

Lecture 4: Sequence Models I

Alan Ritter

(many slides from Greg Durrett, Dan Klein, Vivek Srikumar, Chris Manning, Yoav Artzi)

This Lecture

- ▶ Sequence modeling
- ▶ HMMs for POS tagging
- ▶ HMM parameter estimation
- ▶ Viterbi, forward-backward

Linguistic Structures

- ▶ Language is tree-structured

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I ate the spaghetti with chopsticks

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I ate the spaghetti with chopsticks

I ate the spaghetti with meatballs

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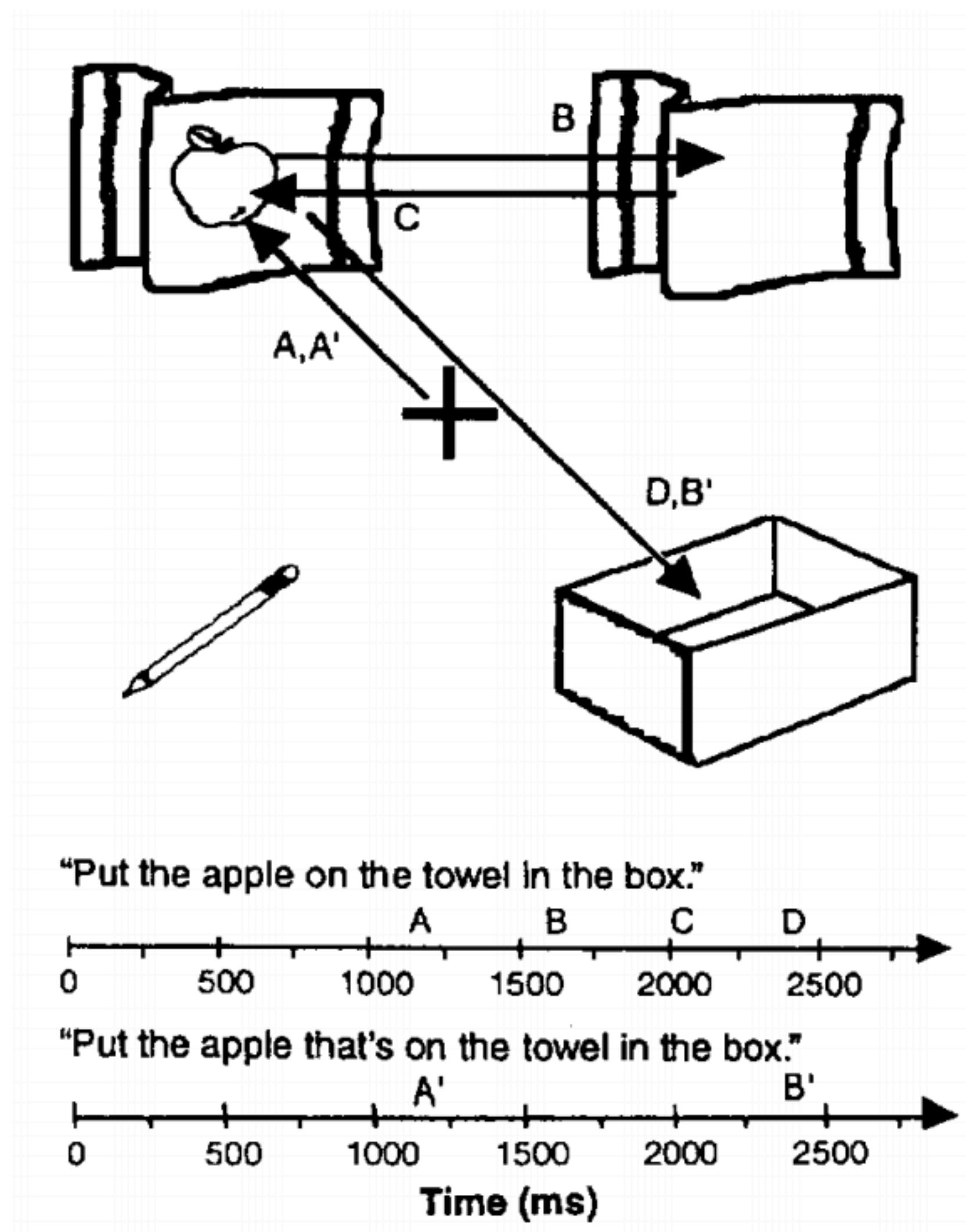
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Linguistic Structures

- ▶ Language is sequentially structured: interpreted in an online way

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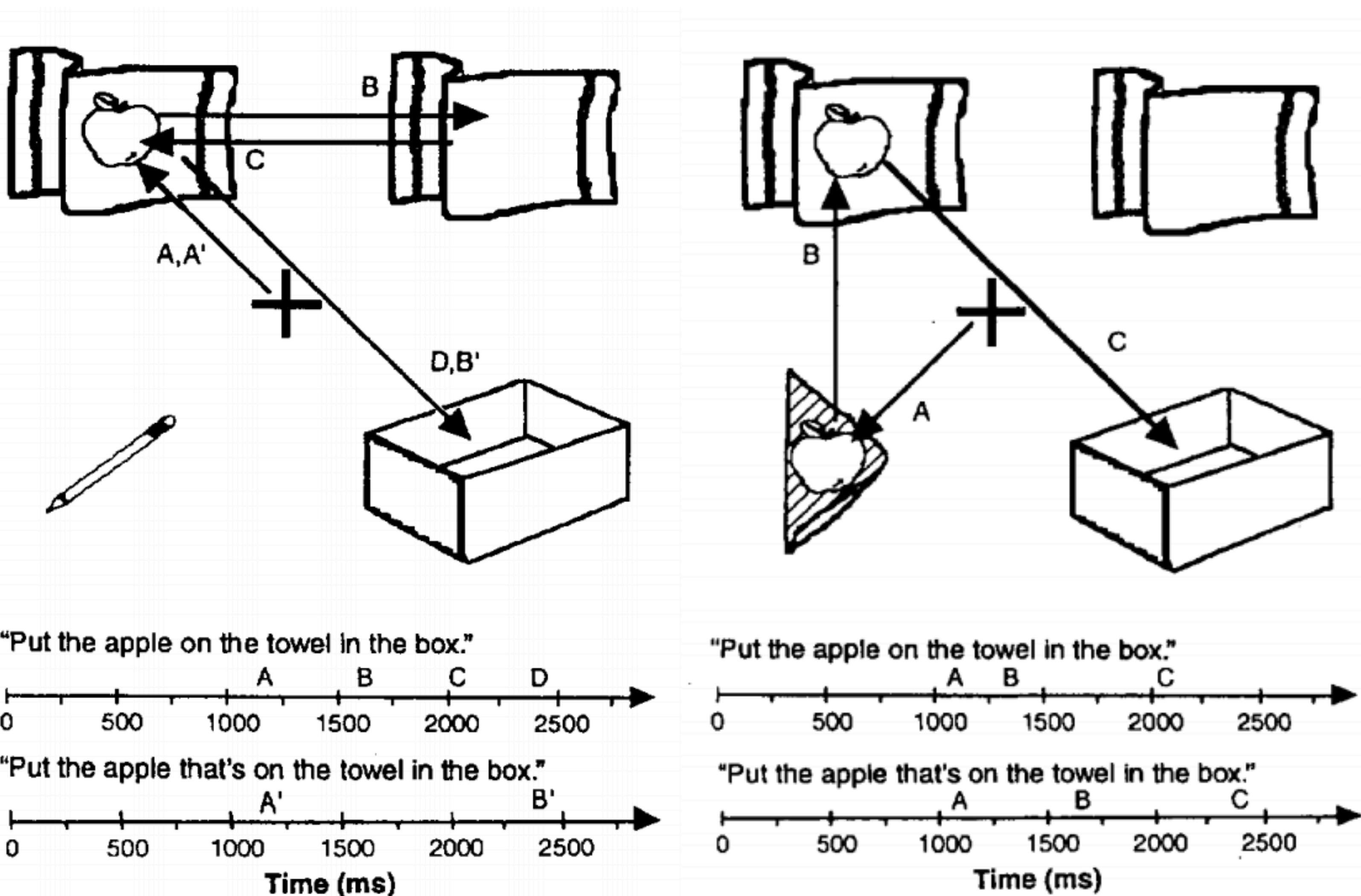
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Tanenhaus et al. (1995)

Linguistic Structures

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POS Tagging

- ▶ What tags are out there?

Ghana's ambassador should have set up the big meeting in DC yesterday .

POS Tagging

Open class (lexical) words

Nouns

Proper

IBM

Italy

Common

cat / cats

snow

Verbs

Main

see

registered

Adjectives

yellow

Adverbs

slowly

Numbers

122,312

one

... more

Closed class (functional)

Determiners

the some

Conjunctions

and or

Pronouns

he its

Auxiliary

can

had

Prepositions

to with

Particles

off up

... more

POS Tagging

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POS Tagging

VBD

VBN

NNP

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POS Tagging

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POS Tagging

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VBN	VBZ	VBP
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- ▶ What governs the correct choice? Word + context

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- ▶ Other paths are also plausible but even more semantically weird...
- ▶ What governs the correct choice? Word + context
- ▶ Word identity: most words have ≤ 2 tags, many have one (*percent, the*)
- ▶ Context: nouns start sentences, nouns follow verbs, etc.

POS Tagging

CC	conjunction, coordinating	and both but either or
CD	numeral, cardinal	mid-1890 nine-thirty 0.5 one
DT	determiner	a all an every no that the
EX	existential there	there
FW	foreign word	gemeinschaft hund ich jeux
IN	preposition or conjunction, subordinating	among whether out on by if
JJ	adjective or numeral, ordinal	third ill-mannered regrettable
JJR	adjective, comparative	braver cheaper taller
JJS	adjective, superlative	bravest cheapest tallest
MD	modal auxiliary	can may might will would
NN	noun, common, singular or mass	cabbage thermostat investment subhumanity
NNP	noun, proper, singular	Motown Cougar Yvette Liverpool
NNPS	noun, proper, plural	Americans Materials States
NNS	noun, common, plural	undergraduates bric-a-brac averages
POS	genitive marker	' s
PRP	pronoun, personal	hers himself it we them
PRP\$	pronoun, possessive	her his mine my our ours their thy your
RB	adverb	occasionally maddeningly adventurously
RBR	adverb, comparative	further gloomier heavier less-perfectly
RBS	adverb, superlative	best biggest nearest worst
RP	particle	aboard away back by on open through
TO	"to" as preposition or infinitive marker	to
UH	interjection	huh howdy uh whammo shucks heck
VB	verb, base form	ask bring fire see take
VBD	verb, past tense	pleaded swiped registered saw
VBG	verb, present participle or gerund	stirring focusing approaching erasing
VBN	verb, past participle	dilapidated imitated reunified unsettled
VBP	verb, present tense, not 3rd person singular	twist appear comprise mold postpone
VBZ	verb, present tense, 3rd person singular	bases reconstructs marks uses
WDT	WH-determiner	that what whatever which whichever
WP	WH-pronoun	that what whatever which who whom
WP\$	WH-pronoun, possessive	whose
WRB	Wh-adverb	however whenever where why

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- ▶ Preprocessing step for syntactic parsers
- ▶ Domain-independent disambiguation for other tasks
- ▶ (Very) shallow information extraction

Sequence Models

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- ▶ POS tagging: x is a sequence of words, y is a sequence of tags
- ▶ Today: generative models $P(x, y)$; discriminative models next time

Hidden Markov Models

- ▶ Input $\mathbf{x} = (x_1, \dots, x_n)$ Output $\mathbf{y} = (y_1, \dots, y_n)$

Hidden Markov Models

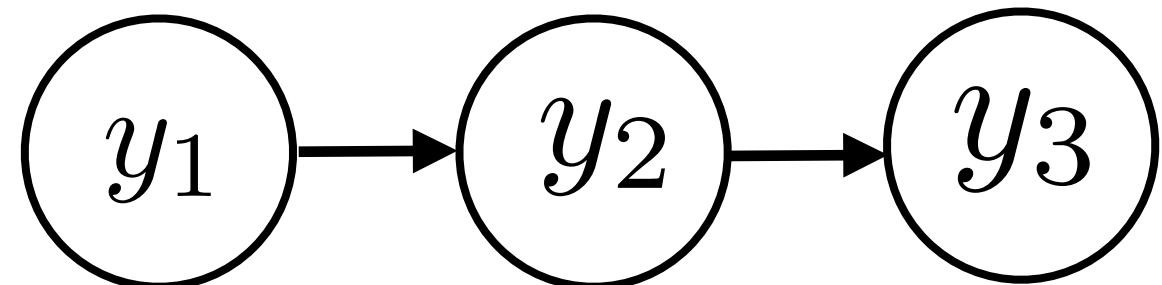
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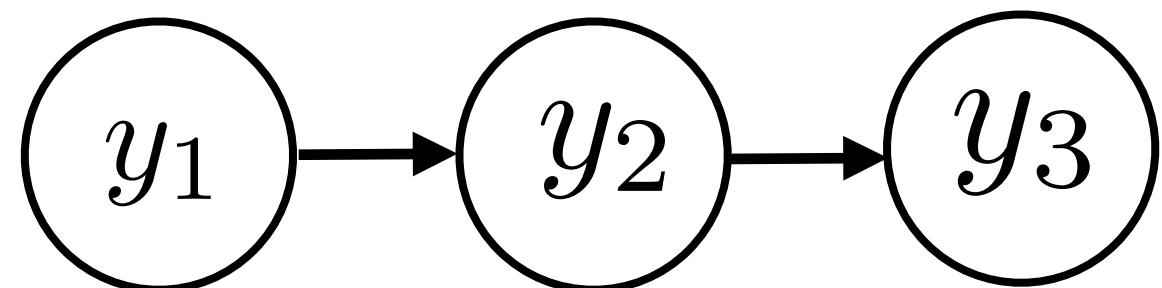
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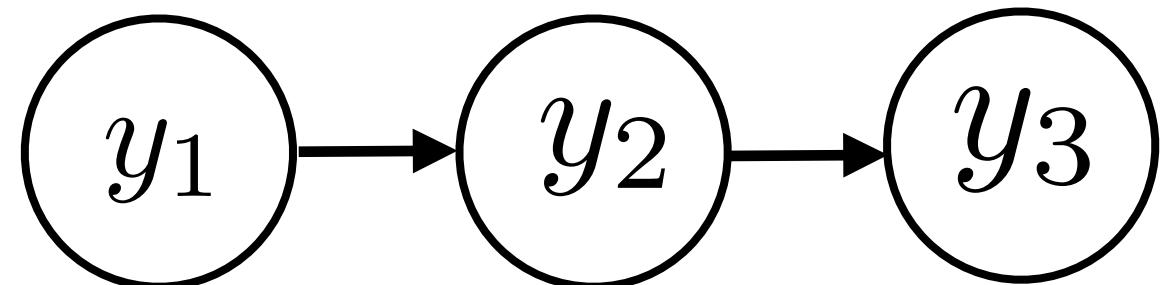


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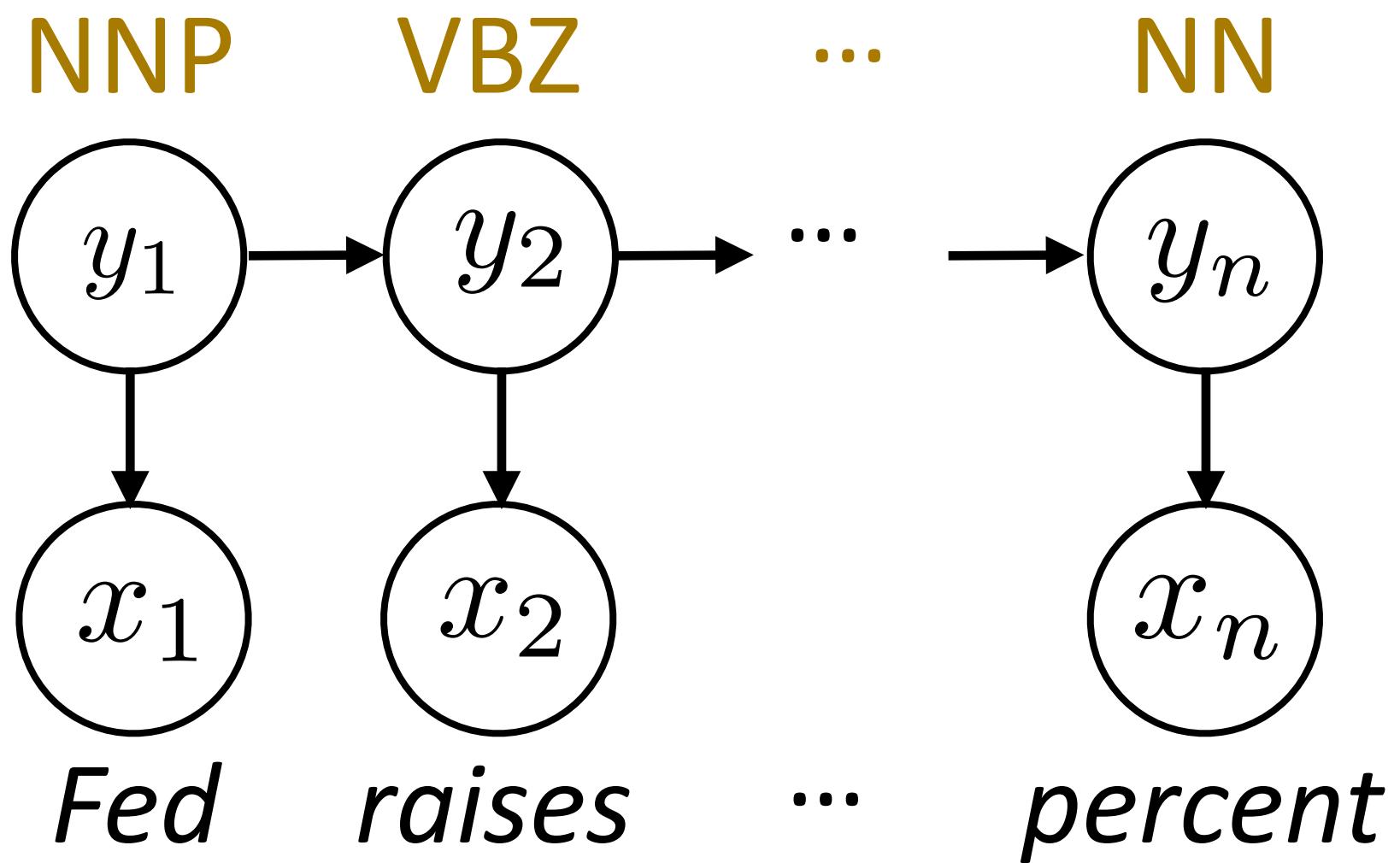
- ▶ Lots of mathematical theory about how Markov chains behave
- ▶ If y are tags, this roughly corresponds to assuming that the next tag only depends on the current tag, not anything before

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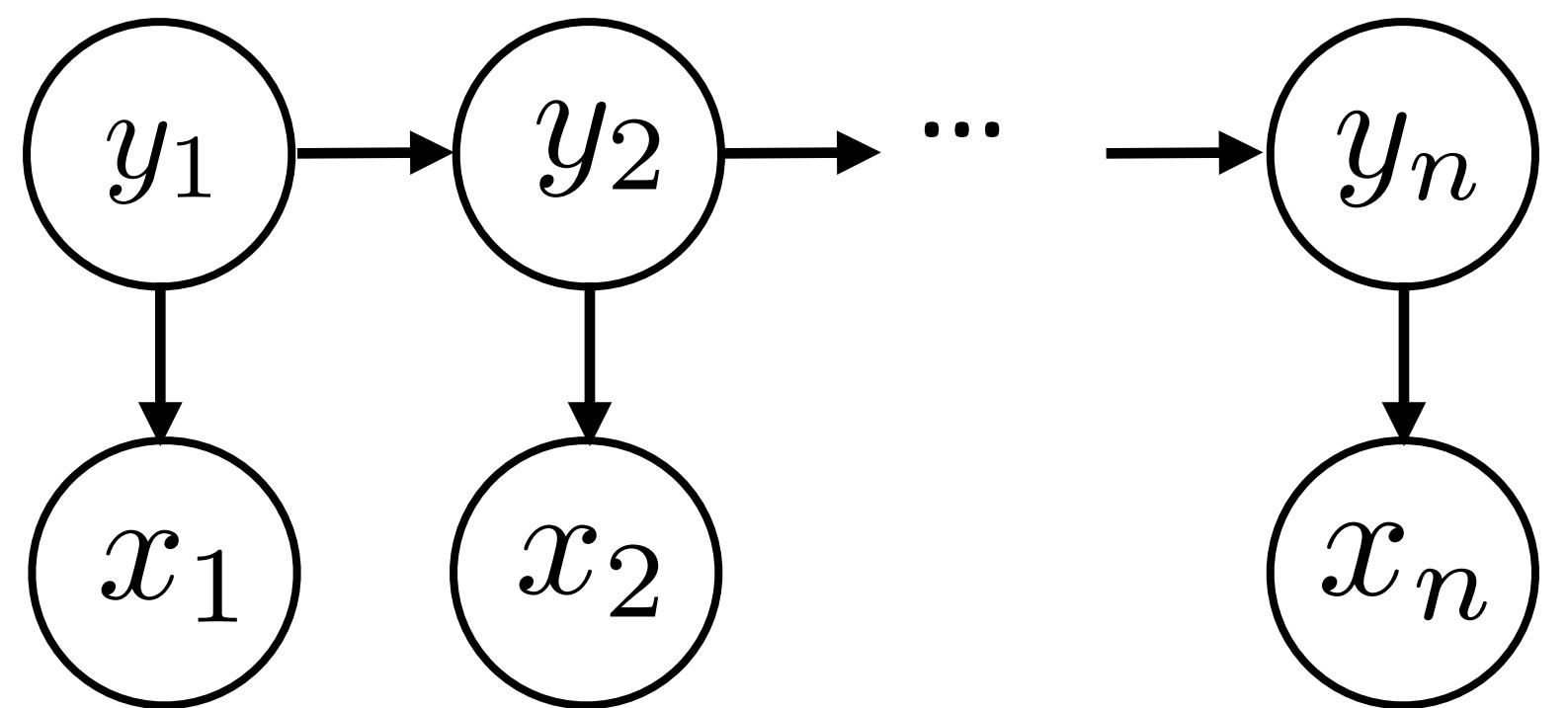
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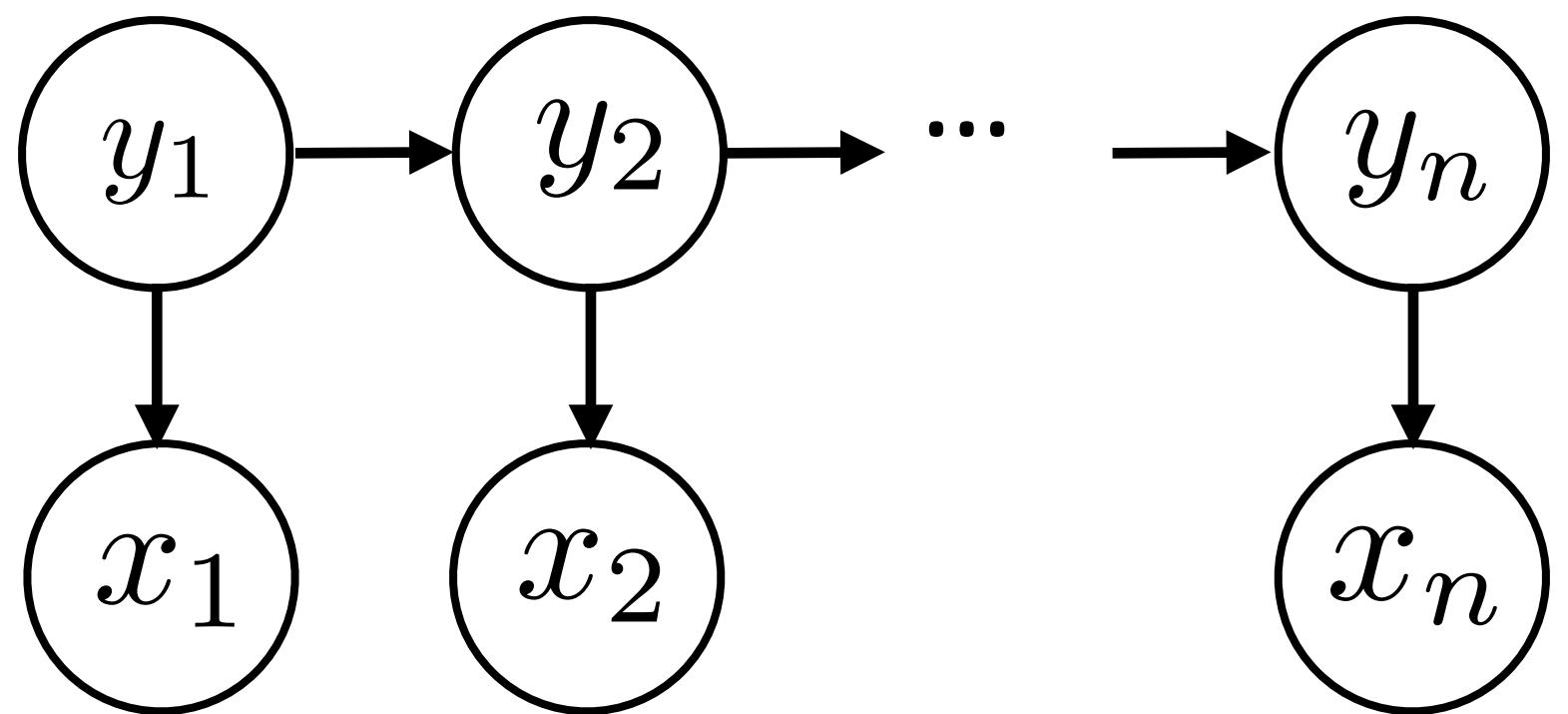
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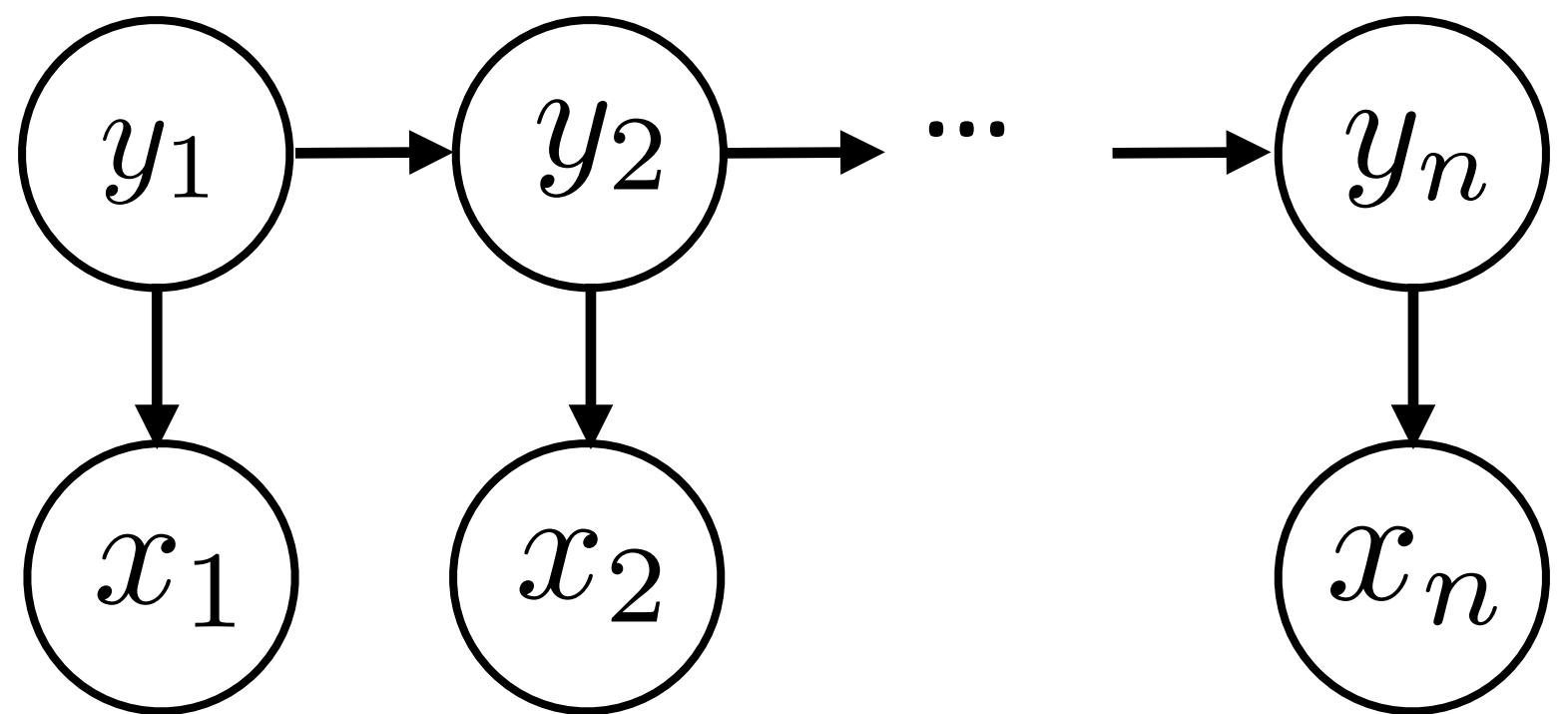
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$$P(\mathbf{y}, \mathbf{x}) = P(y_1) \prod_{i=2}^n P(y_i | y_{i-1}) \prod_{i=1}^n P(x_i | y_i)$$

