CS 138 - Numerical Analysis II

Problem Set 3: Lecture Problems + Feedback

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1. Cascade Kingdom (60 Power Moons)

We have the following SODE,

$$(1+x)y'' + (1-x)y' + ky = 0$$

1.1. Series Substitution

Using the following substitution for y,

$$y = \sum_{n=0}^{\infty} c_n x^n$$

We derive the appropriate series representation for y',

$$y' = \sum_{n=0}^{\infty} n c_n x^{n-1}$$

$$y' = \sum_{n=1}^{\infty} n c_n x^{n-1}$$

$$\boxed{y' = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n}$$

We derive the appropriate series representation for y'',

$$y'' = \sum_{n=0}^{\infty} n(n-1) c_n x^{n-2}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

$$\boxed{y'' = \sum_{n=0}^{\infty} (n+2)(n+1) c_{n+2} x^n}$$

1.2. Recurrence Relation

Substituting our derived series representations to our SODE,

$$(1+x)y'' + (1-x)y' + ky = 0$$

$$(1+x)\left(\sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n\right) + (1-x)\left(\sum_{n=0}^{\infty}(n+1)c_{n+1}x^n\right) + k\left(\sum_{n=0}^{\infty}c_n x^n\right) = 0$$

$$\sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n + \sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^{n+1} + \sum_{n=0}^{\infty}(n+1)c_{n+1}x^n - \sum_{n=0}^{\infty}(n+1)c_{n+1}x^{n+1} + \sum_{n=0}^{\infty}kc_n x^n = 0$$

Transforming each term to yield x^n,

$$\sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n + \sum_{n=1}^{\infty}(n+1)(n)c_{n+1}x^n + \sum_{n=0}^{\infty}(n+1)c_{n+1}x^n - \sum_{n=1}^{\infty}nc_n x^n + \sum_{n=0}^{\infty}kc_n x^n = 0$$

We now need to have a common summation start index for all the terms; we choose $n = 1$. To do this, we compute the 0^{th} term of summation terms that start with $n = 0$.

For the first term,

$$\sum_{n=0}^{\infty}(n+2)(n+1)c_{n+2}x^n = 2c_2 + \sum_{n=1}^{\infty}(n+2)(n+1)c_{n+2}x^n$$

For the second term, we retain. For the third term,

$$\sum_{n=0}^{\infty}(n+1)c_{n+1}x^n = c_1 + \sum_{n=1}^{\infty}(n+1)c_{n+1}x^n$$

For the fourth term, we retain. For the fifth and final term,

$$\sum_{n=0}^{\infty}kc_nx^n = kc_0 + \sum_{n=1}^{\infty}kc_nx^n$$

Putting everything together, we get,

$$2c_2 + c_1 + kc_0 + \sum_{n=1}^{\infty}(n+2)(n+1)c_{n+2}x^n + \sum_{n=1}^{\infty}(n+1)(n)c_{n+1}x^n + \sum_{n=1}^{\infty}(n+1)c_{n+1}x^n - \sum_{n=1}^{\infty}nc_nx^n + \sum_{n=1}^{\infty}kc_nx^n = 0$$

Simplifying,

$$2c_2 + c_1 + kc_0 + x^n\sum_{n=1}^{\infty}[(n+2)(n+1)c_{n+2} + (n+1)(n)c_{n+1} + (n+1)c_{n+1} - nc_n + kc_n] = 0$$

For our starting term, for some k,

$$2c_2 + c_1 + kc_0 = 0$$
$$c_2 = \frac{-c_1 - kc_0}{2}$$

Thus the recurrence relation is given by,

$$(n+2)(n+1)c_{n+2} + (n+1)(n)c_{n+1} + (n+1)c_{n+1} - nc_n + kc_n = 0$$
$$(n+2)(n+1)c_{n+2} + (n+1)(n+1)c_{n+1} - (n-k)c_n = 0$$

Solving for c_{n+2},

$$c_{n+2} = \frac{-(n+1)(n+1)c_{n+1} + (n-k)c_n}{(n+2)(n+1)}$$

for some arbitrary c_0, c_1.

