

Binary Classification

Alan Ritter

(many slides from Greg Durrett and Vivek Srikumar)

This Lecture

- ▶ Linear classification fundamentals
- ▶ Naive Bayes, maximum likelihood in generative models
- ▶ Three discriminative models: logistic regression, perceptron, SVM
 - ▶ Different motivations but very similar update rules / inference!

Classification

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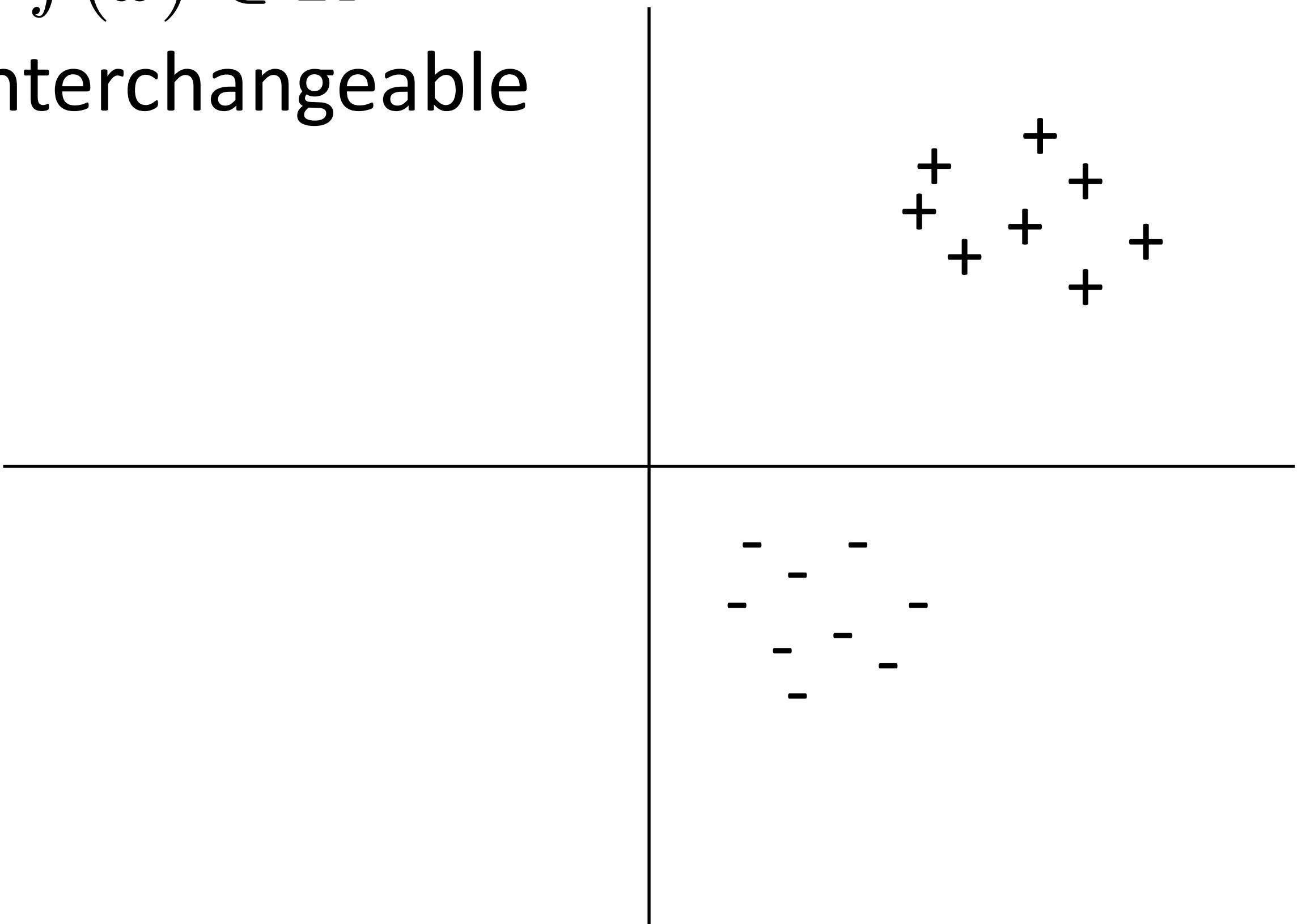
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- ▶ Embed datapoint in a feature space $f(x) \in \mathbb{R}^n$
but in this lecture $f(x)$ and x are interchangeable

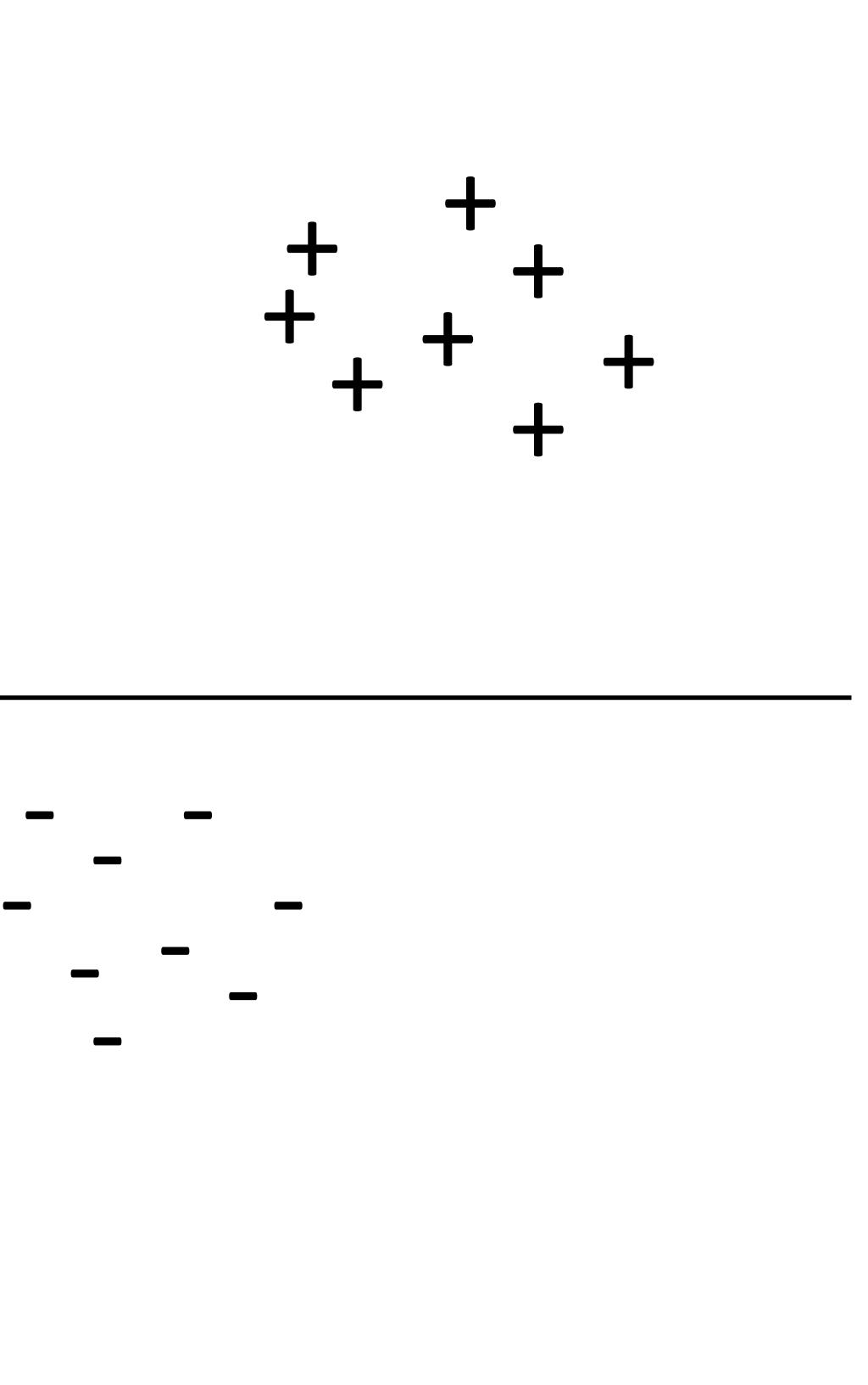
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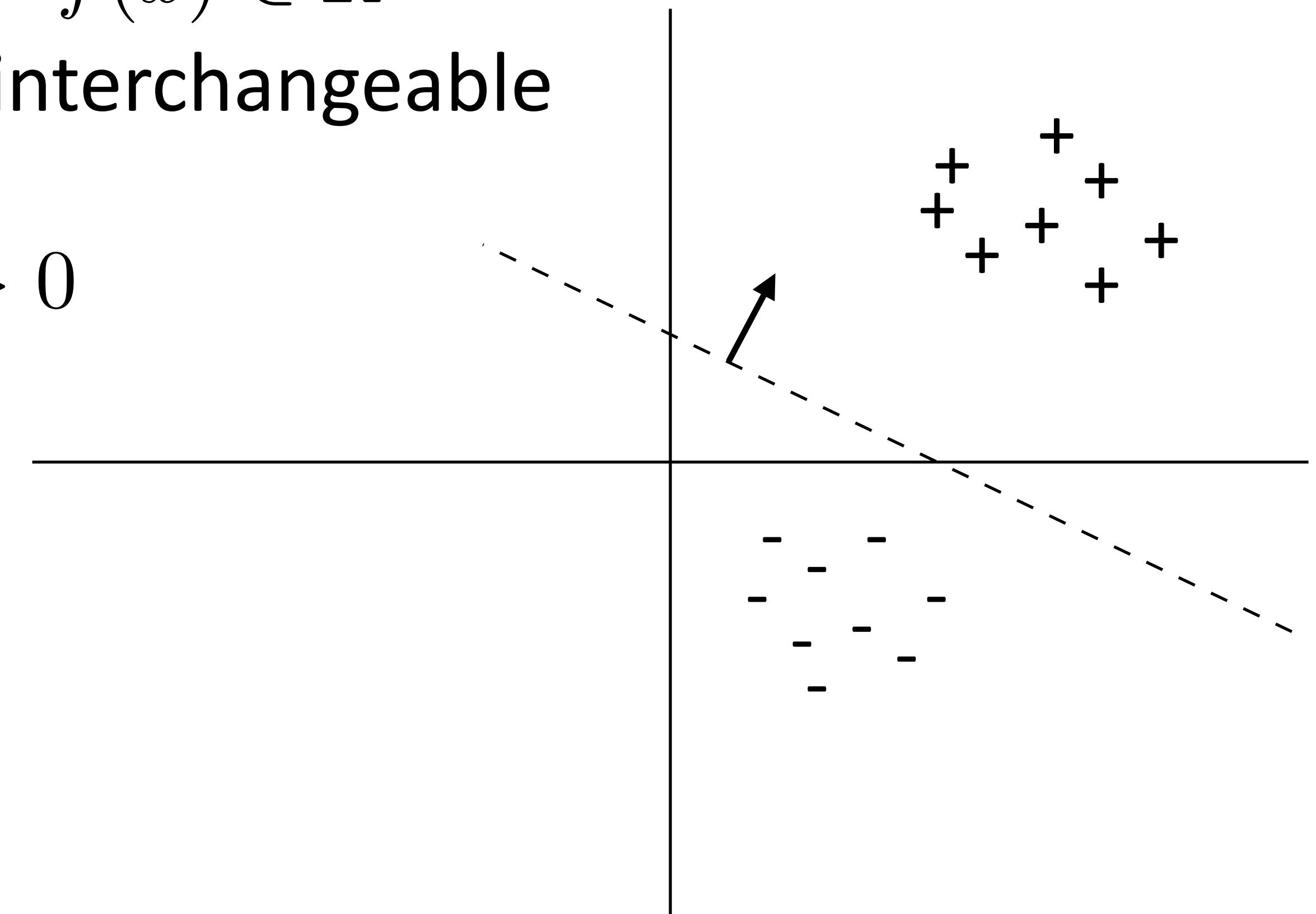
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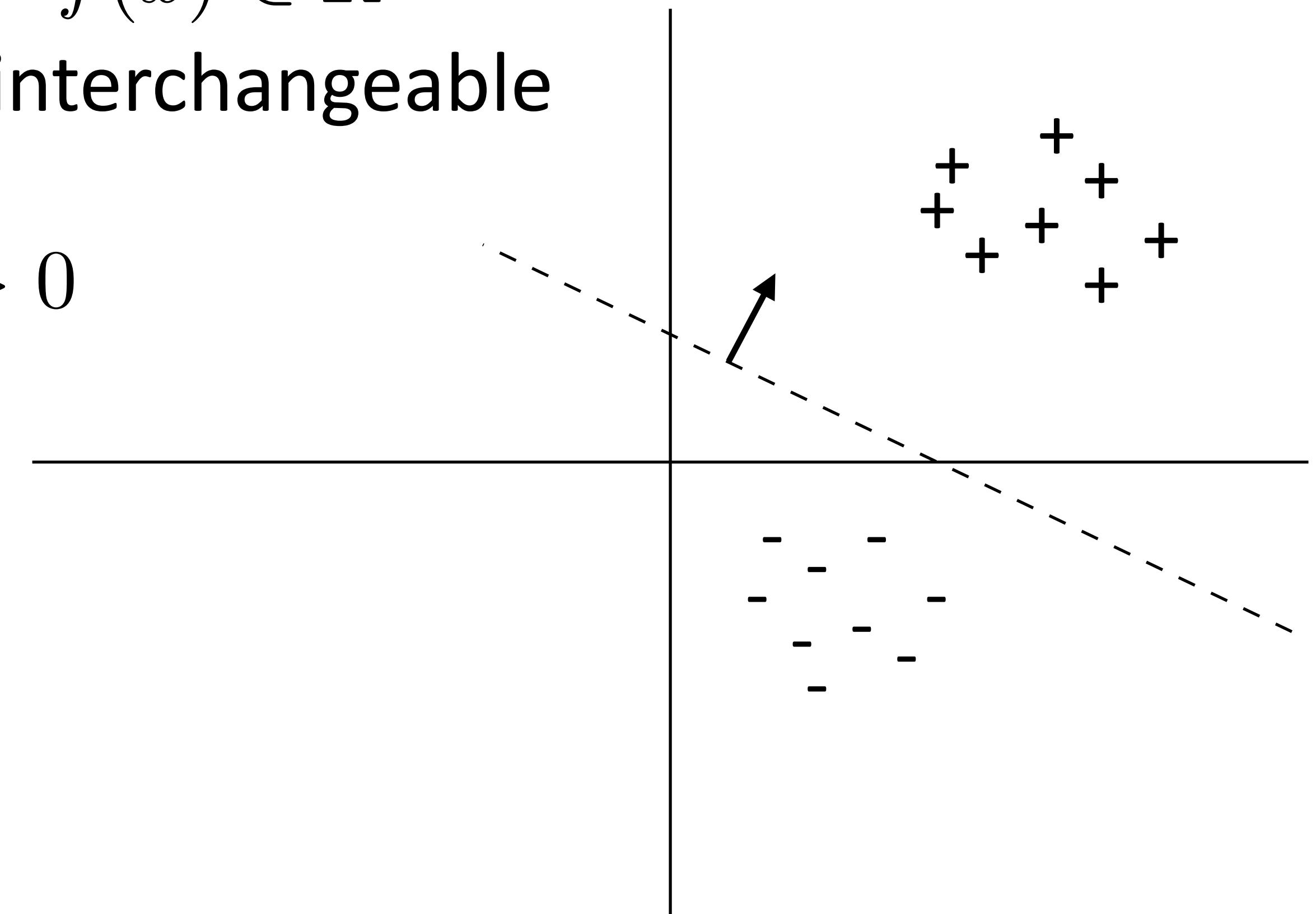
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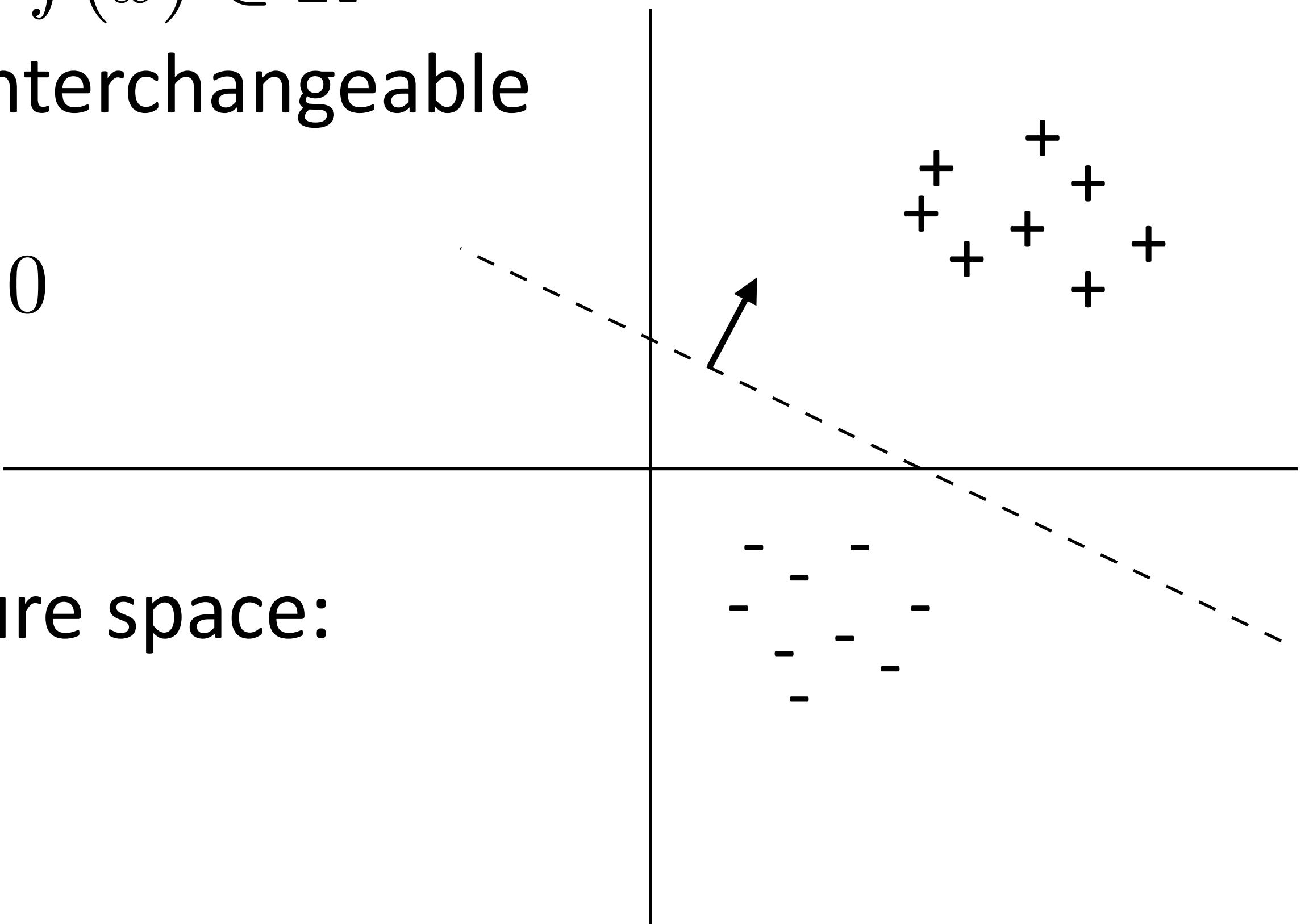
$$w^\top f(x) > 0$$

