

Assignment 1

(27 January, 2026)

Submission Deadline: 07 February, 2026

Course: Algebraic Topology II (KSM4E02)

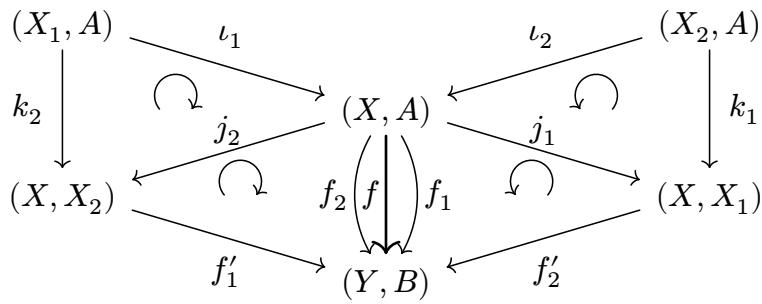
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Q1. (Proper Triad and Summation of Maps) Let $(X; X_1, X_2)$ be a proper triad with $X = X_1 \cup X_2$, and set $A := X_1 \cap X_2$. Let $f, f_1, f_2 : (X, A) \rightarrow (Y, B)$ be maps such that

$$f_1|_{X_1} = f|_{X_1}, \quad f_1(X_2) \subset B \quad f_2|_{X_2} = f|_{X_2}, \quad f_2(X_1) \subset B.$$

Show that $f_\star = f_{1\star} + f_{2\star}$.

Hint : Consider the space-level diagram



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Q2. Let $(X; X_1, X_2)$ be a proper triad. Verify that $(X_1 \cup X_2; X_1, X_2)$ is again a proper triad. Show that the diagram

$$\begin{array}{ccc}
 H_{n+1}(X, X_1 \cup X_2) & \xrightarrow{\partial} & H_n(X_1, X_1 \cap X_2) \\
 \partial_1 \downarrow & & \downarrow \partial_2 \\
 H_n(X_1 \cup X_2) & \xrightarrow{\Delta} & H_{n-1}(X_1 \cap X_2)
 \end{array}$$

commutes, where ∂ is the boundary map of the long exact sequence of the triad $(X; X_1, X_2)$, and Δ is the boundary map of the Mayer-Vietoris sequence of $(X_1 \cup X_2; X_1, X_2)$. The boundary maps ∂_i are from the long exact sequence of the corresponding pairs. 3 + 7 = 10

Q3. Let $(X; X_1, X_2)$ be a proper triad. Consider the diagram

$$\begin{array}{ccc}
 H_{n+1}(X, X_1 \cup X_2) & \xrightarrow{\partial_1} & H_n(X_1, X_1 \cap X_2) \\
 \partial_2 \downarrow & & \downarrow \partial \\
 H_n(X_2, X_1 \cap X_2) & \xrightarrow{\partial} & H_{n-2}(X_1 \cap X_2)
 \end{array}$$

where ∂_i are the boundary maps of the corresponding triad, and the two ∂ 's are the boundary maps of the corresponding pairs. Show that the diagram *anticommutes*, i.e, show that $\partial \circ \partial_1 + \partial \circ \partial_2 = 0$. 10

Q4. Prove the relative Mayer-Vietoris sequence. 10