Two sample test of proportion

Suppose there are two samples with proportion p_1 and p_2 out of samples n_1 and n_2. Here $p_1 = \frac{f_1}{n_1}$ and $p_2 = \frac{f_2}{n_2}$.

We know that $E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2$ and $var(\hat{p}_1 - \hat{p}_2) = var(\hat{p}_1) + var(\hat{p}_2) = \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}$

If sample size is large then

$$\frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}}$$

follows approximately normal distribution with parameter zero and one.

Consider the hypothesis $H_0 : p_1 = p_2$.

So, $E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2 = 0$ and $var(\hat{p}_1 - \hat{p}_2) = p(1-p)(\frac{1}{n_1} + \frac{1}{n_2})$

$\tau = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1} + \frac{1}{n_2})}}$ is used for testing the hypothesis H_0

Here, $\hat{p} = \frac{f_1 + f_2}{n_1 + n_2}$

We will use τ to test the null hypothesis.

Here in this problem,

$n_1 = 46$,$n_2 = 70$,$f_1 = 19$,$f_2 = 45$, $p_1 = 0.4130435$, $p_2 = 0.6428571$, $\hat{p} = 0.5517241$, $\tau = -2.434675$, $Z_{.05} = -1.64$

a)

$$H_0 : p_1 = p - 2 \; against \; H_1 : p_1 < p_2$$

b) let alpha = 0.05.

c) test statistic: $\tau = -2.434675$

d) We, reject the null hypothesis.

e) p-value =0.007452585

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#Rcode
n1=46;n2=70;f1=19;f2=45;p1=0.4130435; p2=0.6428571;
p_hat<-(f1+f2)/(n1+n2)
tau<-(p1-p2)/sqrt((p_hat*(1-p_hat))*((1/n1)+(1/n2)))
pvalue<-pnorm(tau)
```