- ▶ Can delete bias if we augment feature space:

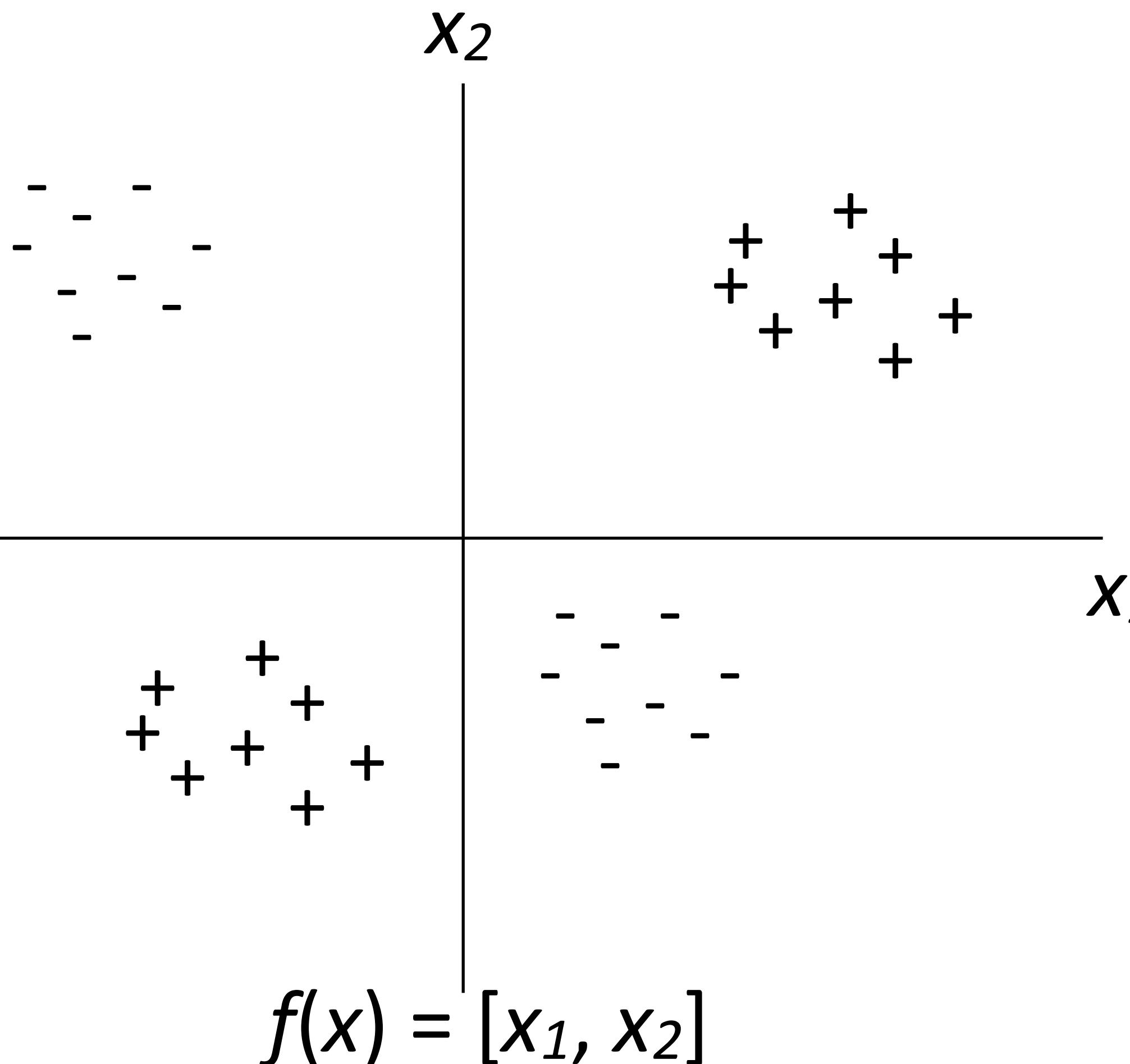
$$f(x) = [0.5, 1.6, 0.3]$$



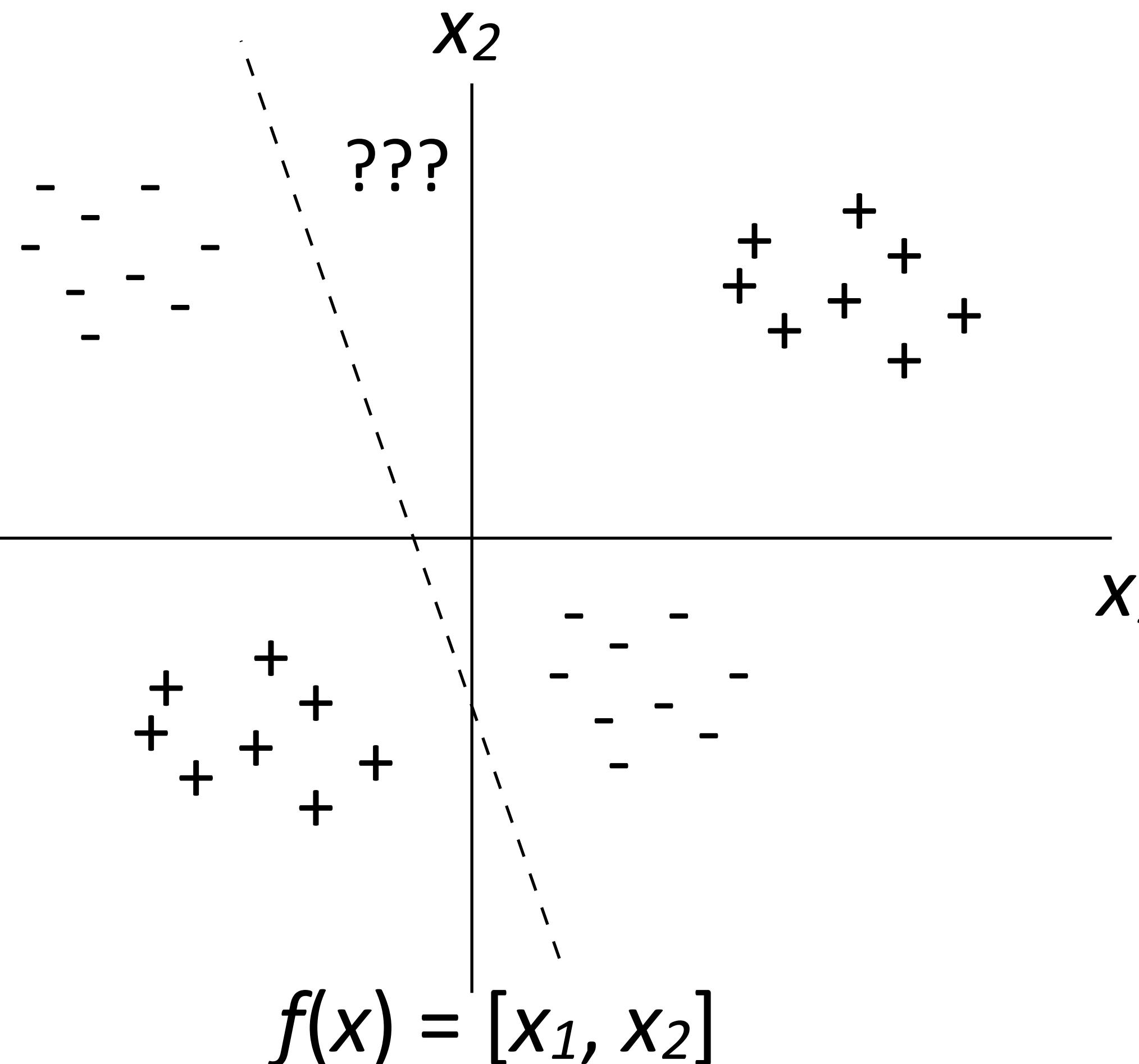
$$[0.5, 1.6, 0.3, 1]$$



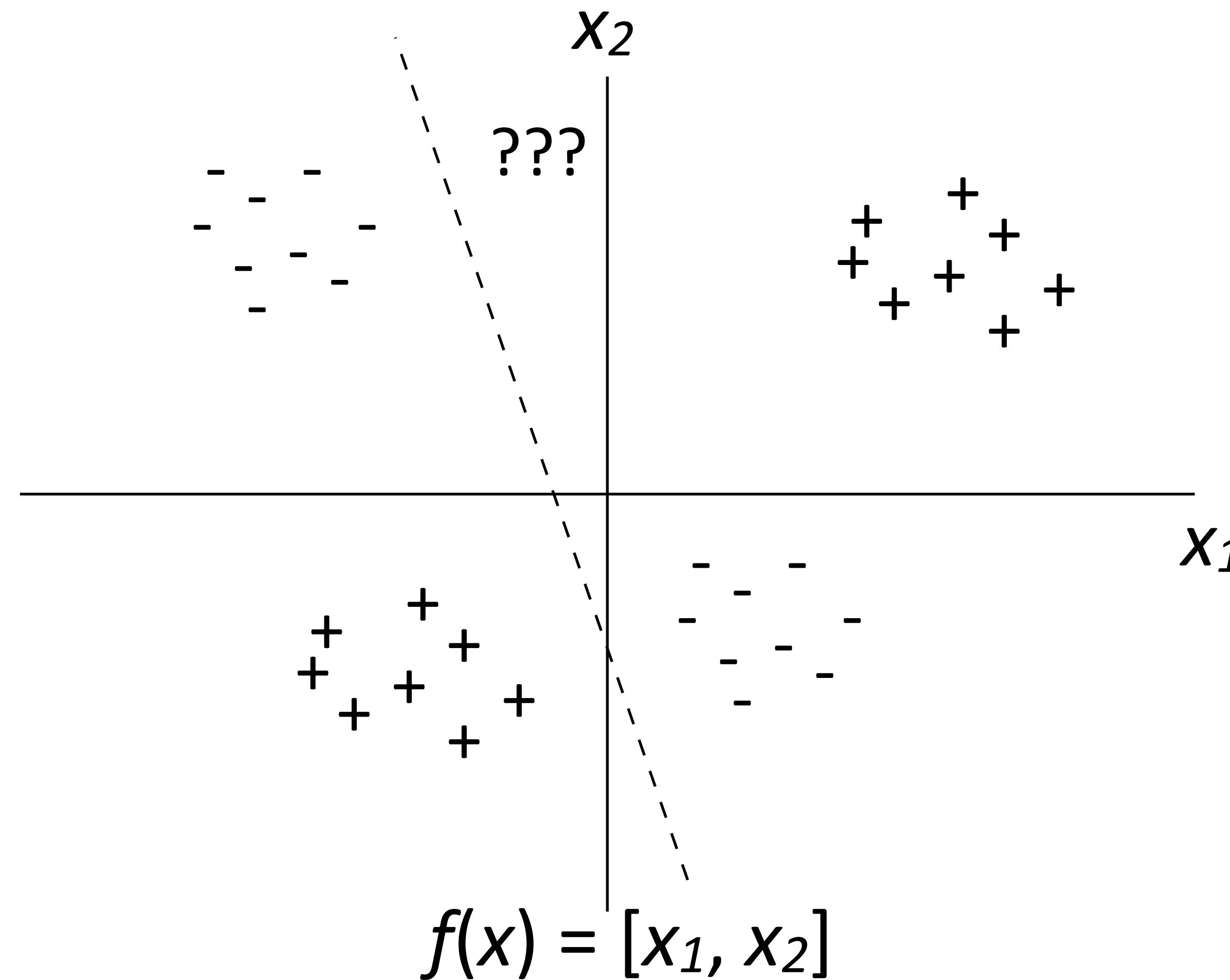
