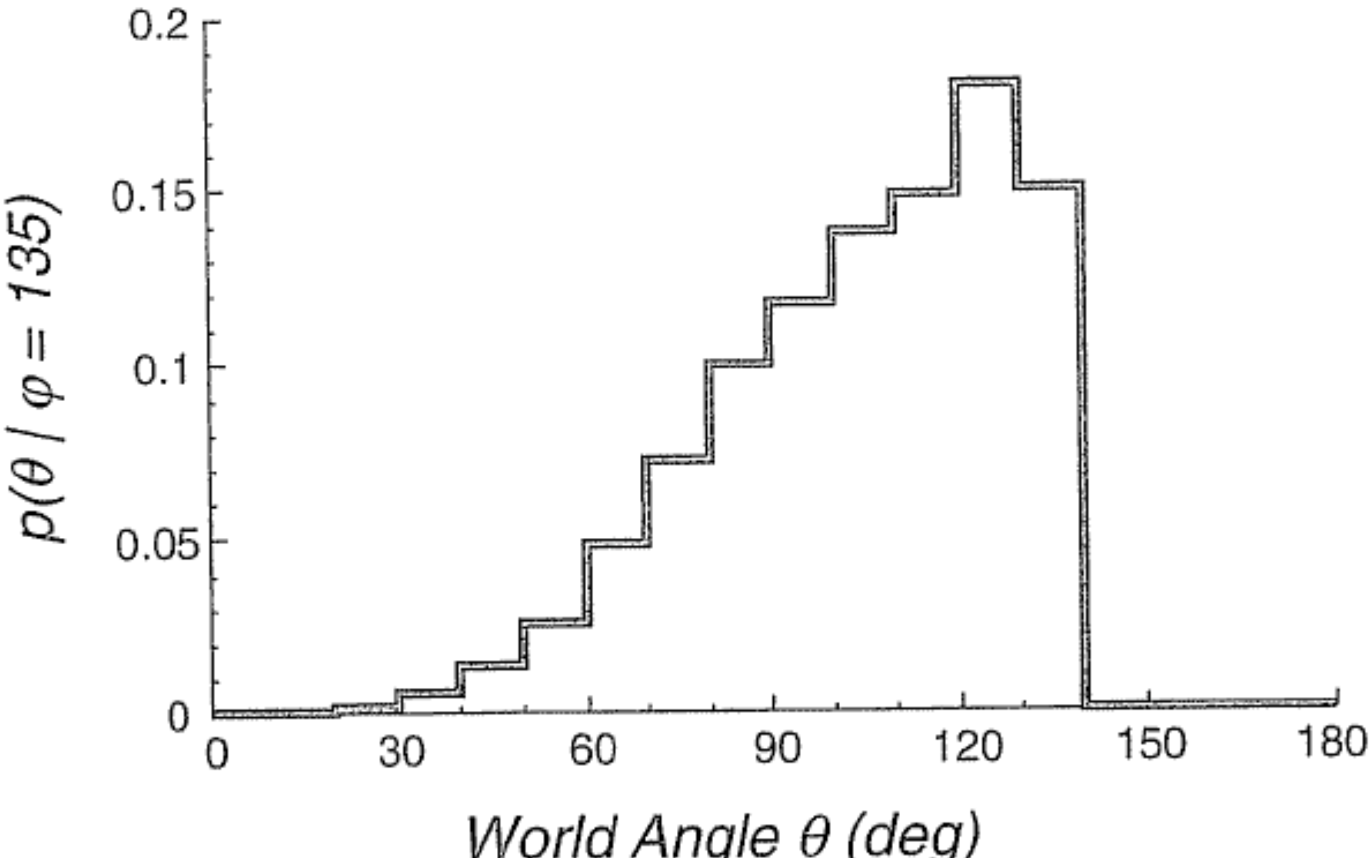
# Example: Spinning demon

- What would happen with a non-uniform prior?
- Depends on the standard deviation of the prior and the likelihood
  - The narrower distribution "wins"
  - From simple multiplication of probabilities

# Example: Spinning demon



# Example: Spinning demon



# Example: Spinning demon

- Final step: The utility function
  - Utility is a function of
    - The scene estimate, $\widehat{S}(Shat)$ or $\alpha$ in the book
    - The actual Scene, *S*
  - Utility is signed
    - The correct estimate has a positive utility (gain)
    - An incorrect estimate has a negative utility (loss)
      - Some incorrect estimates might be worse than others

# Example: Spinning demon

- Utility theory: Maximize the *expected* utility

$$\overline{U}\left(\widehat{S}\right)=\int_S P(S\mid I)U\left(\widehat{S},S\right)dS=\int_S P(\theta\mid\varphi)U(\alpha,\theta)d\theta$$

- Utility theory
  - Fair dice, 6 you win 10€, else you loose 1€
    - Expected utility? Wanna play?
  - Roulette: Pick a number 1-36, if the number comes out you get 36€, if not you loose 1€ and, oh, by the way, you can’t pick zero
    - Expected utility? Wanna play?