1.3. First 3 Polynomial Solutions

1.3.1. Case 1: $k = 0$

For $k = 0$, we have

$$\boxed{c_{n+2} = \frac{-(n+1)(n+1)c_{n+1} + nc_n}{(n+2)(n+1)}}$$

We now obtain the first 3 polynomial solutions,

$$\text{For } n = 0 := c_2 = \frac{-c_1}{2}$$
$$\text{For } n = 1 := c_3 = \frac{-4(c_2) + c_1}{6} = \frac{-4(\frac{-c_1}{2}) + c_1}{6} = \frac{3c_1}{6} = \frac{c_1}{2}$$
$$\text{For } n = 2 := c_4 = \frac{-9c_3 + 2c_2}{12} = \frac{-9(\frac{c_1}{2}) + 2(-\frac{c_1}{2})}{12} = \frac{-11c_1}{24}$$

Hence, we have

$$\boxed{y = c_1\left(-\frac{1}{2} + \frac{1}{2}x - \frac{11}{24}x^2\right)}$$

1.3.2. Case 2: $k = 1$

For $k = 1$,

$$\boxed{c_{n+2} = \frac{-(n+1)(n+1)c_{n+1} + (n-1)c_n}{(n+2)(n+1)}}$$

We now obtain the first 3 polynomial solutions,

$$\text{For } n = 0 := c_2 = \frac{-c_1 - c_0}{2}$$
$$\text{For } n = 1 := c_3 = \frac{-4c_2}{6} = \frac{-2(\frac{-c_1-c_0}{2})}{3} = \frac{c_1 + c_0}{3}$$
$$\text{For } n = 2 := c_4 = \frac{-9c_3 + c_2}{12} = \frac{-9\left(\frac{c_1+c_0}{3}\right) + (\frac{-c_1-c_0}{2})}{12} = \frac{-3(c_1 + c_0) + (\frac{-c_1-c_0}{2})}{12}$$
$$= \frac{-\frac{7}{2}(c_1 + c_0)}{12} = \frac{-7c_1 + c_0}{24}$$

Hence, we have

$$\boxed{y = \frac{-c_1 - c_0}{2} + \frac{c_1 + c_0}{3}x + \frac{-7c_1 + c_0}{24}x^2}$$

1.3.3. Case 3: $k = 2$

For $k = 2$,

$$\boxed{c_{n+2} = \frac{-(n+1)(n+1)c_{n+1} + (n-2)c_n}{(n+2)(n+1)}}$$

We now obtain the first 3 polynomial solutions,

$$\text{For } \ n = 0 \coloneqq c_2 = \frac{-c_1 - 2c_0}{2}$$

$$\text{For } \ n = 1 \coloneqq c_3 = \frac{-4c_2 - c_1}{6} = \frac{-4\left(\frac{-c_1 - 2c_0}{2}\right) - c_1}{6} = \frac{c_1 + 4c_0}{6}$$

$$\text{For } \ n = 2 \coloneqq c_4 = \frac{-9c_3}{12} = \frac{-9\left(\frac{c_1 + 4c_0}{6}\right)}{12} = \frac{-\frac{3}{2}c_1 - 6c_0}{12}$$

Hence, we have

$$\boxed{y = \frac{-c_1 - 2c_0}{2} + \frac{c_1 + 4c_0}{6}x + \frac{-\frac{3}{2}c_1 - 6c_0}{12}x^2}$$

2. Lake Kingdom (70 Power Moons)

2.1. Grading Mario's Functions

To accurately grade Mario's Functions with the least amount of work, we can check the symmetry of a given function, whether it is odd or even.

$$\text{If } f(-x) = f(x), \text{ then the function is even.}$$
$$\text{If } f(-x) = -f(x), \text{ then the function is odd.}$$

In Fourier series, an even function has no sine terms. On the other hand, an odd function has no cosine terms. Thus, our grading scheme can be interpreted as,

- **Grade A:** $b_i = 0$ for $i = 1, 2, 3, ... \Longleftrightarrow$ no sine terms $\Longleftrightarrow$ function is even
- **Grade B:** $a_i = 0$ for $i = 0, 1, 2, ... \Longleftrightarrow$ no cosine terms $\Longleftrightarrow$ function is odd
- **Grade C:** Either A or B $\Longleftrightarrow$ neither an even or odd function

Thus, we only need to show whether a function is even, i.e. $f(-x) = f(x)$ or odd, i.e. $f(-x) = -f(x)$ to determine whether it is graded as A, B or C.