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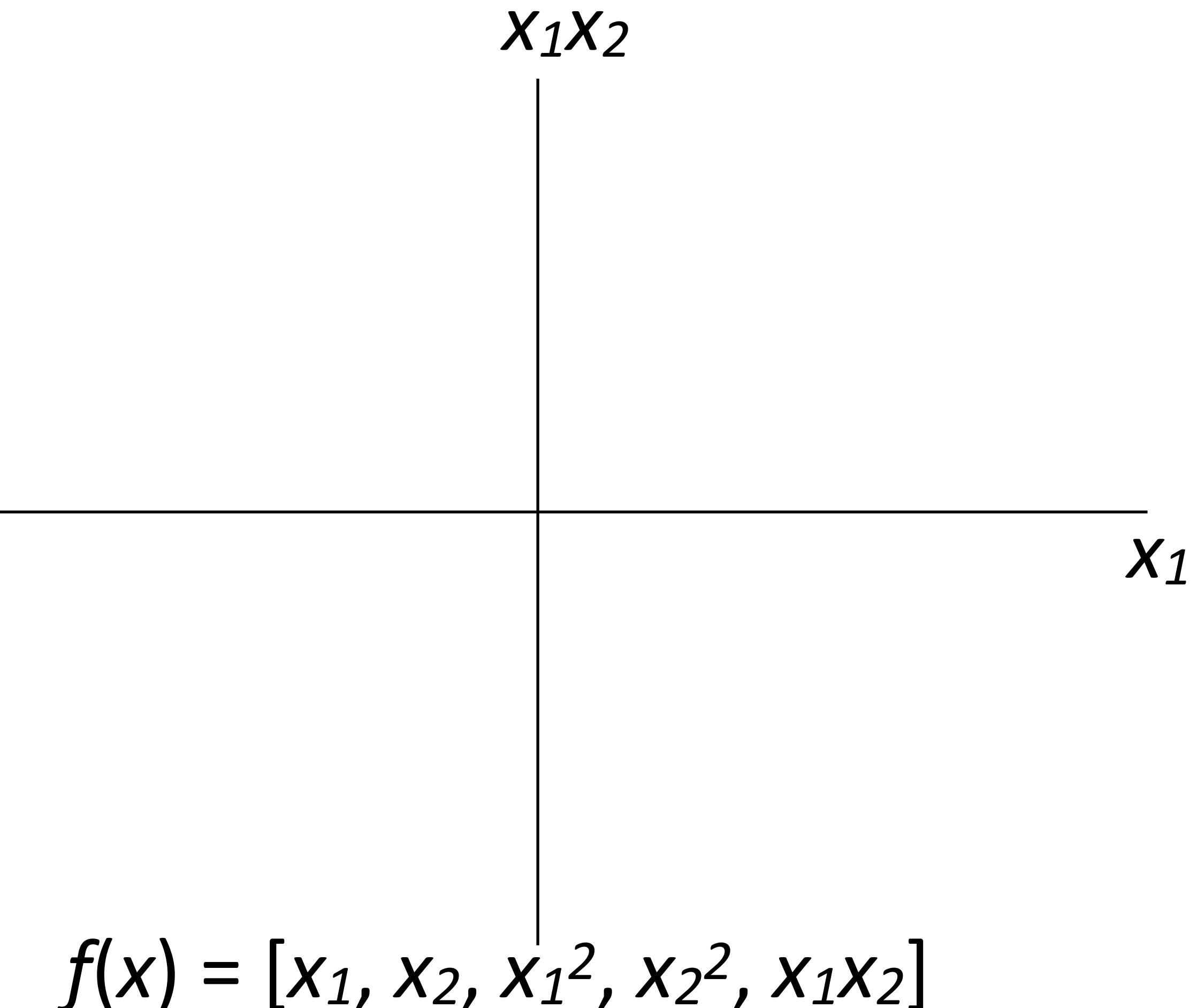
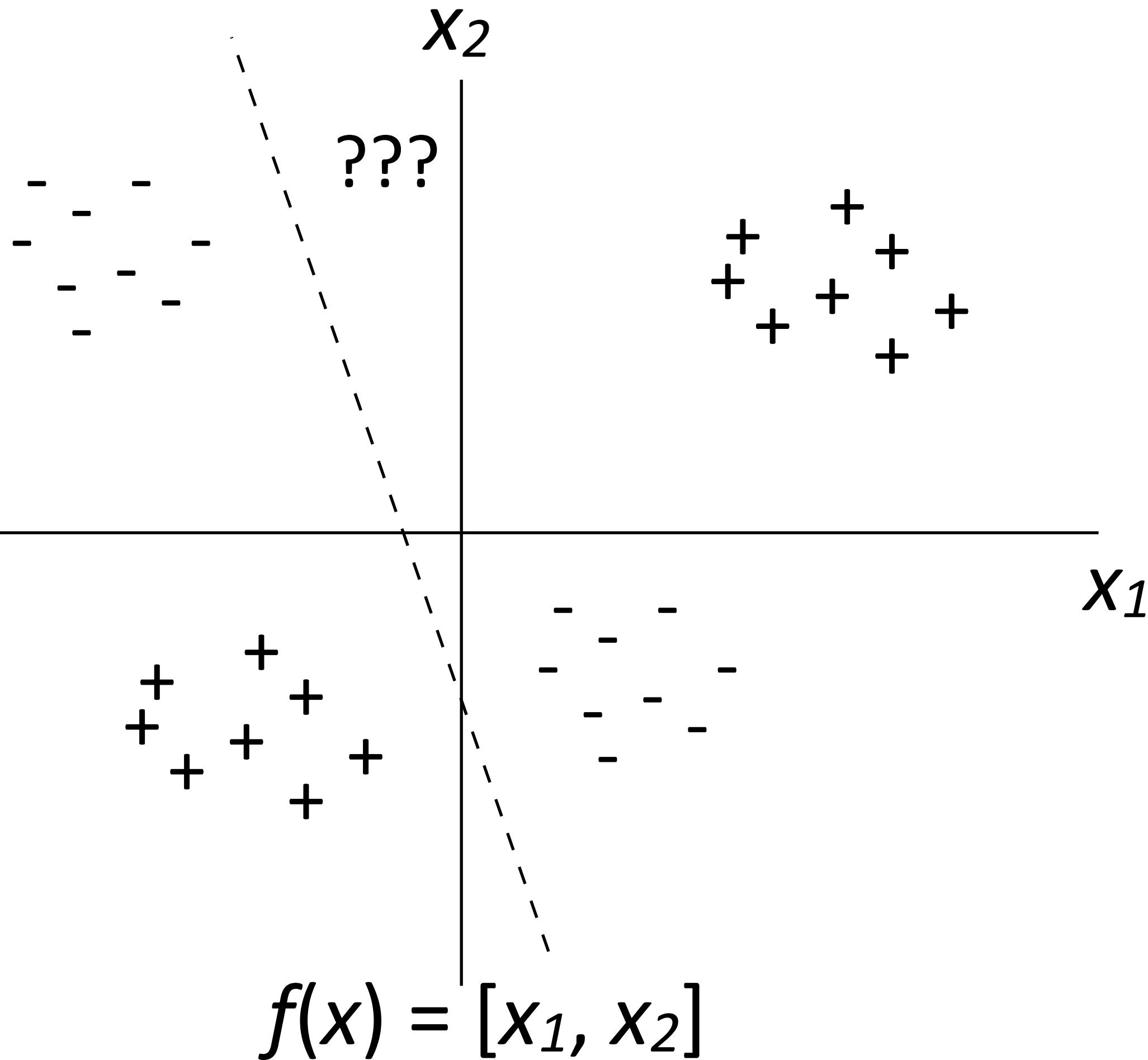


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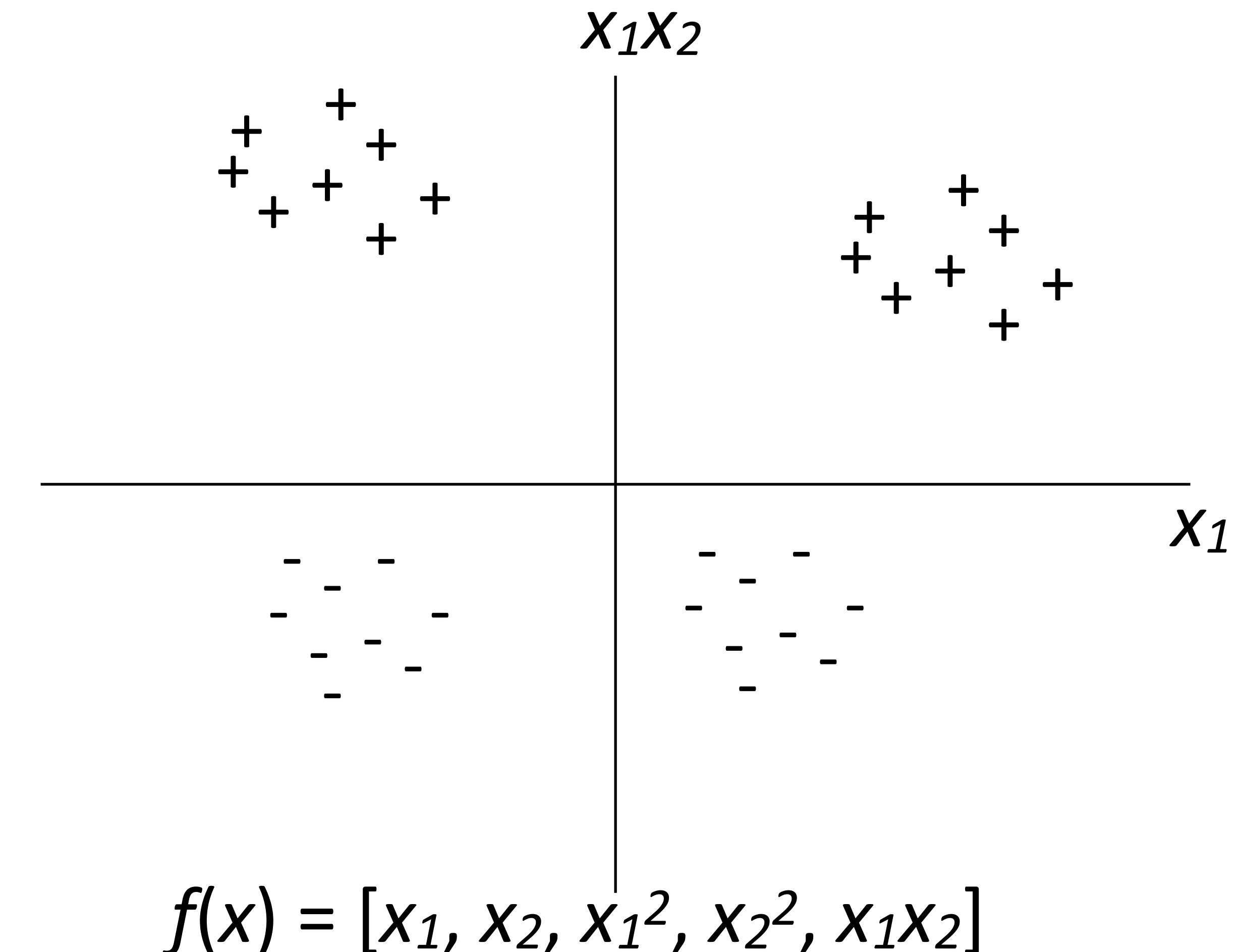
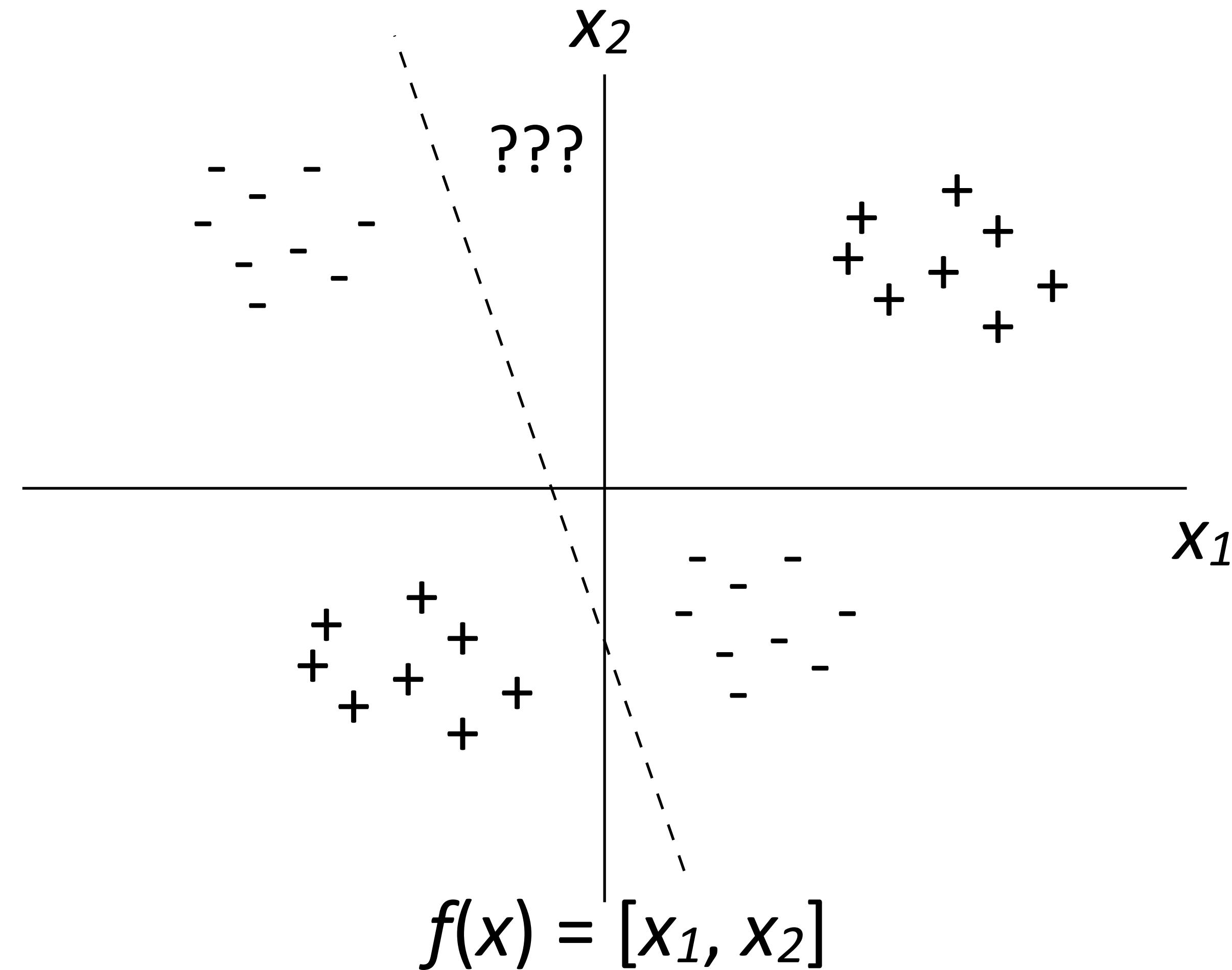