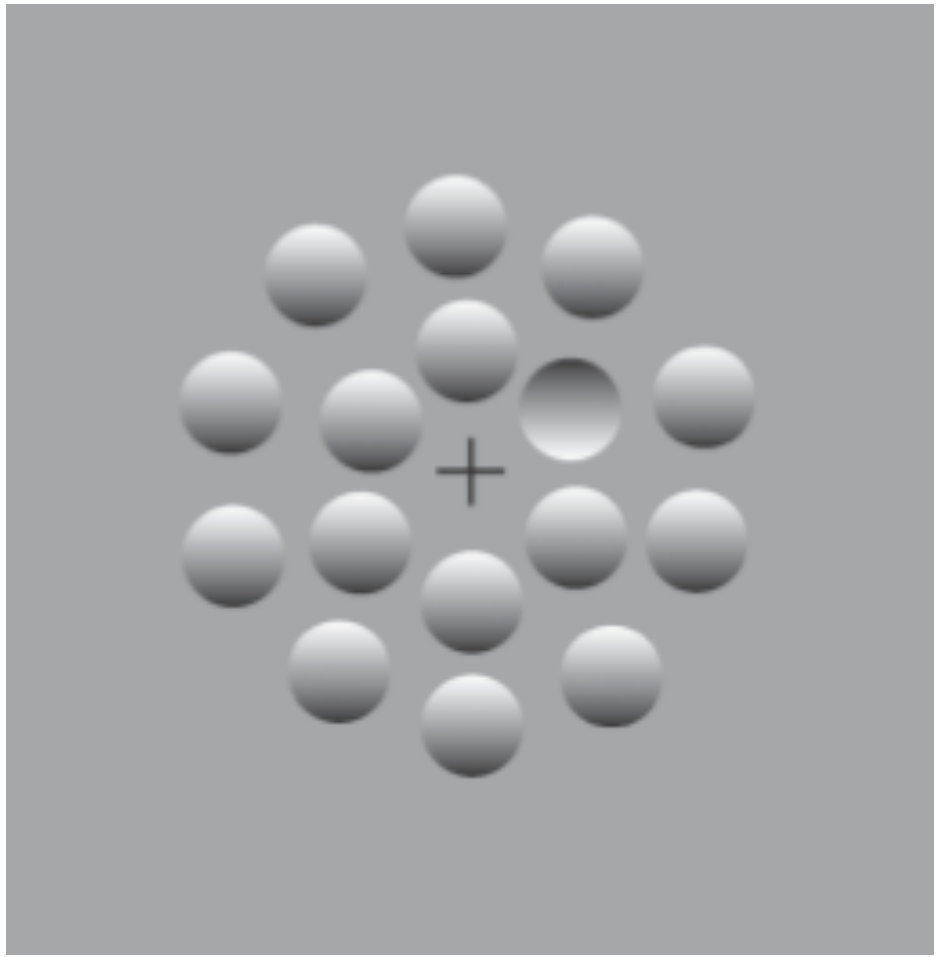
# Example: Spinning demon

- The decision rule depends on the utility function
  - A Dirac delta utility function ->
    - MAP (125 degrees)
  - A squared error loss function ->
    - Mean posterior (105 degrees)

# Example: Spinning demon

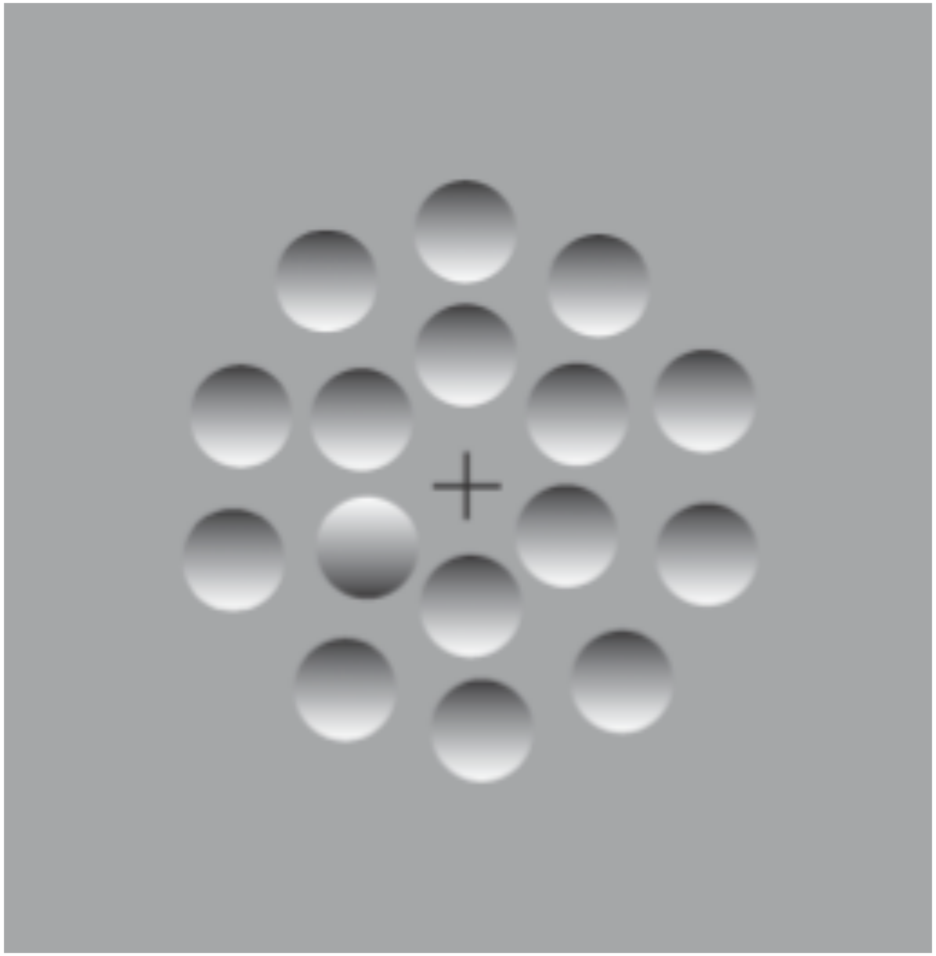
- Response variability?
  - Add noise to the likelihood function
    - E.g. add Gaussian noise to $P(I|S)$
  - Probability matching decision rule
    - Suboptimal decision
    - But may allow exploration and learning