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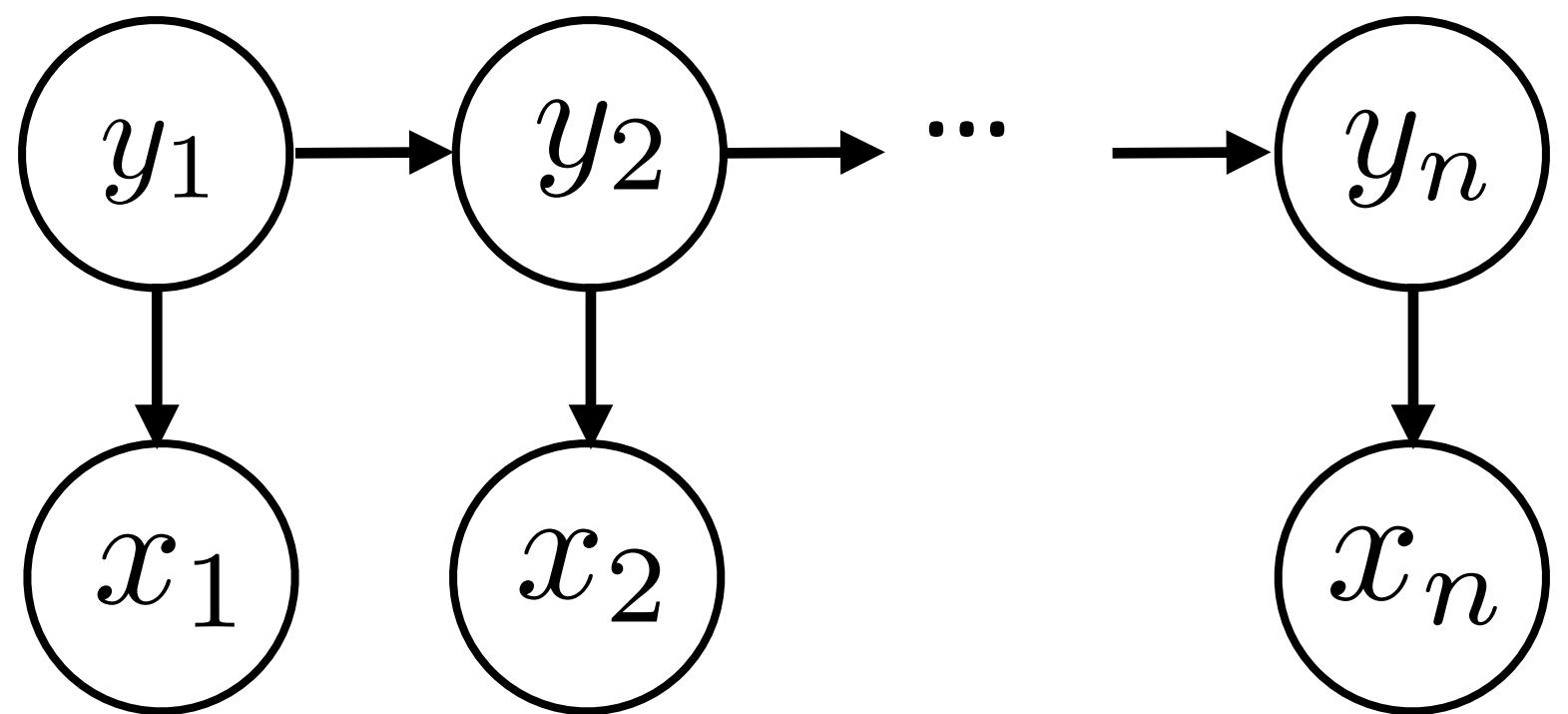
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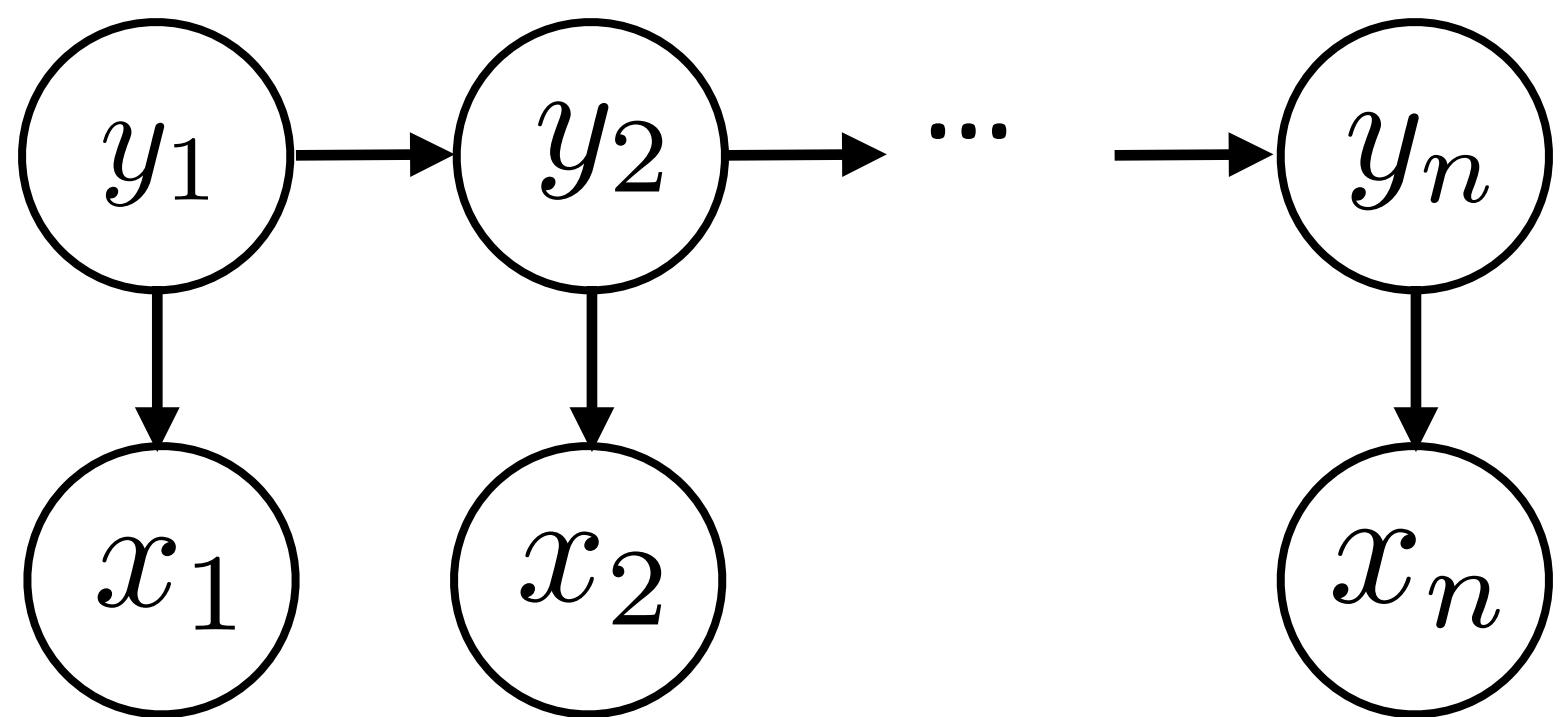
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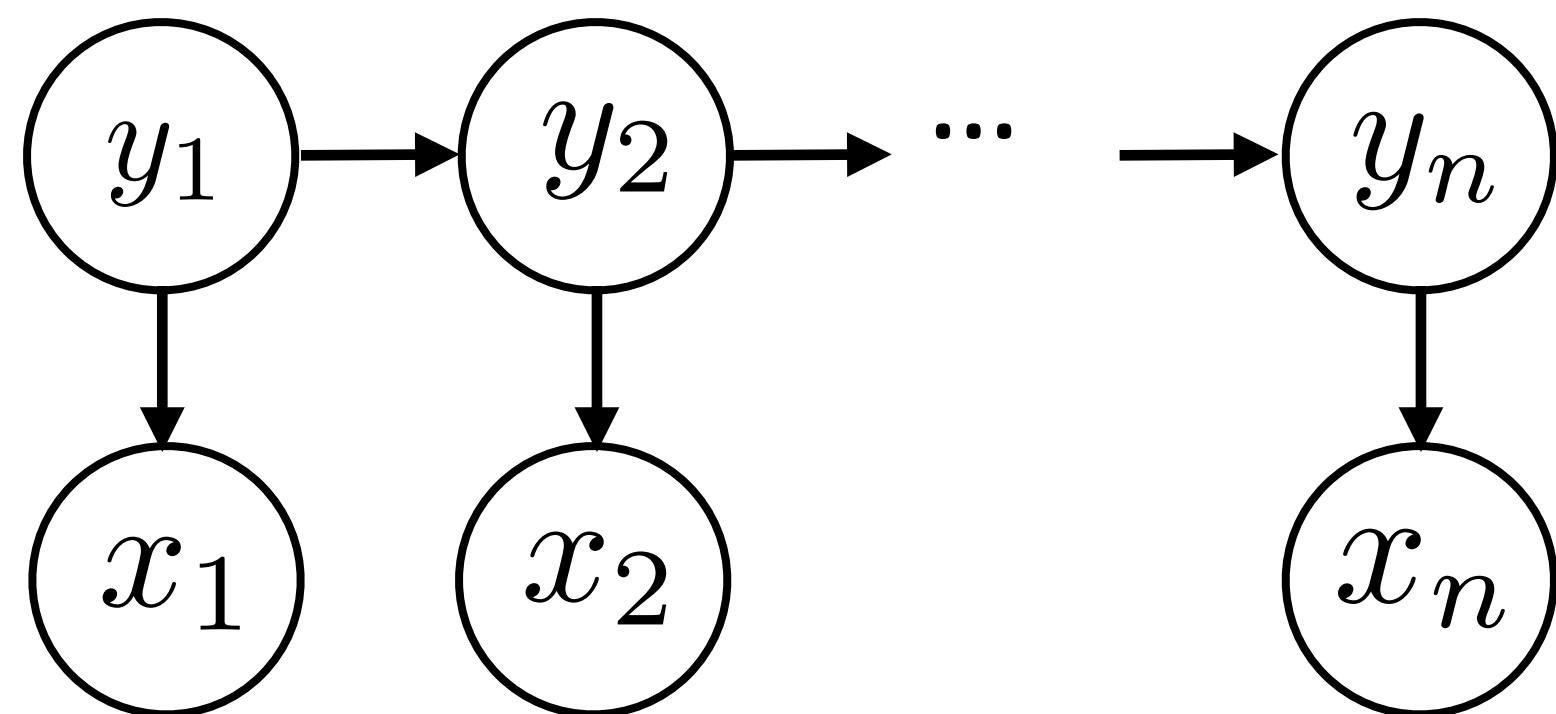
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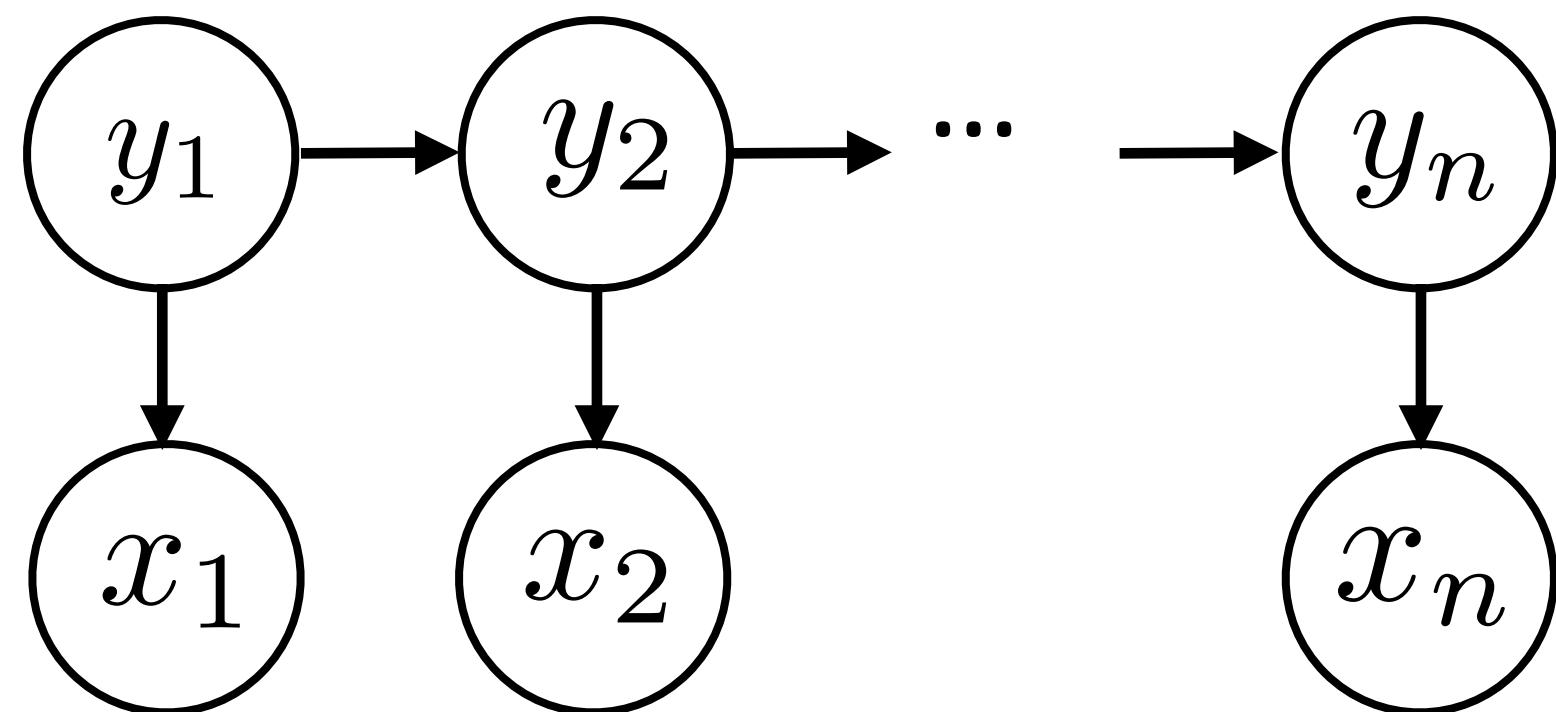


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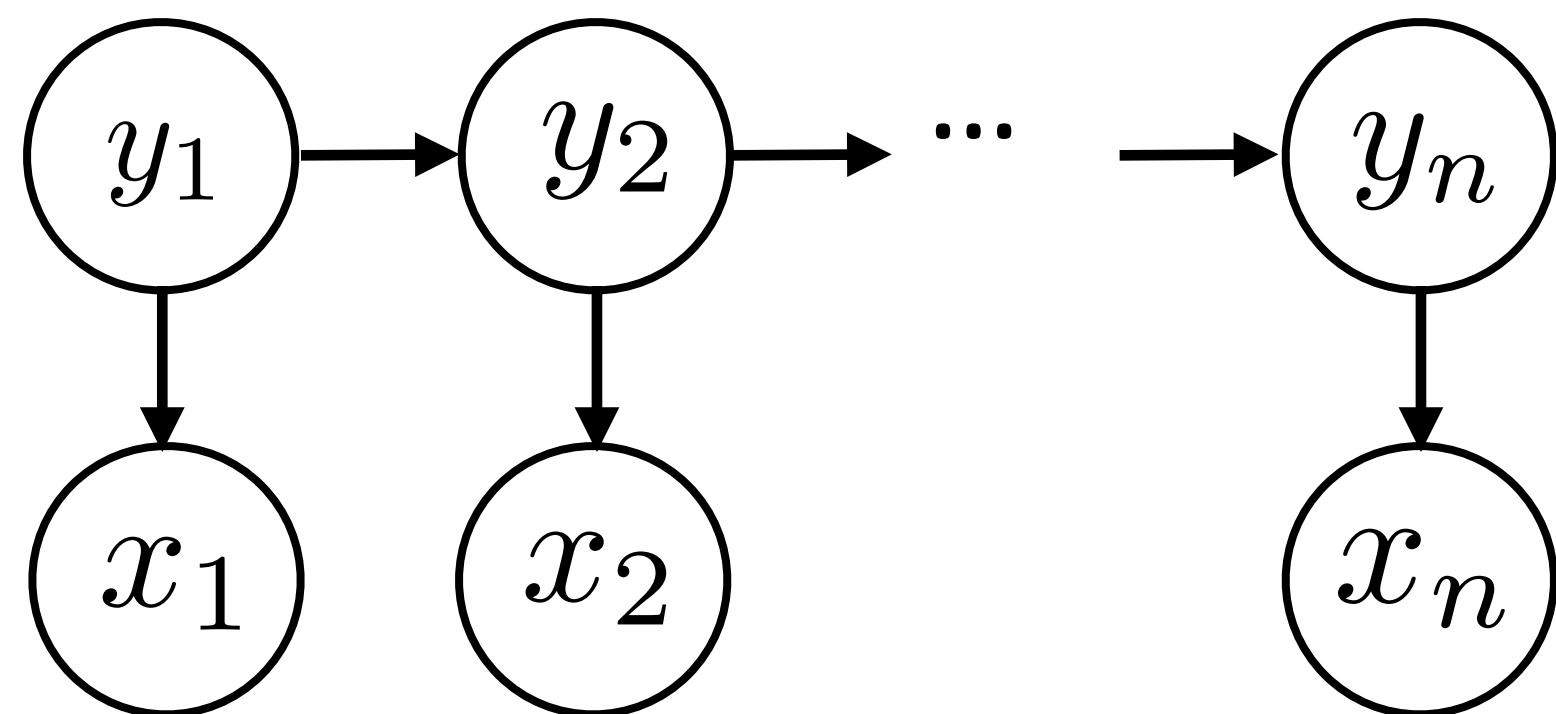


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- Observation (x) depends only on current state (y)
- Multinomials: tag x tag transitions, tag x word emissions
- $P(x|y)$ is a distribution over all words in the vocabulary
 - not a distribution over features (but could be!)

Transitions in POS Tagging

- Dynamics model $P(y_1) \prod_{i=2}^n P(y_i|y_{i-1})$
- VBD VB
VBN **VBZ** VBP VBZ
NNP NNS **NN** **NNS** CD **NN** .

Fed raises interest rates 0.5 percent.

NNP - proper noun, singular
VBZ - verb, 3rd ps. sing. present
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- $P(y_2 = \text{VBZ}|y_1 = \text{NNP})$ likely because verb often follows noun
- $P(y_3 = \text{NN}|y_2 = \text{VBZ})$ direct object follows verb, other verb rarely follows past tense verb (main verbs can follow modals though!)

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$$P(\text{tag} | \text{tag}_{-1}) = (1 - \lambda) \hat{P}(\text{tag} | \text{tag}_{-1}) + \lambda \hat{P}(\text{tag})$$

\hat{P} = empirical distribution (read off from data)

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- ▶ How should we smooth this?

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 $P(\text{word shape} \mid \text{tag})$ (e.g., $P(\text{capitalized word of len} \geq 8 \mid \text{tag})$)

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$$P(\text{word} \mid \text{tag}) = \frac{P(\text{tag} \mid \text{word}) P(\text{word})}{P(\text{tag})}$$

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- $$P(\text{word}|\text{tag}) = \frac{P(\text{tag}|\text{word})P(\text{word})}{P(\text{tag})}$$
- ▶ Fancy techniques from language modeling, e.g. look at type fertility
 - $P(\text{tag}|\text{word})$ is flatter for some kinds of words than for others)

Estimating Emissions

NNP VBZ NN NNS CD NN

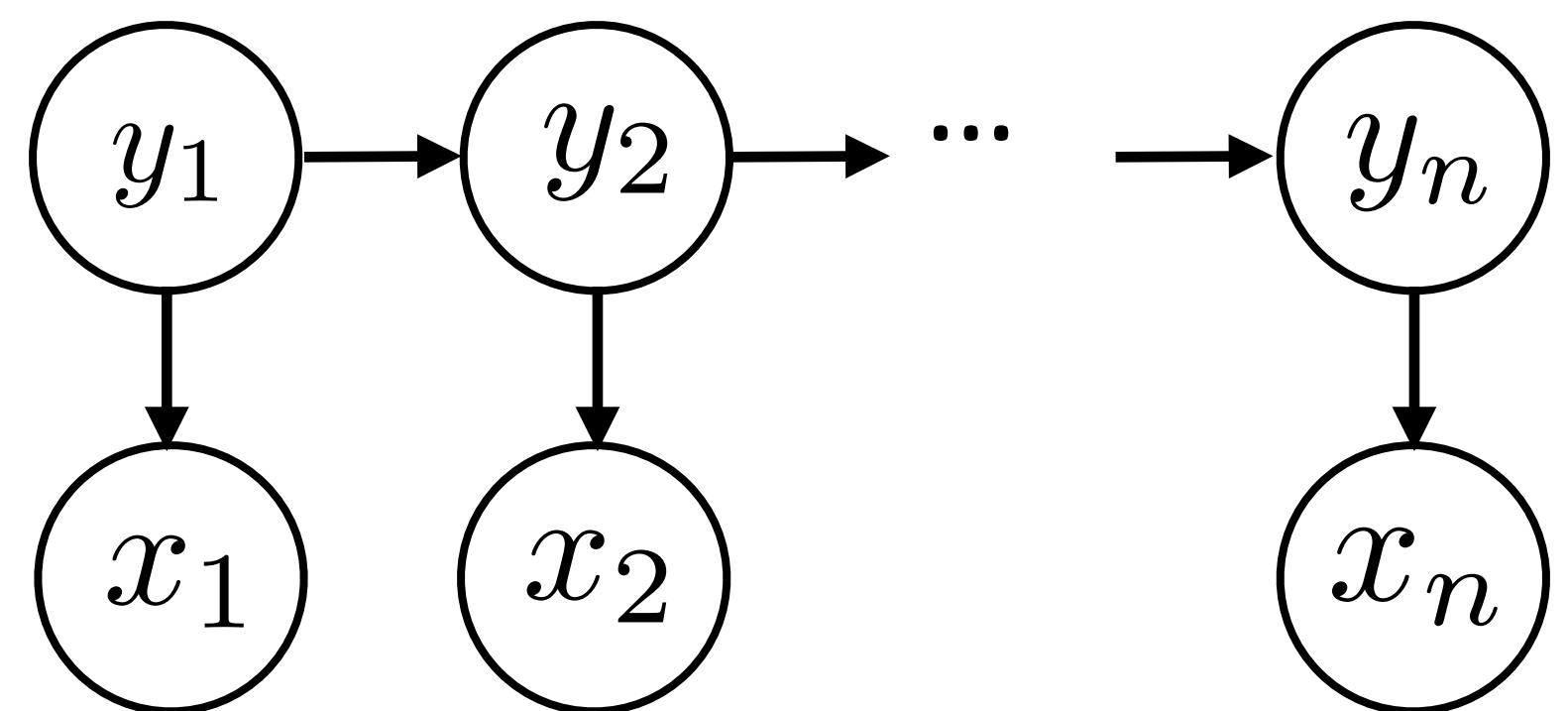
Fed raises interest rates 0.5 percent

- ▶ $P(\text{word} \mid \text{NN}) = (0.5 \textit{interest}, 0.5 \textit{percent})$ — hard to smooth!
 - ▶ Can interpolate with distribution looking at word shape
 $P(\text{word shape} \mid \text{tag})$ (e.g., $P(\text{capitalized word of len } \geq 8 \mid \text{tag})$)
 - ▶ Alternative: use Bayes' rule
- $$P(\text{word}|\text{tag}) = \frac{P(\text{tag}|\text{word})P(\text{word})}{P(\text{tag})}$$
- ▶ Fancy techniques from language modeling, e.g. look at type fertility
 - $P(\text{tag}|\text{word})$ is flatter for some kinds of words than for others)
 - ▶ $P(\text{word}|\text{tag})$ can be a log-linear model — we'll see this in a few lectures

