Conference Program

Arithmetic Geometry in Cabourg

Organizers: François Ballaÿ, Jérôme Poineau, and Robert Wilms
(Université de Caen Normandie)

12.–16. Mai 2025

Schedule

Arrival: 11 May 2025 **Departure:** 16 May 2025 (after lunch)

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9:00	Pazuki (9:30)	Schmidt	Burgos Gil	Dill	Mavraki
10:00		Coffee break	Coffee break	Coffee break	Coffee break
10:30	Coffee break	Sedillot	Sombra	Javanpeykar	Gauthier (10:15)
11:45	de Jong (11:00)	Szachniewicz*	Biswas*	Bartsch*	
12:30	Lunch	Lunch	Lunch	Lunch	Lunch (12:00)
15:00	Pengo	Nicolussi	Free Afternoon	Habegger	
16:00	Coffee break	Coffee break		Coffee break	
16:30	Hultberg	Pille-Schneider		Mehmeti	

All talks last 1 hour; only talks marked with $*$ last 45 minutes.

Talks and Abstracts

Finn Bartsch (Radboud Universiteit Nijmegen)

Symmetric products and puncturing Campana-special varieties

In 2001, Hassett and Tschinkel posed the following "puncturing problem": If X is a projective variety with at most canonical singularities such that no finite étale cover of X dominates a variety of general type and Z is a closed subset of X of codimension at least 2, does it follow that no finite étale cover of $X \setminus Z$ dominates a variety of log-general type? Following the philosophy that maps to varieties of general type should be the main obstruction to density of rational points, they also suggested the following "arithmetic puncturing problem": With X and Z as above, if the rational points on X are potentially dense, are the integral points on $X \setminus Z$ potentially dense? In this talk, I will explain how symmetric powers of products of curves provide counterexamples to both of these puncturing problems. On the other hand, conjectures of Campana suggest that the arithmetic puncturing problem has a positive answer if we additionally assume X to be smooth. This is joint work with Ariyan Javanpeykar and Aaron Levin.

Debam Biswas (Universität Regensburg)

Differentiability of volumes and equidistribution on quasi-projective varieties

In this talk, I will speak about the differentiability of arithmetic and geometric volumes in the quasi-projective setting of Yuan and Zhang. If time permits I will mention how the results can be extended to obtain logarithmic equidistribution for a subclass of logarithmically singular functions on projective varieties.

José Ignacio Burgos Gil (ICMAT Madrid)

The arithmetic degree of the line bundle of Siegel–Jacobi modular forms

The singularities of the line bundle of Siegel–Jacobi modular forms are not logarithmic and the usual techniques to deal with automorphic vector bundles on pure Shimura varieties do not apply. Yuan and Zhang have proved that this singular line bundle is an example of adelic line bundle on a quasi-projective variety. Nevertheless it is not clear whether this line bundle is integrable, so the techniques of Yuan and Zhang also do not apply. We will show how to use the recent development on relative finite energy singularities by Darvas, Di Nezza and Lu to extend Yuan and Zhang intersection product to cover the line bundle of Siegel-Jacobi modular forms and using its functorial properties we will compute its arithmetic degree. This is joint work with J. Kramer.

Gabriel Dill (Université de Neuchâtel)

Arithmetic Unlikely Intersections in Powers of the Multiplicative Group (or: the Deeper Meaning of 1, 1, 4, 25, 11, 153664, . . .)

In joint work in progress with Francesco Campagna, we formulate a conjecture about unlikely intersections in split semiabelian schemes over some ring of S-integers in a number field. Broadly speaking, if an intersection with a subgroup scheme is unlikely for dimension reasons, its "size" should not be too big compared to the "complexity" of the subgroup scheme. Focusing on the case of powers of the multiplicative group over the rational integers, I will show some evidence that we have acquired so far for our conjecture and discuss in some detail the example of a certain arithmetic surface inside the fibered cube of the multiplicative group over the integers, which leads to the sequence of integers from the title.

Thomas Gauthier (Université Paris-Saclay)

Uniformity in arithmetic dynamics

The dynamical Bogomolov conjecture is a dynamical counterpart of the classical Bogomolov conjecture. Roughly speaking, it states that given a polarized endomorphism $f : X \to X$ of a projective variety and a subvariety $Z \subset X$, all defined over a number field, the subvariety contains a Zariski dense and small sequence for an appropriate canonical height function if and only if it is preperidoic – except for obvious counter-examples.

In a joint work with Johan Taflin and Gabriel Vigny, we study uniform versions of this conjecture. We prove several results. As a particular case, we obtain a dynamical proof of a uniform version of a Bogomolov type statement for algebraic tori.

Philipp Habegger (Universität Basel)

The transfinite diameter of finite trees and the dynamical Schinzel–Zassenhaus Conjecture

In 2019, Dimitrov proved the Schinzel–Zassenhaus Conjecture. Harry Schmidt and I showed how his general strategy can be adapted to cover some dynamical variants of this conjecture. One common tool in both results is Dubinin's Theorem on the transfinite diameter of hedgehogs.