# Protrusions or crevasses?

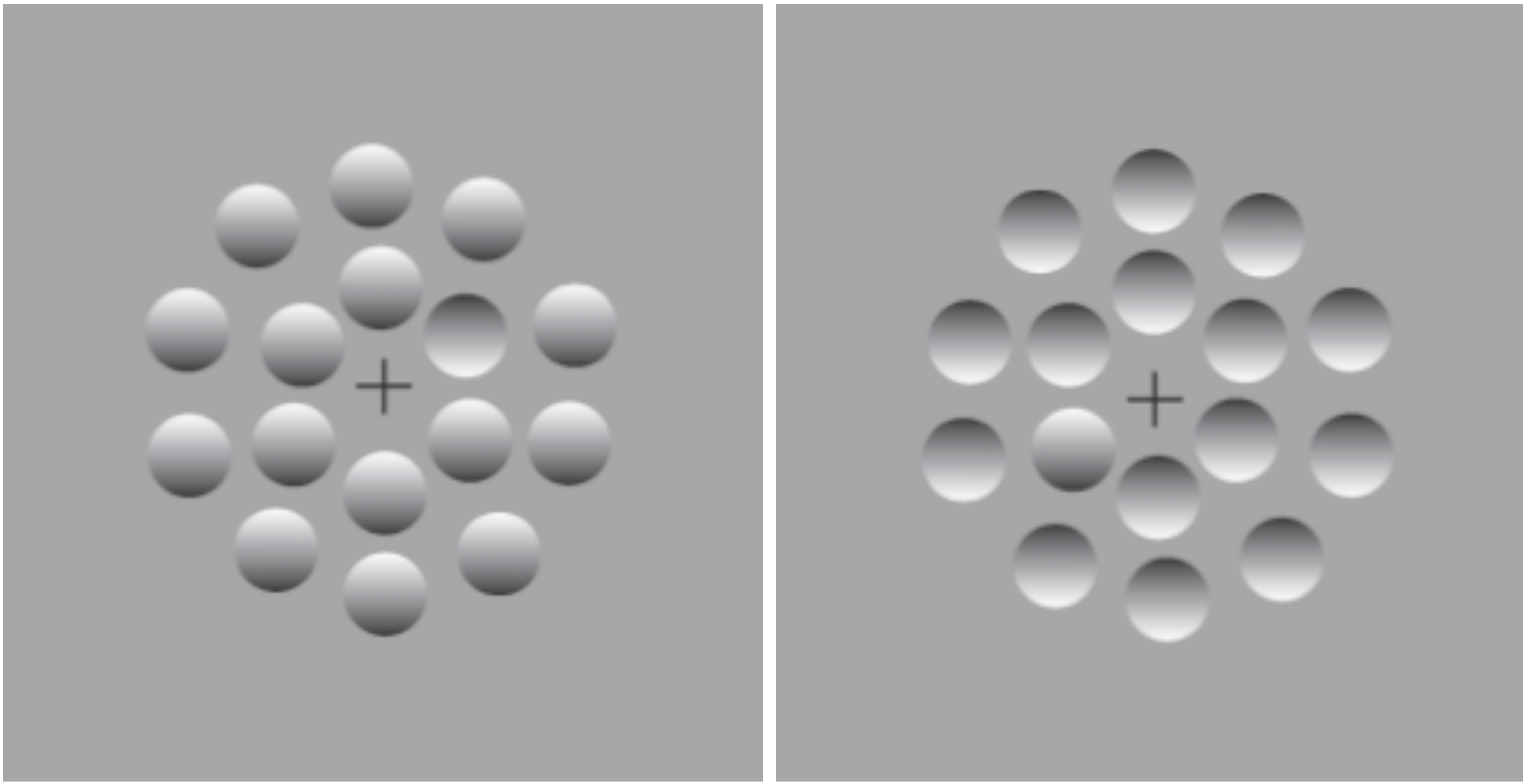


Champions and Adams, JOV, 2007

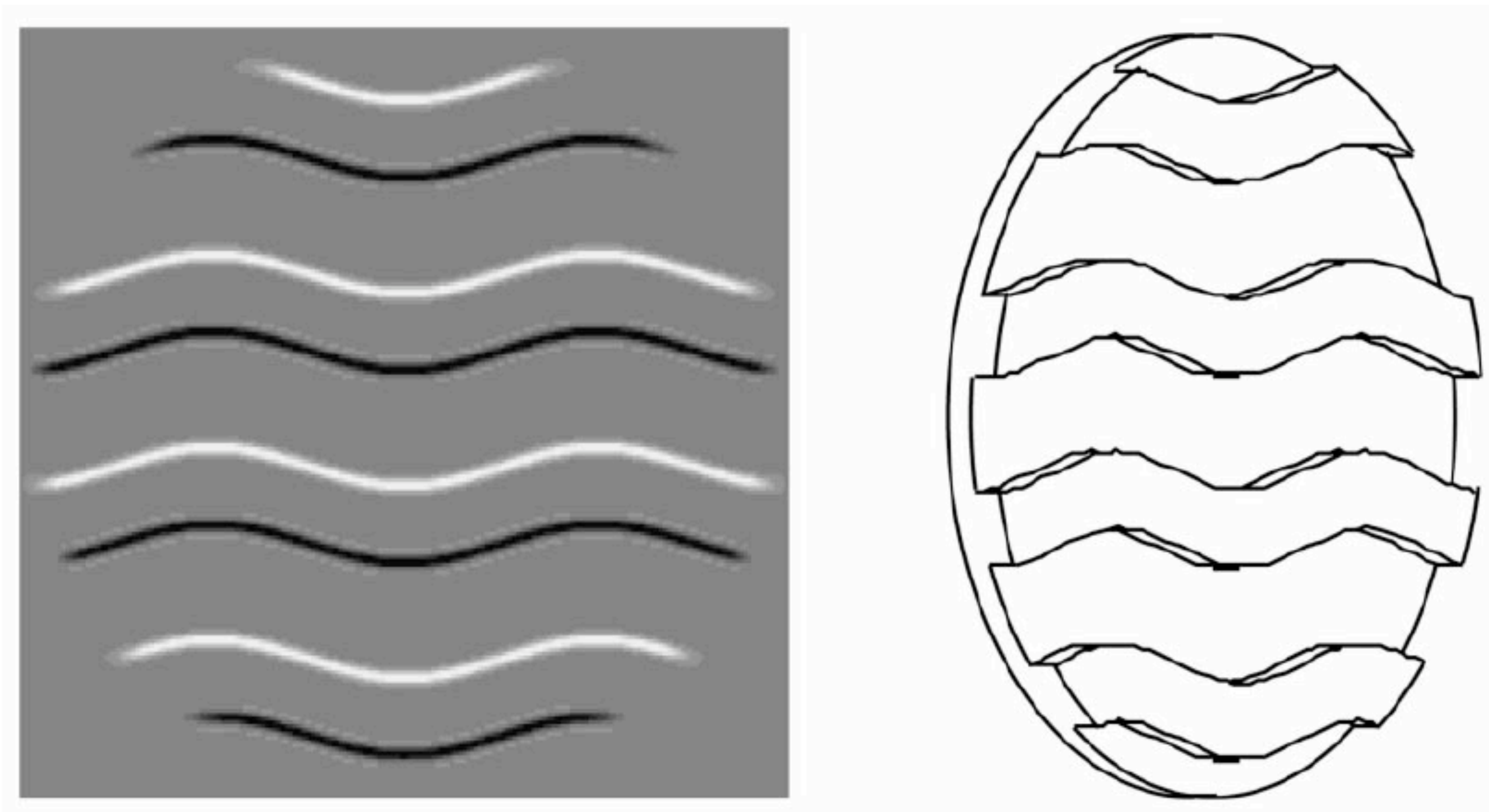
# Protrusions or crevasses?