2.2. Grading Each Function

1. $y = x$: **B**
2. $y = e^x$: **C**
3. $y = (1 - x^{2024})(1 + x^{2024})$: **A**
4. $y = \sin(\cos(-\cos(-\sin(x))))$: **A**
5. $y = |x + 138|$: **C**
6. $y = 2024^{x^{2024}}$: **A**
7. $y = \frac{1}{x+1}$: **C**
8. $y = \sin(x)\cos(x)\tan(x)\sinh(x)\cosh(x)\tanh(x)$: **A**
9. $y = \frac{138}{\sqrt{e}} \int_0^x e^{-t^2} dt$: **B**
10. $y = \frac{d}{dx}\left(\frac{138}{x^2 + x^0 + x^2 + x^4}\right)$: **B**

3. Cloud Kingdom (40 Power Moons)

We have the following differential equation

$$(1 + x)y'' + (1 - x)y' + ky = 0$$

with bounds $[-1, 1]$.

3.1. Sturm-Liouville Form

We want to rewrite the problem in the form,

$$(p(x)y')' + q(x)y = -\lambda r(x)y$$
$$p(x)y'' + p'(x)y' + q(x)y = -\lambda r(x)y$$
$$y'' + \frac{p'(x)}{p(x)}y' + \frac{q(x)}{p(x)}y = -\lambda\frac{r(x)}{p(x)}y$$

From our given differential equation, we can infer that,

$$(1+x)y'' + (1-x)y' + ky = 0$$
$$y'' + \frac{1-x}{1+x}y' + \frac{k}{1+x}y = 0$$

Solving for $p(x)$,

$$\frac{p'(x)}{p(x)} = \frac{1-x}{1+x}$$
$$\int \frac{p'(x)}{p(x)}dx = \int \frac{1-x}{1+x}dx$$
$$\ln(p(x)) = \int \frac{1-x}{1+x} = -\int \frac{x-1}{x+1}$$

$$\text{Let } u = x+1 \Rightarrow du = dx$$
$$\ln(p(x)) = -\int \frac{u-2}{u}du$$
$$= -\int du + 2\int \left(\frac{1}{u}\right)du$$
$$= -u + 2\ln(u) \Longrightarrow$$
$$\ln(p(x)) = -(x+1) + 2\ln(x+1)$$
$$p(x) = e^{2\ln(x+1)-(x+1)}$$
$$p(x) = (x+1)^2\big(e^{-(x+1)}\big)$$

Solving for $q(x)$,

$$\frac{q(x)}{p(x)} = \frac{k}{1+x}$$
$$q(x) = \frac{k}{x+1}p(x)$$
$$q(x) = \frac{k(x+1)^2\big(e^{-(x+1)}\big)}{x+1}$$
$$q(x) = k(x+1)\big(e^{-(x+1)}\big)$$

Solving for $r(x)$,

$$\frac{r(x)}{p(x)} = 1$$
$$r(x) = p(x)$$
$$r(x) = (x+1)^2\big(e^{-(x+1)}\big)$$

Thus our Sturm-Liouville form is,

$$\big(p(x)y'\big)' + q(x)y = -\lambda r(x)y$$
$$\Big((x+1)^2\big(e^{-(x+1)}\big)y'\Big)' + k(x+1)\big(e^{-(x+1)}\big)y = -k(x+1)^2\big(e^{-(x+1)}\big)y$$
$$\boxed{\Big((x+1)^2\big(e^{-(x+1)}\big)y'\Big)' = -2k(x+1)\big(e^{-(x+1)}\big)y}$$