$$f(x) = [x_1, x_2, x_1^2, x_2^2, x_1 x_2]$$

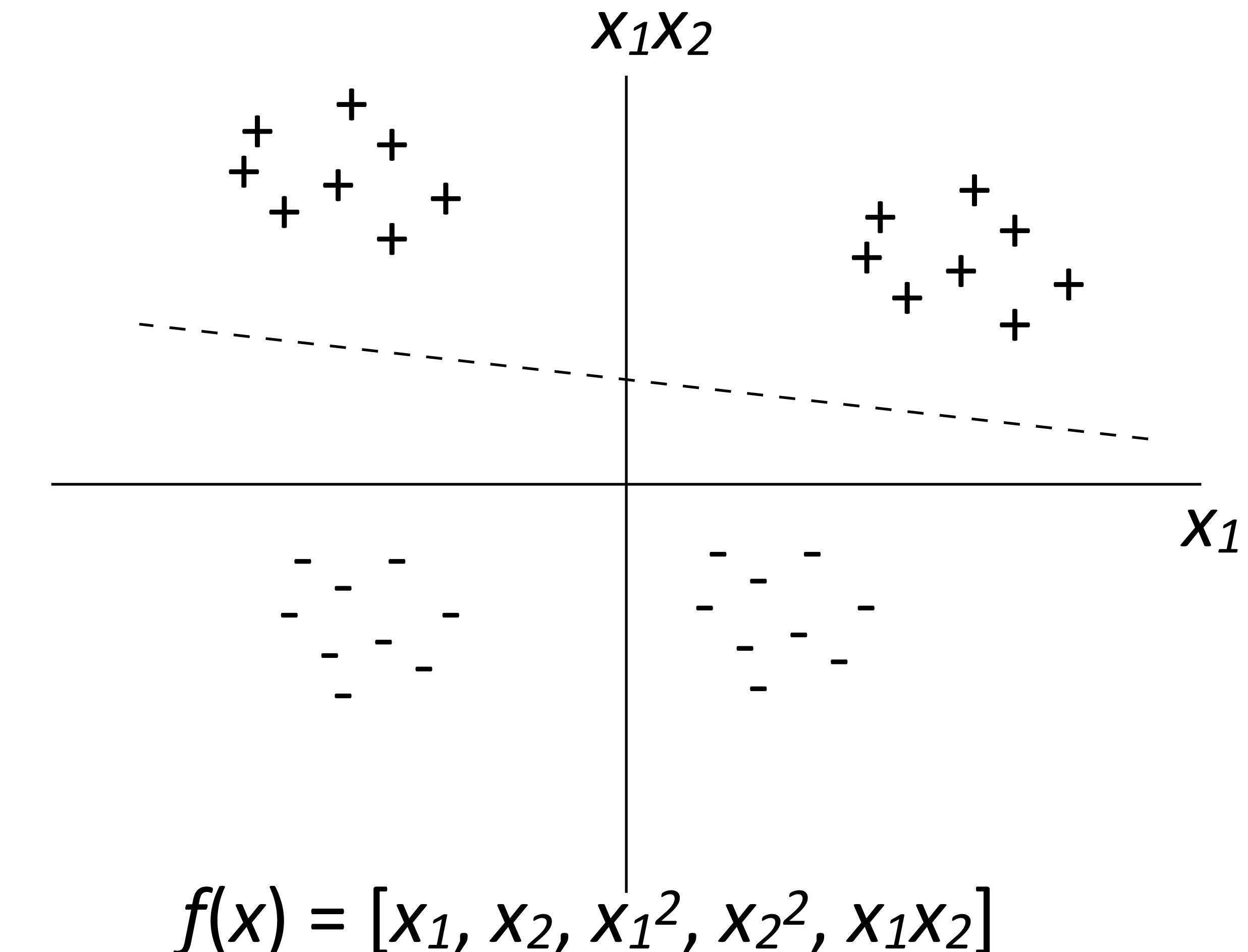
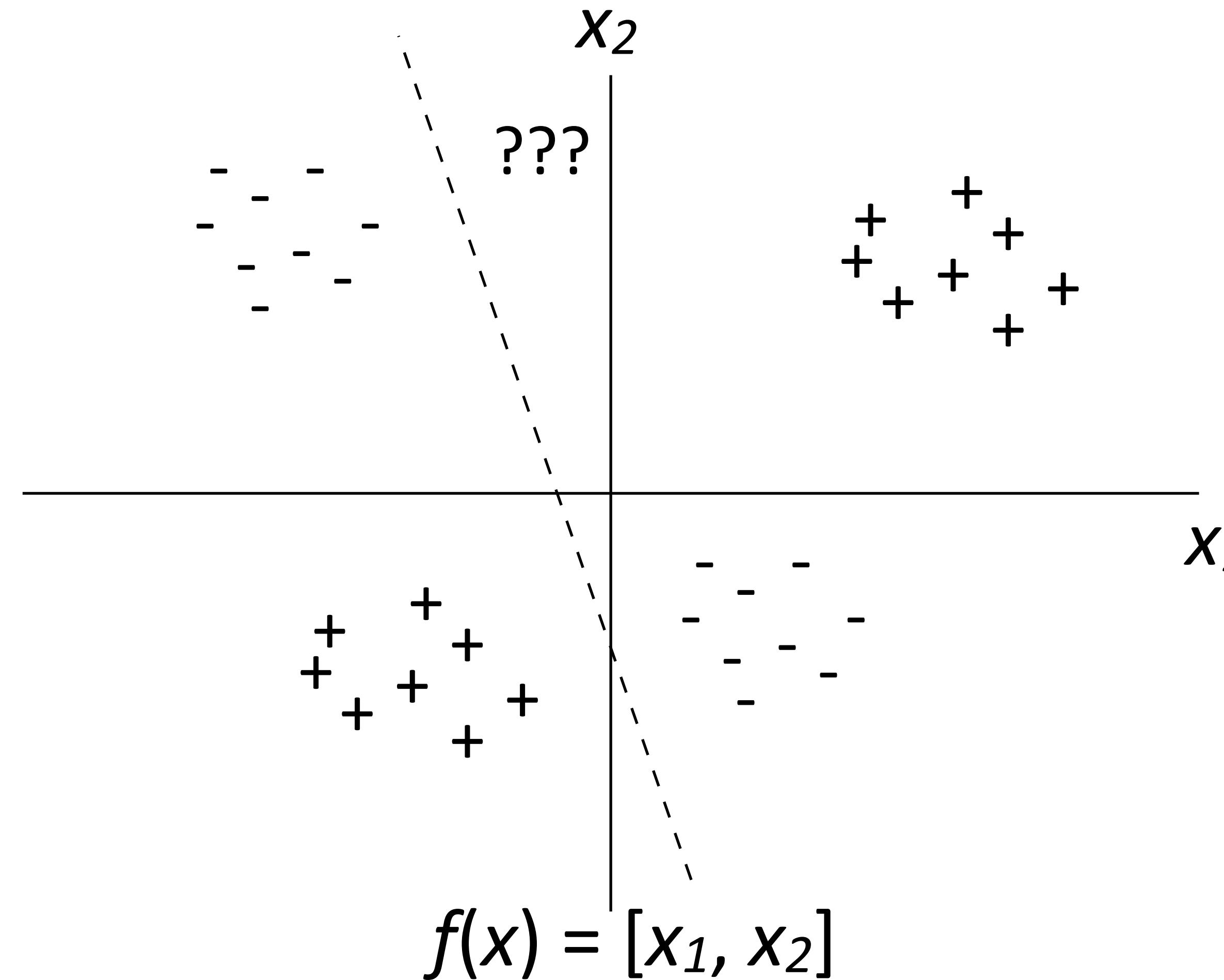
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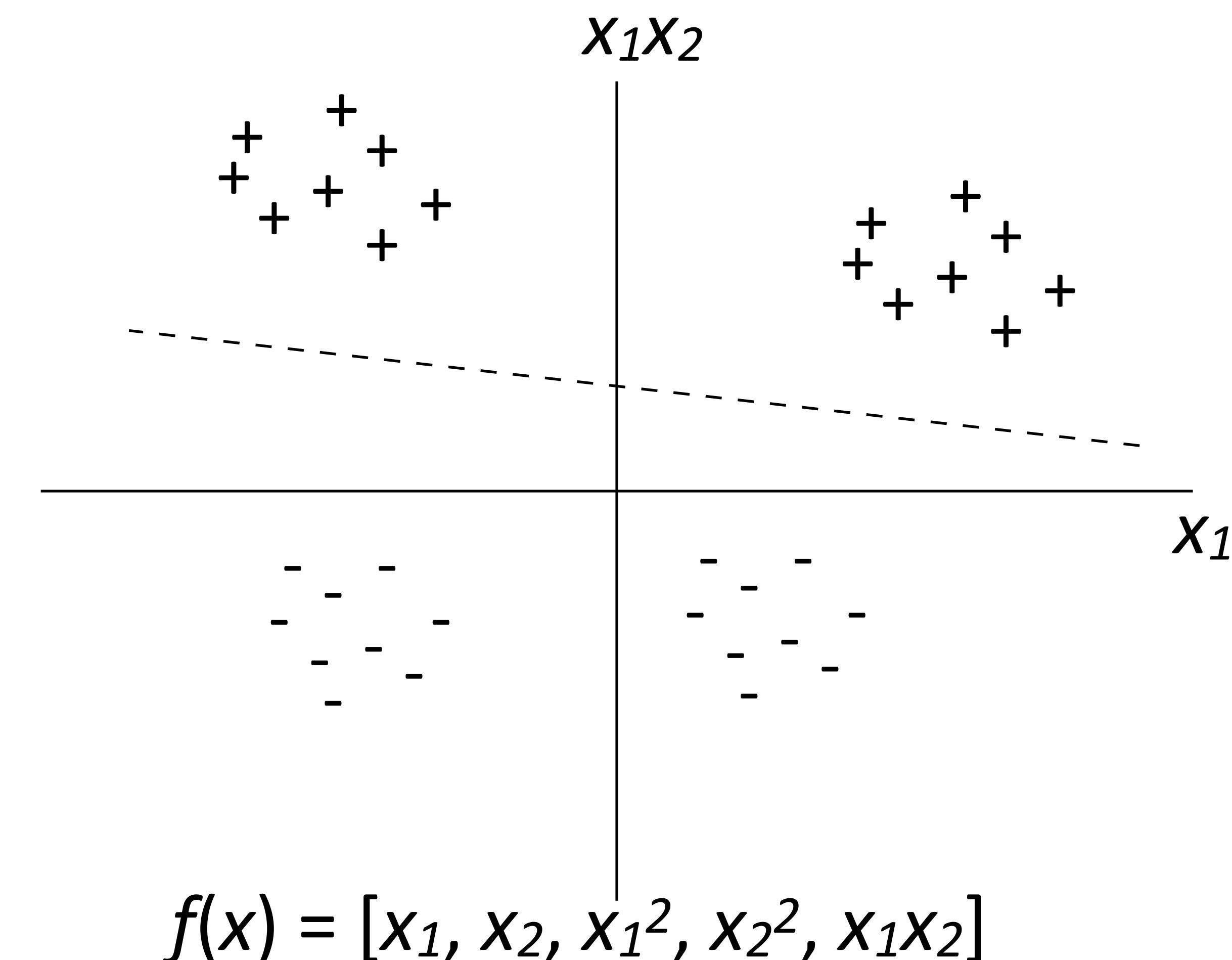
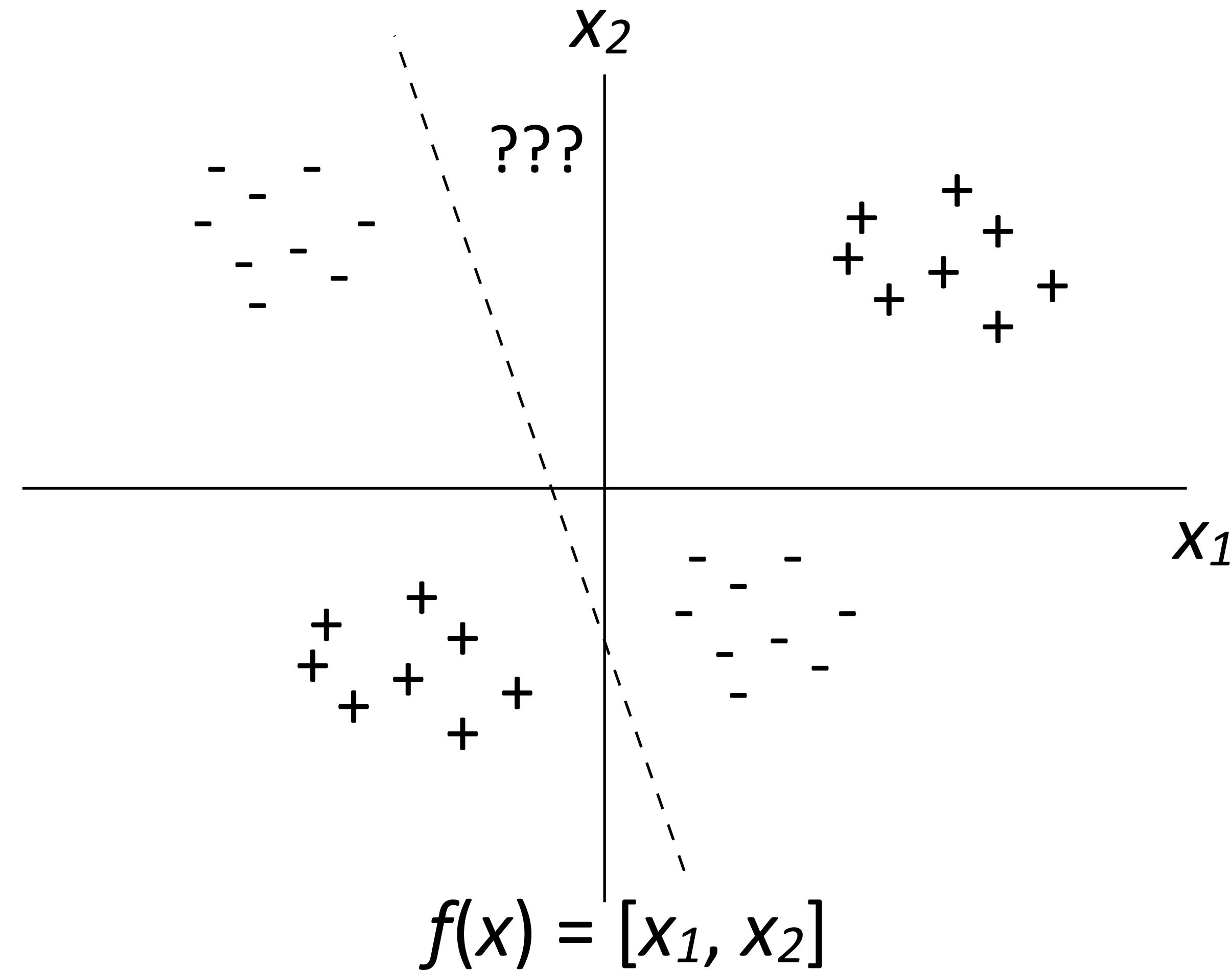
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- ▶ “Kernel trick” does this for “free,” but is too expensive to use in NLP applications, training is $O(n^2)$ instead of $O(n \cdot (\text{num feats}))$