Inference in HMMs

► Input $\mathbf{x} = (x_1, \dots, x_n)$

Output $\mathbf{y} = (y_1, \dots, y_n)$

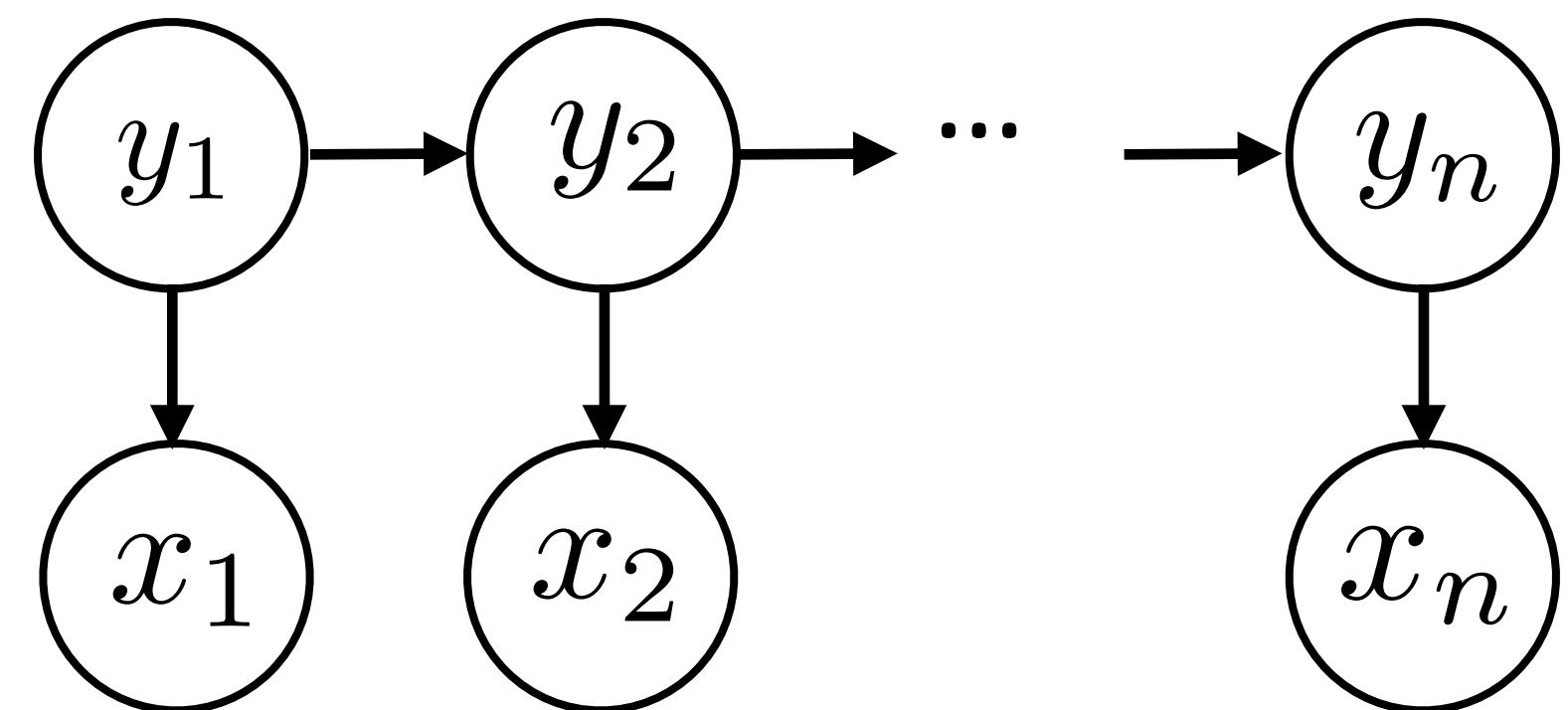


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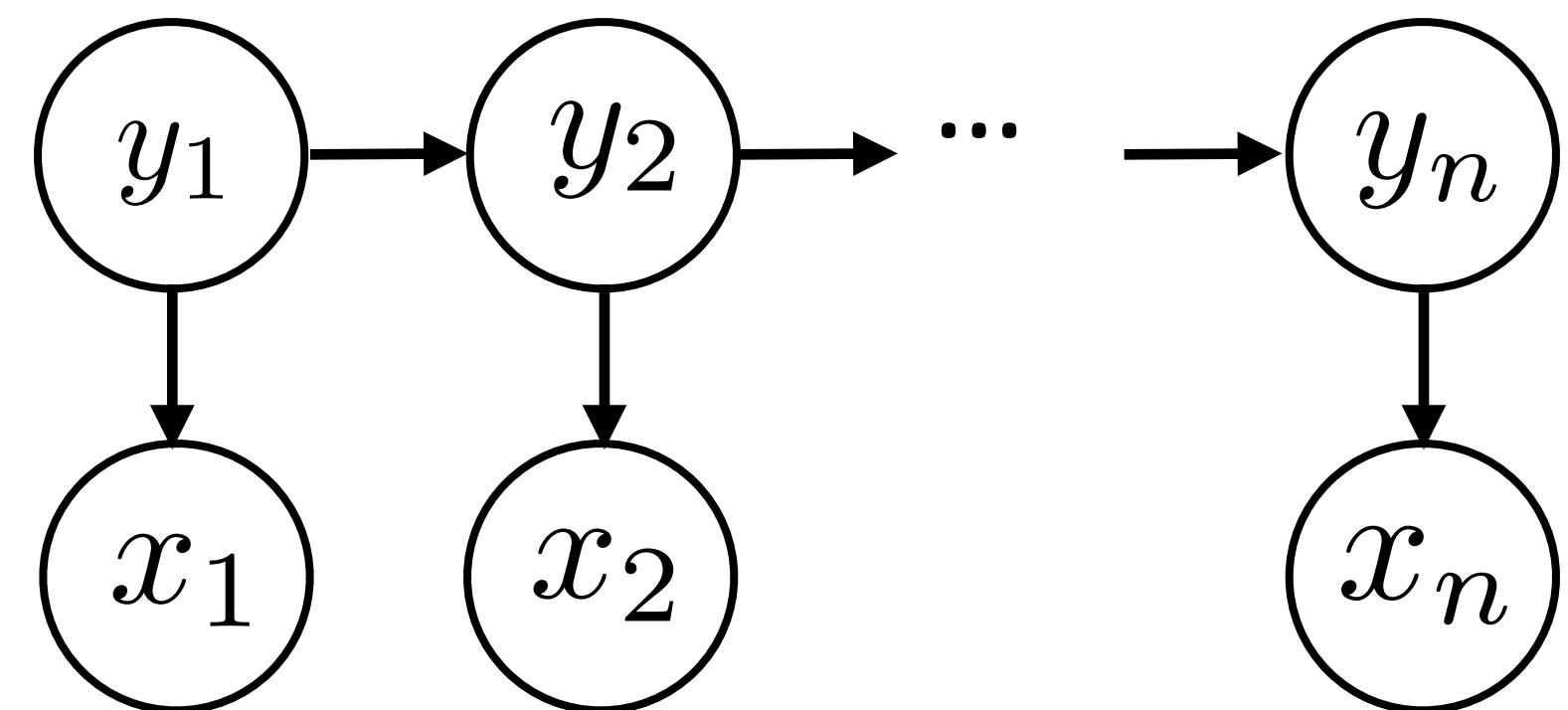
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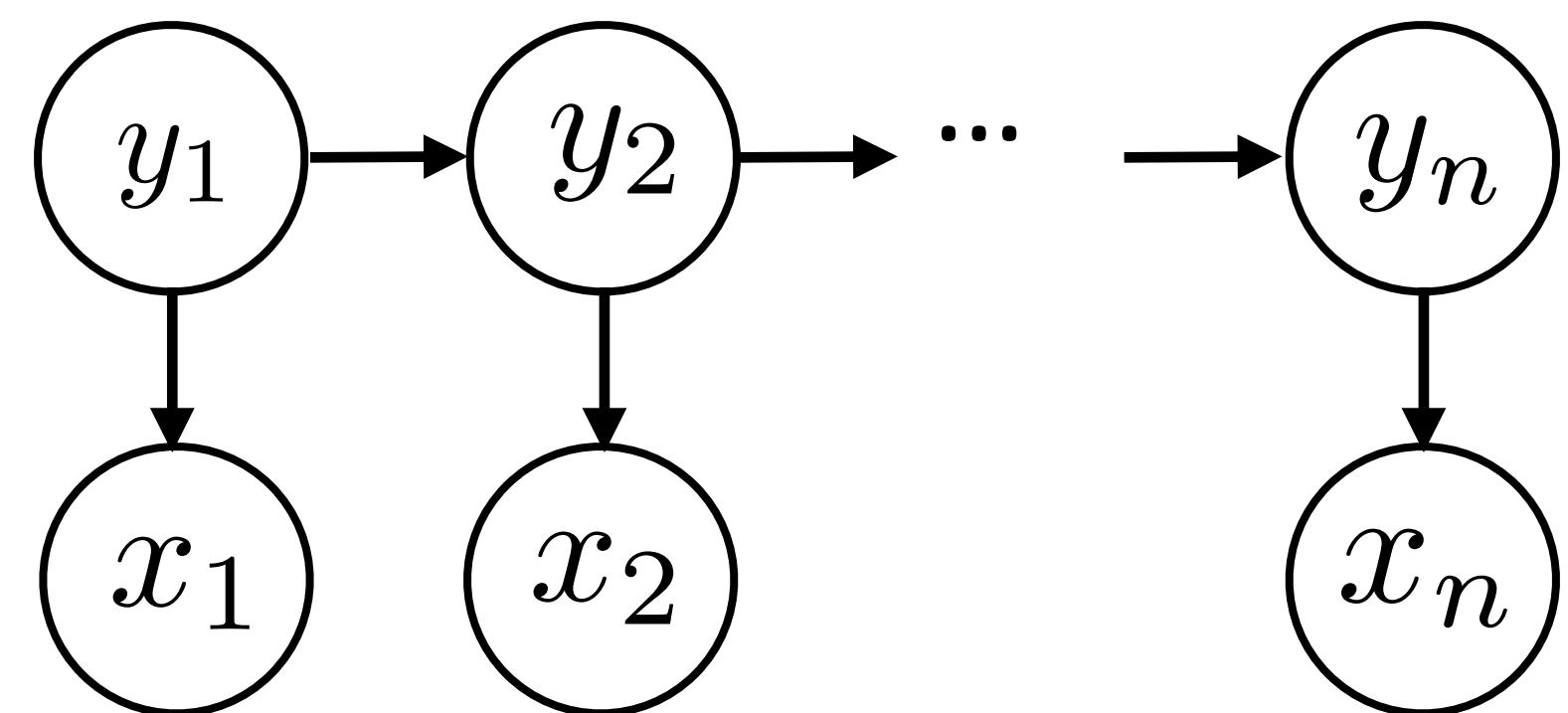
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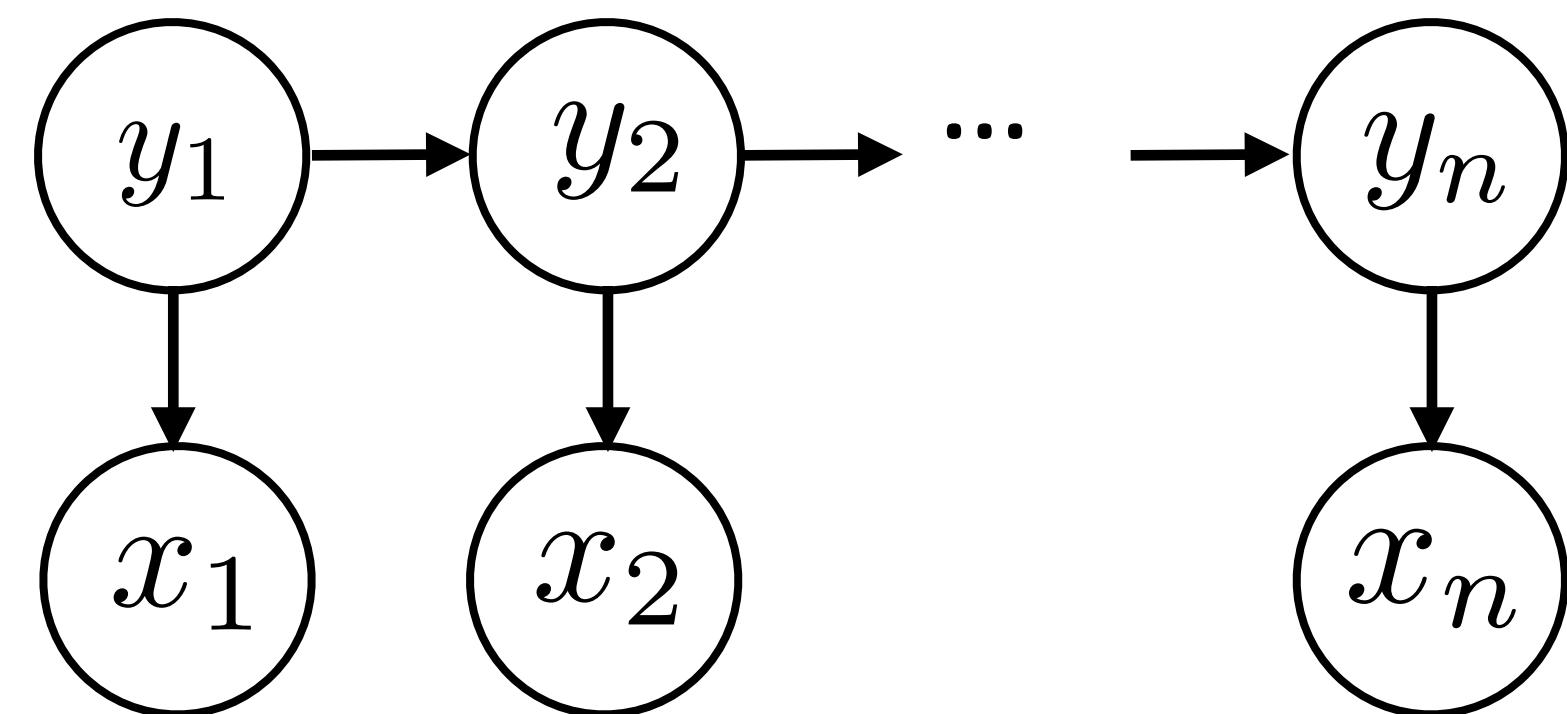
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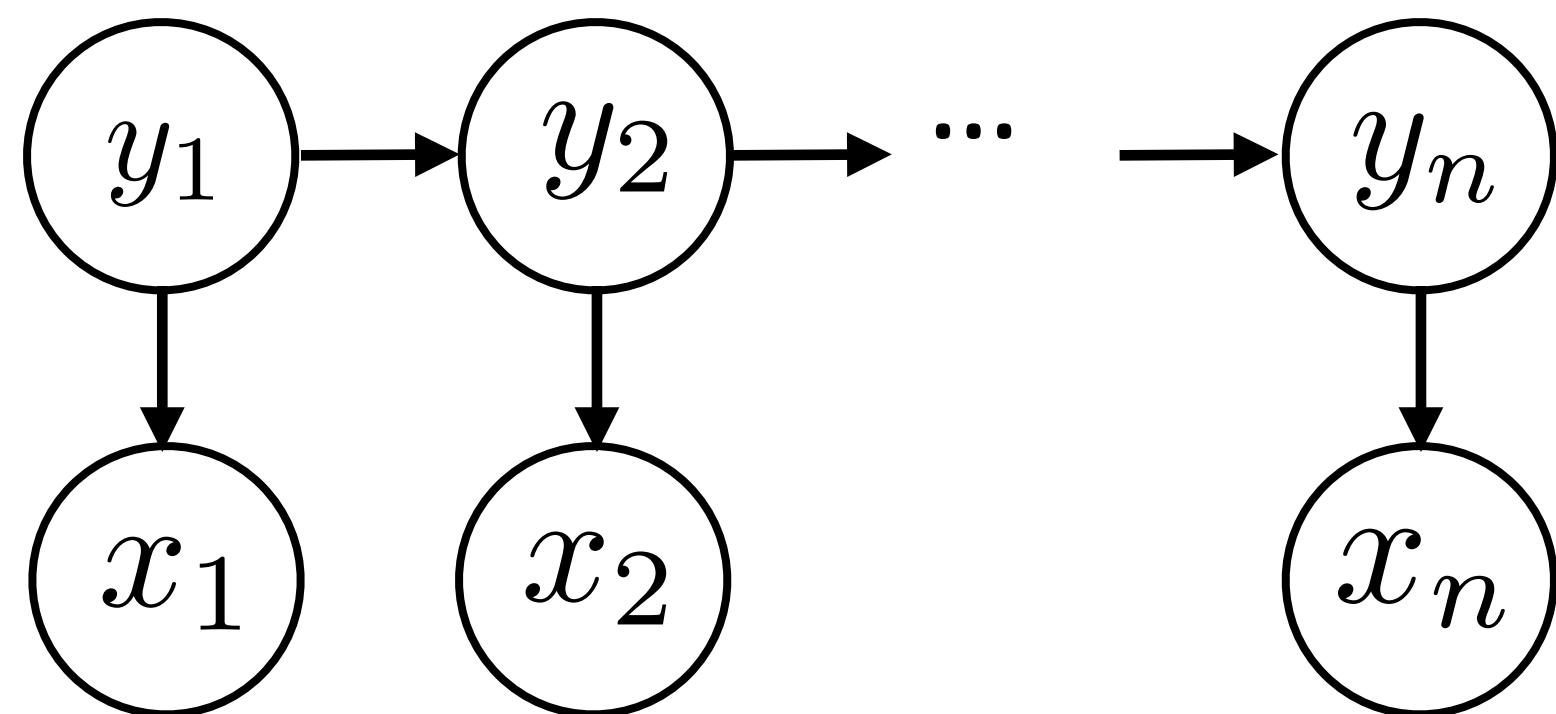
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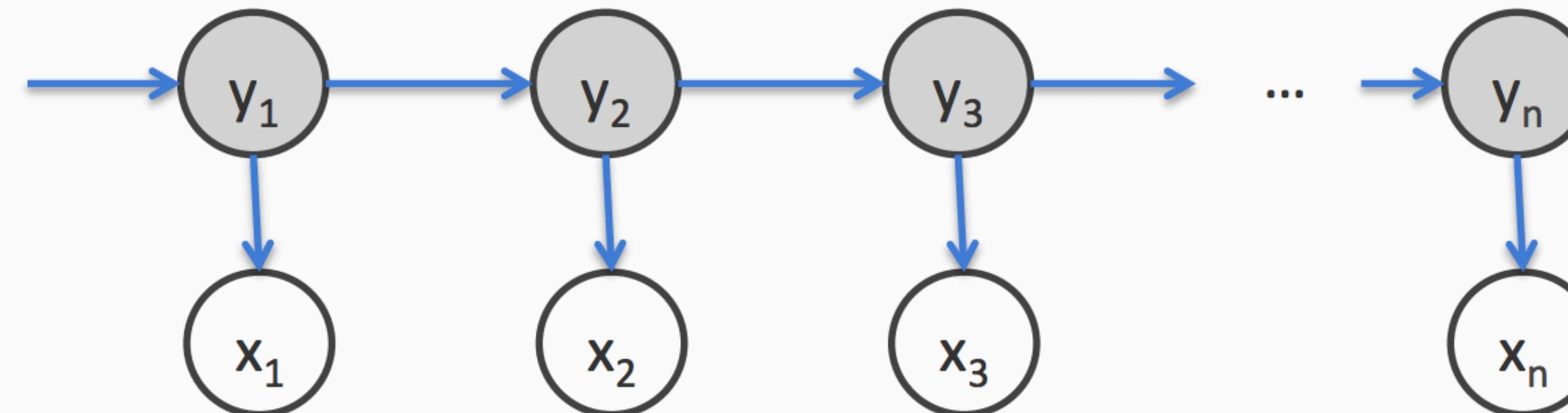
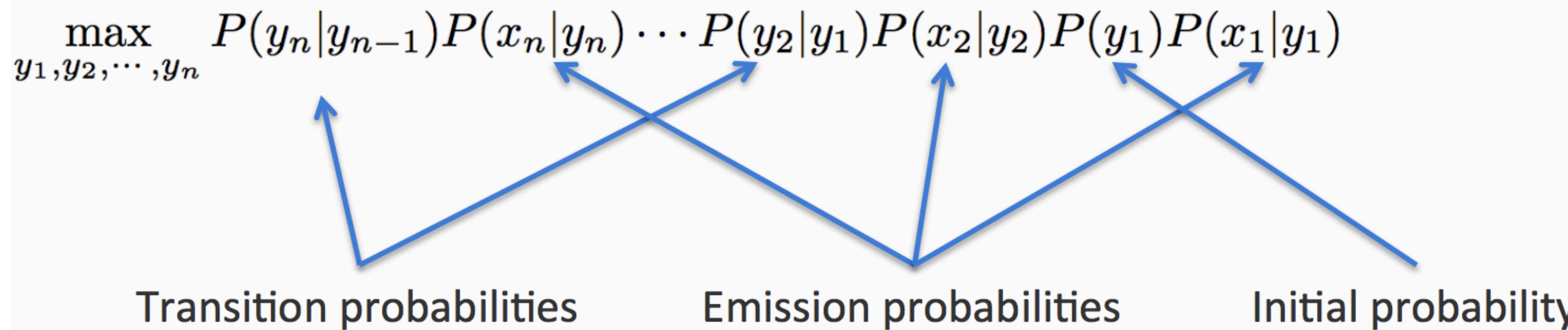
► Exponentially many possible \mathbf{y} here!

► Solution: dynamic programming (possible because of **Markov structure!**)

► Many neural sequence models depend on entire previous tag sequence, need to use approximations like beam search

Viterbi Algorithm

$$P(x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n) = P(y_1) \prod_{i=1}^{n-1} P(y_{i+1}|y_i) \prod_{i=1}^n P(x_i|y_i)$$



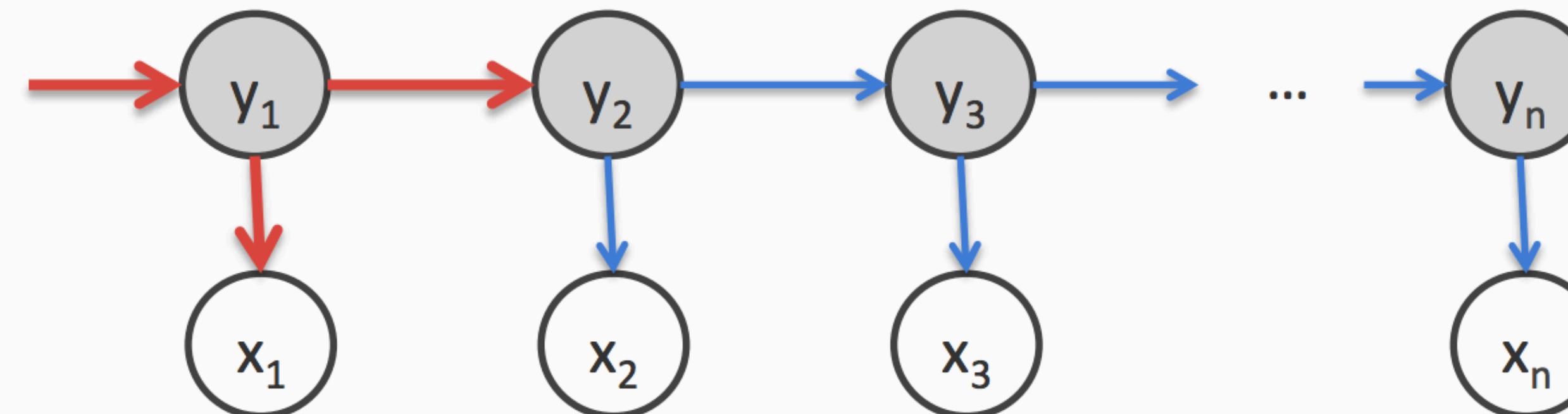
slide credit: Vivek Srikumar

Viterbi Algorithm

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The only terms that depend on y_1



slide credit: Vivek Srikumar

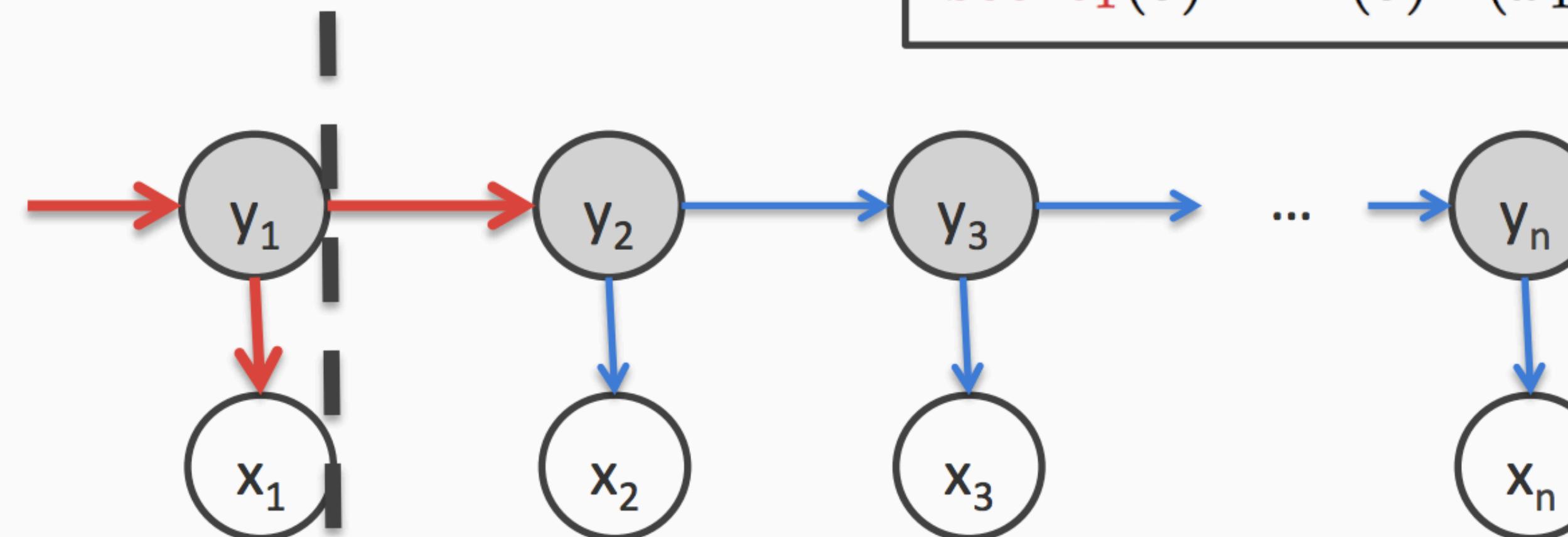
Viterbi Algorithm

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Abstract away the score for all decisions till here into **score**

$$\text{score}_1(s) = P(s)P(x_1|s)$$



Viterbi Algorithm

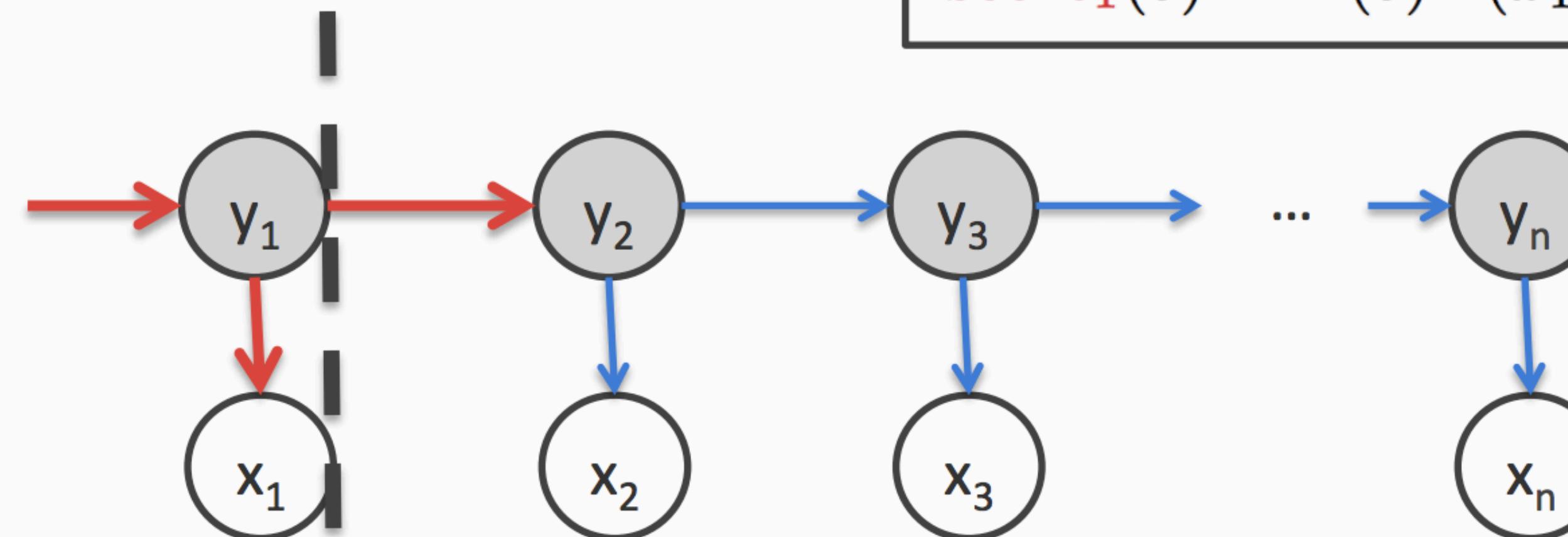
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best (partial) score for
a sequence ending in state s