Our inner product is given by,

$$< u, v \underset{r}{>}= \int_{-1}^{1} r(x)u(x)v(x)dx$$
$$\boxed{< u, v \underset{r}{>}= \int_{-1}^{1} u(x)v(x)dx}$$

3.2. Gram-Schmidt Orthogonalization

Let the basis be $\{1, x, x^2\}$. For the first poylnomial,

$$y_0(x) = 1$$

Our normalized first polynomial is,

$$\|y_0\| = \sqrt{\langle y_0, y_0\rangle} = \sqrt{\int_{-1}^{1}(1)(1)dx} = \sqrt{2}$$
$$\boxed{y_0(x) = \frac{1}{\sqrt{2}}}$$

For the second polynomial,

$$y_1(x) = x - \frac{\langle x, y_0\rangle}{\langle y_0, y_0\rangle}y_0(x)$$

$$\langle x, y_0\rangle = \int_{-1}^{1}\frac{x}{\sqrt{2}}dx = \frac{1}{\sqrt{2}}\int_{-1}^{1}xdx = 0$$
$$\implies y_1(x) = x$$

Our normalized second polynomial is,

$$\|y_1\| = \sqrt{\langle y_1, y_1 \rangle} = \sqrt{\int_{-1}^{1} (x)(x) dx} = \sqrt{\frac{2}{3}}$$

$$\boxed{y_1(x) = \sqrt{\frac{3}{2}} x}$$

For the third polynomial,

$$y_2(x) = x^2 - \frac{\langle x^2, y_0 \rangle}{\langle y_0, y_0 \rangle} y_0(x) - \frac{\langle x^2, y_1 \rangle}{\langle y_1, y_1 \rangle} y_1(x)$$

$$\langle x^2, y_0 \rangle = \int_{-1}^{1} x^2 \left(\frac{1}{\sqrt{2}} \right) dx = \frac{\sqrt{2}}{3}$$

$$\langle x^2, y_1 \rangle = \int_{-1}^{1} x^2 \left(x \sqrt{\frac{3}{2}} \right) dx = \sqrt{\frac{3}{2}} \int_{-1}^{1} x^3 dx = 0$$

$$\implies y_2(x) = x^2 - \left(\frac{\sqrt{2}}{3} \right) \left(\frac{1}{\sqrt{2}} \right) = x^2 - \frac{1}{3}$$

Our normalized third polynomial is,

$$\|y_2\| = \sqrt{\langle y_2, y_2 \rangle} = \int_{-1}^{1} \left(x^2 - \frac{1}{3} \right)^2 dx = \frac{8}{45}$$

$$\boxed{y_2(x) = \frac{x^2 - 1}{\sqrt{\frac{8}{45}}}}$$

4. Metro Kingdom (70 Power Moons)

We have the advection equation,

$$u_t = -1.38 u_x$$

with $x \in [0, 1]$ and $t \geq 0$.

4.1. Solution to PDE

Given $u(0, x) = f(x)$, need to show $u(t, x) = f(x - 1.38t)$,

$$u_t = -1.38 f'(x - 1.38t)$$
$$u_x = f'(x - 1.38t)$$

Since we have,

$$u_t = -1.38 u_x$$

Substituting u_t and u_x,

$$-1.38 f'(x - 1.38t) = -1.38 f'(x - 1.38t)$$

which holds true. Thus, $u(t, x) = f(x - 1.38t)$ is a solution.

4.2. Characteristic of PDE

I give up :(

5. Seaside Kingdom (50 Power Moons)

We have the following wave equation,

$$u_{tt} = 1.38u_{xx}$$

where

$$u(0, x) = x$$
$$u(t, 1) = 1 - x$$

and

$$u(t, 0) = u(t, 1) = 1.5$$

5.1. Find Two ODEs

Let $u = f(x)g(t)$. Then we have $u_{tt} = f(x)g''(t)$ and $u_{xx} = f''(x)g(t)$. Substituting to our wave equation,

$$f(x)g''(t) = 1.38f''(x)g(t)$$
$$f(x)g''(t) = 1.38f''(x)g(t) = -\lambda, \text{for some } \lambda \geq 0$$

