$THH \quad TC^- \quad TP$

even on $\mathrm{QRSPerfd}_R$

$HH(-/R;\mathbb{Z}_p), HC^-, HP$

In fact,

$THH(-;\mathbb{Z}_p), TC^-(-;\mathbb{Z}_p), TP(-;\mathbb{Z}_p) \in \mathrm{Shv}(\mathrm{QSyn}, \mathrm{Sp})$

$HH(-/R;\mathbb{Z}_p), HC^-, HP$

Moreover, they are "hyper-complete sheaves":

t-structure on $\mathrm{Shv}(\mathrm{QSyn}, \mathrm{Sp})$ (not specific to QSyn)

$\mathrm{Shv}(\mathrm{QSyn}, \mathrm{Sp})_{\geq 0}$: $\mathcal{F} \in \mathrm{Shv}(\mathrm{QSyn}, \mathrm{Sp})_{\geq 0}$

iff: $\forall A \in \mathrm{QSyn}$

$x \in \pi_j(\mathcal{F}(A))$, $j<0$.

$\exists$ quasisyntomic cover

$A \to B$ s.t. $\underset{\overset{\|}{0}}{x}$ in

$\pi_j(\mathcal{F}(B))$

"locally connective".

$\mathrm{Shv}(\mathrm{QSyn}, \mathrm{Sp})_{\leq 0}$: $\mathcal{F} \in \mathrm{Shv}(\mathrm{QSyn}, \mathrm{Sp})_{\leq 0}$

iff $\forall A \in \mathrm{QSyn}$

$\mathcal{F}(A) \in \mathrm{Sp}_{\leq 0}$.

$\heartsuit \simeq$ sheaves of ab gps (locally discrete)

Hypercomplete Sheaves:

A sheaf $\mathcal{F}$ is hypercomplete if

$$\mathcal{F} \xrightarrow{\sim} \lim_n \tau_{\leq n} \mathcal{F}$$

Fact $(L_{-/\mathbb{R}})^\wedge_p[-1]$

$(L_{-/\mathbb{Z}_p})^\wedge_p[-1]$ are hypercomplete in $\mathrm{Shv}(\mathrm{QSyn}, \mathrm{Sp})$

$(L_{-/\mathbb{Z}_p})^\wedge_p$

concentrated in

cohomological degrees $[-1, 0]$

$\Rightarrow$ (morning talk)

THH, TC^-, TP, TC

— - (even also subtle!).

are hypercomplete

Motivic filtration

$$\mathrm{Fil}^*_M \mathrm{THH}(-;\mathbb{Z}_p) := \tau_{\geq 2*} \mathrm{THH}(-;\mathbb{Z}_p)$$

$$\mathrm{Fil}^*_M \mathrm{TC}^-(-;\mathbb{Z}_p) := \tau_{\geq 2*} \mathrm{TC}^-(-;\mathbb{Z}_p)$$

$$\mathrm{Fil}^*_M \mathrm{TP}(-;\mathbb{Z}_p) := \tau_{\geq 2*} \mathrm{TP}(-;\mathbb{Z}_p)$$

$$\hat{\Delta}\_\{i\} = \mathrm{gr}^i_M \mathrm{TP}(-;\mathbb{Z}_p)[-2i]$$

$$N^{\geq i} \hat{\Delta}\_\{i\} = \mathrm{gr}^i_M \mathrm{TC}^-(-;\mathbb{Z}_p)[-2i]$$

$S \in \mathrm{QRSPerfd}$

$$\mathrm{TC}^-(S;\mathbb{Z}_p) \xrightarrow{\mathrm{can}} \mathrm{TP}(S;\mathbb{Z}_p)$$

$$\begin{tikzcd}
\mathrm{TC}^-(S;\mathbb{Z}_p) \arrow[r, "\varphi_p^{hT}"] \arrow[d] & \mathrm{TP}(S;\mathbb{Z}_p) \arrow[d] \\
\mathrm{THH}(S;\mathbb{Z}_p) \arrow[r, "\varphi_p"] & \mathrm{THH}(S;\mathbb{Z}_p)^{tC_p}
\end{tikzcd}$$

HPPSS Tate SS

$$\hat{\Delta}_S\{-1\} \oplus \hat{\Delta}_S \oplus N^{\geq 1}\hat{\Delta}_S\{1\} \xrightarrow{can} \hat{\Delta}_S\{-1\} \oplus \hat{\Delta}_S \oplus \hat{\Delta}_S\{1\}$$

$$\begin{tikzcd}
\hat{\Delta}_S \oplus N^{\geq 1}\hat{\Delta}_S\{1\} \arrow[r, "\varphi_p^{h\mathbb{T}}"] \arrow[d] & \hat{\Delta}_S \oplus \hat{\Delta}_S\{1\} \oplus \arrow[d] \\
S \oplus \mathrm{gr}^1_N \hat{\Delta}_S \arrow[r] &
\end{tikzcd}$$