Abstract away the score for all
decisions till here into **score**

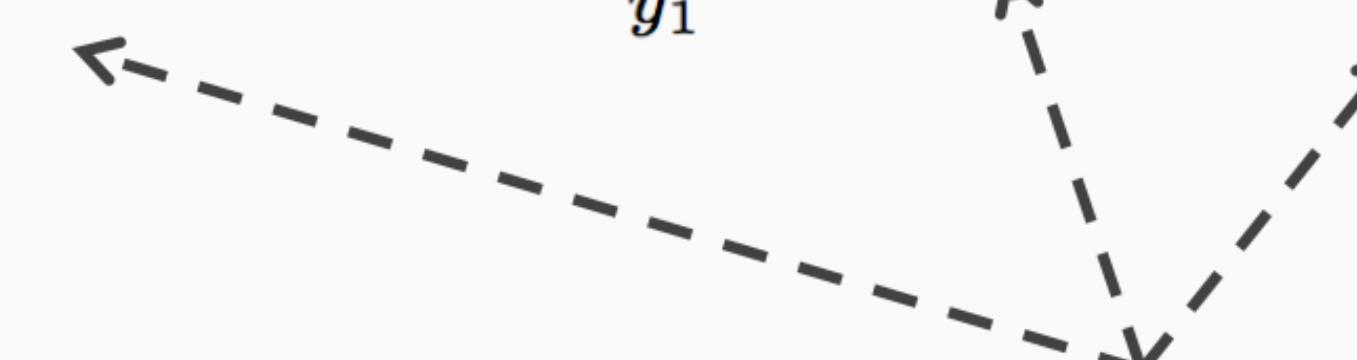
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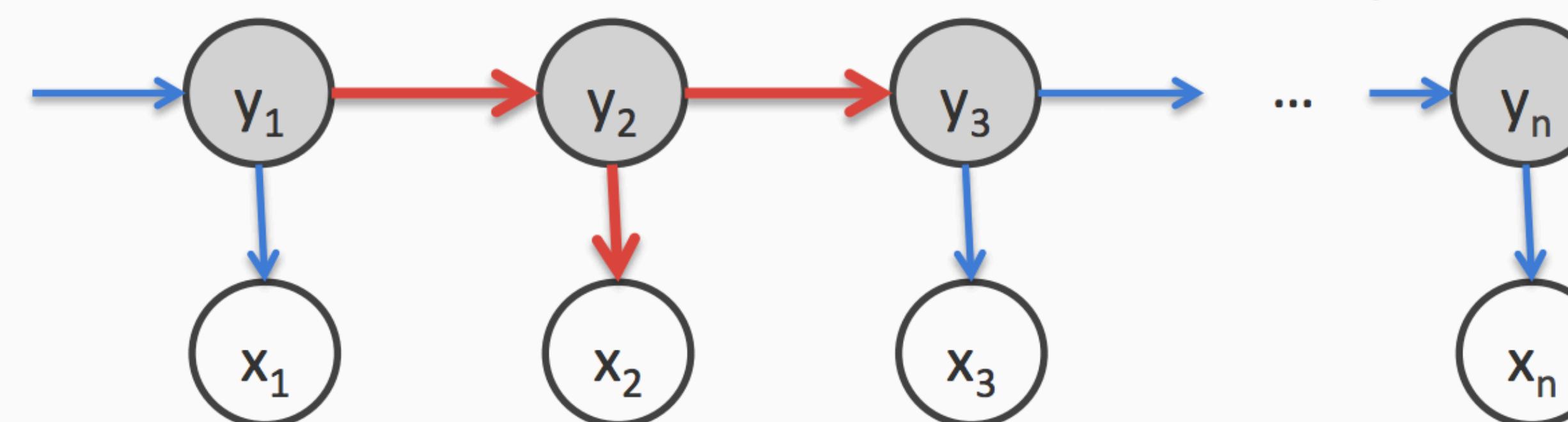
Viterbi Algorithm

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Only terms that depend on y_2

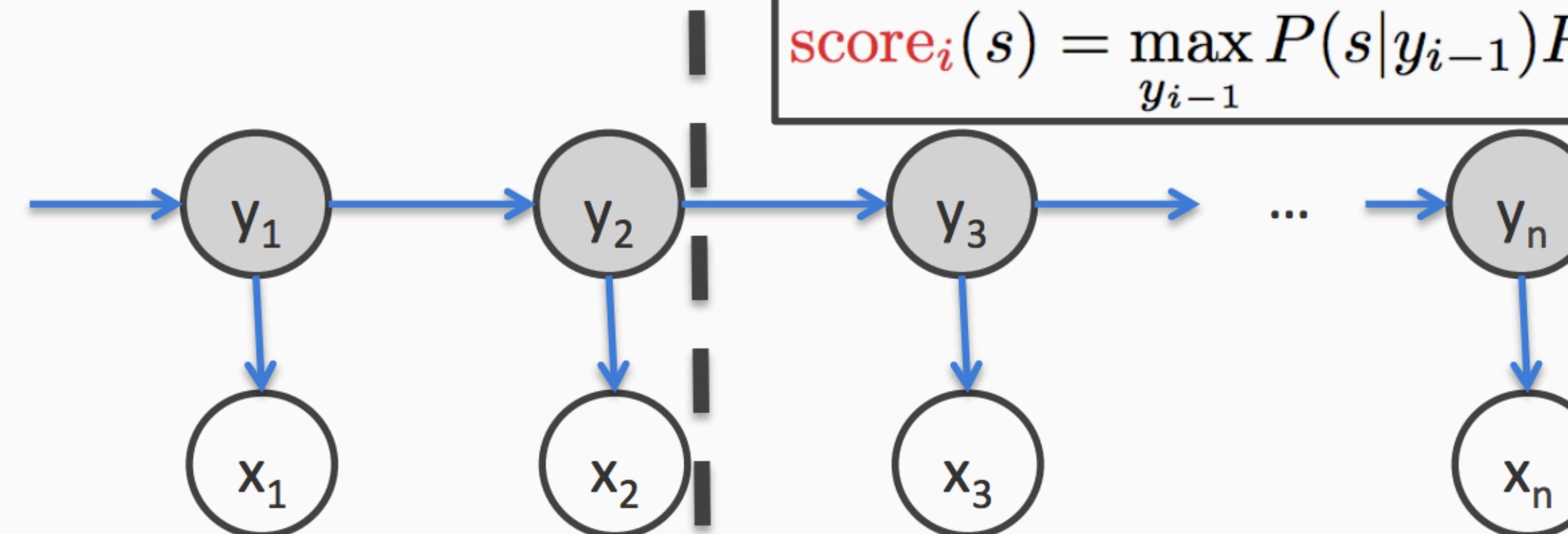


Viterbi Algorithm

$$P(x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n) = P(y_1) \prod_{i=1}^{n-1} P(y_{i+1}|y_i) \prod_{i=1}^n P(x_i|y_i)$$

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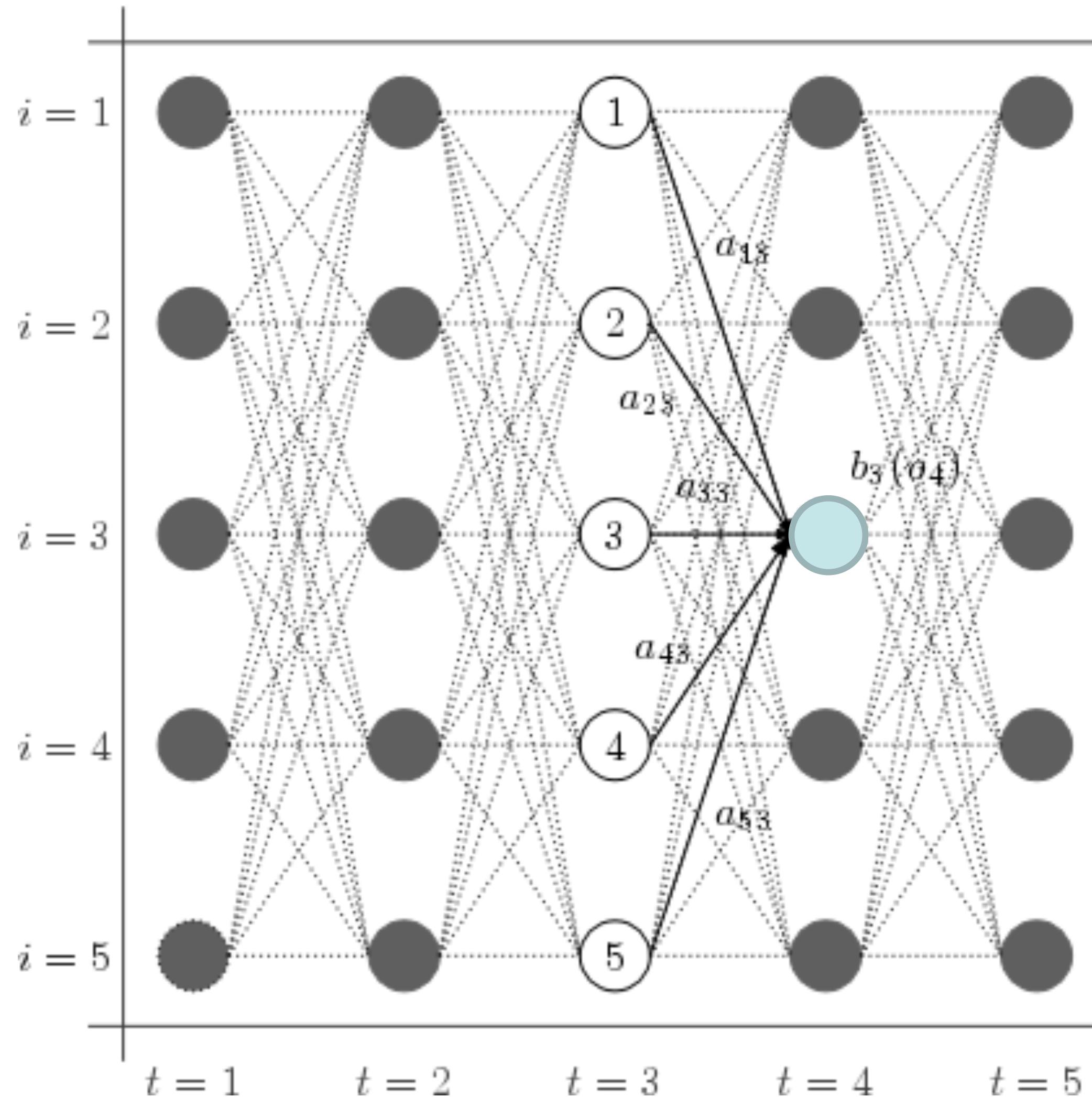
$$\text{score}_i(s) = \max_{y_{i-1}} P(s|y_{i-1})P(x_i|s) \text{score}_{i-1}(y_{i-1})$$



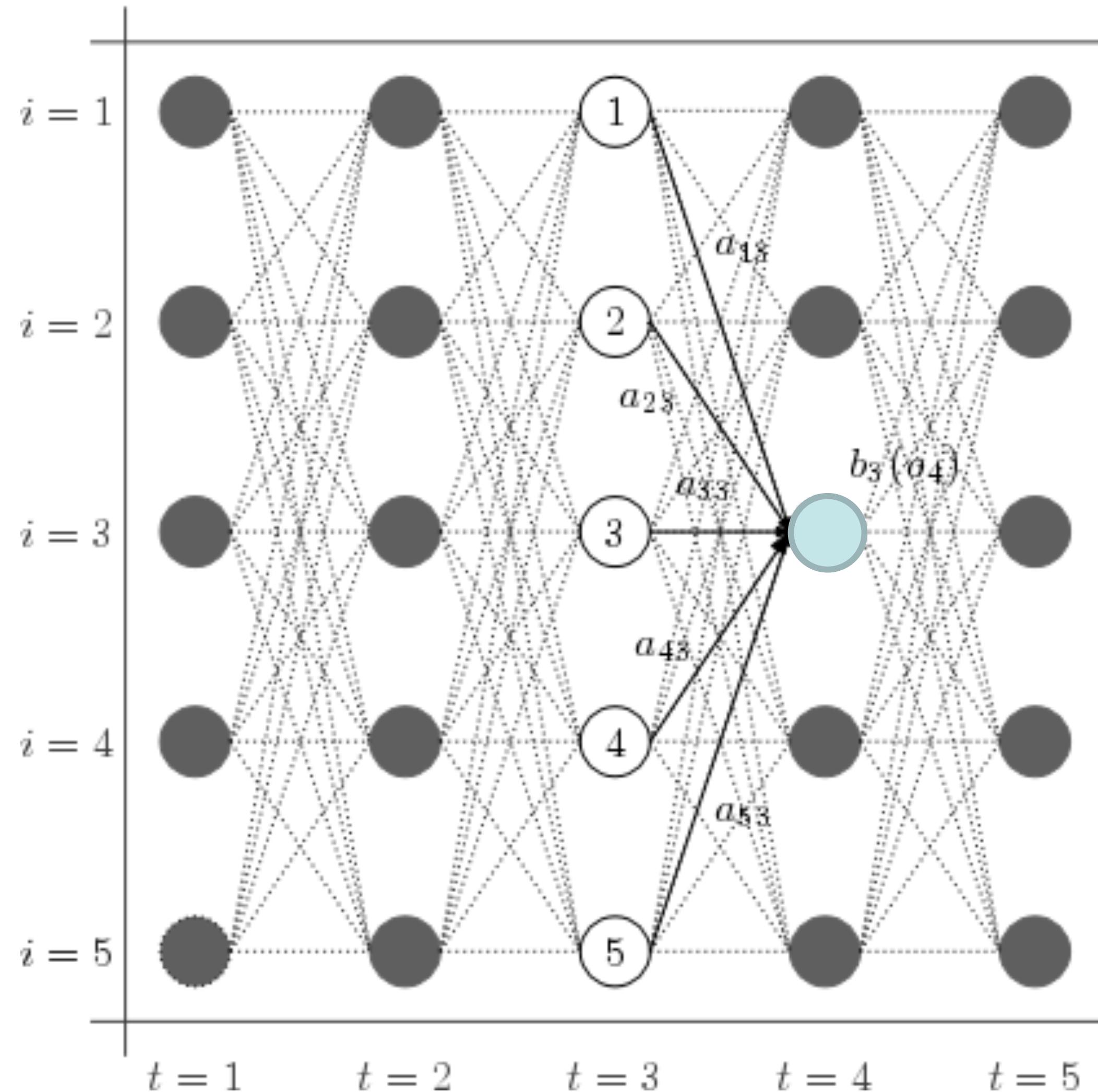
Abstract away the score for all decisions till here into **score**

slide credit: Vivek Srikumar

Viterbi Algorithm



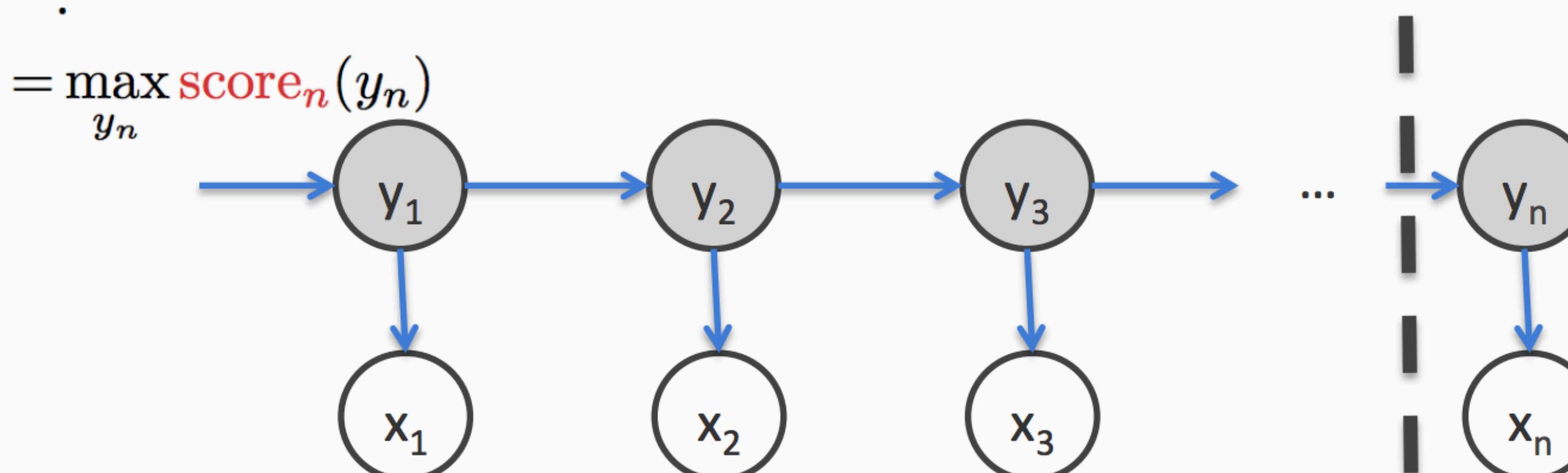
Viterbi Algorithm



► “Think about” all possible immediate prior state values. Everything before that has already been accounted for by earlier stages.

Viterbi Algorithm

$$\begin{aligned} P(x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n) &= P(y_1) \prod_{i=1}^{n-1} P(y_{i+1}|y_i) \prod_{i=1}^n P(x_i|y_i) \\ &\max_{y_1, y_2, \dots, y_n} P(y_n|y_{n-1})P(x_n|y_n) \cdots P(y_2|y_1)P(x_2|y_2)P(y_1)P(x_1|y_1) \\ &= \max_{y_2, \dots, y_n} P(y_n|y_{n-1})P(x_n|y_n) \cdots \max_{y_1} P(y_2|y_1)P(x_2|y_2)P(y_1)P(x_1|y_1) \\ &= \max_{y_2, \dots, y_n} P(y_n|y_{n-1})P(x_n|y_n) \cdots \max_{y_1} P(y_2|y_1)P(x_2|y_2)\text{score}_1(y_1) \\ &= \max_{y_3, \dots, y_n} P(y_n|y_{n-1})P(x_n|y_n) \cdots \max_{y_2} P(y_3|y_2)P(x_3|y_3) \max_{y_1} P(y_2|y_1)P(x_2|y_2)\text{score}_1(y_1) \\ &= \max_{y_3, \dots, y_n} P(y_n|y_{n-1})P(x_n|y_n) \cdots \max_{y_2} P(y_3|y_2)P(x_3|y_3)\text{score}_2(y_2) \\ &\vdots \\ &= \max_{y_n} \text{score}_n(y_n) \end{aligned}$$



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$$\text{score}_1(s) = P(s)P(x_1|s)$$

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Viterbi Algorithm

1. **Initial:** For each state s , calculate

$$\text{score}_1(s) = P(s)P(x_1|s) = \pi_s B_{x_1,s}$$

2. **Recurrence:** For $i = 2$ to n , for every state s , calculate

$$\begin{aligned}\text{score}_i(s) &= \max_{y_{i-1}} P(s|y_{i-1})P(x_i|s)\text{score}_{i-1}(y_{i-1}) \\ &= \max_{y_{i-1}} A_{y_{i-1},s}B_{s,x_i}\text{score}_{i-1}(y_{i-1})\end{aligned}$$

3. **Final state:** calculate

$$\max_{\mathbf{y}} P(\mathbf{y}, \mathbf{x}|\pi, A, B) = \max_s \text{score}_n(s)$$

π : Initial probabilities
A: Transitions
B: Emissions

This only calculates the max. To get final answer (*argmax*),

- keep track of which state corresponds to the max at each step
- build the answer using these back pointers

Forward-Backward Algorithm

Forward-Backward Algorithm

- ▶ In addition to finding the best path, we may want to compute marginal probabilities of paths $P(y_i = s | \mathbf{x})$

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$$P(y_i = s | \mathbf{x}) = \sum_{y_1, \dots, y_{i-1}, y_{i+1}, \dots, y_n} P(\mathbf{y} | \mathbf{x})$$

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- ▶ What did Viterbi compute? $P(\mathbf{y}_{\max} | \mathbf{x}) = \max_{y_1, \dots, y_n} P(\mathbf{y} | \mathbf{x})$

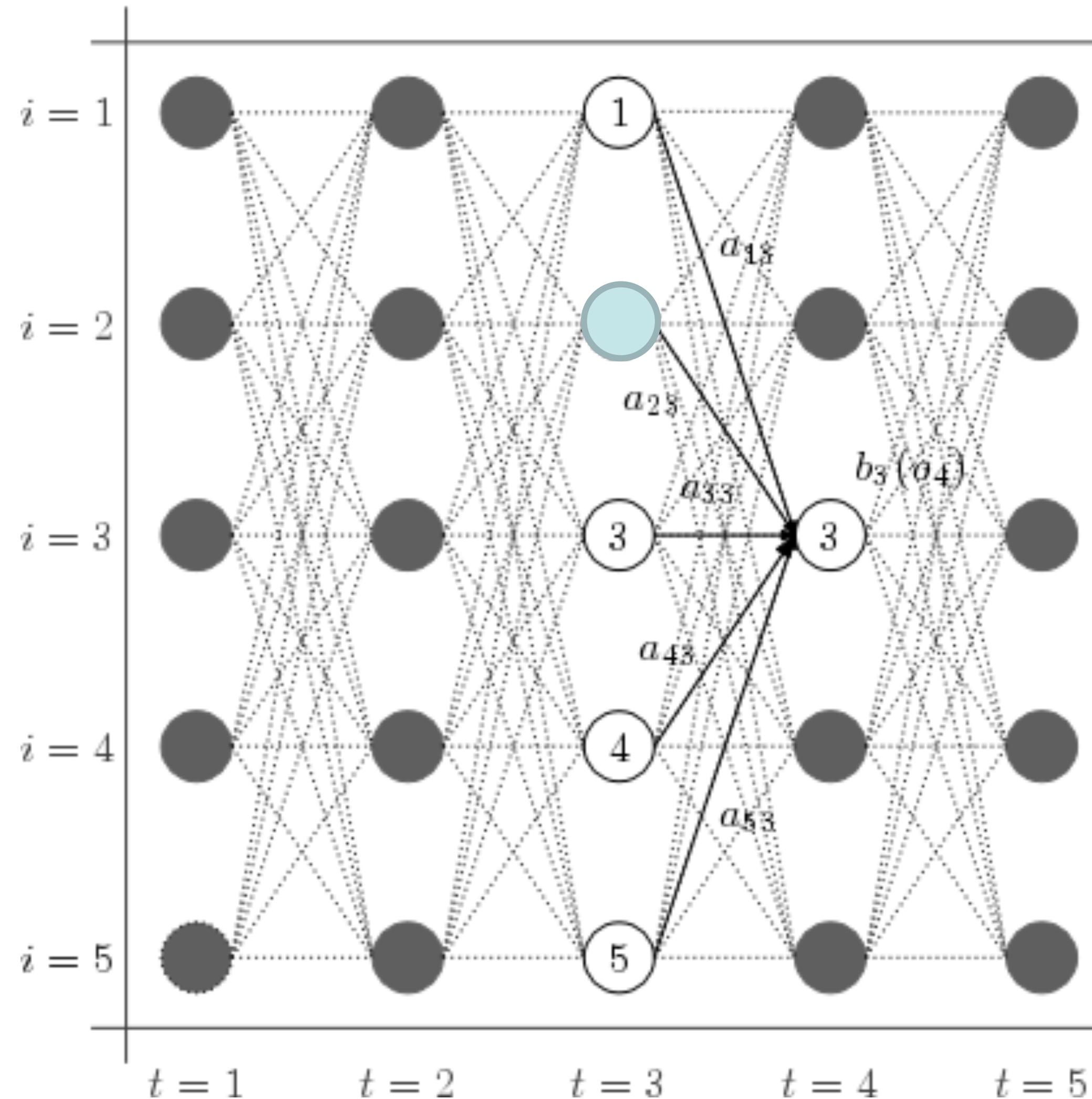
Forward-Backward Algorithm

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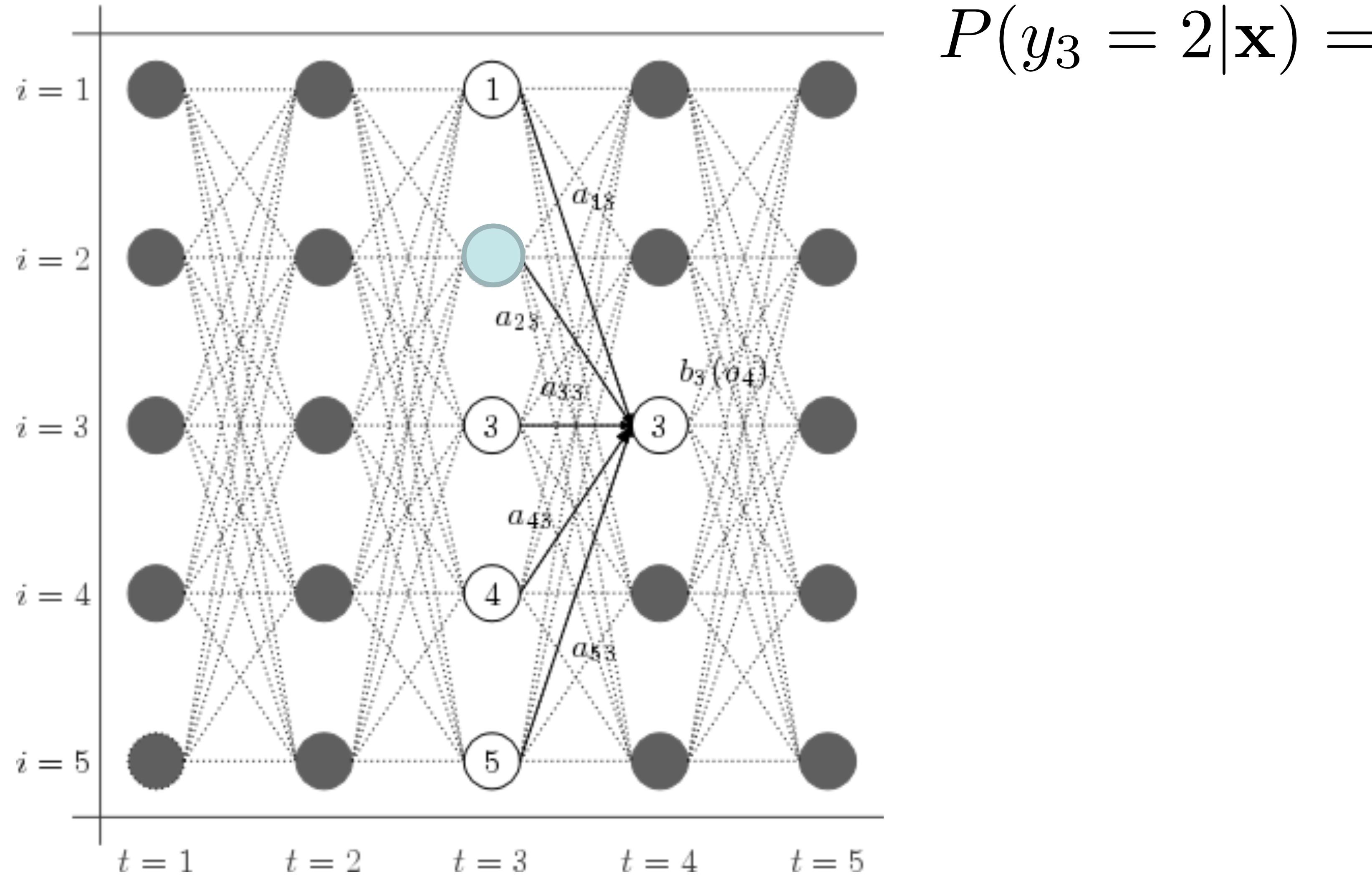
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- ▶ What did Viterbi compute? $P(\mathbf{y}_{\max} | \mathbf{x}) = \max_{y_1, \dots, y_n} P(\mathbf{y} | \mathbf{x})$
- ▶ Can compute marginals with dynamic programming as well using an algorithm called forward-backward