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 - ▶ Train weights on data to get our classifier

Feature Representation

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- ▶ Convert this example to a vector using *bag-of-words features*

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- ▶ Very large vector space (size of vocabulary), sparse features
- ▶ Requires *indexing* the features (mapping them to axes)
- ▶ More sophisticated feature mappings possible (tf-idf), as well as lots of other features: character n-grams, parts of speech, lemmas, ...

Naive Bayes

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$$\begin{aligned} P(y|x) &= \frac{P(y)P(x|y)}{P(x)} && \text{Bayes' Rule} \\ &\propto P(y)P(x|y) && \text{constant: irrelevant} \\ &= P(y) \prod_{i=1}^n P(x_i|y) && \text{for finding the max} \end{aligned}$$

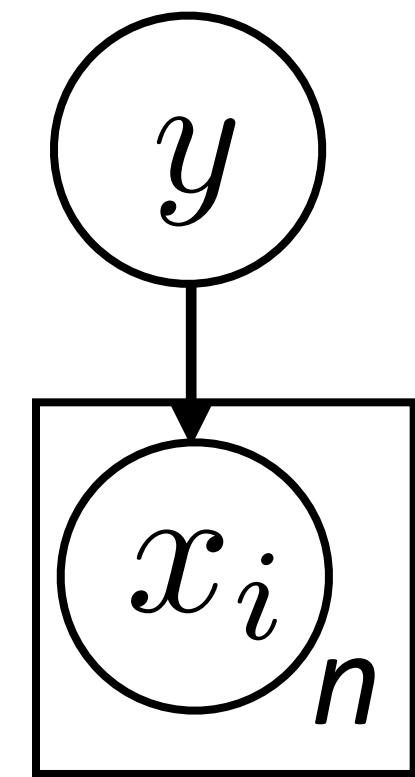
“Naive” assumption:

The diagram consists of two arrows. One arrow points from the term $P(x)$ in the first equation to the text "constant: irrelevant for finding the max". Another arrow points from the term n in the third equation to the text "‘Naive’ assumption:".

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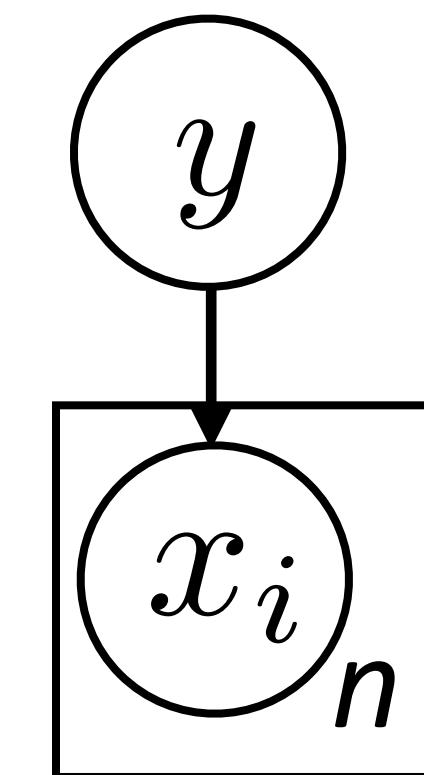
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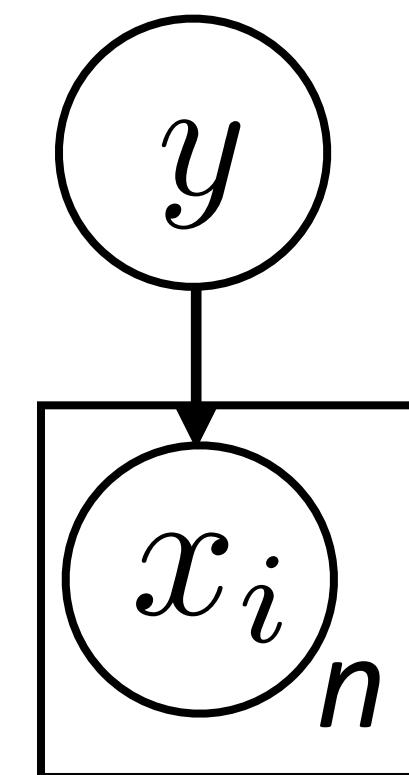
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linear model!

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Naive Bayes Example

$$it\ was\ great \longrightarrow P(y|x) \propto []$$

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$$\operatorname{argmax}_y \log P(y|x) = \operatorname{argmax}_y \left[\log P(y) + \sum_{i=1}^n \log P(x_i|y) \right]$$

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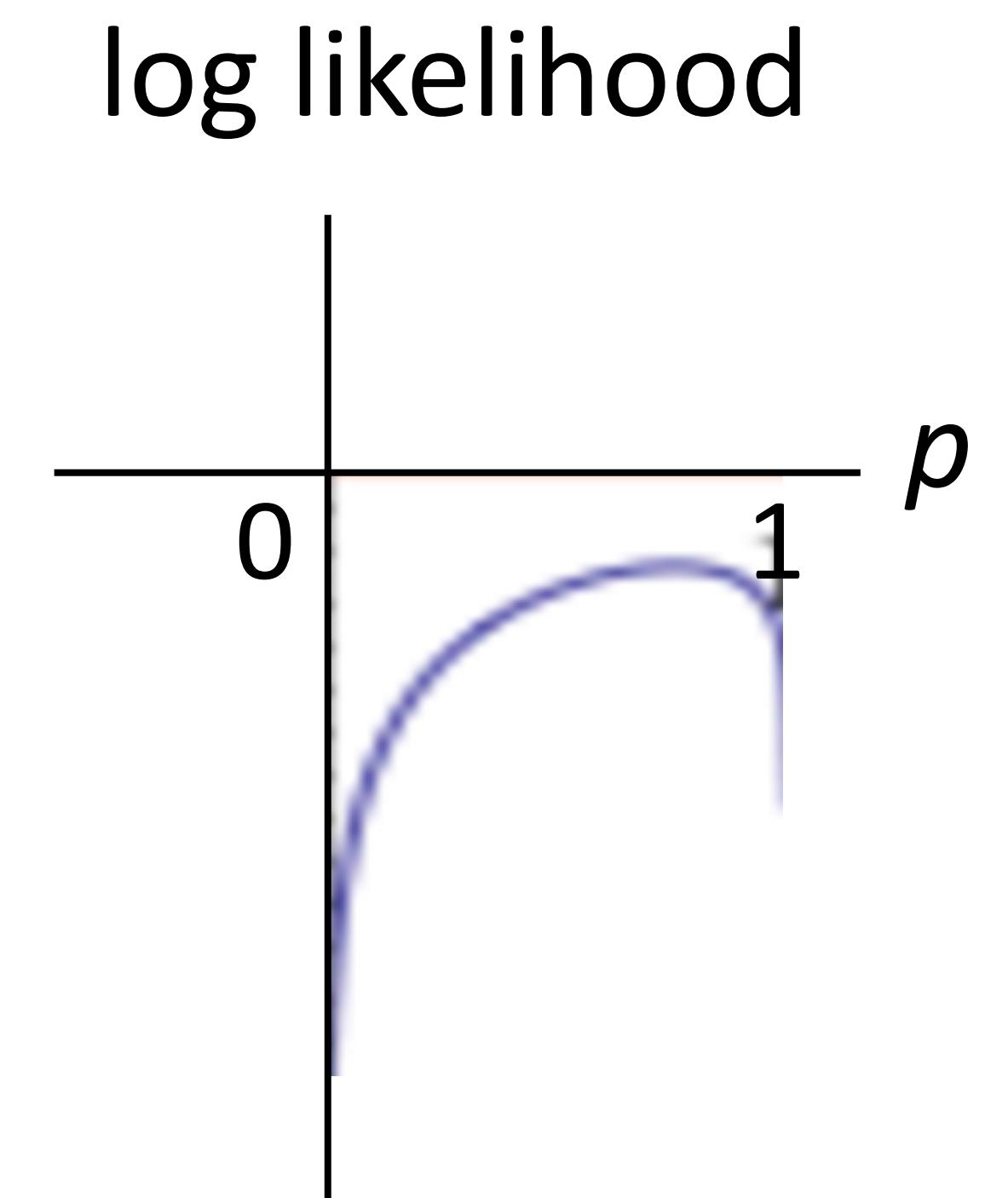
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- ▶ Easier: maximize *log* likelihood

