# PRELIMINARY SCHEDULE AND SYLLABUS, VERSION 2023.7.21

Website: `https://aritopo.github.io/2023-08/`
Preliminary version as of July 21, 2023. Will be finalized before the workshop.
Feel free to contact any of the organizers for any comments or suggestions.

The syllabus (i.e., the math content) is mainly compiled by Guo Jingbang.
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| | Aug 7, Monday | Aug. 8 | Aug. 9 |
|---|---|---|---|
| 09:30–10:30 | Gao Hui | Yu Xiao/Liang | Zhao Luming |
| 10:30–11:00 | *tea break* | *tea break* | *tea break* |
| 11:00–12:00 | Guo Jingbang | Liang Tongtong | ?? |
| 12-14 | ∼ *lunch break* ∼ | ∼ *lunch break* ∼ | ∼ *lunch break* ∼ |
| 14:00–15:00 | Liang Tongtong | Guo Jingbang | Guo Jingbang |
| 15:00–15:30 | *tea break* | *tea break* | *tea break* |
| 15:30–16:30 | Liang Tongtong | Guo Jingbang | Guo Jingbang |
| 16:30–16:45 | *short break* | *short break* | *short break* |
| 16:45–17:30 (17:45?) | Yu Xiao | Guo Jingbang | *END?* |

Syllabus The main reference is [BMS19]; some parts of Sections 8–10 might be selectively omitted.

Day 1.

- Morning.
- **Introduction (a), Gao**. Briefly introduce Breuil-Kisin modules and their role in number theory. This is the main motivation of BMS's work. Briefly mention Breuil-Kisin twist.
- **Introduction (b), Guo**. State the main theorems, discuss the structure of the paper and the workshop.
- Afternoon
- **Introduction (c), Liang**. Briefly introduce the language of $\infty$-categories. This is the foundational framework used in NS18 and BMS19. Briefly mention Postnikov tower.
- **Introduction (d), Liang**. Introduce the formulation of THH, $\mathrm{TC}^-$, TP and TC. The key items include the Tate construction, the $S^1$-action, cyclotomic structure, related spectral sequences and Tate periodicity. Briefly mention HH, HP, $\mathrm{HC}^-$, contangent complex, and HKR filtration.
- **Descent problem and the quasi-syntomic site, Yu Xiao**. section 3-4. Discuss flat descent; then introduce the powerful quasi-syntomic site. Briefly introduce pefectoid rings, quasi-regular semi-perfectoid rings, and their properties.

Day 2.

- Morning
- **Descent problem and the quasi-syntomic site, Yu Xiao/Liang**. Discuss flat descent; then introduce the powerful quasi-syntomic site. Briefly introduce pefectoid rings, quasi-regular semi-perfectoid rings, and their properties.

*Date*: July 21, 2023.

- **THH of perfectoid, Liang**. section 6.1-6.2 Based on Bökstedt periodicity and some homological properties of perfectoid rings discussed yesterday, we compute THH of perfectoid rings. Furthermore, we introduce how to use homotopy fixed point spectral sequences and Tate spectral sequences to compute $\mathrm{TC}^-$ and TP of perfectoid rings. We try to dig out more essential hidden relations among Tate periodicity, Bökstedt periodicity, Nygaard filtration, and Breuil-Kisin twist.
- Afternoon,
- **Motivic filtrations, Guo**. section 6.3, section 7, main task is to explain the construction of the motivic filtrations on THH and its variants. The following topics might be touched upon: (1) the evenness of THH and its variants, more precisely, as sheaves on the site of quasiregular semiperfectoid rings, THH and its variants are even with respect to the canonical t-structure; (2) Nygaard filtration as the abutment filtration of the homotopy fixed point spectral sequence; (3) filtered derived categories; (4) unfold THH and its variants, together with their double speed Postnikov towers, from the site of quasiregular semiperfectoid rings to the site of quasisyntomic rings; (5) derived de Rham cohomology; (6) $\mathrm{TC}^-$ in characteristic p: the crystalline.

## Day 3.

- Morning
- **Crystalline Galois representations and Breuil-Kisin modules, Zhao**. Define crystalline representations. Discuss its classification by weakly admissible filtered $\varphi$-modules. Briefly discuss the construction of Breuil-Kisin modules associated to crystalline representations. State Kisin's fully faithfulness theorem. Time permitting, say a few more words on applications of these modules.
- **Prismatic cohomology**. (To be determined later.) We expect to have some speaker to briefly review prismatic cohomology of Bhatt-Scholze.
- Afternoon.
- **Breuil-Kisin cohomology, Guo**. section 11, Breuil-Kisin cohomology. This might be treated with a general theory of relative THH. The following topics might be (im)possible: (1) Tate valued Frobenius and the Segal conjecture; (2) cyclotomic bases for THH; (3) flat $\mathbb{S}$-algebra as cyclotomic bases for THH; (4) the comparisons/specializations of the prismatic theory; (5) Breuil-Kisin cohomology through relative THH; (6) q-de Rham ($\widetilde{p}$-de Rham) through relative THH.

## References

- **The BMS2**. The goal of this workshop is to study the article [BMS19], focusing on the interconnection between topological Hochschild homology and prismatic cohomology (in particular the Breuil-Kisin cohomology).
- **Survey** & **Lecture Notes**. There is a good survey [Mat22], which gives an account of [BMS19] in equal characteristic. Also a series of lecture notes emphasizing the arithmetic point of view [Mor19].
- **Prismatic Cohomology/Integral $p$-adic Hodge Theory**. The integral $p$-adic Hodge theory has its first incarnation in [BMS18], which is also a good reference on the algebraic preliminaries for (integral) perfectoid rings.

  In [BS22], the integral $p$-adic Hodge theory is more systematically actualized as the theory of prismatic cohomology. There are corresponding lecture notes [Bha18] and [Ked21].

  The current culmination is the *geometrization/transmutation* of the prismatic cohomology theory, achieved in [BL22a], [BL22b], and [Bha22].
- **Topological Hochschild Homology**. The modern realization of the theory of topological Hochschild homology is provided by [NS18], related to which there is a series of lecture notes [KN18]. There is also a modern survey [HN20] from the *handbook of homotopy theory*.
- **Algebraic $K$-theory**. The workshop requires no essential familiarity with algebraic $K$-theory. However, it would be helpful to slightly acquaint oneself with the motivic filtration for algebraic $K$-theory and the cyclotomic trace from algebraic $K$-theory to topological cyclic homology, and for this purpose the introduction of [Mat22] would suffice. See also [Mor21].
- **The Language of $\infty$-Categories and Higher Algebra**. Everyone should be brave enough to manipulate the language of $\infty$-categories without certainty. There are suitable surveys [Gep20] and [Gro20]. If one *has time*, the endless writings such as [Lur09], [Lur17], [Lur18b], [Lur23], and so on, are recommended. *Wish* that everyone would have time to *submerge* oneself in [Gro21].

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