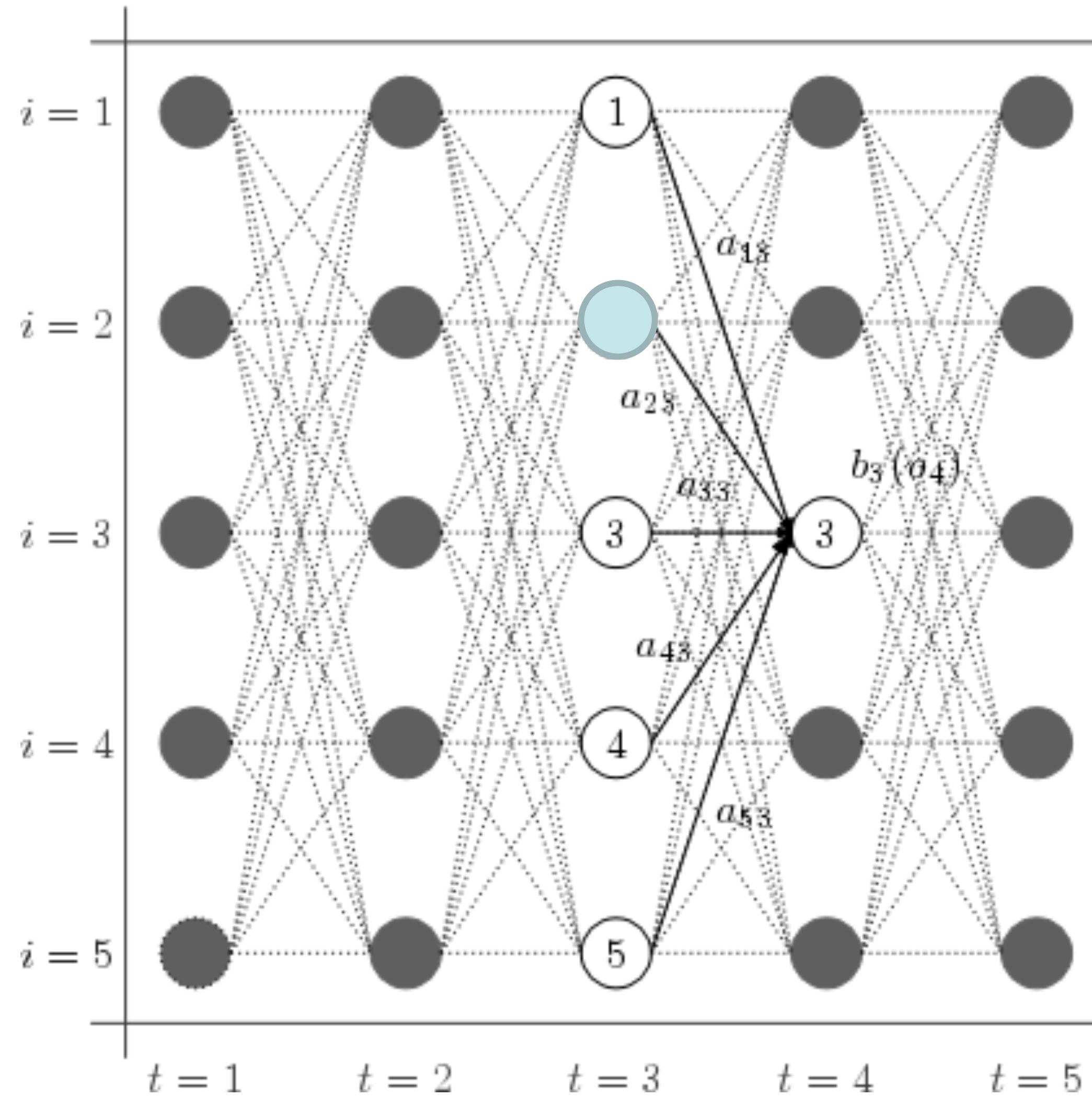
Forward-Backward Algorithm



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Forward-Backward Algorithm

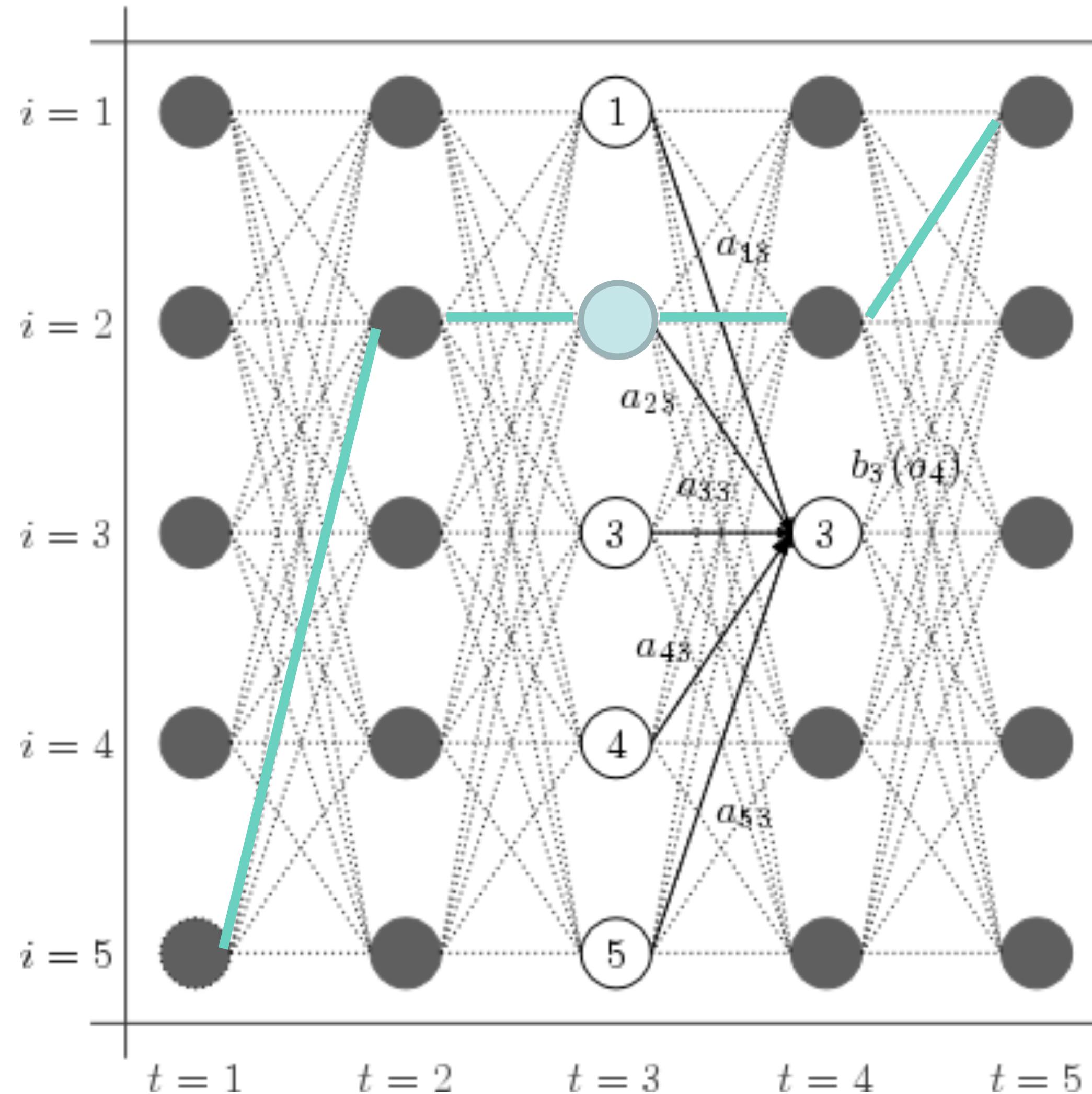


$$P(y_3 = 2 | \mathbf{x}) =$$

sum of all paths through state 2 at time 3

sum of all paths

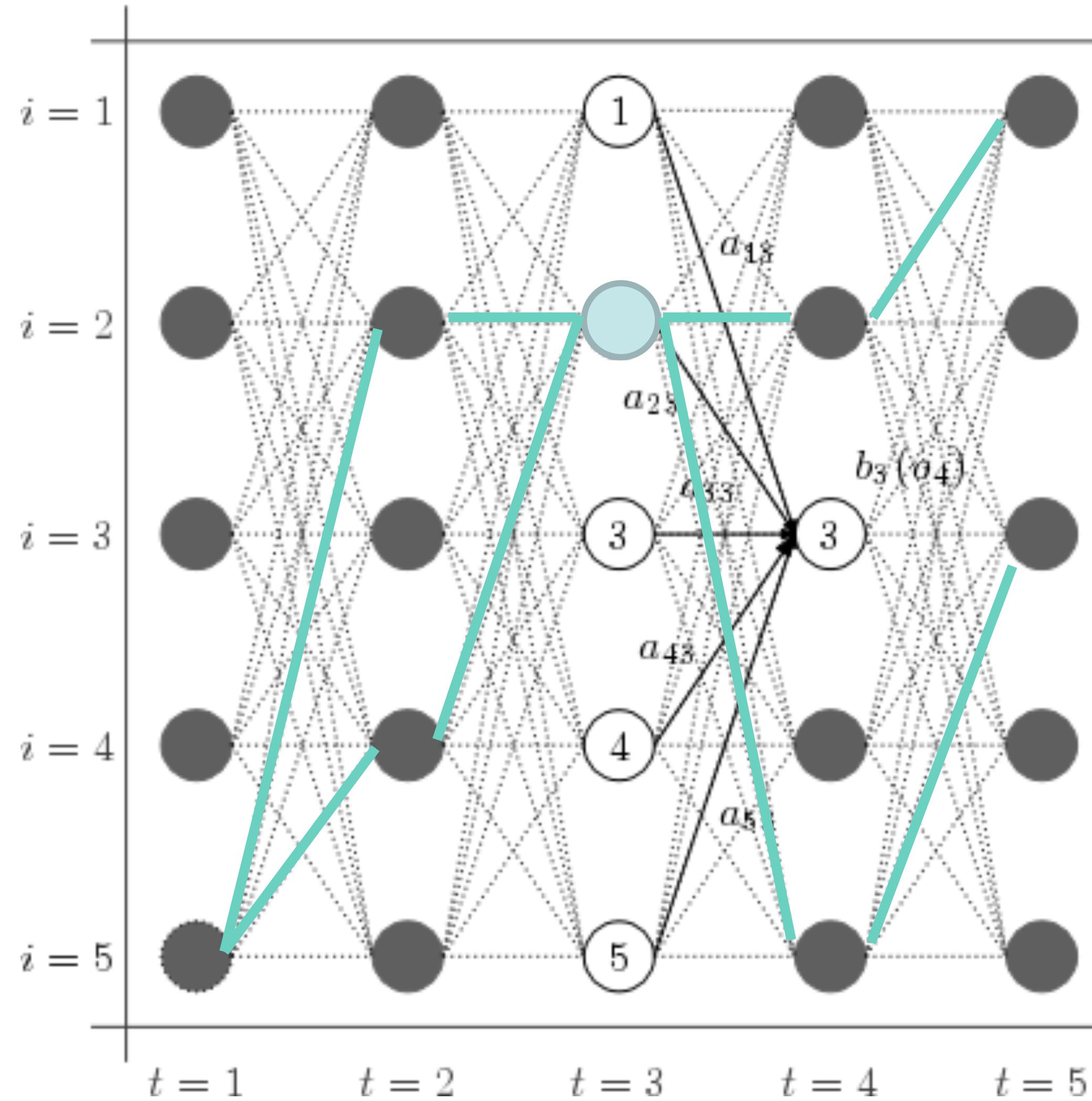
Forward-Backward Algorithm



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Forward-Backward Algorithm

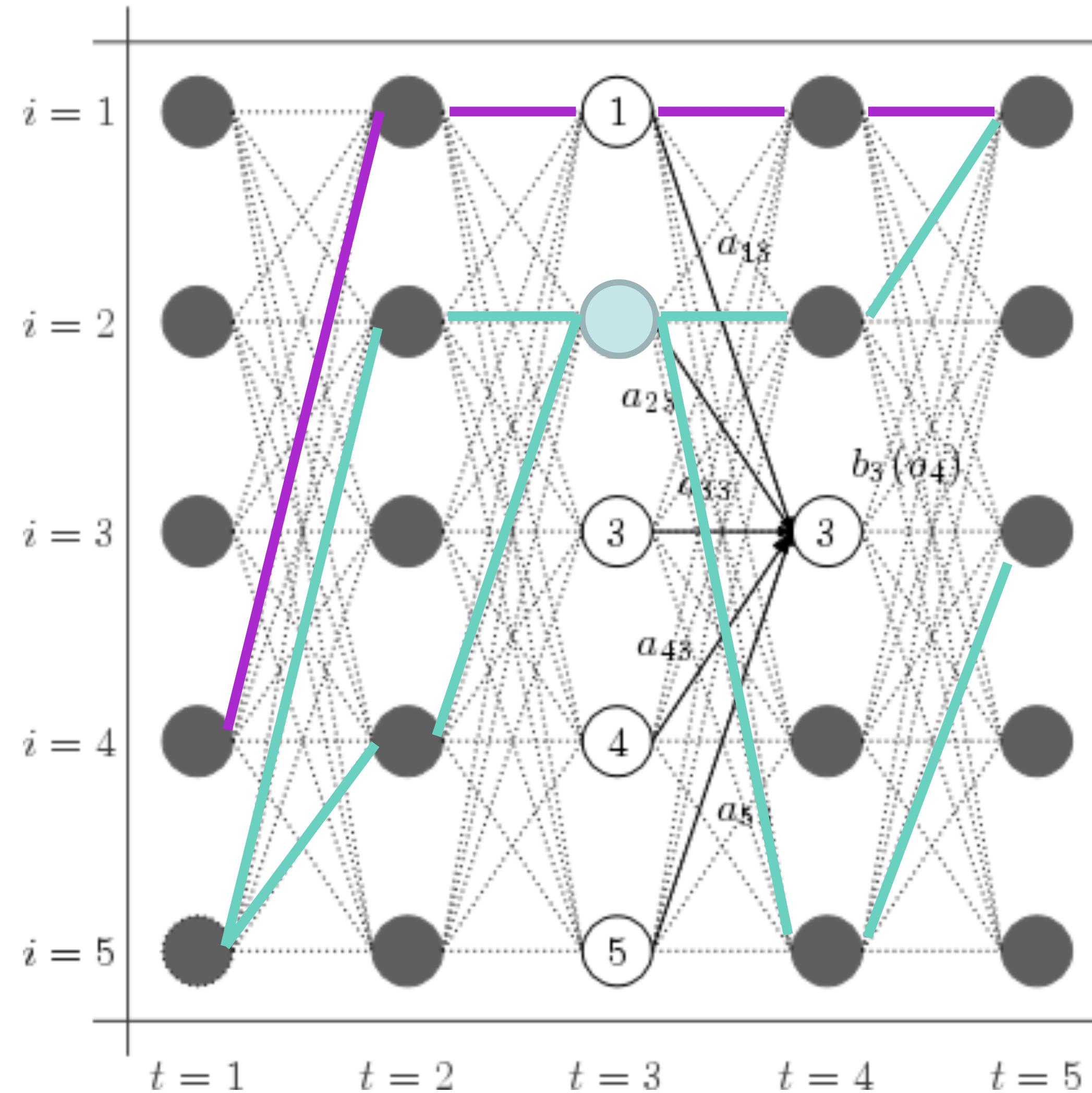


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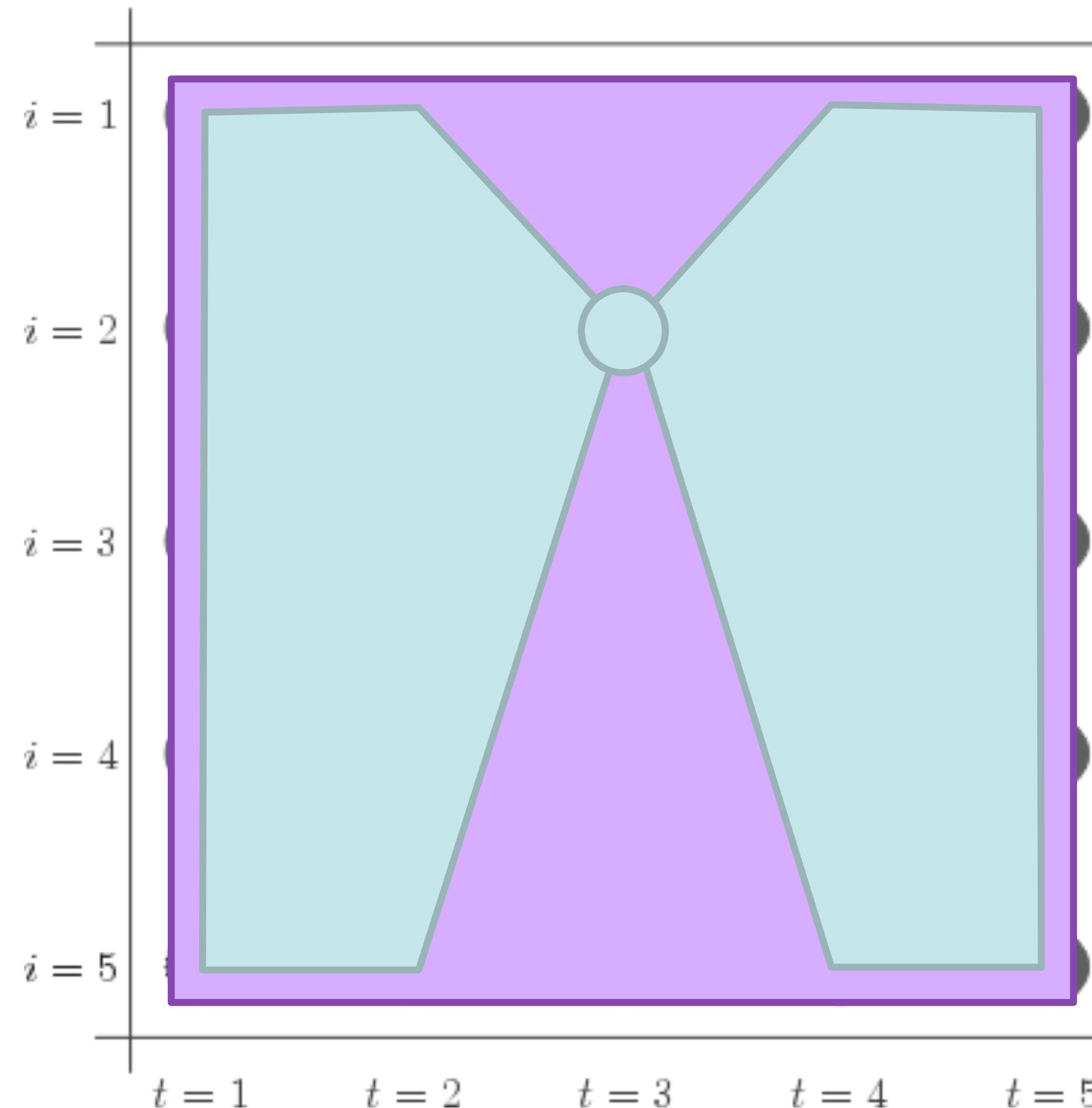
Forward-Backward Algorithm



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Forward-Backward Algorithm

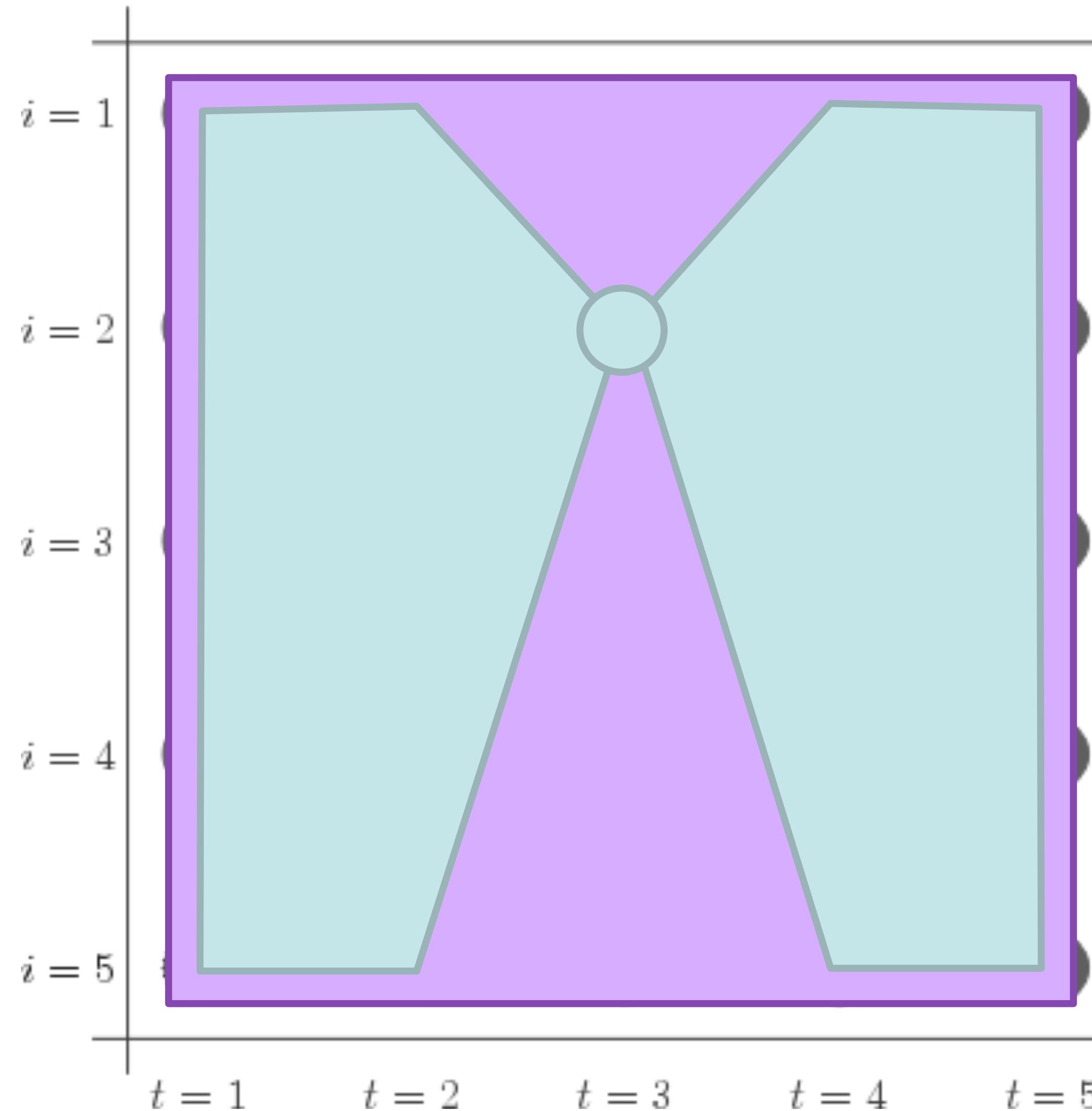


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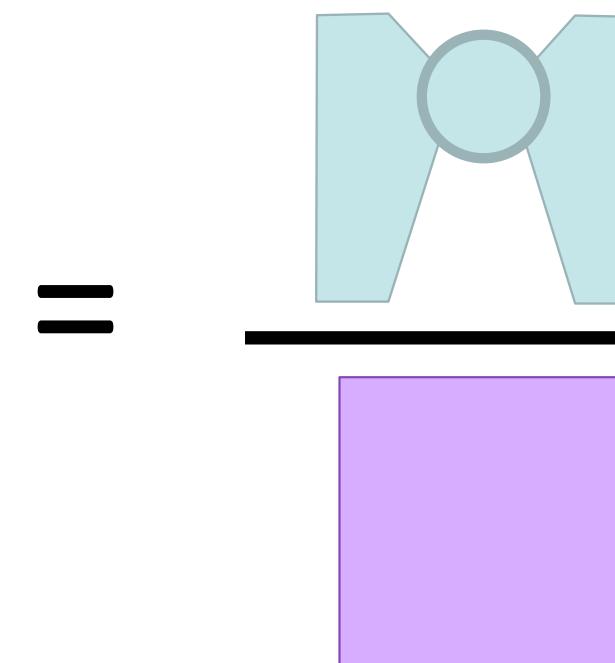
Forward-Backward Algorithm