$$\sum_{j=1}^m \log P(y_j) = 3 \log p + \log(1 - p)$$

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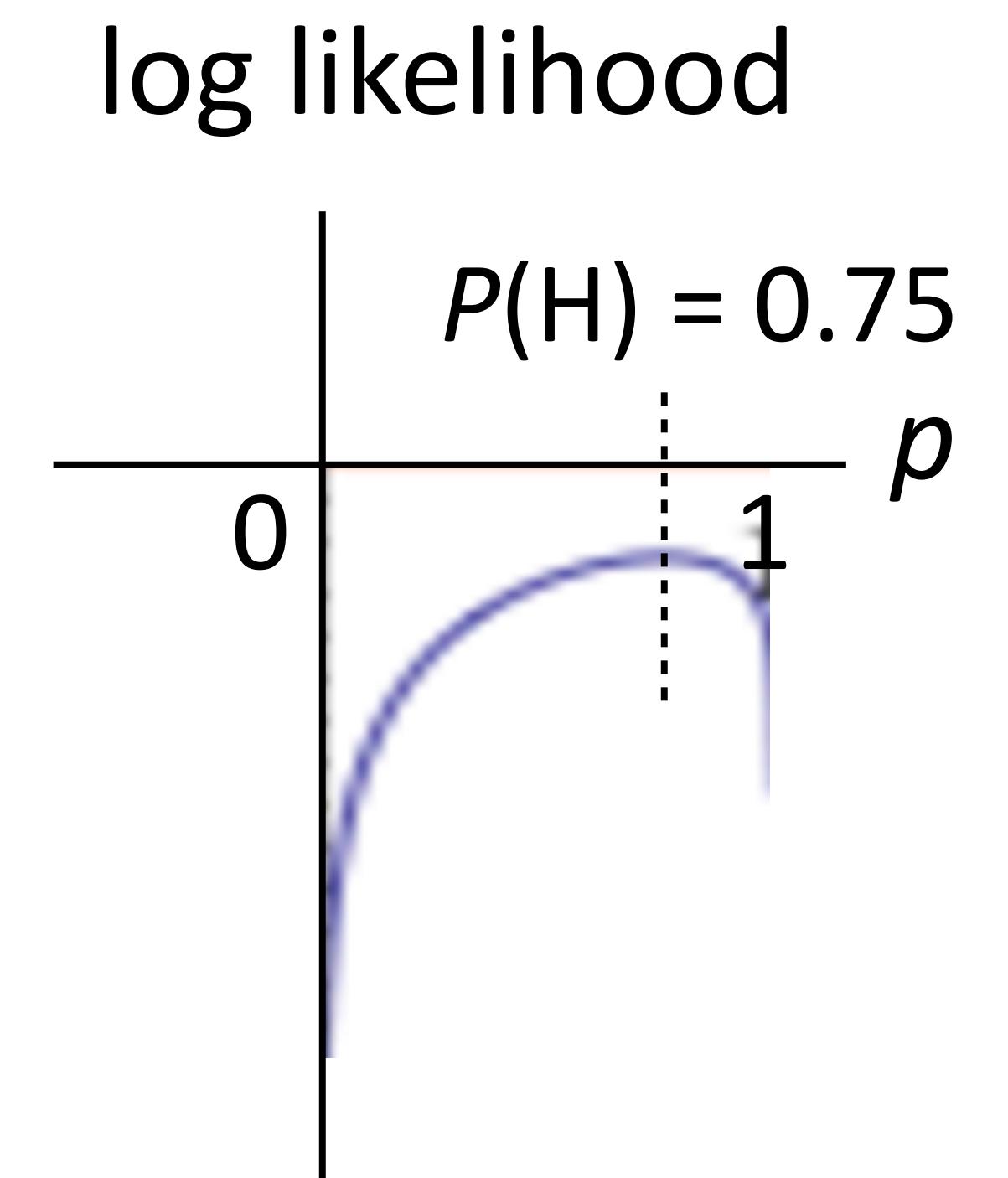
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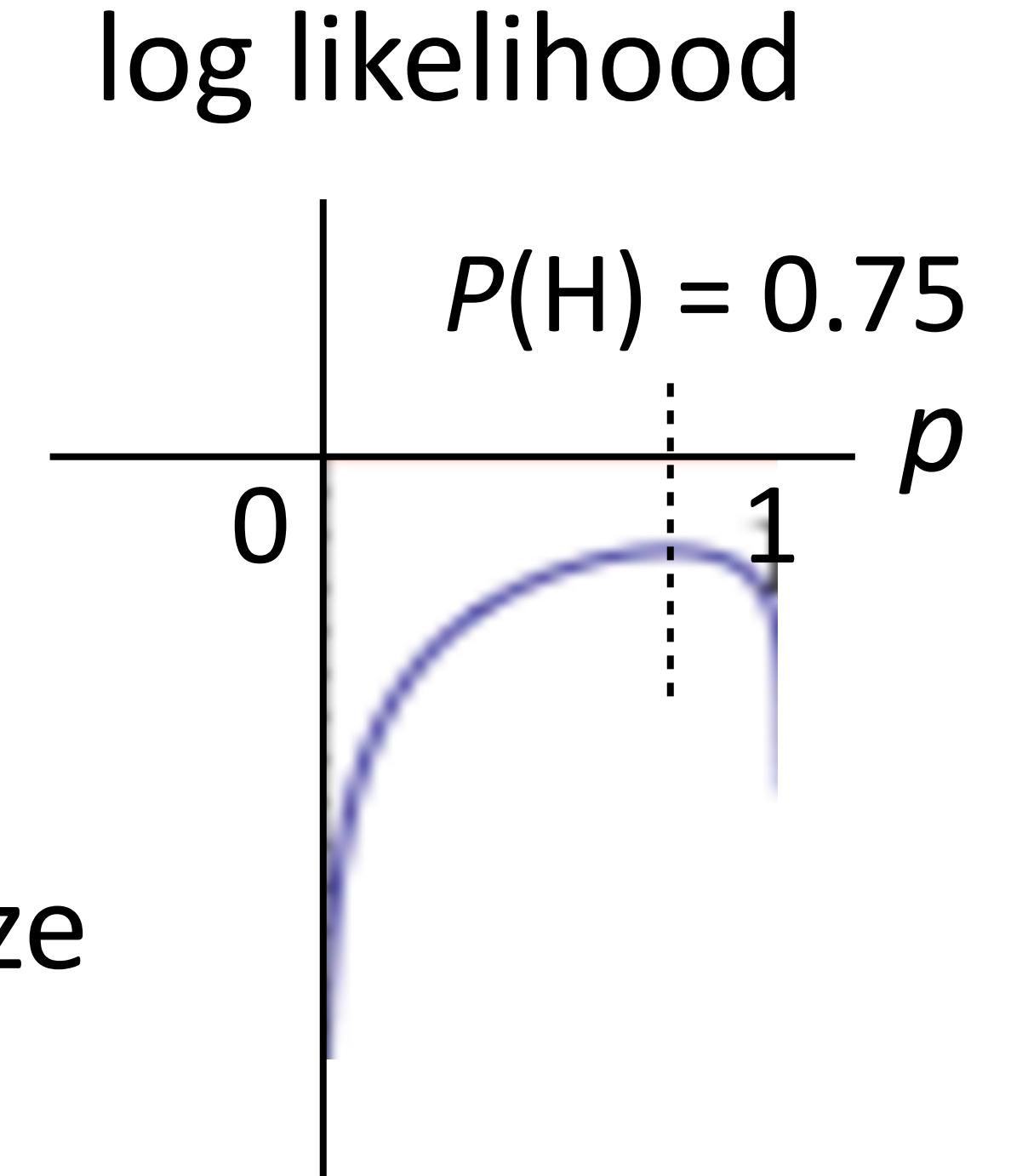
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- ▶ Maximum likelihood parameters for binomial/multinomial = read counts off of the data + normalize



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data points (j) features (i) i th feature of j th example

- ▶ Equivalent to maximizing logarithm of data likelihood:

$$\sum_{j=1}^m \log P(y_j, x_j) = \sum_{j=1}^m \left[\log P(y_j) + \sum_{i=1}^n \log P(x_{ji}|y_j) \right]$$

Maximum Likelihood for Naive Bayes

this movie was great! would watch again

+

I liked it well enough for an action flick

+

I expected a great film and left happy

+

brilliant directing and stunning visuals

+

that film was awful, I'll never watch again

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I didn't really like that movie

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dry and a bit distasteful, it misses the mark

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I didn't really like that movie

—

dry and a bit distasteful, it misses the mark

—

great potential but ended up being a flop

—

$$P(+) = \frac{1}{2}$$

$$P(-) = \frac{1}{2}$$

$$P(\text{great}|+) = \frac{1}{2}$$

Maximum Likelihood for Naive Bayes

this movie was great! would watch again

+

I liked it well enough for an action flick

+

I expected a great film and left happy

+

brilliant directing and stunning visuals

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it was great —→

$$P(y|x) \propto \begin{bmatrix} P(+)\text{P(great|+)} \\ P(-)\text{P(great|-)} \end{bmatrix}$$

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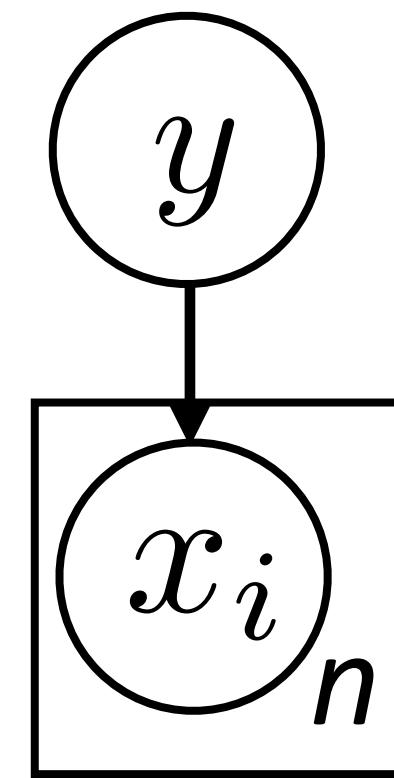
$$P(\text{great}|-) = \frac{1}{4}$$

$$\text{it was great} \longrightarrow P(y|x) \propto \begin{bmatrix} P(+)\text{P(great|+)} \\ P(-)\text{P(great|-)} \end{bmatrix} = \begin{bmatrix} 1/4 \\ 1/8 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$$

Naive Bayes: Summary

- Model

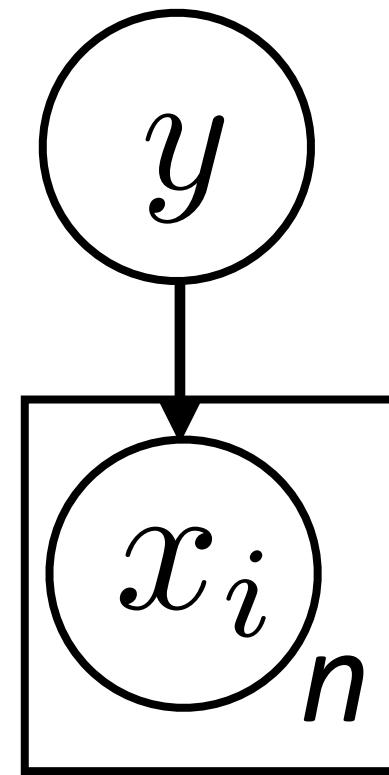
$$P(x, y) = P(y) \prod_{i=1}^n P(x_i | y)$$



Naive Bayes: Summary

- Model

$$P(x, y) = P(y) \prod_{i=1}^n P(x_i|y)$$



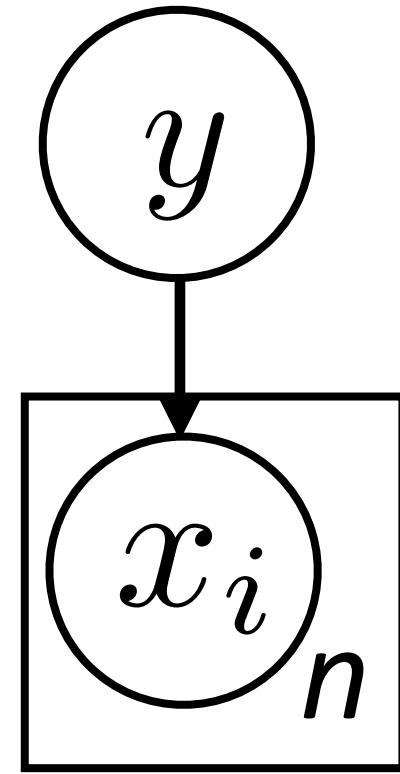
- Inference

$$\operatorname{argmax}_y \log P(y|x) = \operatorname{argmax}_y \left[\log P(y) + \sum_{i=1}^n \log P(x_i|y) \right]$$

Naive Bayes: Summary

- ▶ Model

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- ▶ Inference

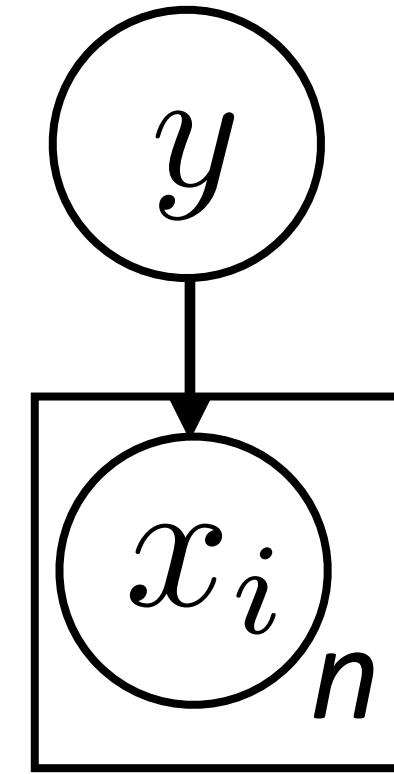
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- ▶ Alternatively: $\log P(y=+|x) - \log P(y=-|x) > 0$

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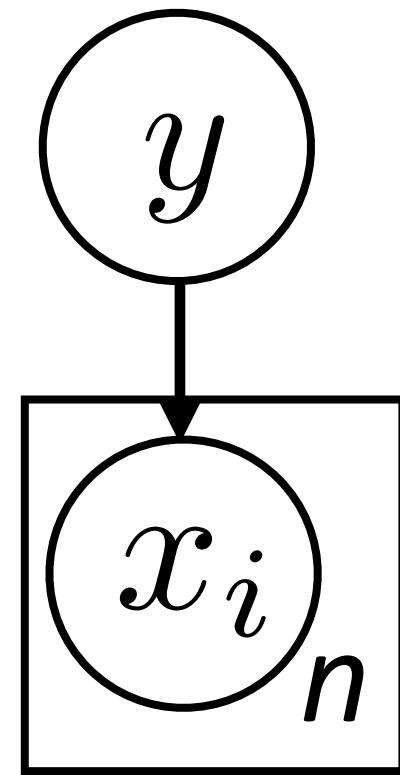
- Alternatively: $\log P(y=+|x) - \log P(y=-|x) > 0$

$$\Leftrightarrow \log \frac{P(y=+)}{P(y=-)} + \sum_{i=1}^n \log \frac{P(x_i|y=+)}{P(x_i|y=-)} > 0$$

Naive Bayes: Summary

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- Learning: maximize $P(x, y)$ by reading counts off the data

Problems with Naive Bayes

the film was beautiful, stunning cinematography and gorgeous sets, but boring —

$$P(x_{\text{beautiful}}|+) = 0.1$$

$$P(x_{\text{beautiful}}|-) = 0.01$$

$$P(x_{\text{stunning}}|+) = 0.1$$

$$P(x_{\text{stunning}}|-) = 0.01$$

$$P(x_{\text{gorgeous}}|+) = 0.1$$

$$P(x_{\text{gorgeous}}|-) = 0.01$$

$$P(x_{\text{boring}}|+) = 0.01$$

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- ▶ Correlated features compound: *beautiful* and *gorgeous* are not independent!