# Protrusions or crevasses?



# Example: Embossed surfaces



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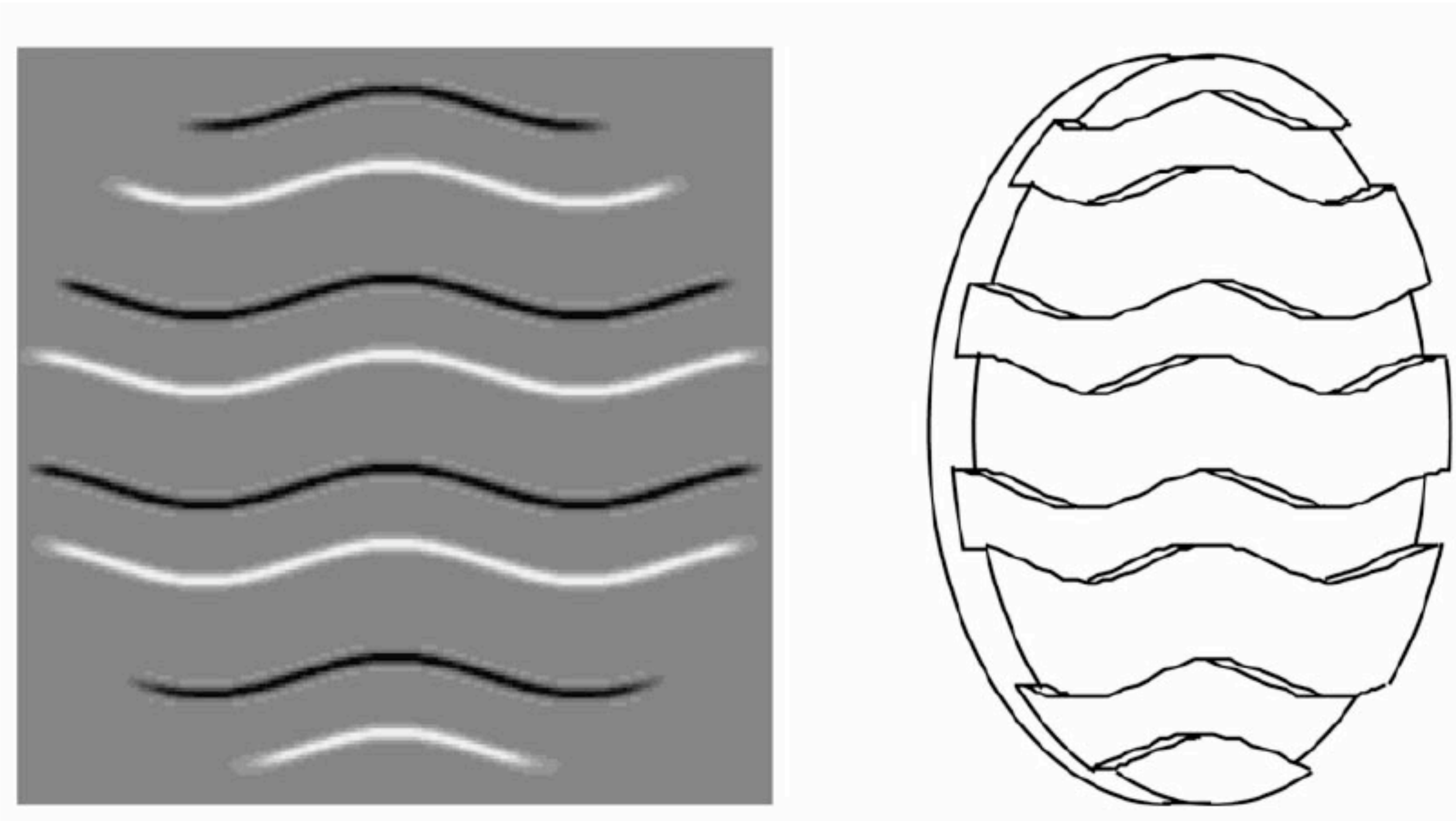
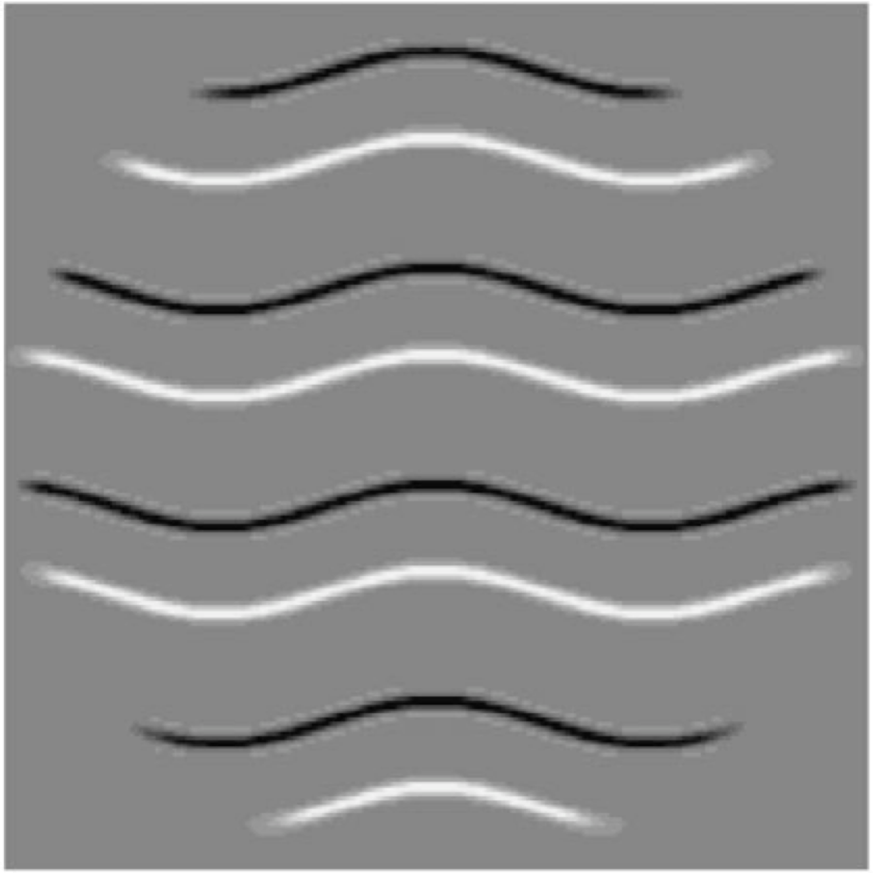
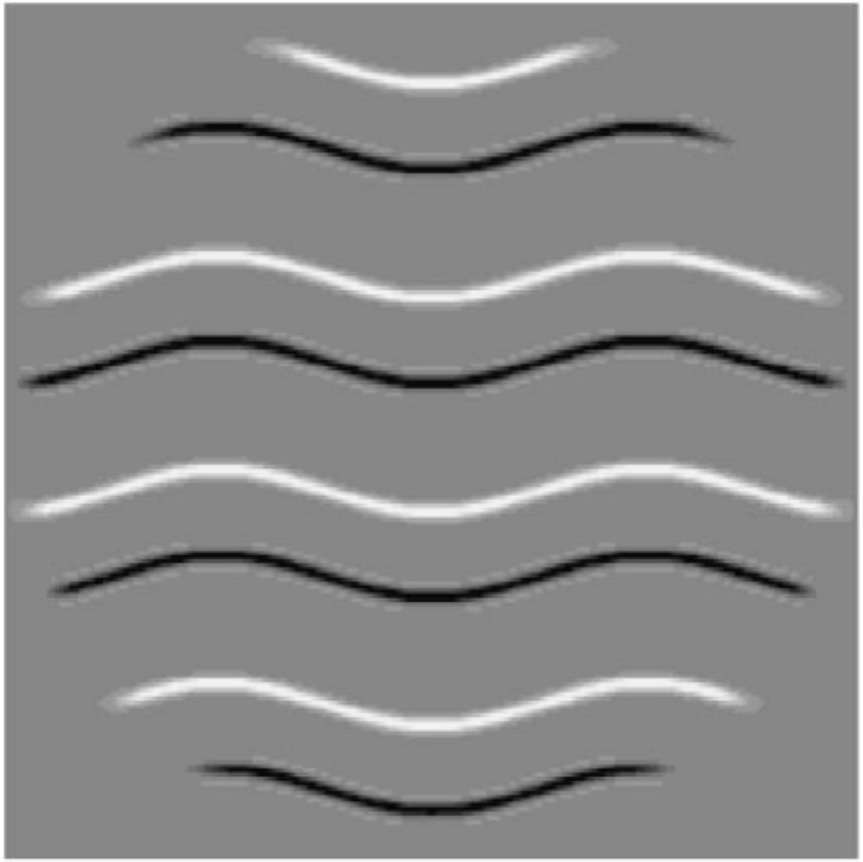