For the first ODE derived from $g(t)$,

$$\frac{g''(t)}{1.38g(t)} = -\lambda$$
$$g''(t) = -1.38\lambda g(t)$$
$$\boxed{g''(t) + 1.38\lambda g(t) = 0}$$

For the second ODE derived from $f(x)$,

$$\frac{f''(x)}{f(x)} = -\lambda$$
$$f''(x) = -\lambda f(x)$$
$$\boxed{f''(x) + \lambda f(x) = 0}$$

5.2. Identify BVP

The BVP is $f''(x) + \lambda f(x) = 0$. Rewriting this,

$$(f'(x))' = -\lambda f(x)$$

We can compare this with the Sturm-Liouville form,

$$(p(x)y')' + q(x)y = -\lambda r(x)y$$

and we yield the following values,

$$p(x) = 1$$
$$q(x) = 0$$
$$r(x) = 1$$

We can also yield the Boundary Conditions as follows,

$$u(t, 0) = 1.5$$
$$u(t, 1) = 1.5$$

$$f(0)g(t) = 1.5$$
$$f(1)g(t) = 1.5$$
$$\implies f(0) = f(1) = 1.5$$

Thus, forming our inner product,

$$\langle f, g \rangle = \int_0^1 f(x)g(x)r(x)dx$$

$$\boxed{\langle f, g \rangle = \int_0^1 f(x)g(x)dx}$$

6. Luncheon Kingdom (70 Power Moons)

Given the heat equation,

$$u_t = u_{xx}, x \in [0, 1] \text{ for } t \geq 0$$

where

$$u(0, x) = 1$$

and

$$u(t, 0) = u(t, 1)$$
$$u_x(t, 0) = u_x(t, 1)$$

as well as the ODEs

$$f''(x) = -kf(x), \text{for } k \geq 0$$
$$\text{BC: } f(0) = f(1), f'(0) = f'(1)$$

$$g'(t) = -kg(t), \text{for } k \geq 0$$

6.1. Finding Eigenfunctions

6.1.1. Space Dependent ODE

Given the equation

$$f''(x) = -kf(x)$$

7. Ruined Kingdom (40 Power Moons)

Let $u(t, x)$ be the value of solution at time t and position x, and use the notation u_j^n to represent the value of u at the j-th spatial grid point and the n-th time step.

$$u_j^n = u(n\Delta t, j\Delta x)$$

where $\Delta t = 0.1$ and $\Delta x = 0.5$.

7.1. Forward Approximation

The forward difference approximation in t is given by,

$$u_t \approx \frac{u_j^{n+1} - u_j^n}{\Delta t}$$

The cented difference approximation in x is given by,

$$u_{xx} \approx \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}$$

Substituting in to the heat equation and solving for u_j^{n+1},

$$u_t = u_{xx}$$

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}$$

$$u_j^{n+1} = u_j^n + \left(\frac{\Delta t}{\Delta x^2}\right)\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)$$

$$u_j^{n+1} = u_j^n + (0.4)\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)$$

$$\boxed{u_j^{n+1} = (0.2)u_j^n + (0.4)u_{j+1}^n + (0.4)u_{j-1}^n}$$

7.2. Backward Approximation

The backward difference approximation for t is given by,

$$u_t \approx \frac{u_j^n - u_j^{n-1}}{\Delta t}$$

The centered difference approximation for x is given by,

$$u_{xx} \approx \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}$$

Substituting in to the heat equation and solving for u_j^{n-1},

$$u_t = u_{xx}$$

$$\frac{u_j^n - u_j^{n-1}}{\Delta t} = \frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2}$$

$$u_j^{n-1} = u_j^n - \left(\frac{\Delta t}{\Delta x^2}\right)\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)$$

$$u_j^{n-1} = u_j^n - (0.4)\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)$$

$$\boxed{u_j^{n-1} = (1.8)u_j^n - (0.4)u_{j+1}^n - (0.4)u_{j-1}^n}$$

7.3. Crank-Nicholson

The Crank-Nicholson method is given by,

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{1}{2}\left[\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2} + \frac{u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}}{\Delta x^2}\right]$$

$$u_j^{n+1} - u_j^n = \frac{\Delta t}{2\Delta x^2}\left[\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right) + \left(u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}\right)\right]$$