"Sheaf of spectral sequences" ?

relevant

$\hat{\Delta}_-\{1\}$. $\hat{\Delta}_-$ invertible sheaf

$S \in \mathrm{QRSPerfd}_R$ $\quad TC^-(R;\mathbb{Z}_p)$ $\quad TP(R;\mathbb{Z}_p)$

u, v $\quad \sigma, \xi$

$$THH(S,\mathbb{Z}_p)^{tC_p} \simeq TP(S;\mathbb{Z}_p)/\varphi(\xi)$$

$TP(S;\mathbb{Z}_p) \otimes THH(R;\mathbb{Z}_p)$

$$TC^-(R;\mathbb{Z}_p)$$
$$\wr|$$
$$THH(S;\mathbb{Z}_p)^{tC_p}$$

$$TC^-(S;\mathbb{Z}_p)[2] \xrightarrow{u} TC^-(S;\mathbb{Z}_p) \to HC^-(S;\mathbb{Z}_p)$$

$\rightsquigarrow$ LES

$$\hat{\Delta}_S/\xi \simeq HC^-_0(S;\mathbb{Z}_p)$$
$$\wr|$$
$$(L\Omega_{S/R})^\wedge_{(p,\mathrm{Hod})}$$

relative de Rham comparison

$x \in TC^-_{2n}(S;\mathbb{Z}_p)$ $\quad v \in TC^-_{-2}(R;\mathbb{Z}_p)$

$$(\pi_0\varphi_p^{h\mathbb{T}})(v^n x) = \varphi(\xi)^n \sigma^{-n} \cdot (\pi_{2n}\varphi_p^{h\mathbb{T}})(x)$$

or

$$\sigma^n(\pi_0\varphi_p^{h\mathbb{T}})(v^n x) = \varphi(\xi)^n (\pi_{2n}\varphi_p^{h\mathbb{T}})(x)$$

divided Frob

$\underline{v^n x}$ $\quad x \in N^{\geq n} TC^-_0(S;\mathbb{Z}_p) = N^{\geq n}\hat{\Delta}_S$

modular forms?

Geometry???
behind this

$$HC_0^-(S;\mathbb{Z}_p) \simeq (L\Omega_{S/R})^{\wedge}_{p,\text{Hod}})$$

Beilinson t-structure

$$DF(\mathbb{Z}) = \text{Fun}((\mathbb{N}_{\geq})^{op}, D(\mathbb{Z}))$$

$$DF(\mathbb{Z})^{\heartsuit} = Ch(\mathbb{Z})$$

$$M \in DF(\mathbb{Z})$$

$$\text{Fil}^1 M \to \text{Fil}^0 M \to \text{Fil}^{-1} M \to \text{Fil}^{-2} M$$

$$H^0_{\text{Beil}}(\text{Fil}^\cdot M) = \left(H^n(\text{gr}^n(M)), \partial\right)$$

d

$$\frac{\text{Fil}^{n+1}M}{\text{Fil}^{n+2}M} \to \frac{\text{Fil}^{n}M}{\text{Fil}^{n+2}M} \to \frac{\text{Fil}^{n}M}{\text{Fil}^{n+1}M}$$

$HC_0^-(S/R;\mathbb{Z}_p)$ $\quad$ $HH(S/R;\mathbb{Z}_p)^{h\mathbb{T}}$

$\mathrm{Fil}^*_{h\mathbb{T}} HC_0^-(S/R;\mathbb{Z}_p)$ $\quad$ $\pi_{2n} = (\Lambda^n L_{S/R})^\wedge_p[-n]$

$$\mathrm{gr}^n_{h\mathbb{T}} HC_0^-(S/R;\mathbb{Z}_p) = \pi_{2n} = (\Lambda^n L_{S/R})^\wedge_p[-n]$$

Filtered sheaf on QSyn site

$$\mathrm{gr}^0_M HC^-(-/R;\mathbb{Z}_p) \simeq (L\Omega_{A/R})^\wedge_{(p,\mathrm{Hod})}$$

$$\mathrm{Fil}^*_{h\mathbb{T}}\, \mathrm{gr}^0_M HC^-(-/R;\mathbb{Z}_p)$$

A a polynomial R-algebra

$$\mathrm{gr}^n_{h\mathbb{T}}\, \mathrm{gr}^0_M HC^-(A/R;\mathbb{Z}_p) = (\Omega^n_{A/R})^\wedge_p[-n]$$

order of poles

prismatic coh level? height?

$$\mathrm{Fil}^*_{h\mathbb{T}}\, \mathrm{gr}^0_M HC^-(A/R;\mathbb{Z}_p)$$

Beilinson t-structure $\quad A \xrightarrow{d} (\Omega^1_{A/R})^\wedge_p \to (\Omega^2_{A/R})^\wedge_p \to$

- BMS2 Ch5 笔记
- Absolute Prismatic cohomology Ch6

HKR-fil

- Raskit

Antieau