# Example: Embossed surfaces





# Example: Embossed surfaces

- Posterior:
  - P(narrow, illumination, viewpoint|stimulus)
- Variable of interest: Narrow
  - Illumination and viewpoint are nuisances
- Discounting nuisances:

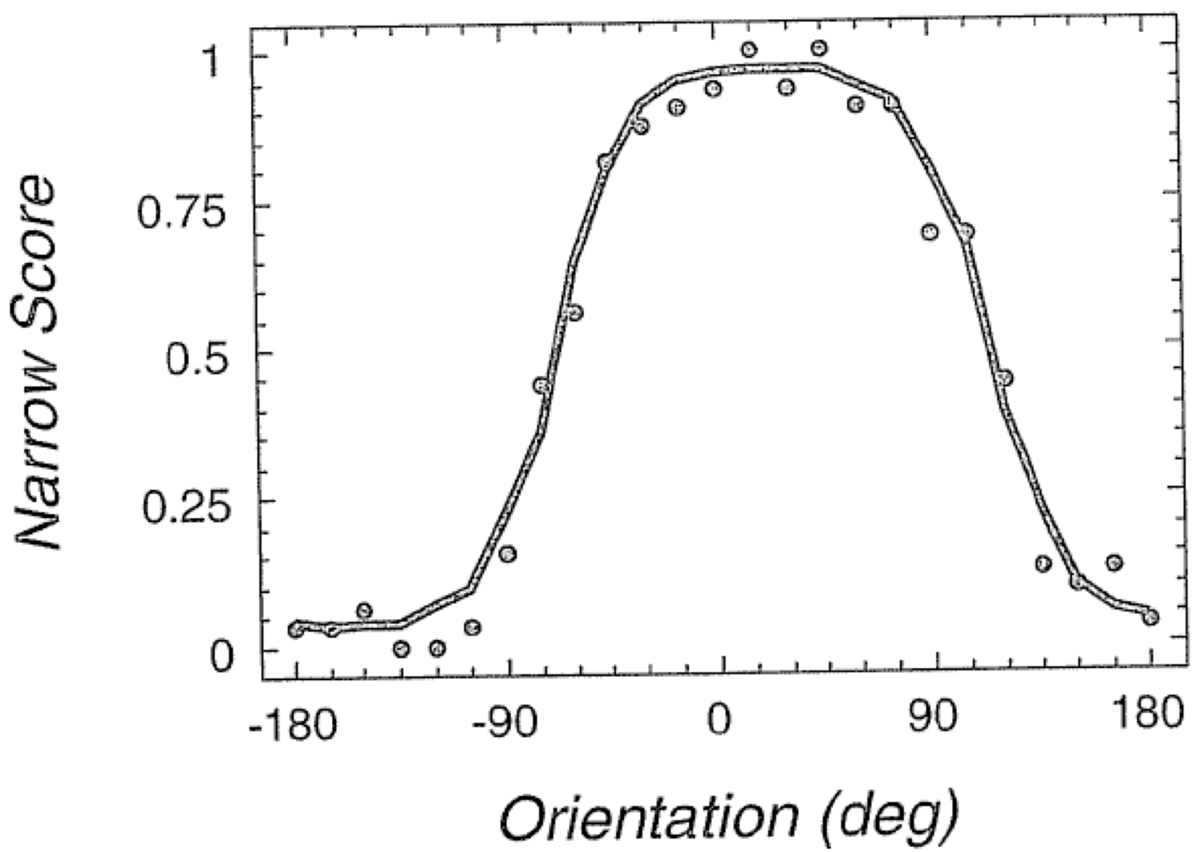
$$P(narrow \mid stimulus) = \int_{viewpoin\ and\ illum} P(narrow, viewpoin, illum \mid stimulus)\, d(viewpoin)\, d(illum)$$

# Example: Embossed surfaces

- Likelihood
  - P(stimulus|narrow, illumination, viewpoint)
  - Modeled as an illumination model
    - Bevel angle (depth of embossments)
    - Light source (point or distributed)
    - Lambertian reflectance
    - Viewpoint
- Prior: Gaussian on illumination
- Decision rule: Probability matching