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- ▶ Correlated features compound: *beautiful* and *gorgeous* are not independent!
- ▶ Naive Bayes is naive, but another problem is that it's *generative*: spends capacity modeling $P(x,y)$, when what we care about is $P(y|x)$
- ▶ Discriminative models model $P(y|x)$ directly (SVMs, most neural networks, ...)

Logistic Regression

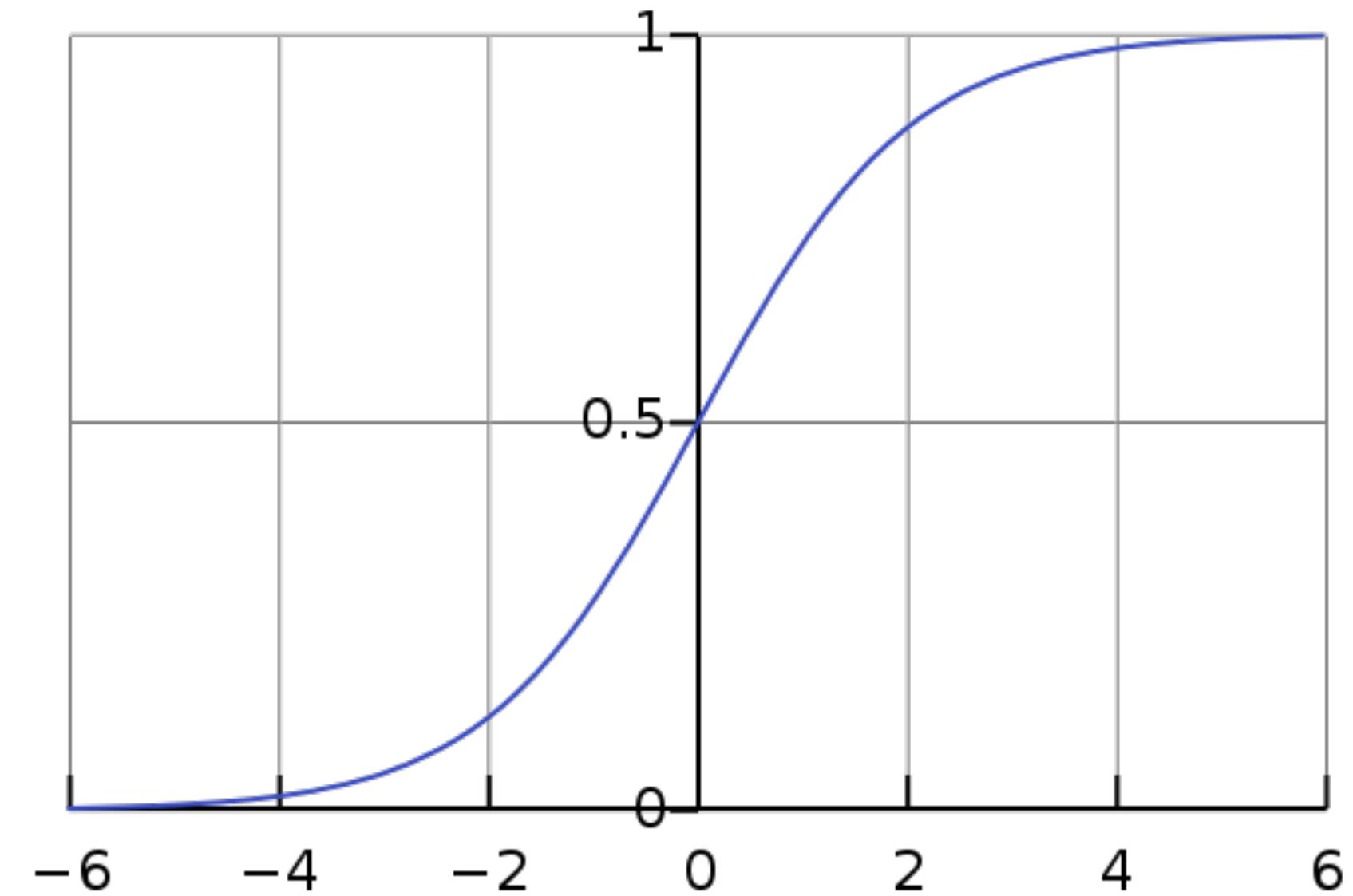
Logistic Regression

Logistic Regression

$$P(y = +|x) = \text{logistic}(w^\top x)$$

Logistic Regression

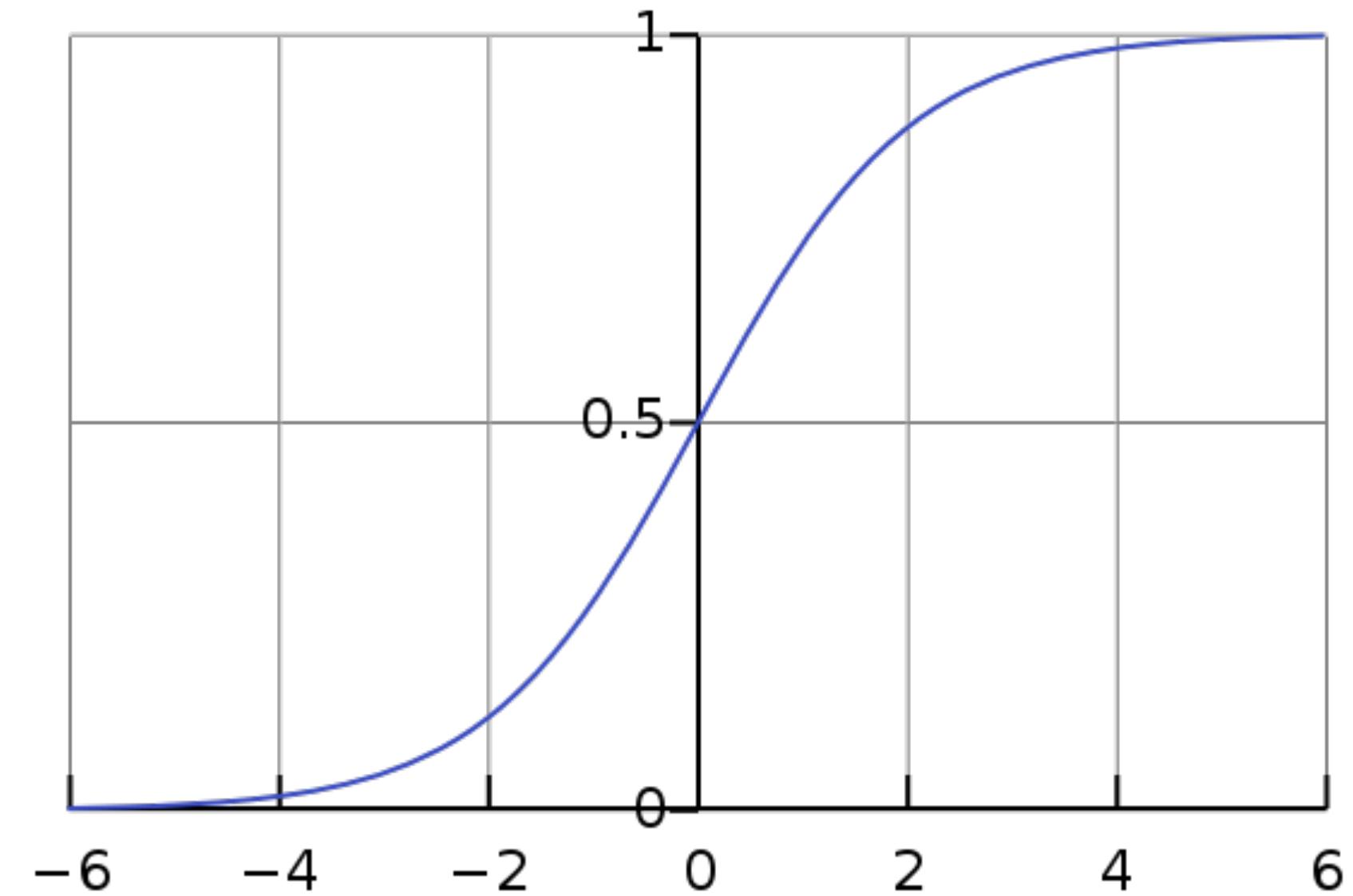
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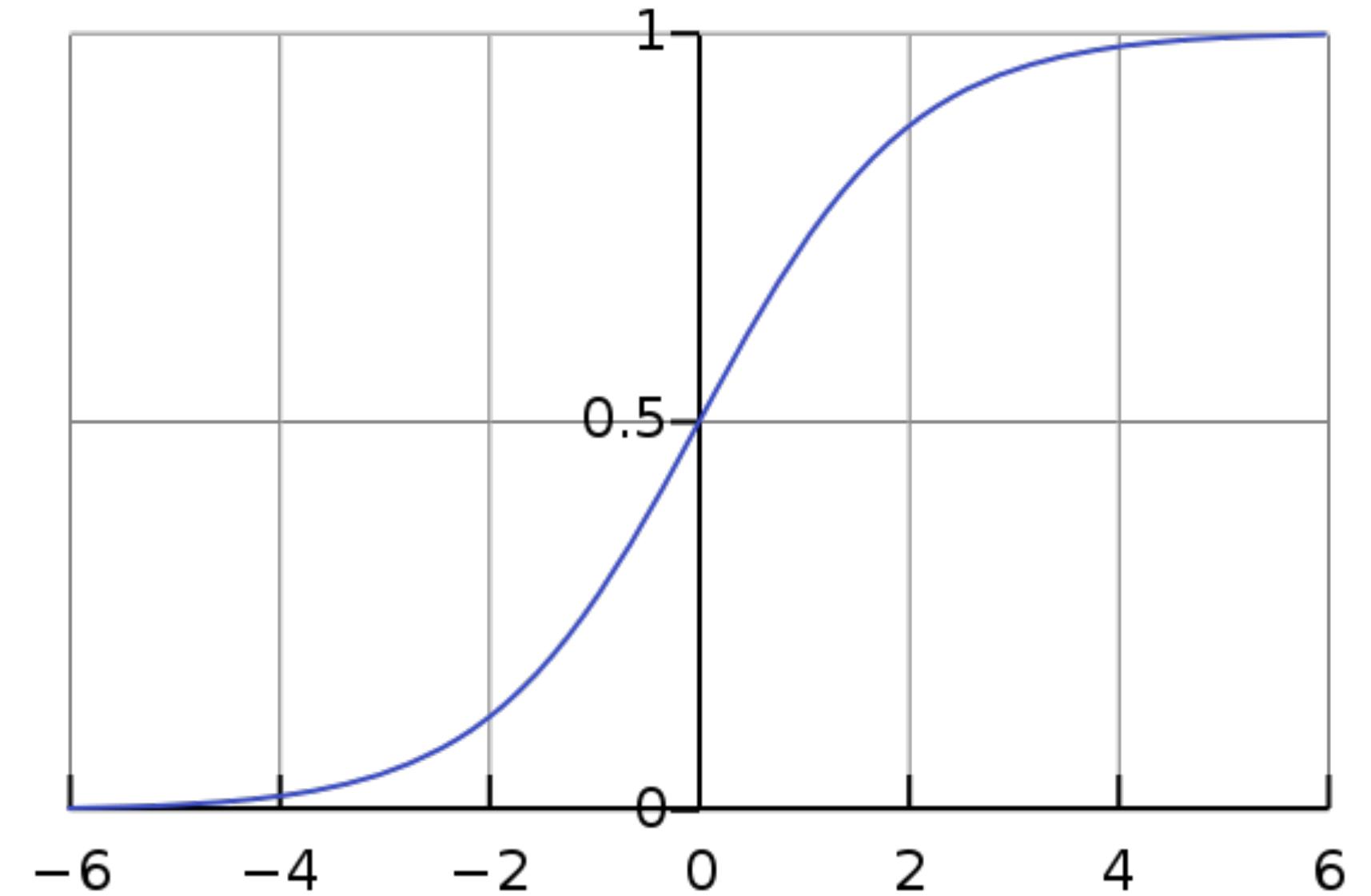
$$P(y = +|x) = \frac{\exp(\sum_{i=1}^n w_i x_i)}{1 + \exp(\sum_{i=1}^n w_i x_i)}$$



Logistic Regression

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$$P(y = +|x) = \frac{\exp(\sum_{i=1}^n w_i x_i)}{1 + \exp(\sum_{i=1}^n w_i x_i)}$$