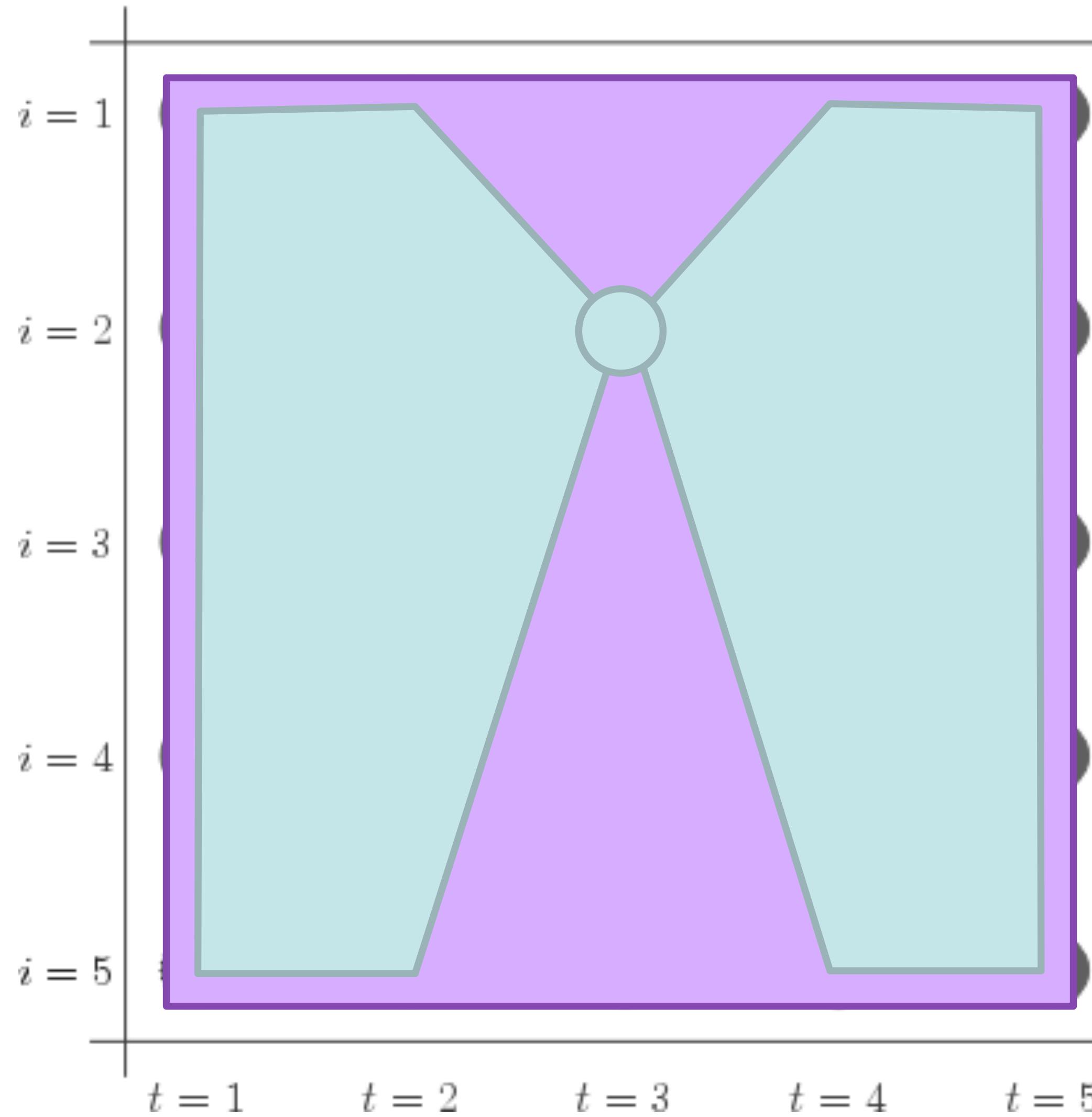
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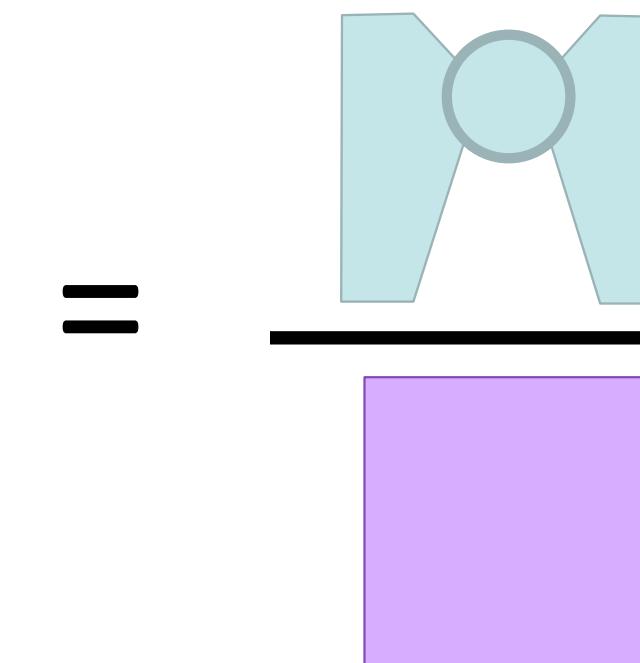


Forward-Backward Algorithm



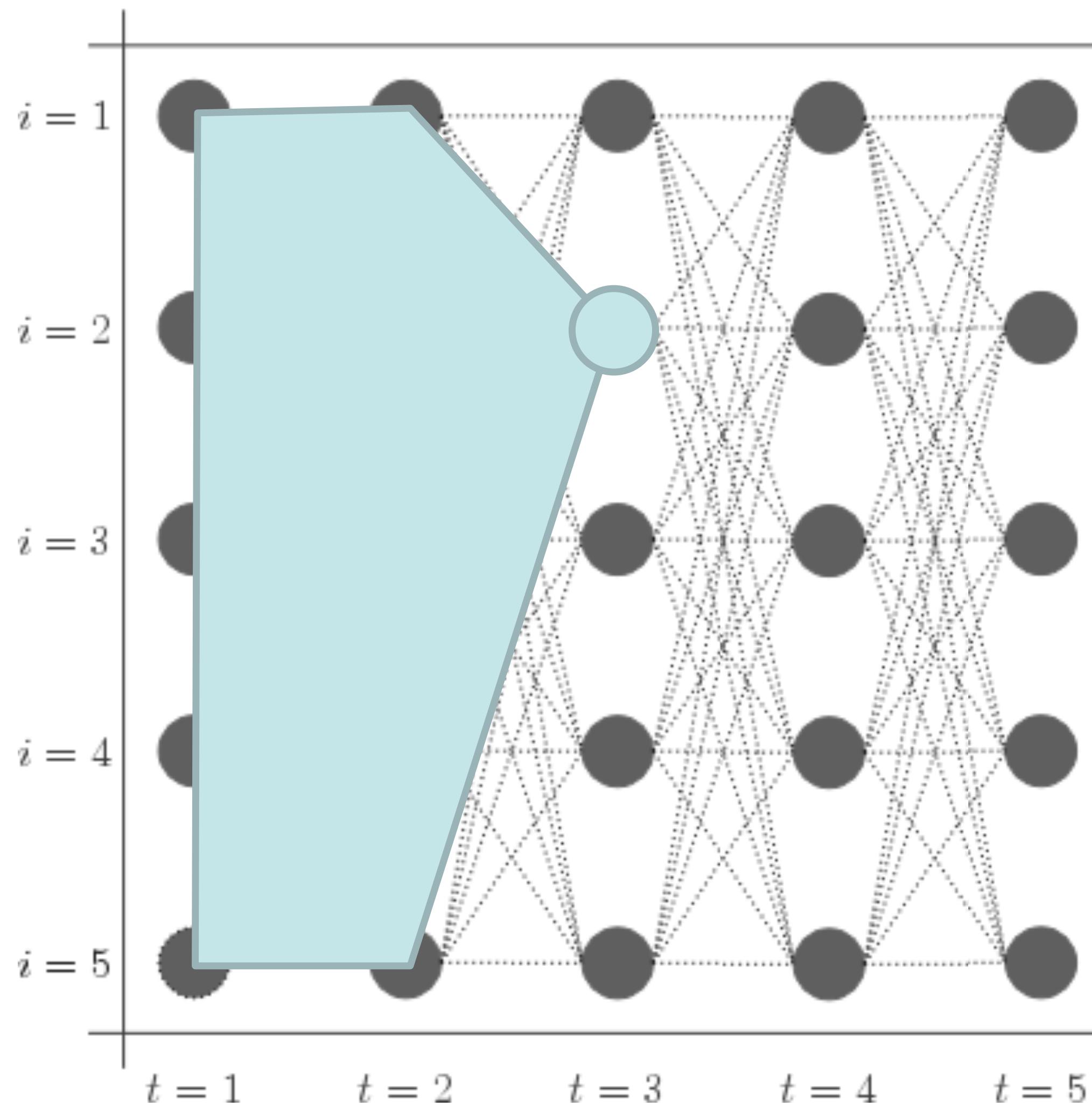
$$P(y_3 = 2 | \mathbf{x}) =$$

$$\frac{\text{sum of all paths through state 2 at time 3}}{\text{sum of all paths}}$$

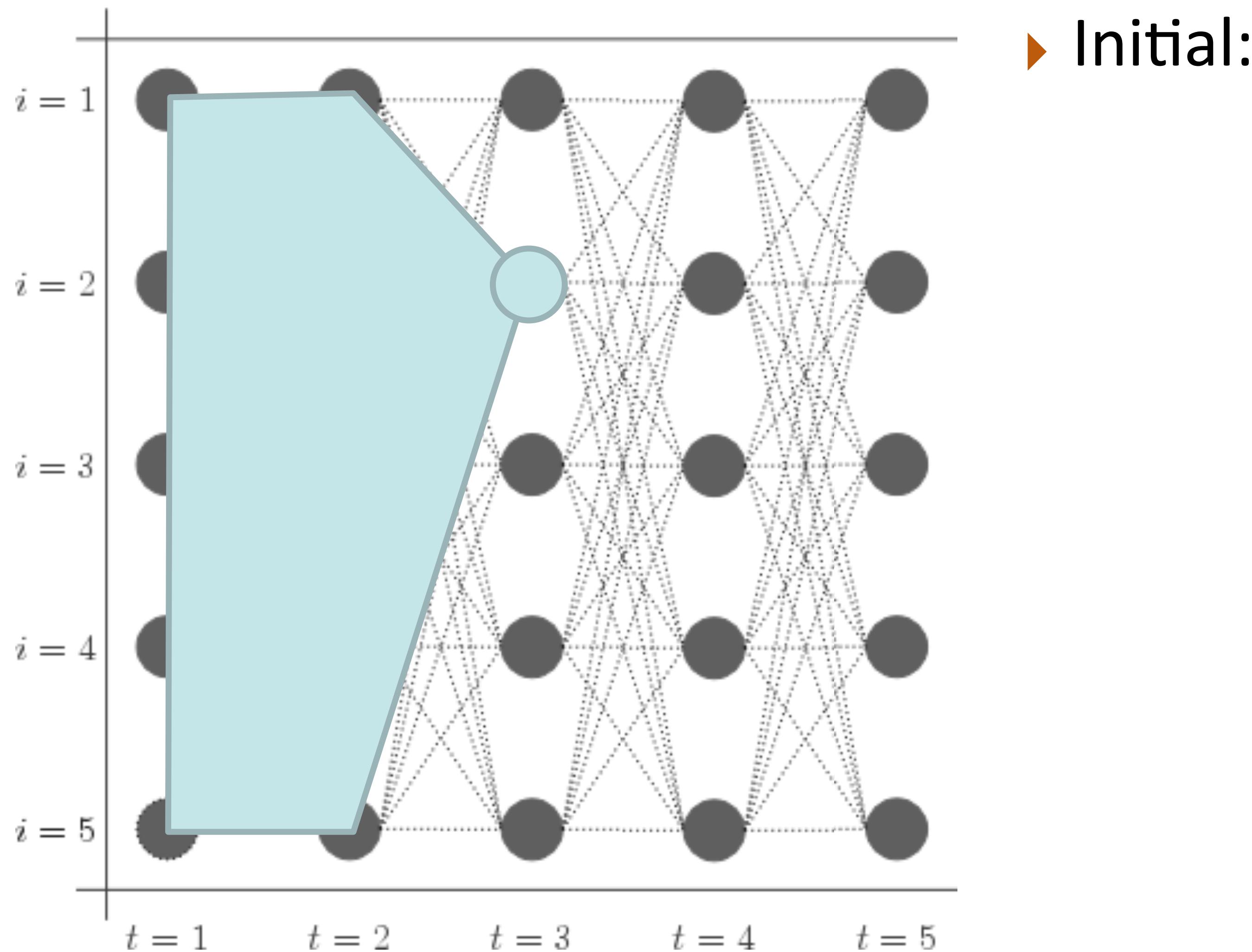


- ▶ Easiest and most flexible to do one pass to compute  and one to compute 

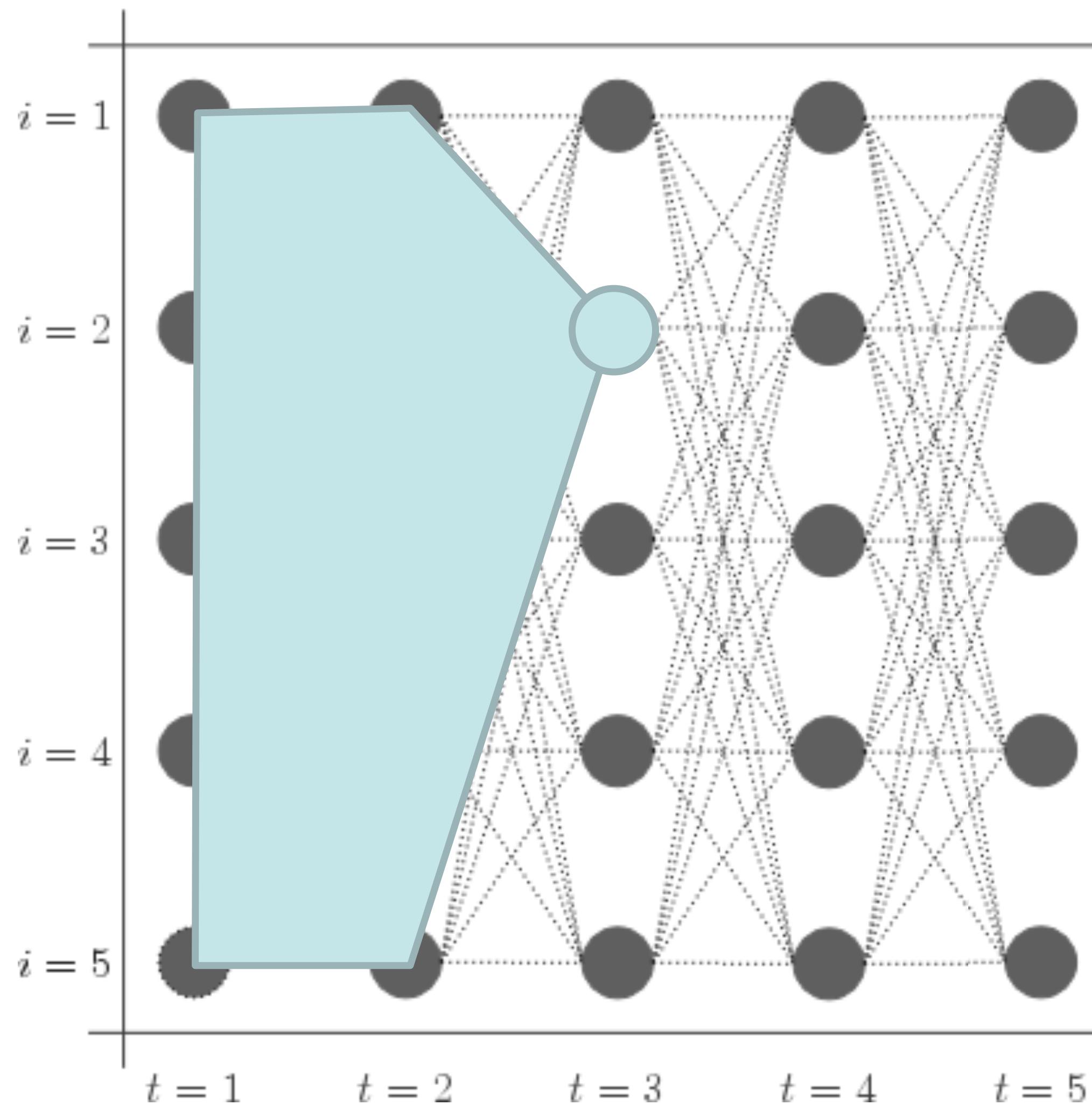
Forward-Backward Algorithm



Forward-Backward Algorithm



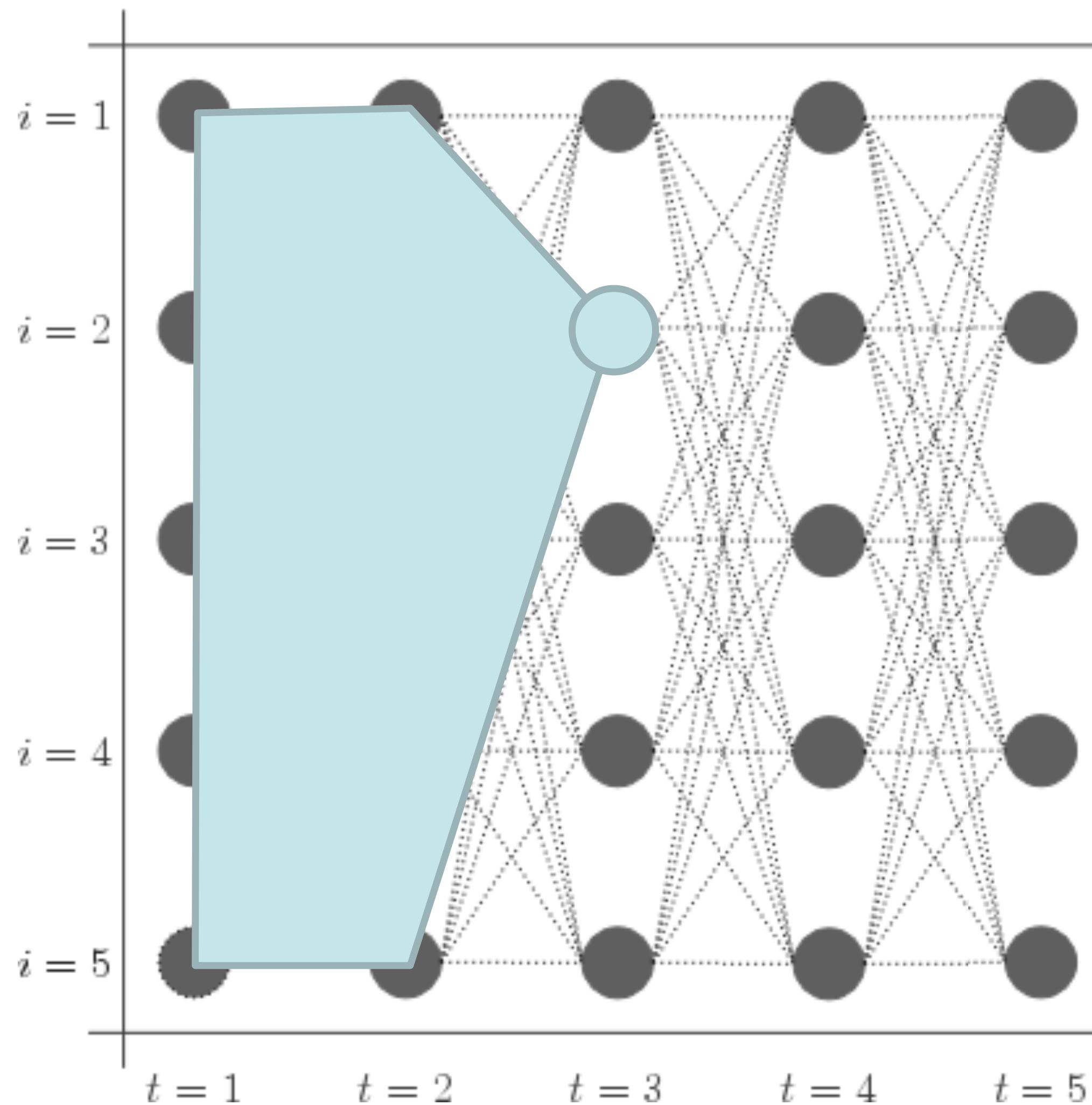
Forward-Backward Algorithm



► Initial:

$$\alpha_1(s) = P(s)P(x_1|s)$$

Forward-Backward Algorithm

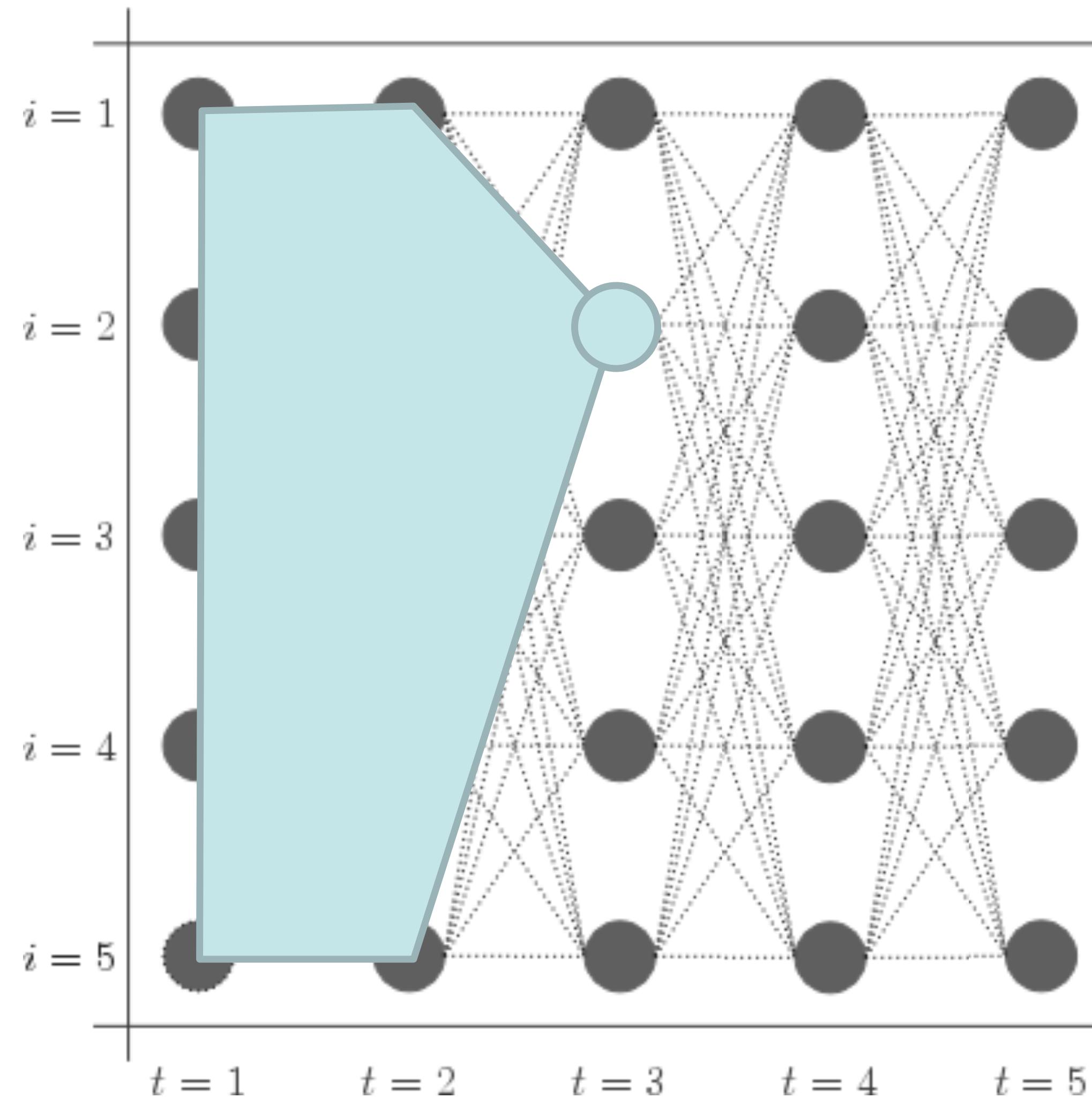


► Initial:

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► Recurrence:

Forward-Backward Algorithm



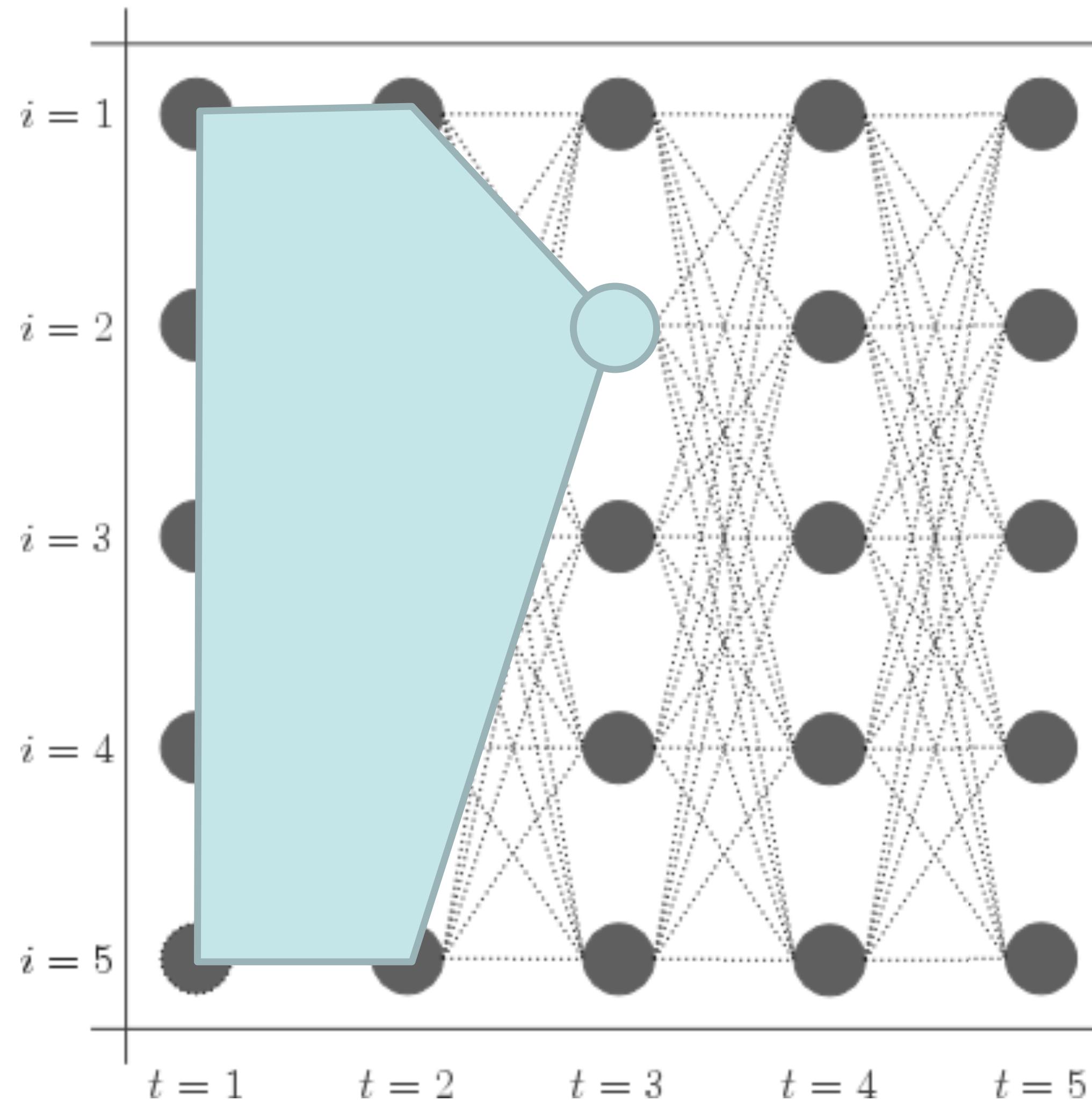
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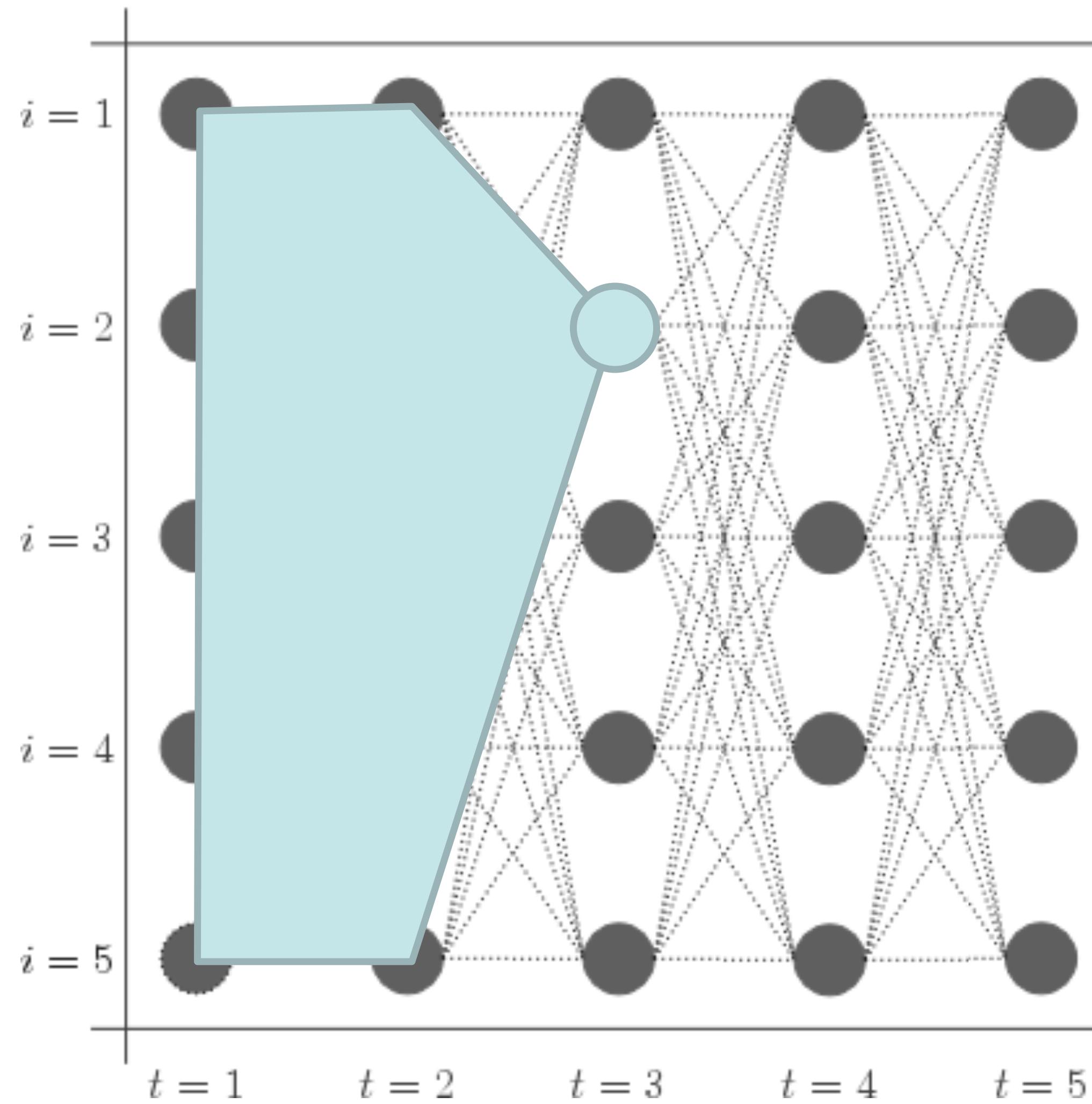
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► Same as Viterbi but summing instead of maxing!

Forward-Backward Algorithm



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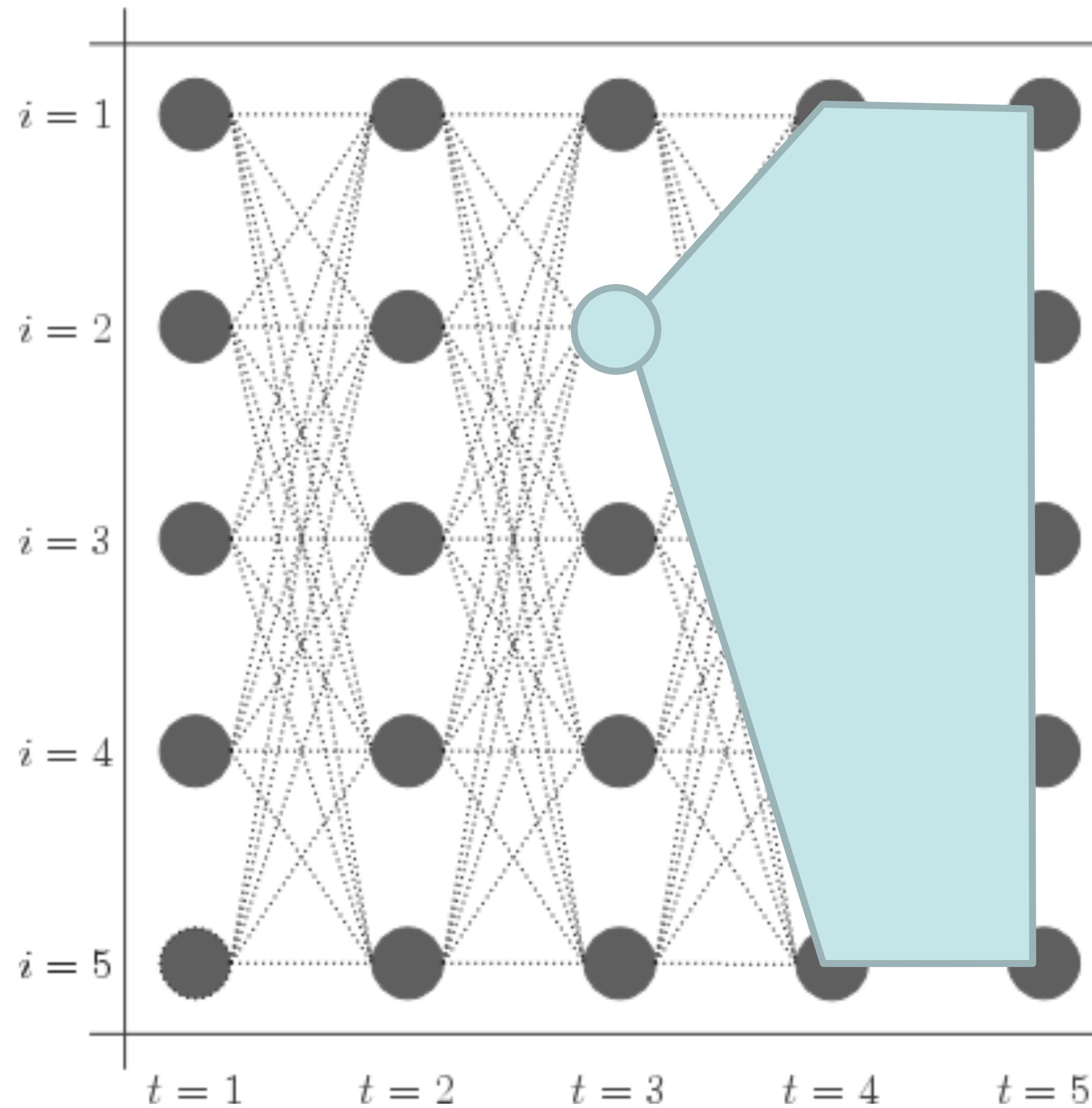
► Recurrence:

$$\alpha_t(s_t) = \sum_{s_{t-1}} \alpha_{t-1}(s_{t-1})P(s_t|s_{t-1})P(x_t|s_t)$$

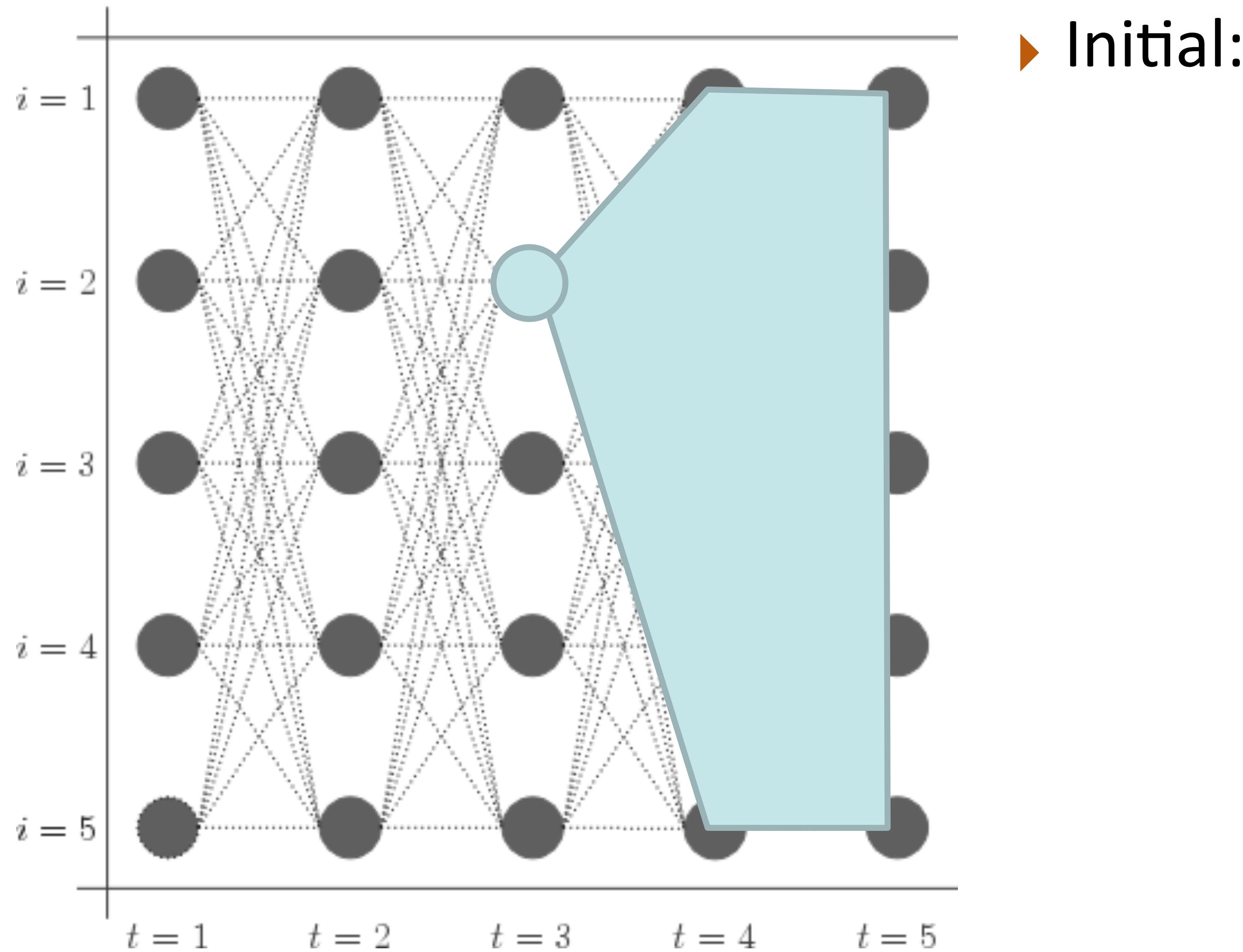
► Same as Viterbi but summing instead of maxing!

► These quantities get very small!
Store everything as log probabilities

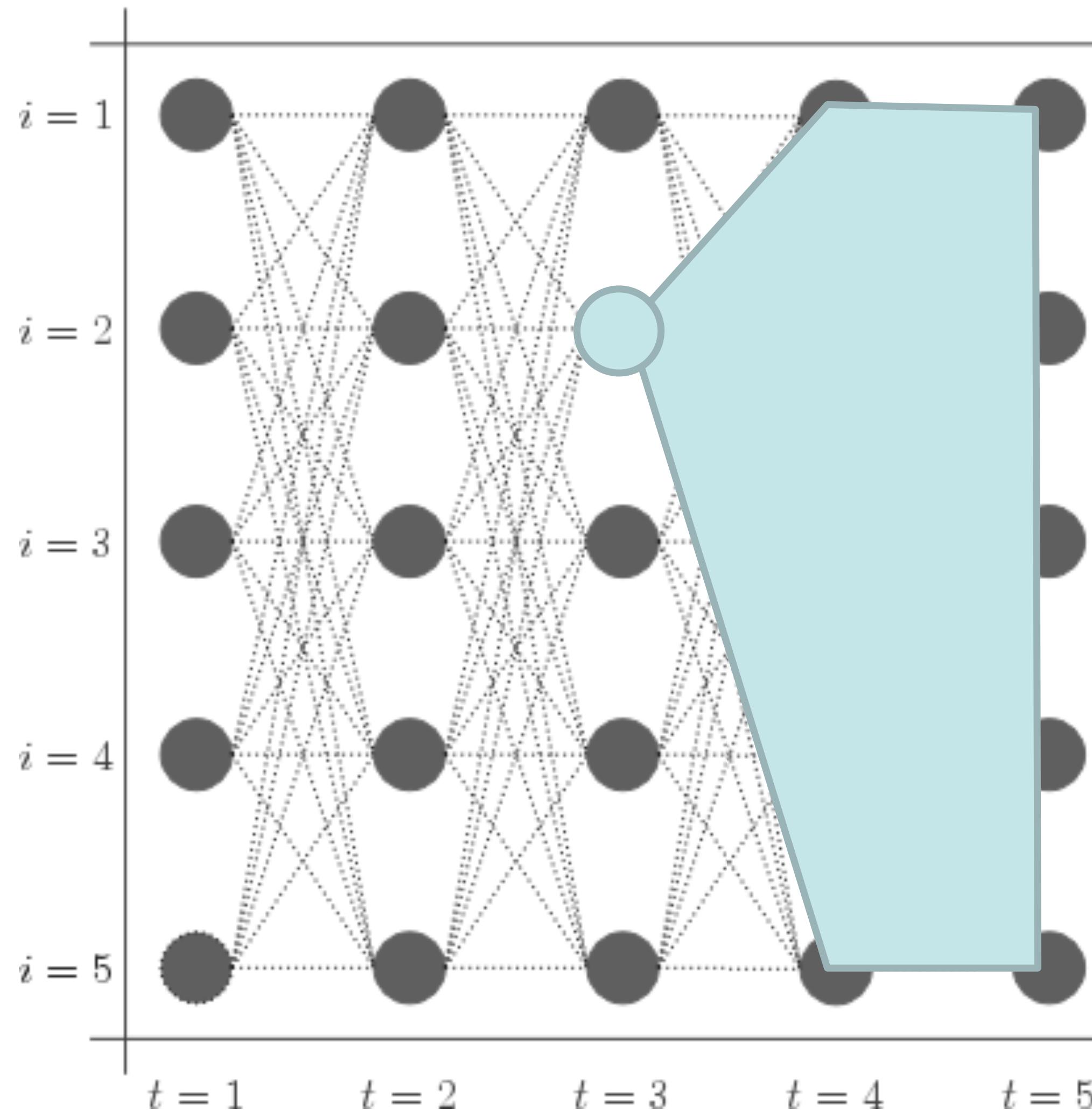
Forward-Backward Algorithm



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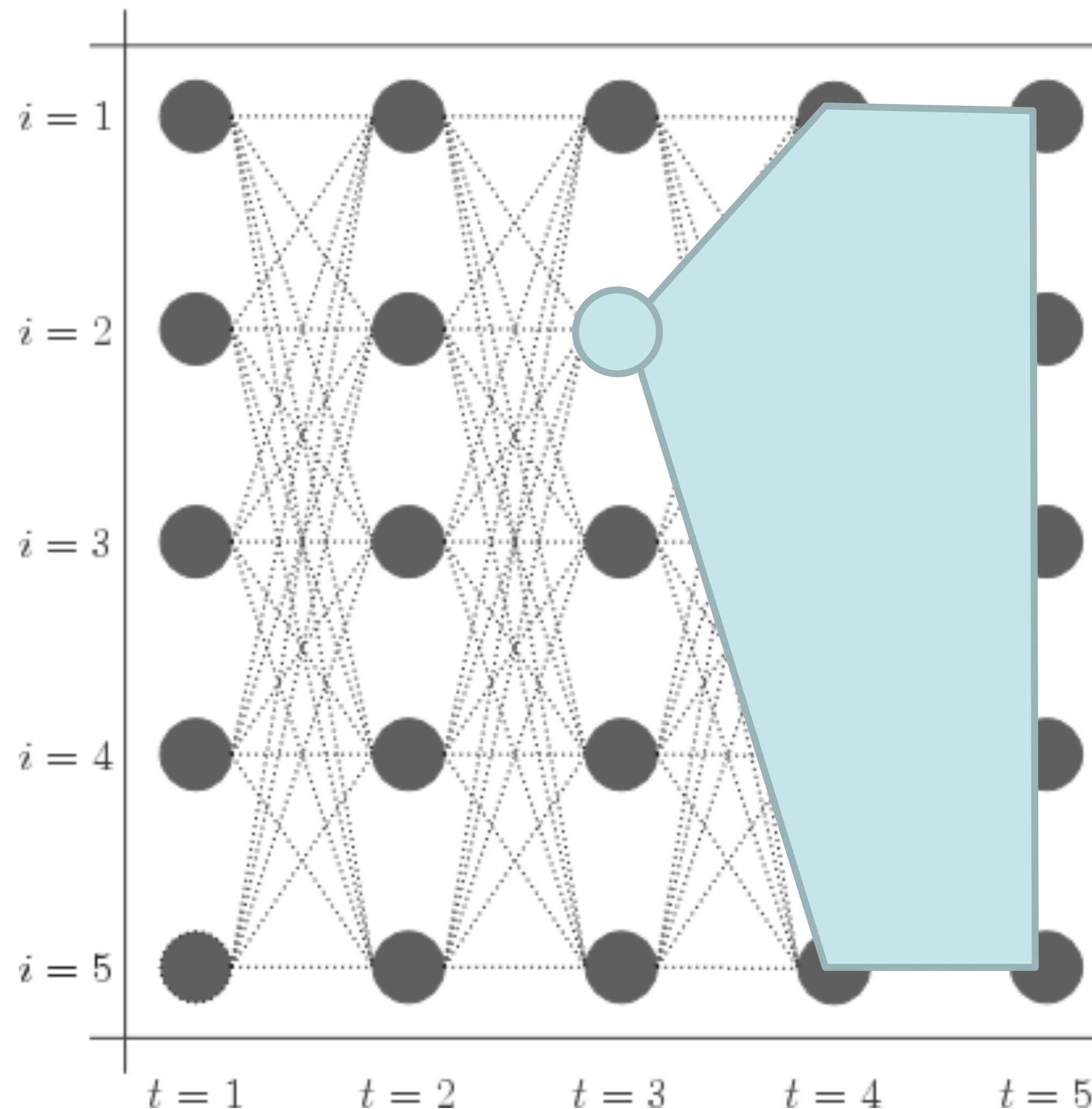
Forward-Backward Algorithm



► Initial:

$$\beta_n(s) = 1$$

Forward-Backward Algorithm

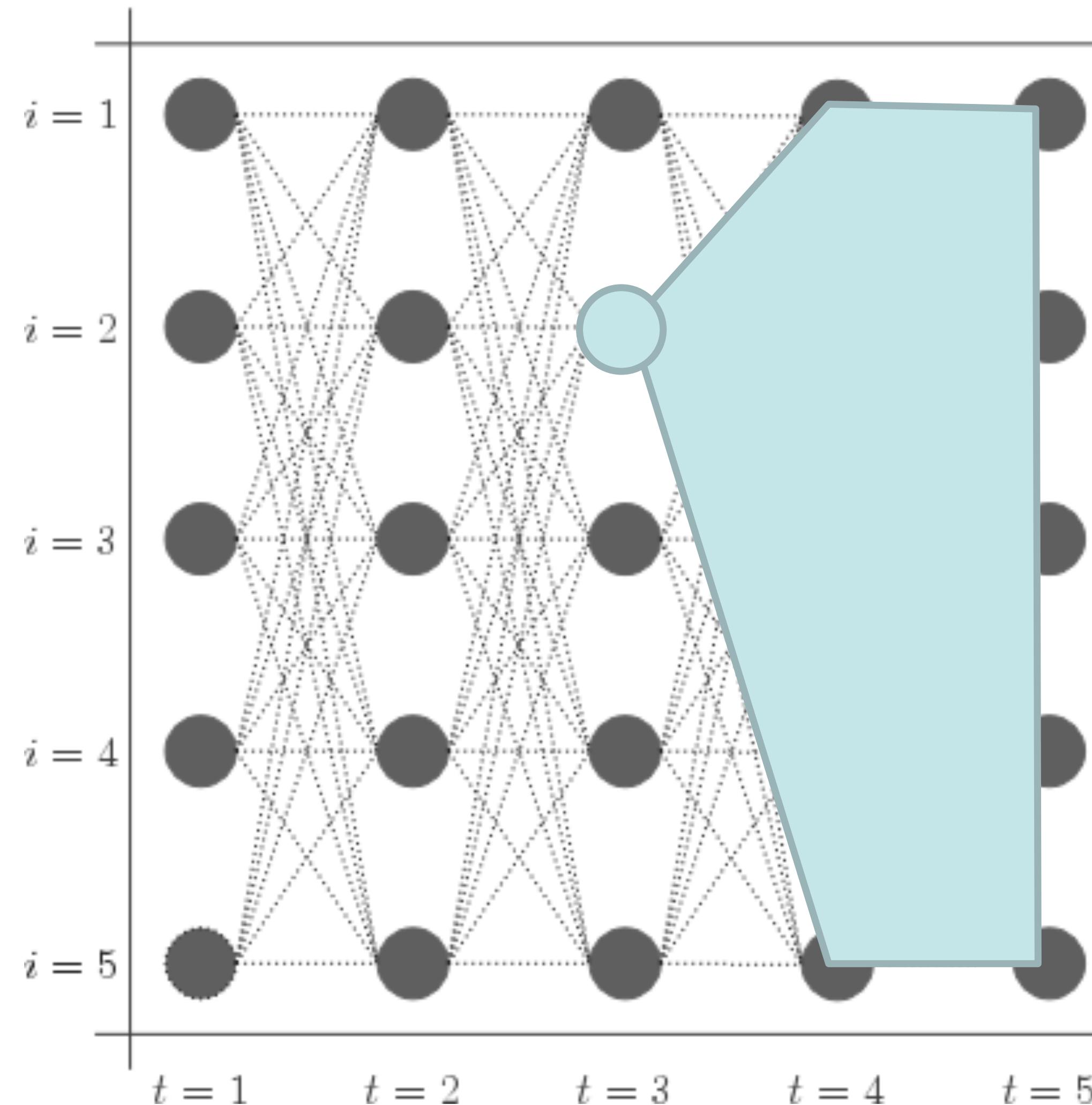


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Forward-Backward Algorithm



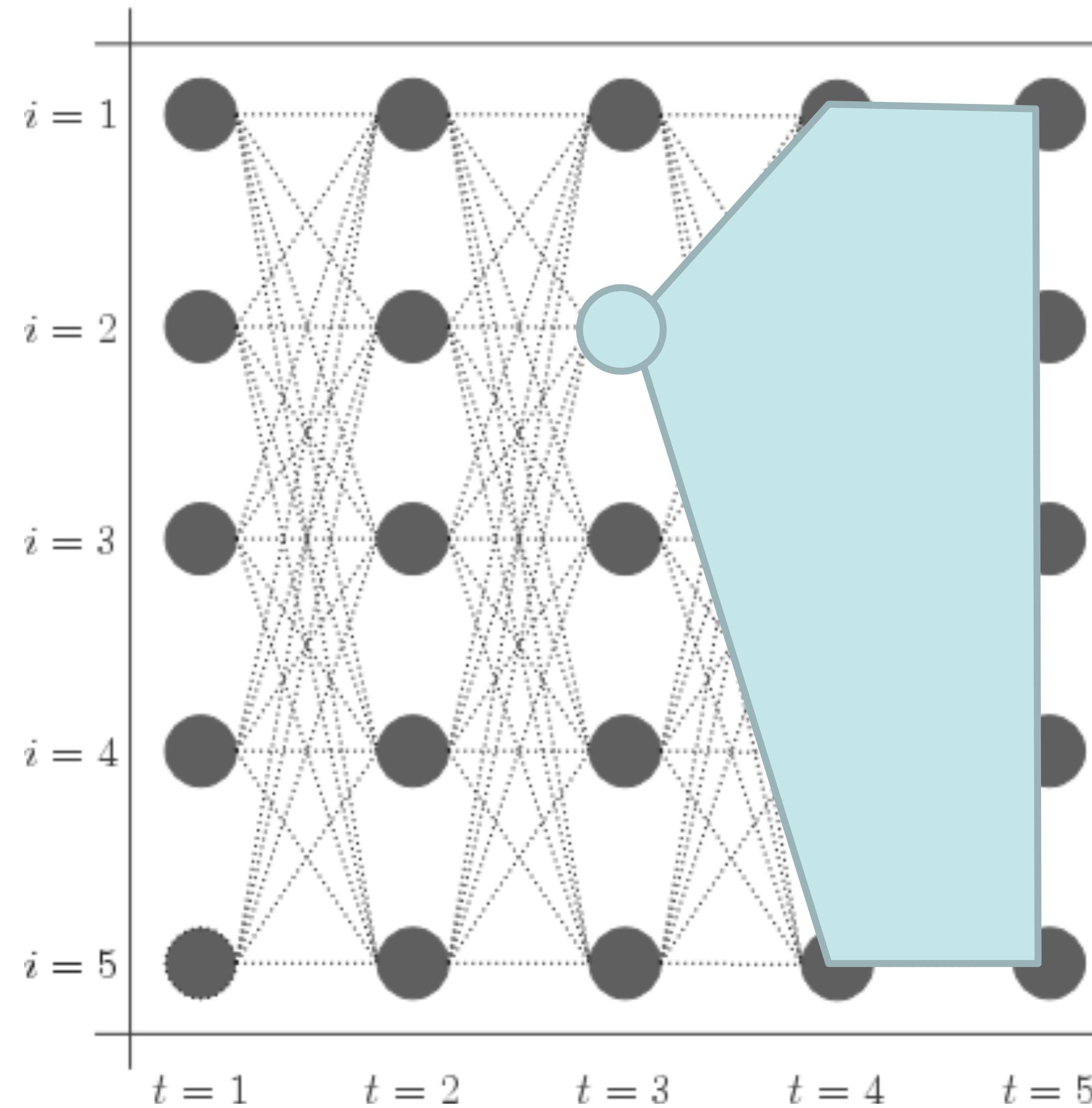
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Forward-Backward Algorithm



