UNIVERSITY OF CALIFORNIA, SAN DIEGO
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Homework #1

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Course: *CSE 251B, WI 21* – Professor: *Dr. Gary Cottrell*
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Question 1

Work problems 1-4 (5 points each) on pages 28-30 of Bishop.

Answer.

(*a*) From (1.41) we could derive

$$\begin{aligned}
&\int_{-\infty}^{\infty} exp\{-\frac{\lambda}{2}x^2\}dx = (\frac{2\pi}{\lambda})^{1/2} \\
\Rightarrow &\int_{-\infty}^{\infty} exp\{-(\sqrt{\frac{\lambda}{2}}x)^2\}d\sqrt{\frac{\lambda}{2}}x = \pi^{1/2} \\
\Rightarrow &\int_{-\infty}^{\infty} exp\{-x^2\}dx = \pi^{1/2}
\end{aligned} \tag{1}$$

Therefore, the result of the L.H.S of (1.42) is $\pi^{d/2}$, thus we can convert the 1.42 into following form

$$\begin{aligned}
&\pi^{d/2} = S_d \int_0^{\infty} e^{-r^2} r^{d-1} dr \\
\Leftrightarrow &S_d = \frac{\pi^{d/2}}{\int_0^{\infty} e^{-r^2} r^{d-1} dr} \\
\Leftrightarrow &S_d = \frac{2\pi^{d/2}}{\int_0^{\infty} e^{-r^2} r^{d-2} 2r dr} \\
\Leftrightarrow &S_d = \frac{2\pi^{d/2}}{\int_0^{\infty} e^{-r^2} r^{d-2} dr^2} \\
\Leftrightarrow &S_d = \frac{2\pi^{d/2}}{\int_0^{\infty} e^{-u} u^{\frac{d}{2}-1} du} \\
\Leftrightarrow &S_d = \frac{2\pi^{d/2}}{\Gamma(d/2)} (Inserting(1.44))
\end{aligned} \tag{2}$$

Hence we have proved the (1.43).

When $d = 2$, $S_d = \frac{2\pi}{\Gamma(1)} = 2\pi$ and when $d = 3$ we have $S_d = \frac{2\pi^{3/2}}{\Gamma(3/2)} = 4\pi$, which are equal to well-know expressions of circle and 3D sphere.

(*b*) By utilizing the (1.43)

$$\begin{aligned} S_d(r) &= S_d(1)r^d = S_d r^{d-1} \\ V_d &= \int_0^a S_d(r)dr \\ \Leftrightarrow &= S_d \int_0^a r^{d-1}dr \\ \Leftrightarrow &= \frac{S_d a^d}{d} \end{aligned} \tag{3}$$

It's easy to know that the volume of hypercube is $(2a)^d$, so the ratio would be

$$\begin{aligned} \frac{VS_d}{VC_d} &= \frac{S_d a^d}{d(2a)^d} \\ &= \frac{2\pi^{d/2}a^d}{d2^d a^d \Gamma(d/2)} \\ &= \frac{\pi^{d/2}}{d2^{d-1}\Gamma(d/2)} \end{aligned} \tag{4}$$

When $d \to \infty$, we could use Stirling approx(1.47) to derive that

$$\begin{aligned} \lim_{d\to\infty} \Gamma(x+1) &\approx (2\pi)^{1/2}e^{-x}(x)^{x+1/2} \\ &> e^{-x}x^x \\ &= (\frac{x}{e})^x \to \infty \end{aligned} \tag{5}$$

Right now we now when d is large enough the Γfunction would be close to infinity, for the rest part of the eq4 we have

$$\lim_{d\to\infty} \frac{\pi^{d/2}}{d2^{d-1}} < \frac{\pi^{d/2}}{2^d} = (\frac{\pi}{4})^{d/2} \to 0 \tag{6}$$

By combining eq5 and eq6 we can draw a conclution that

$$\lim_{d\to\infty} \frac{VS_d}{VC_d} \to \frac{0}{\infty} \to 0 \tag{7}$$

The distance from center to corner is $\sqrt{\Sigma_{i=1}^d a^2} = \sqrt{d}a$ while the height(center to hypersurface distance) is simply a, therefore the ratio is $\sqrt{d}$, and when d is large enough this ratio goes to infinity.

(*c*)

(*d*)

Question 2

Logistic Regression

Answer.