In this talk, I will report on joint work in progress with Harry Schmidt. We find new upper bounds for the transfinite diameter of some finite topological trees. We construct these trees using the Hubbard tree of a postcritically finite map. They are better suited to the dynamical setting than hedgehogs. We establish bounds for the transfinite diameter of such trees. One such bound is ultimately motivated by Mahler's work on root separation and a new proof of Dubinin's Theorem (with a worse constant). As a consequence, we can cover new cases of the dynamical Schinzel–Zassenhaus Conjecture and obtain new lower bounds for the Call–Silverman height.

Nuno Hultberg (Westlake University)

Globally valued fields as generic points in Arakelov geometry

After giving a gentle introduction to globally valued fields and GVF analytifications I present 3 applications of globally valued fields. The density of $h(x) + h(x+1)$ above the essential minimum, an alternative proof of the uniform Bogomolov conjecture by reducing it to a result of Chen and Moriwaki over adelic curves and the conjecture of Roberto Gualdi on limit heights. This is partly based on joint work with Pablo Destic and Michal Szachniewicz.

Ariyan Javanpeykar (Radboud Universiteit Nijmegen)

The weakly special conjecture contradicts Orbifold Mordell (and hence abc)

Which varieties over a number field have a potentially dense set of rational points? Lang conjectured that varieties of general type over a number field have very few rational points. In 2000, guided by Lang's conjecture and in search of a converse statement, Abramovich, Colliot-Thélène, Harris, and Tschinkel formulated the "Weakly Special Conjecture": every weakly special variety over a number field has a potentially dense set of rational points. In this talk I will explain how this conjecture contradicts the abc conjecture, and more precisely Campana's "Orbifold Mordell" conjecture. This is joint work with Finn Bartsch, Frederic Campana, and Olivier Wittenberg.

Robin de Jong (Universiteit Leiden)

On Néron's canonical local heights for abelian varieties

This is based on joint work with Farbod Shokrieh. We review Néron's canonical local heights on abelian varieties over non-archimedean valued fields from the point of view of Berkovich analytic spaces. Using the canonical skeleton of an abelian variety we present a refinement of Néron's classical result relating his local heights with intersection multiplicities on the Néron model. Our results can be viewed as an extension of Tate's explicit formulas for elliptic curves to higher dimensions. If time permits we will discuss some applications.

Myrto Mavraki (University of Toronto)

Uniformity in the dynamical André–Oort conjecture

Many problems in arithmetic geometry have the following form: given a subvariety X of a variety M and a subset Ξ of M, can one describe the structure of the components of the Zariski closure of $X \cap \Xi$? These questions become particularly interesting when the set Ξ has some 'special' structure (perhaps related to a group law in M). The expectation is then that the components of $\overline{X \cap \Xi}$ will inherit this structure and be 'Ξ-special' themselves. Examples of problems in this form, called 'unlikely intersections', include the Manin–Mumford conjecture, the Mordell–Lang conjecture and the André–Oort conjecture.

Post Critically finite maps (PCF) are those whose critical points are preperiodic – they play a special role within the moduli space $\mathcal{M}_d$ of degree d rational maps. In this talk we will discuss the dynamical André–Oort Conjecture (DAO), which asks for a classification of the PCF-special subvarieties in $\mathcal{M}_d$. DAO was recently proven in the case of curves by Ji–Xie, following works by many authors, but remains open in higher dimensions. We will discuss results obtained with L. DeMarco and H. Ye, on bounding the geometry of the PCF-special subvarieties. Our results can be thought of as a 'uniform DAO'.

Vlerë Mehmeti (Sorbonne Université)

Variation of the Hausdorff dimension and degenerating Schottky groups

I will speak of the Hausdorff dimension of the limit set of a Schottky group defined over an arbitrary complete valued field. In 2021, Poineau and Turchetti constructed a moduli space for these groups using Berkovich spaces over $\mathbb{Z}$. I will present a result on the continuity of the Hausdorff dimension of limit sets on this moduli space, and conclude with an application to degenerating families of complex Schottky groups. This is based on joint work with Nguyen-Bac Dang.

Noema Nicolussi (TU Graz)

Hybrid curves and their moduli spaces

This talk provides an introduction to hybrid curves, a geometric object which mixes Riemann surfaces and graphs, and their moduli spaces.

The moduli space of hybrid curves refines the classical Deligne–Mumford compactification of the moduli space of Riemann surfaces. It provides a framwork to answer several analytic questions on degenerating Riemann surfaces (e.g., asymptotics of canonical measures and the Arakelov–Green function) which cannot be studied in the Deligne–Mumford compactification due to discontinuity phenomena. In addition, hybrid curves enjoy analogs of fundamental theorems on smooth compact Riemann surfaces, e.g. a Riemann–Roch and Abel–Jacobi theorem.

In this talk, we introduce hybrid curves and overview several results.

Based on joint work with Omid Amini.