# Example: Embossed surfaces

- Posterior fitted against data

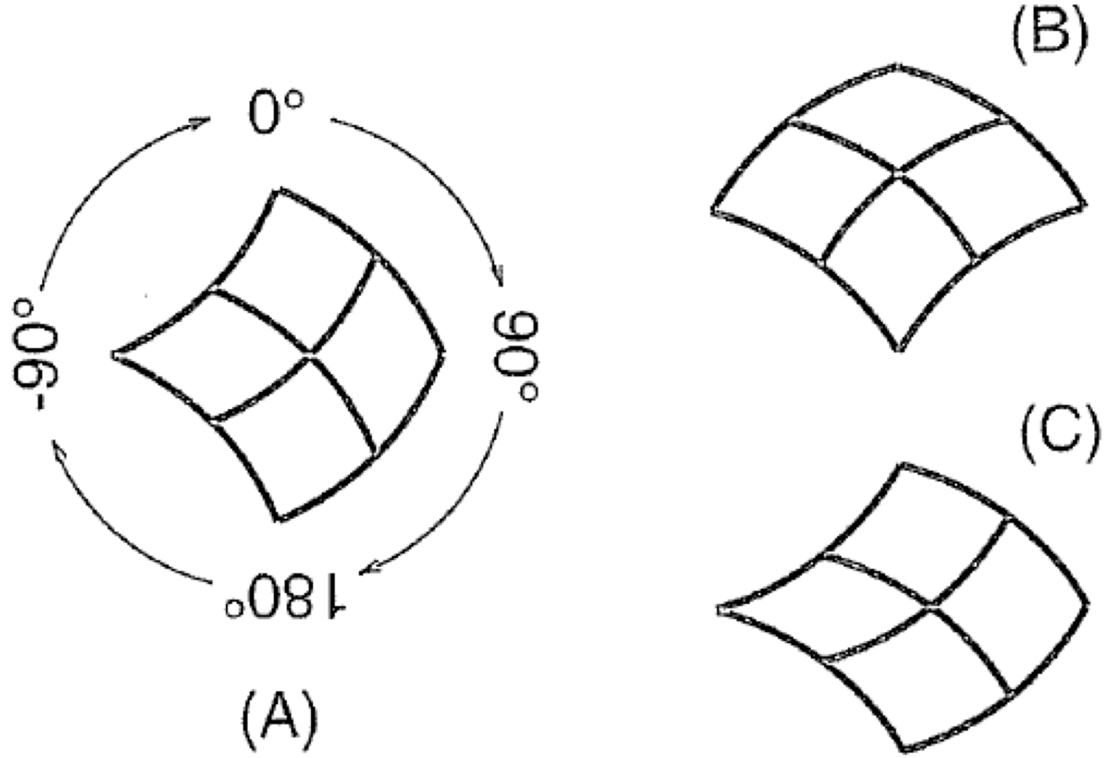


# Example: Embossed surfaces

- Nice fit to data, but
  - That is not the main point
  - Lots of free parameters could account for that
- Light from above (and slightly to the left) prior
  - Main point!