Note that $\frac{\Delta t}{2\Delta x^2} = 0.2$. Rearranging, we get,

$$u_j^{n+1} - (0.2)\left(u_{j+1}^{n+1} - 2u_j^{n+1} + u_{j-1}^{n+1}\right) = u_j^n + (0.2)\left(u_{j+1}^n - 2u_j^n + u_{j-1}^n\right)$$

$$(1.4)u_j^{n+1} - (0.2)u_{j+1}^{n+1} - (0.2)u_{j-1}^{n+1} = (0.6)u_j^n + (0.2)u_{j+1}^n + (0.2)u_{j-1}^n$$

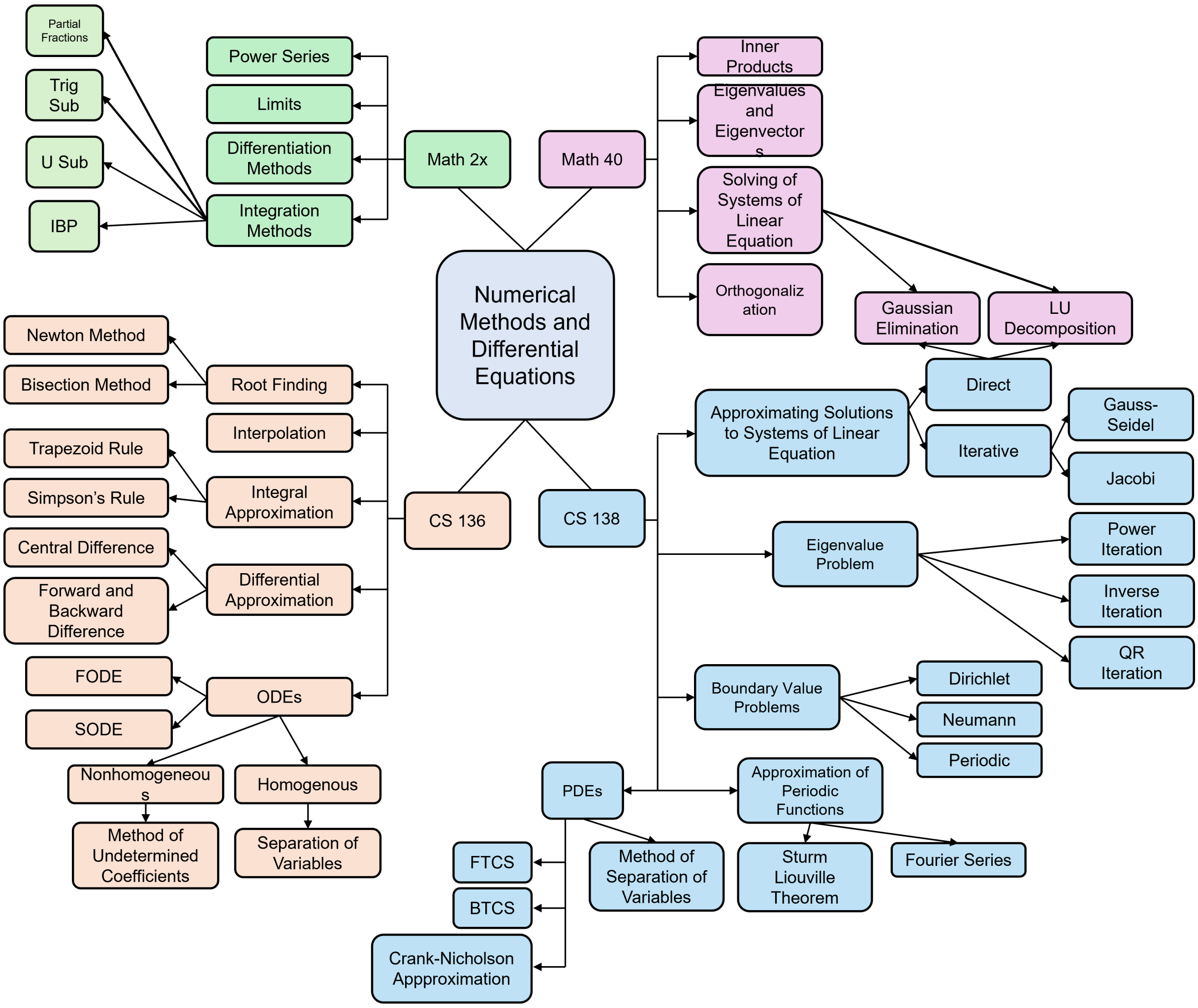
Thus we can form our matrix of the form $Au^{n+1} = Bu^n$ with 6 spatial points,