Fabien Pazuki (Københavns Universitet)

Parallelogram inequality for abelian varieties and applications

Let A be an abelian variety defined over a number field. A theorem of Rémond states that for any two finite subgroup schemes G, H, the Faltings height of the four isogenous abelian varieties $A/G, A/H, A/(G+H), A/(G\cap H)$ are linked by an elegant inequality, which has important applications in diophantine geometry. The goal of the talk is to present an analogous inequality for abelian varieties defined over function fields (in any characteristic). This is joint work with Richard Griffon and Samuel Le Fourn.

Riccardo Pengo (Università degli Studi di Messina)

Diophantine properties of special values of L-functions

According to Northcott's theorem, each set of algebraic numbers whose height and degree are bounded is finite. Analogous finiteness properties are also satisfied by many other heights, such as Faltings's celebrated height. Given the many (expected and proven) links between heights and special values of L-functions (with the BSD conjecture as the most remarkable example), it is natural to ask whether the special values of an L-function satisfy a Northcott property. In this talk, based on a joint work with Fabien Pazuki, and on another joint work with Jerson Caro and Fabien Pazuki, we will explain how this Northcott property is often satisfied for special values taken at the left of the critical strip, and not satisfied on the right. We will also overview the links between these Northcott properties and those of the motivic heights defined by Kato, and also some effective aspects of our work, which aim at giving some explicit bounds for the cardinality of the finite sets that we come across.

Leonard Pille-Schneider (Universität Regensburg)

Continuity of families of Monge–Ampère measures

Let R be a discrete valuation ring, and $(X, L)/R$ a normal integral polarized scheme. We prove that if φ is a continuous family of semi-positive metrics on the Berkovich analytification X^{an}, varying in a semi-positive way with respect to the base $M(R)$ (the Berkovich spectrum of R), then the associated family of fiberwise Monge-Ampère measures is weakly continuous on the total space.

I will also try to discuss some applications.

Harry Schmidt (University of Warwick)

On Galois orbits of pre-periodic points

In joint work with Philipp Habegger we study points of small canonical height for certain polynomial dynamical systems. For example, we prove that pre-periodic points of post-critically finite quadratic polynomials enjoy Galois properties analogous to those of torsion points of elliptic curves with complex multiplication. Our work has consequences for the study of dynamical portraits, and I will explain this and some other analogies between families of polynomials and elliptic curves.

Antoine Sedillot (Universität Regensburg)

Pseudo-absolute values and topological adelic curves

In this talk, we will introduce an approach to Arakelov geometry over possibly uncountable base fields. To do so, we introduce objects called topological adelic curves. Roughly speaking, a topological adelic curve is a family of pseudo-absolute values (a generalisation of the usual notion of absolute value) on a given field parametrised by a topological space and such that a product formula holds. An important feature available in this context is some (embryonic) attempt to implement the analogy between Nevanlinna theory and Diophantine geometry. Moreover, this framework shares many similarities with the framework of adelic curves introduced by Chen and Moriwaki. It is also related to the theory of globally valued fields introduced by Ben Yaacov-Hrushovski.

It turns out that the space of all pseudo-absolute values on a given field can be seen as an analytic space living over the Zariski-Riemann space of the field, namely the set of its valuations rings. If time permits, we will say a few words on some work in progress in the direction of understanding how analytic geometry on the space of pseudo-absolute values could help to understand adelic geometry.

Martin Sombra (ICREA and Universitat de Barcelona)

Approximation of adelic divisors and equidistribution of small points

Let $\overline{D}$ be an adelic divisor on a projective variety X over a number field. We say that a generic sequence of algebraic points of X is small if their heights with respect to $\overline{D}$ converge to the smallest possible value, namely the essential minimum of the height function. Yuan's equidistribution theorem (2008) describes the asymptotic distribution of the Galois orbits of points in a generic small sequence, under the assumption that the essential minimum coincides with the normalized height of X. This hypothesis holds in important cases such as Néron–Tate heights on abelian varieties and dynamical heights on projective varieties, but it fails for most choices of $(X, \overline{D})$. A theorem of Burgos Gil, Philippon, Rivera-Letelier and Sombra (2019) enlightens many new cases of equidistribution phenomena on toric varieties, that fall outside the scope of Yuan's theorem. In this talk, I will present a generalization of this theorem valid for all projective varieties. This result applies in particular to canonical heights on semi-abelian varieties, and thus permits to recover Kühne's equidistribution theorem (2019) in this setting.

Michał Szachniewicz (University of Oxford)

Globally Valued Fields – foundations and perspectives

I will advertise a model theoretic formalism for studying heights, defined by Ben Yaacov and Hrushovski. I will compare it to other approaches for global geometry, notably with adelic curves of Chen, Moriwaki and with adelic line bundles of Yuan, Zhang. I will mention an application of GVFs from a joint work with Pablo Destic and Nuno Hultberg. If time permits, I will pose some questions regarding the arithmetic Siu inequality and its relevance to the model theory of GVFs (based on a work in progress with Antoine Sedillot).