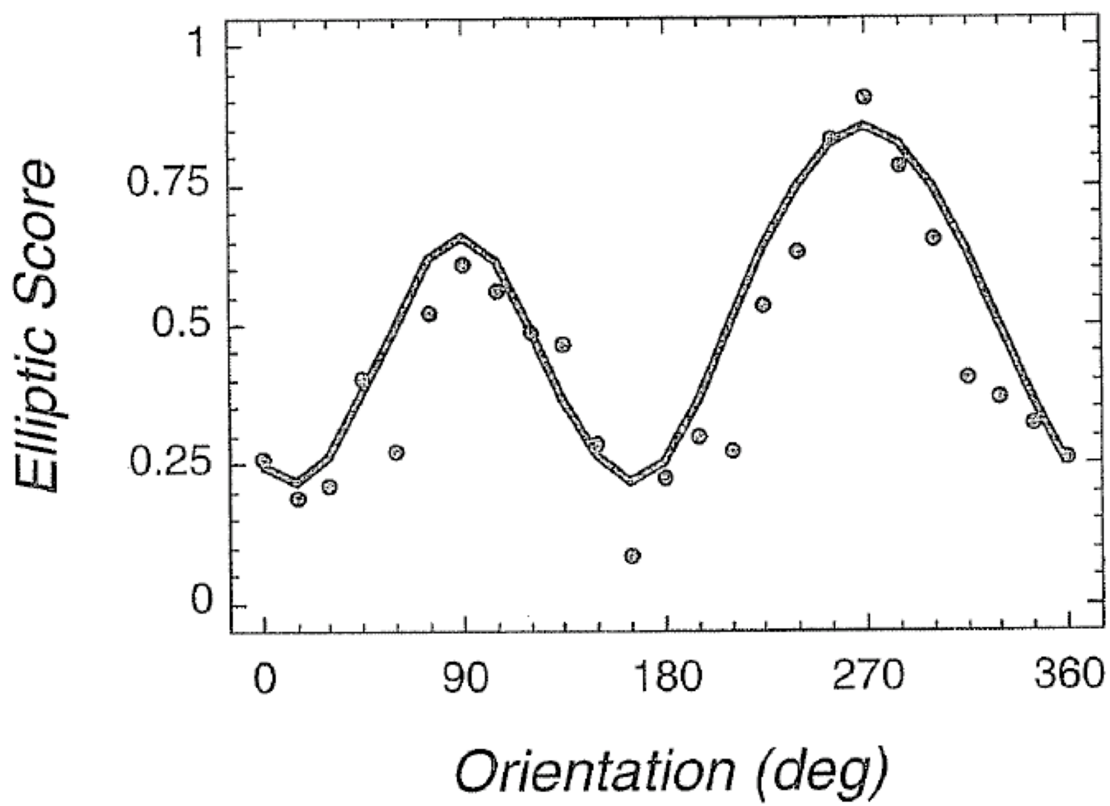
# Example: Viewpoint from above

- Shape depends on orientation
  - Saddle vs. elliptic

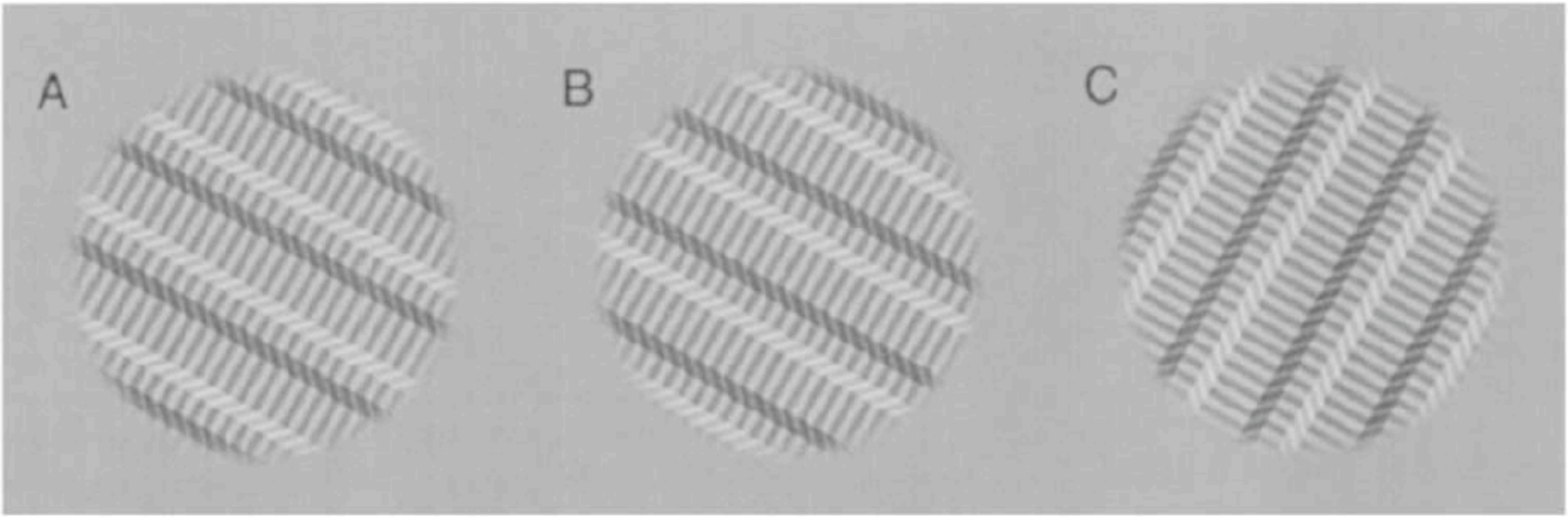


# Example: Viewpoint from above

- Bimodal posterior
- Fits data well
- Based on a viewpoint from above prior
- Transferable hypothesis



# Example: Opposing priors

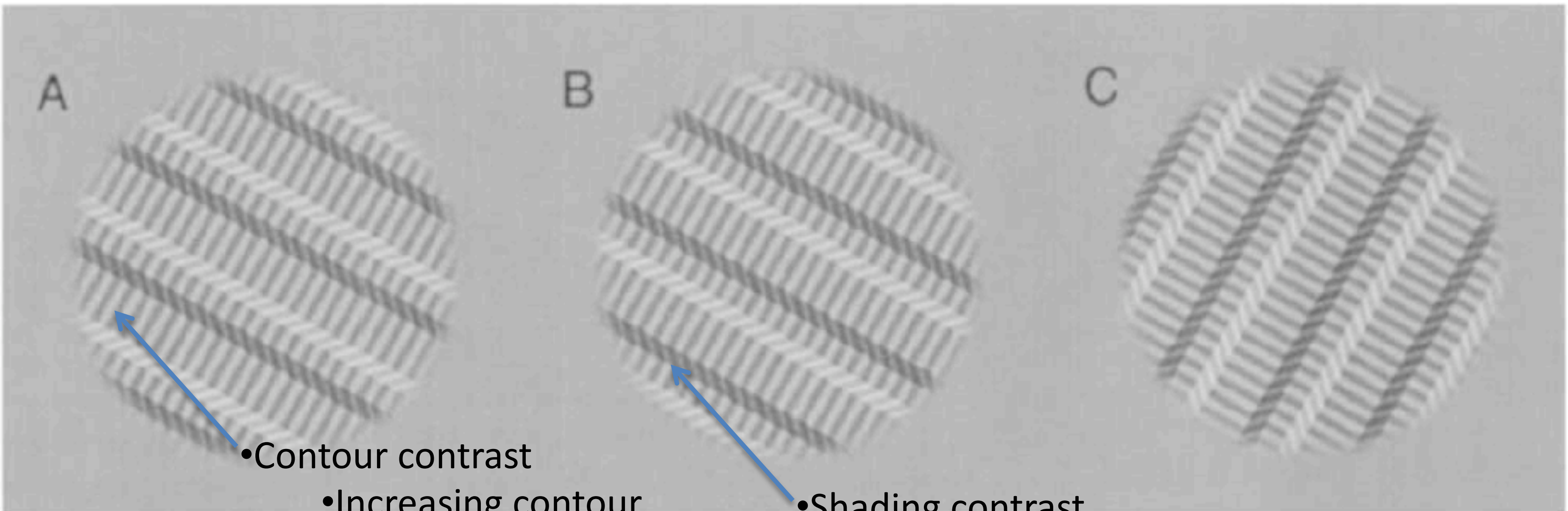



Agreeing priors make the narrow stripes bulge. Light and viewpoint is from above.

Agreeing priors make the wide stripes bulge. Light and viewpoint is from above.

Opposing priors. The figure is ambiguous. Light can be seen as coming from above and below.

# Example: Opposing priors




- Contour contrast
  - Increasing contour contrast makes the contours clearer and increase the influence of the viewpoint prior
- Shading contrast
  - Increasing shading contrast makes the shading clearer and increase the influence of the light from above prior

# Perspective

- Bayesian models are rather complex
  - Likelihood function / Generative model
  - Prior distributions (form, variance, effect of context)
  - Utility function
  - Decision rule
- Does it make sense to test such complex models?
  - Very complex models are “always right”
  - “A good scientist can draw an elephant with 3 parameters, with 4 he can draw a knot on its tail”

# Perspective

- Bayesian models is not about testing *whether* vision is Bayesian
- Bayesian models is a *framework* for formulating models
- In Bayesian models the components relates directly to the observer, the task, the stimulus
  - There are no “mystery parameters”
  - The models are directly comparable