$$\boxed{Au^{n+1} = \begin{bmatrix} 1.4 & -0.2 & 0 & 0 & 0 & 0 \\ -0.2 & 1.4 & -0.2 & 0 & 0 & 0 \\ 0 & -0.2 & 1.4 & -0.2 & 0 & 0 \\ 0 & 0 & -0.2 & 1.4 & -0.2 & 0 \\ 0 & 0 & 0 & -0.2 & 1.4 & -0.2 \end{bmatrix} \begin{bmatrix} u_1^{n+1} \\ u_2^{n+1} \\ u_3^{n+1} \\ u_4^{n+1} \\ u_5^{n+1} \\ u_6^{n+1} \end{bmatrix}}$$

$$\boxed{Bu^n = \begin{bmatrix} 0.6 & 0.2 & 0 & 0 & 0 & 0 \\ 0.2 & 0.6 & 0.2 & 0 & 0 & 0 \\ 0 & 0.2 & 0.6 & 0.2 & 0 & 0 \\ 0 & 0 & 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 & 0.6 & 0.2 \end{bmatrix} \begin{bmatrix} u_1^n \\ u_2^n \\ u_3^n \\ u_4^n \\ u_5^n \\ u_6^n \end{bmatrix}}$$

8. Moon Kingdom (BONUS: 70 Power Moons)

8.1. Mind Map



8.2. Connection of Related Terms and Mind Map Explainer

The mind map shows how topics from Math 2x and Math 40, such as integration, differentiation, and linear algebra, serve as the foundation for advanced numerical methods in CS 136 and differential equations in CS 138. Integration and differentiation methods connect directly to numerical integration (e.g., Trapezoid Rule) and derivative approximations (e.g., Central Differences), which are essential for solving ODEs and PDEs. Linear algebra concepts like Gaussian Elimination and eigenvalue problems underpin solving systems of equations and analyzing eigenfunctions in boundary value problems. Numerical methods, such as Newton's Method and interpolation, enable approximations for root-finding and integral solutions. In CS 138, these methods extend to solving FOODE, SODE, and PDEs using boundary conditions (Dirichlet, Neumann) and techniques like Fourier Series and Crank-Nicholson for numerical PDE solutions. Together, these topics form a cohesive framework for solving real-world computational problems.

As for a suggestion, I believe it make much more sense to relegate some CS 138 topics down to CS 136, specifically the ones that concern with Math 40 topics. This is so that we can start the Numerical Analysis series with Linear Algebra topics still fresh in mind.

9. Mushroom Kingdom (BONUS: 80 Power Moons)

9.1. Meme

9.2. Problem Set 3 Feedback

I had a hard time answering the problems, but I think that's just the nature of the subject; it's really hard. It was also a bit overwhelming seeing the number of problems we have for this Problem Set, but making the denominator lower helped alleviate the pressure. Otherwise, the website aspect of PS3 is pretty cool, and I appreciate the theming not only for this PS3 but also for the previous ones. Although, I think it might be better to just have the website roll out all the problems at once, since it would be useful for studying for the exam ahead of time. Other than that, I say continue the website implementation of future Problem Sets!

9.3. BGM Prediction

ブルーアーカイブ Blue Archive OST 138. Utaha No Uta.
Because 138.