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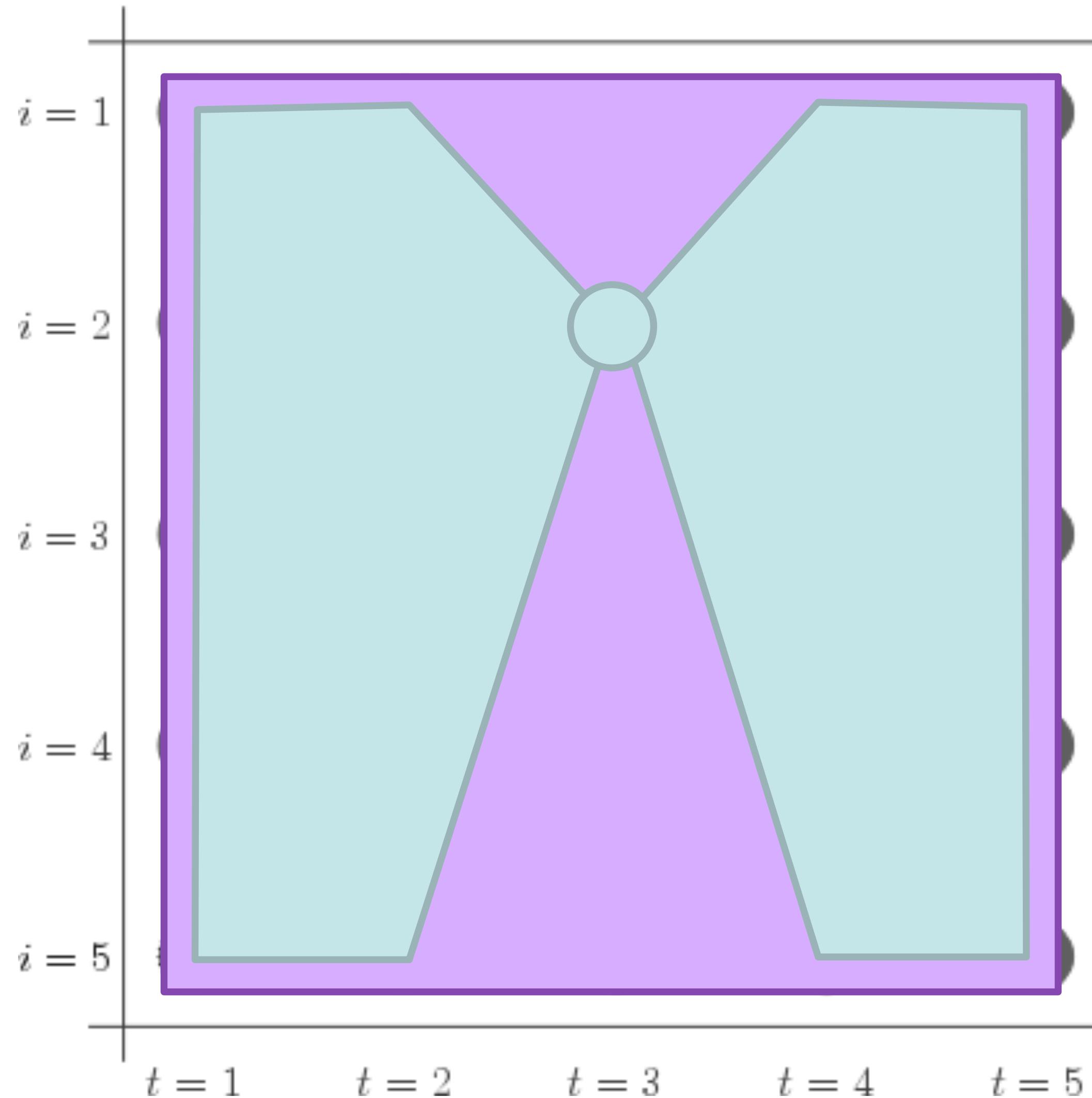
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► Big differences: count emission for the *next timestep* (not current one)

Forward-Backward Algorithm



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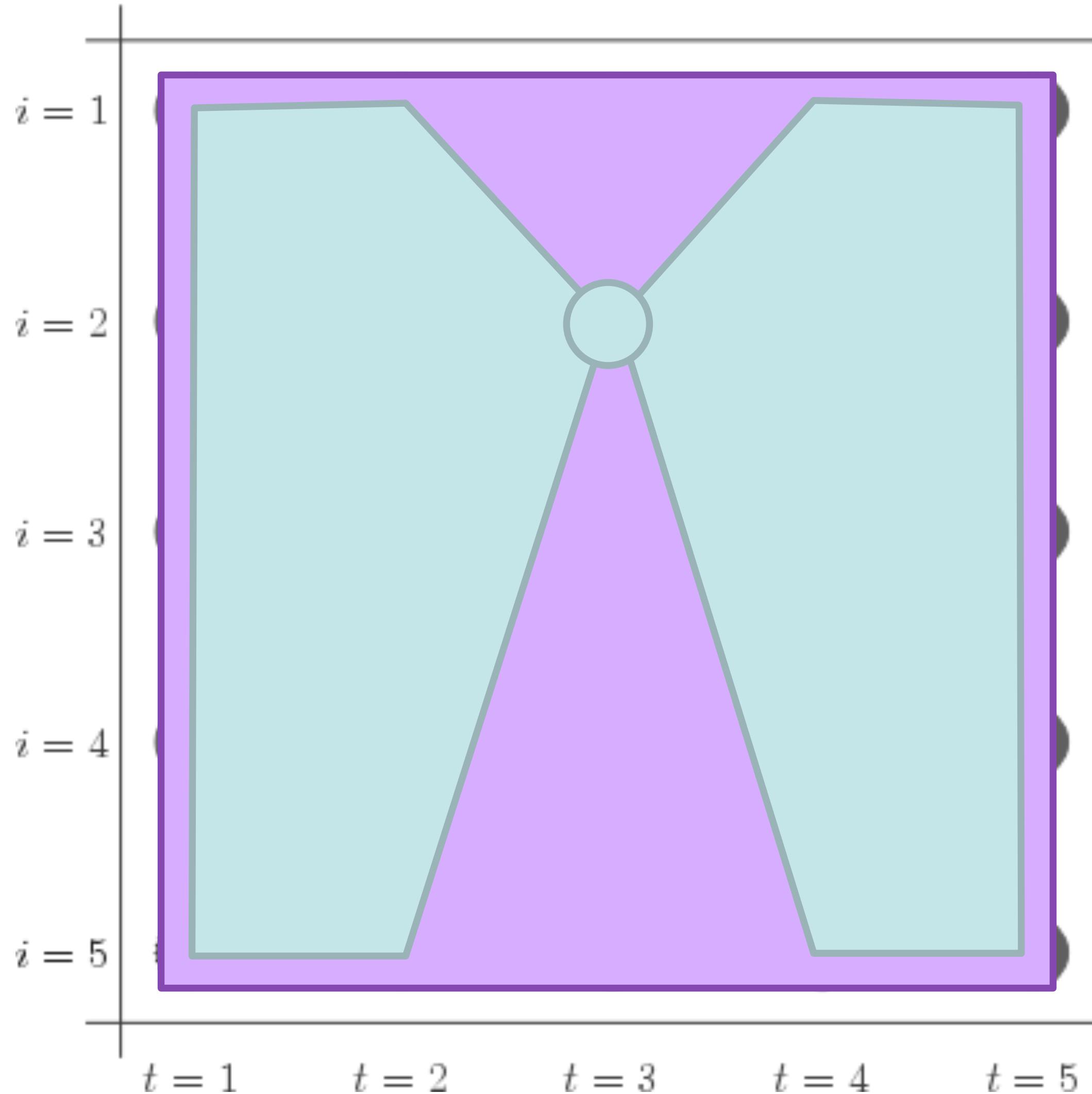
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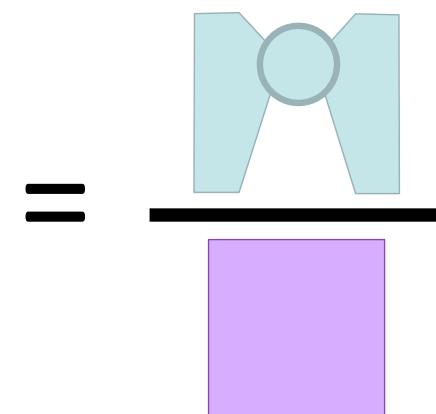


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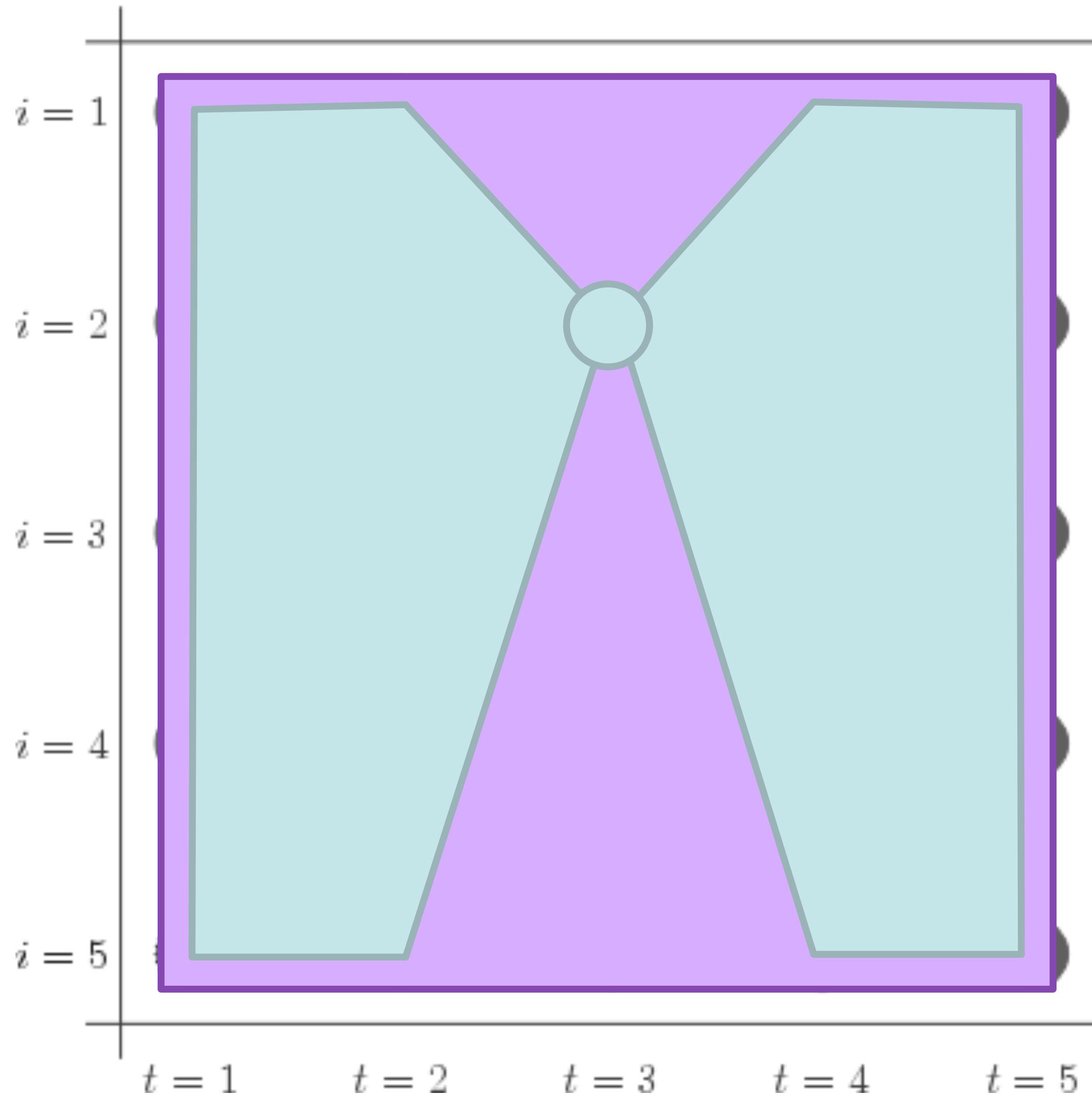
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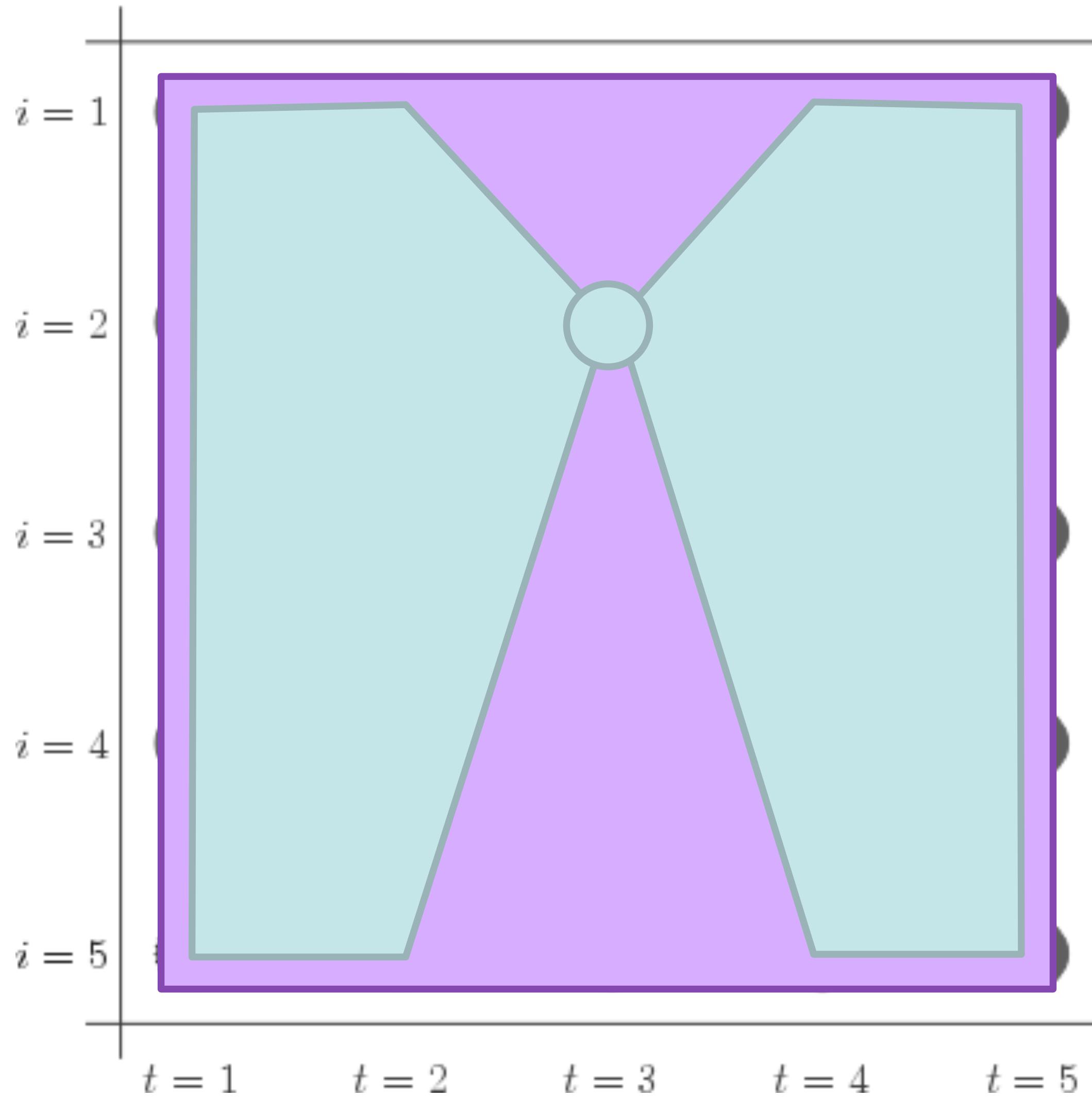
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$$P(s_3 = 2 | \mathbf{x}) = \frac{\alpha_3(2)\beta_3(2)}{\sum_i \alpha_3(i)\beta_3(i)} = \frac{\text{light blue area}}{\text{light purple area}}$$

Forward-Backward Algorithm



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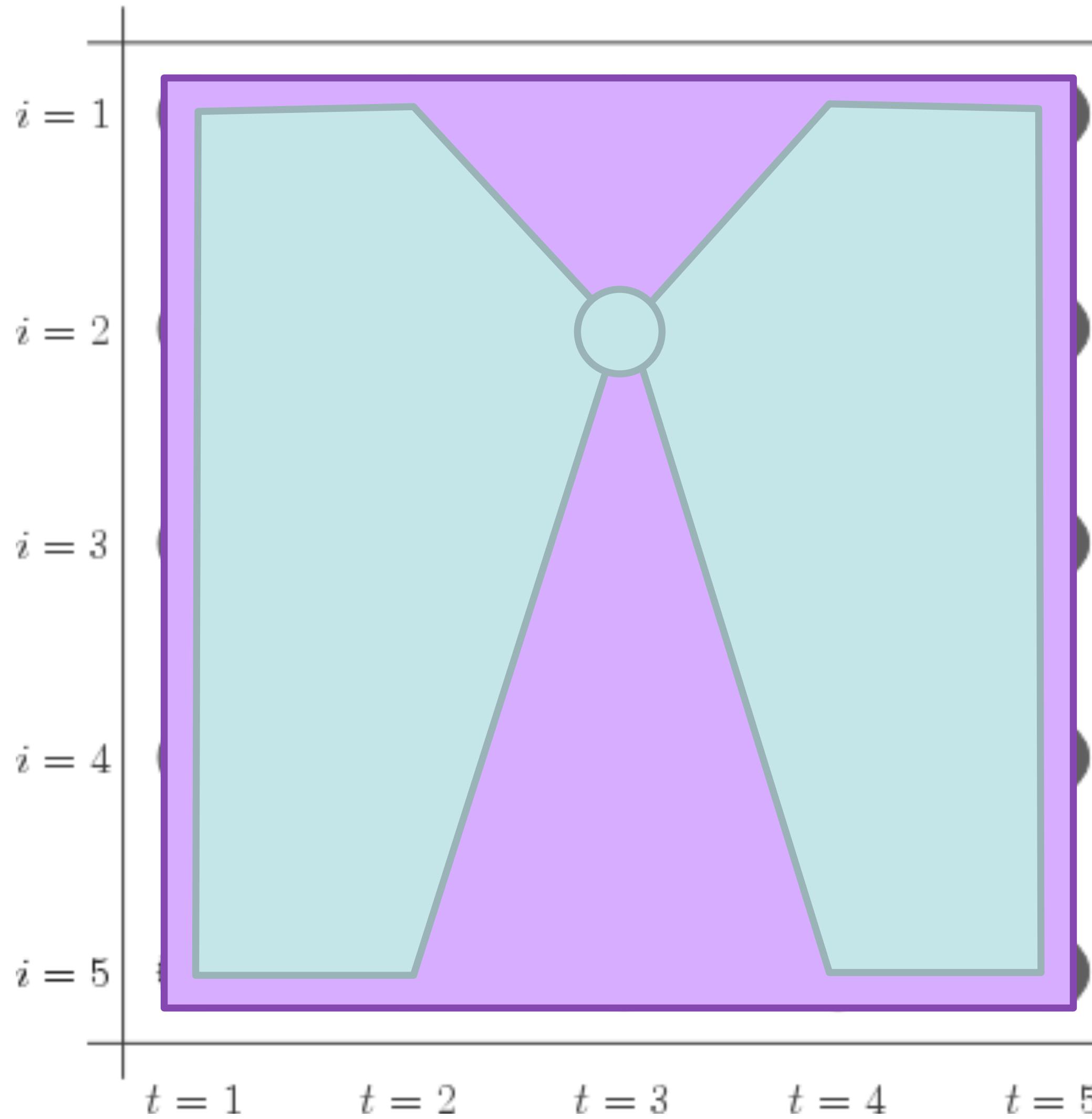
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► What is the denominator here?

Forward-Backward Algorithm



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$$P(s_3 = 2|\mathbf{x}) = \frac{\alpha_3(2)\beta_3(2)}{\sum_i \alpha_3(i)\beta_3(i)} = \frac{\text{[Diagram showing a light blue trapezoid divided by a vertical line, with a small circle at the top right representing } \alpha_3(2)\beta_3(2)]}{\text{[Diagram showing a pink trapezoid divided by a vertical line]}}$$

► What is the denominator here? $P(\mathbf{x})$

HMM POS Tagging

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NNP VBZ NN NNS CD NN

Fed raises interest rates 0.5 percent

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- ▶ Tradeoff between model capacity and data size — trigrams are a “sweet spot” for POS tagging

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Errors

	JJ	NN	NNP	NNPS	RB	RP	IN	VB	VBD	VBN	VBP	Total
JJ	0	177	56	0	61	2	5	10	15	108	0	488
NN	244	0	103	0	12	1	1	29	5	6	19	525
NNP	107	106	0	132	5	0	7	5	1	2	0	427
NNPS	1	0	110	0	0	0	0	0	0	0	0	142
RB	72	21	7	0	0	16	138	1	0	0	0	295
RP	0	0	0	0	39	0	65	0	0	0	0	104
IN	11	0	1	0	169	103	0	1	0	0	0	323
VB	17	64	9	0	2	0	1	0	4	7	85	189
VBD	10	5	3	0	0	0	0	3	0	143	2	166
VBN	101	3	3	0	0	0	0	3	108	0	1	221
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JJ/**NN** NN
official knowledge

VBD RP/**IN** DT NN
made up the story

RB VBD/**VBN** NNS
recently sold shares

(NN NN: *tax cut, art gallery, ...*)

Remaining Errors

Manning 2011 “Part-of-Speech Tagging from 97% to 100%: Is It Time for Some Linguistics?”

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VBD / VBP? (past or present?)

They set up absurd situations, detached from reality

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adjective or verbal participle? JJ / VBN?

a \$ 10 million fourth-quarter charge against discontinued operations

Other Languages

Language	Source	# Tags	O/O	U/U	O/U
Arabic	PADT/CoNLL07 (Hajič et al., 2004)	21	96.1	96.9	97.0
Basque	Basque3LB/CoNLL07 (Aduriz et al., 2003)	64	89.3	93.7	93.7
Bulgarian	BTB/CoNLL06 (Simov et al., 2002)	54	95.7	97.5	97.8
Catalan	CESS-ECE/CoNLL07 (Martí et al., 2007)	54	98.5	98.2	98.8
Chinese	Penn ChineseTreebank 6.0 (Palmer et al., 2007)	34	91.7	93.4	94.1
Chinese	Sinica/CoNLL07 (Chen et al., 2003)	294	87.5	91.8	92.6
Czech	PDT/CoNLL07 (Böhmová et al., 2003)	63	99.1	99.1	99.1
Danish	DDT/CoNLL06 (Kromann et al., 2003)	25	96.2	96.4	96.9
Dutch	Alpino/CoNLL06 (Van der Beek et al., 2002)	12	93.0	95.0	95.0
English	PennTreebank (Marcus et al., 1993)	45	96.7	96.8	97.7
French	FrenchTreebank (Abeillé et al., 2003)	30	96.6	96.7	97.3
German	Tiger/CoNLL06 (Brants et al., 2002)	54	97.9	98.1	98.8
German	Negra (Skut et al., 1997)	54	96.9	97.9	98.6
Greek	GDT/CoNLL07 (Prokopidis et al., 2005)	38	97.2	97.5	97.8
Hungarian	Szeged/CoNLL07 (Cséndes et al., 2005)	43	94.5	95.6	95.8
Italian	ISST/CoNLL07 (Montemagni et al., 2003)	28	94.9	95.8	95.8
Japanese	Verbmobil/CoNLL06 (Kawata and Bartels, 2000)	80	98.3	98.0	99.1
Japanese	Kyoto4.0 (Kurohashi and Nagao, 1997)	42	97.4	98.7	99.3
Korean	Sejong (http://www.sejong.or.kr)	187	96.5	97.5	98.4
Portuguese	Floresta Sintá(c)tica/CoNLL06 (Afonso et al., 2002)	22	96.9	96.8	97.4
Russian	SynTagRus-RNC (Boguslavsky et al., 2002)	11	96.8	96.8	96.8
Slovene	SDT/CoNLL06 (Džeroski et al., 2006)	29	94.7	94.6	95.3
Spanish	Ancora-Cast3LB/CoNLL06 (Civit and Martí, 2004)	47	96.3	96.3	96.9
Swedish	Talbanken05/CoNLL06 (Nivre et al., 2006)	41	93.6	94.7	95.1
Turkish	METU-Sabancı/CoNLL07 (Oflazer et al., 2003)	31	87.5	89.1	90.2

Next Time

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- ▶ CRFs: feature-based discriminative models

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- ▶ Structured SVM for sequences

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- ▶ CRFs: feature-based discriminative models
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