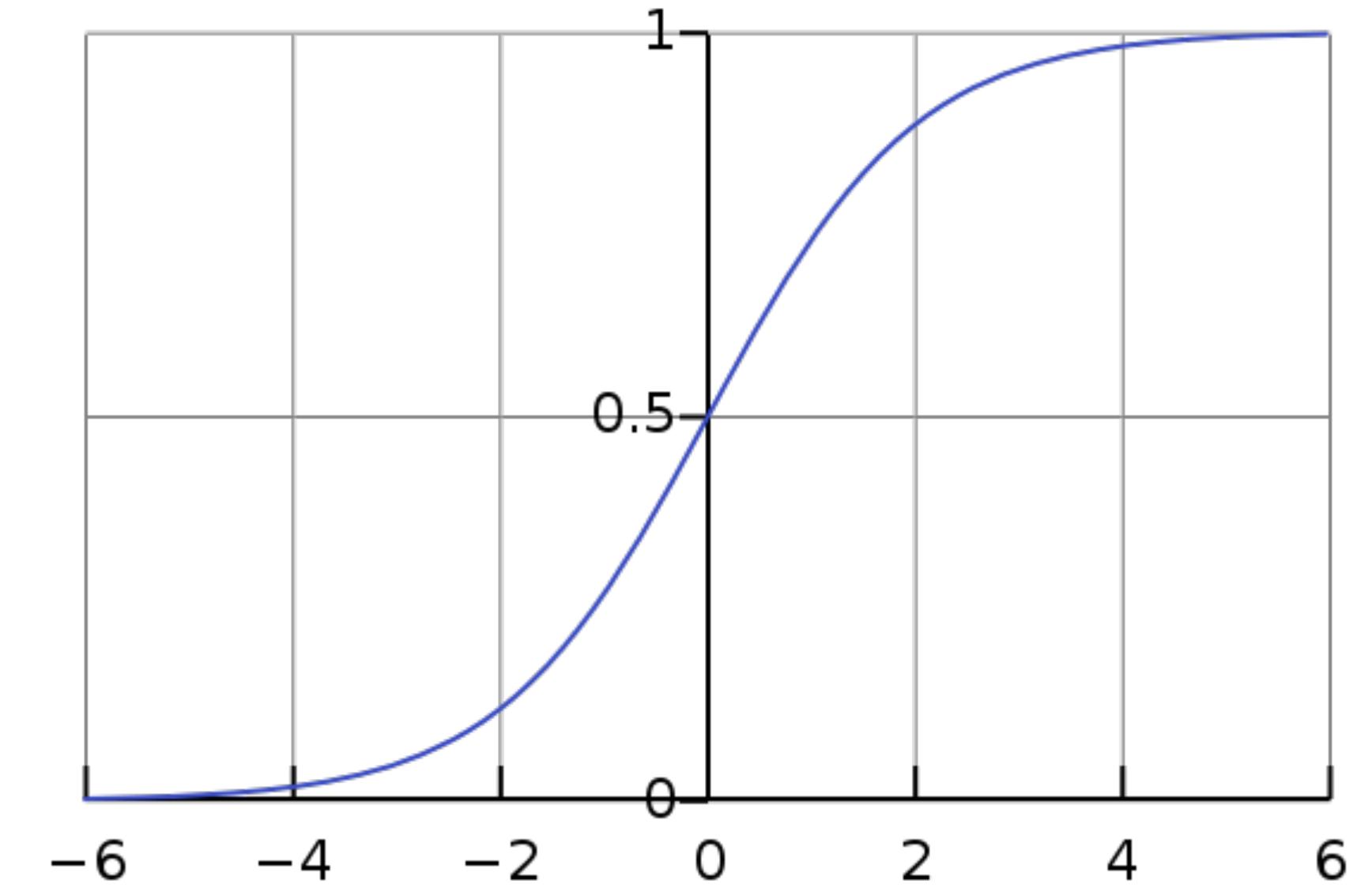


- ▶ To learn weights: maximize discriminative log likelihood of data $P(y|x)$

Logistic Regression

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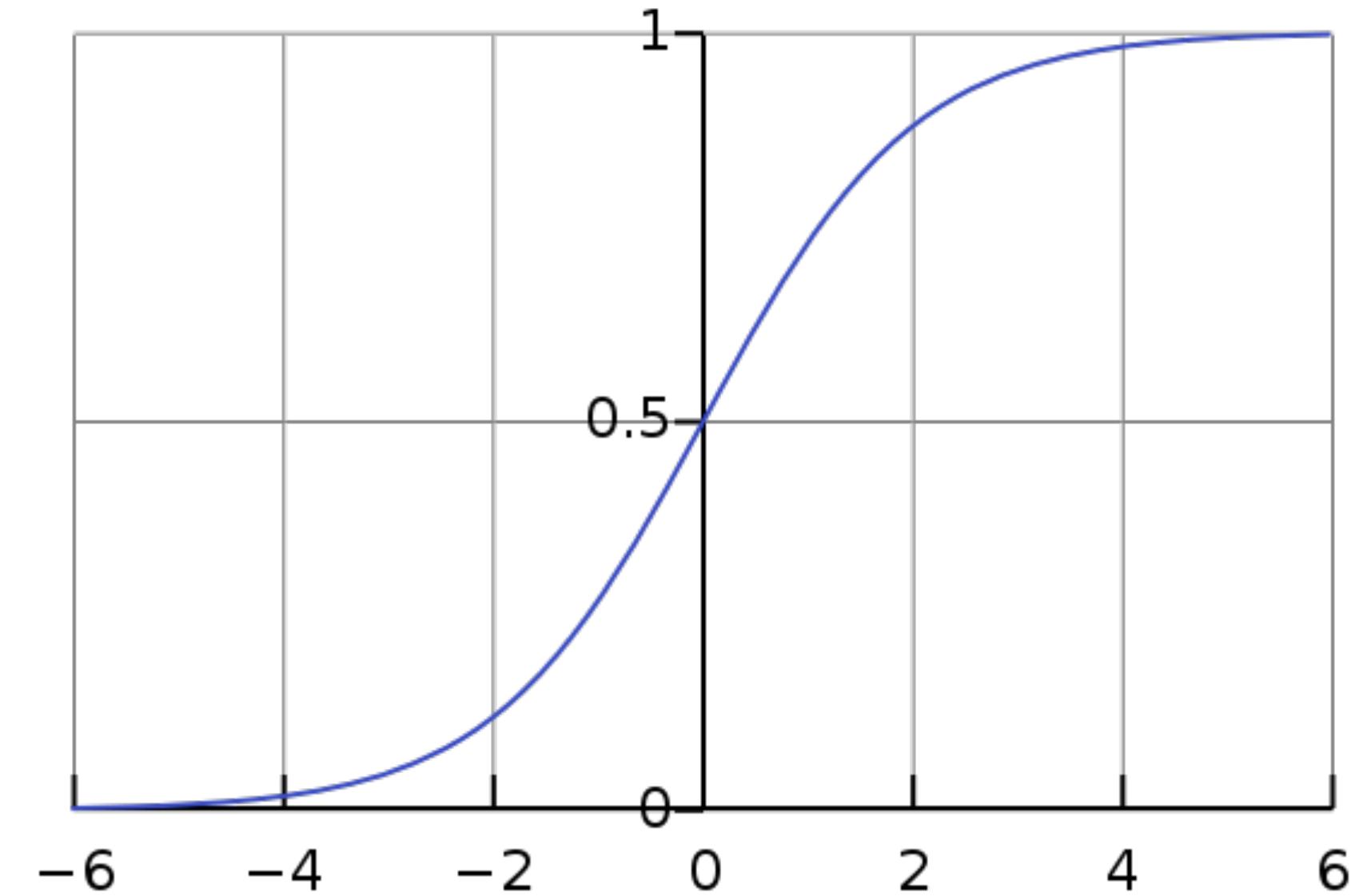
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$$\mathcal{L}(x_j, y_j = +) = \log P(y_j = +|x_j)$$

Logistic Regression

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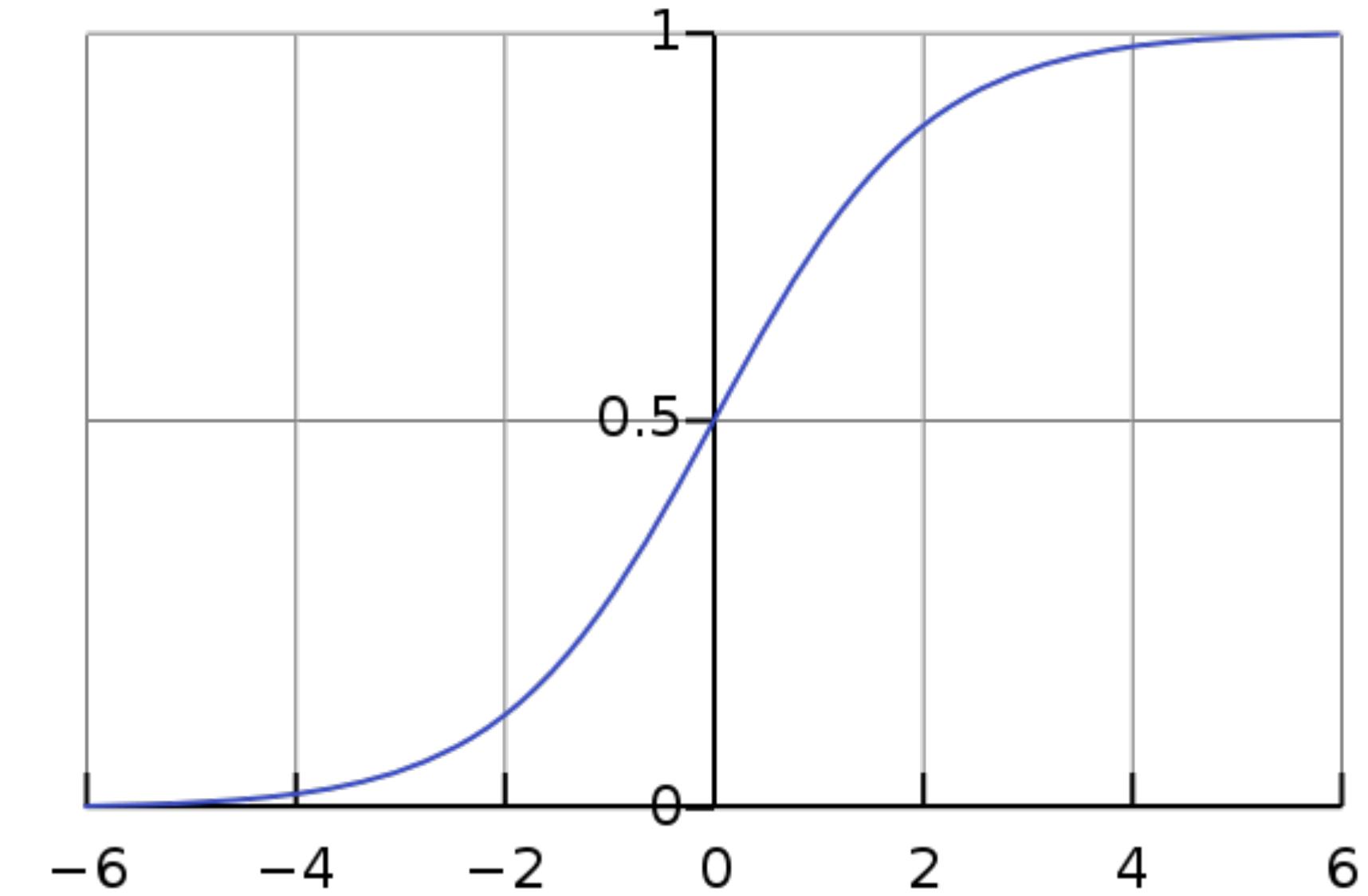
$$\mathcal{L}(x_j, y_j = +) = \log P(y_j = +|x_j)$$

$$= \sum_{i=1}^n w_i x_{ji} - \log \left(1 + \exp \left(\sum_{i=1}^n w_i x_{ji} \right) \right)$$

Logistic Regression

$$P(y = +|x) = \text{logistic}(w^\top x)$$

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$$\begin{aligned}\mathcal{L}(x_j, y_j = +) &= \log P(y_j = +|x_j) \\ &= \sum_{i=1}^n w_i x_{ji} - \log \left(1 + \exp \left(\sum_{i=1}^n w_i x_{ji} \right) \right)\end{aligned}$$

sum over features →

Logistic Regression

Logistic Regression

$$\mathcal{L}(x_j, y_j = +) = \log P(y_j = + | x_j) = \sum_{i=1}^n w_i x_{ji} - \log \left(1 + \exp \left(\sum_{i=1}^n w_i x_{ji} \right) \right)$$

Logistic Regression

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$$\frac{\partial \mathcal{L}(x_j, y_j)}{\partial w_i} =$$

Logistic Regression

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Logistic Regression

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$$= x_{ji} - \frac{1}{1 + \exp \left(\sum_{i=1}^n w_i x_{ji} \right)} \frac{\partial}{\partial w_i} \left(1 + \exp \left(\sum_{i=1}^n w_i x_{ji} \right) \right)$$

deriv
of log

Logistic Regression

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deriv
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deriv
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Logistic Regression

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$$= x_{ji} - \frac{1}{\boxed{1 + \exp \left(\sum_{i=1}^n w_i x_{ji} \right)}} \frac{\partial}{\partial w_i} \left(\boxed{1 + \exp \left(\sum_{i=1}^n w_i x_{ji} \right)} \right)$$

$$= x_{ji} - \frac{1}{1 + \exp \left(\sum_{i=1}^n w_i x_{ji} \right)} x_{ji} \exp \left(\sum_{i=1}^n w_i x_{ji} \right)$$

deriv
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$$= x_{ji} - x_{ji} \frac{\exp \left(\sum_{i=1}^n w_i x_{ji} \right)}{1 + \exp \left(\sum_{i=1}^n w_i x_{ji} \right)} = x_{ji} (1 - P(y_j = + | x_j))$$

deriv
of log

deriv
of exp

Logistic Regression

- ▶ Recall that $y_j = 1$ for positive instances, $y_j = 0$ for negative instances.
- ▶ Gradient of w_i on positive example $= x_{ji}(1 - P(y_j = +|x_j))$

Logistic Regression

- ▶ Recall that $y_j = 1$ for positive instances, $y_j = 0$ for negative instances.
- ▶ Gradient of w_i on positive example $= x_{ji}(1 - P(y_j = +|x_j))$
 - If $P(+)$ is close to 1, make very little update
 - Otherwise make w_i look more like x_{ji} , which will increase $P(+)$

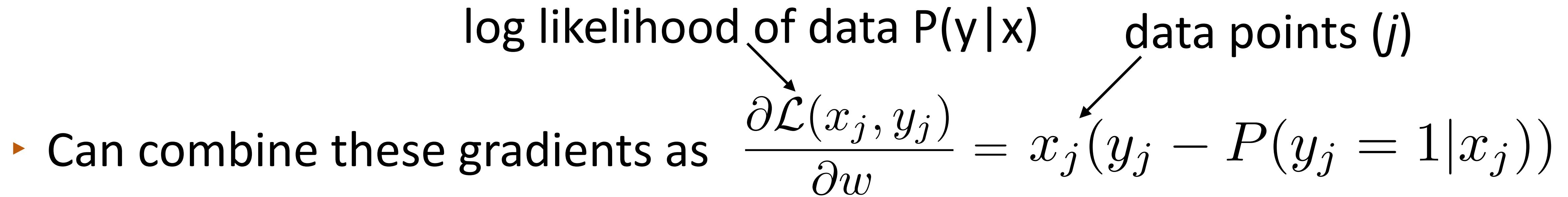
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- ▶ Gradient of w_i on negative example $= x_{ji}(-P(y_j = +|x_j))$

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- ▶ Gradient of w_i on negative example $= x_{ji}(-P(y_j = +|x_j))$
 - If $P(+)$ is close to 0, make very little update
 - Otherwise make w_i look less like x_{ji} , which will decrease $P(+)$
- ▶ Can combine these gradients as $x_j(y_j - P(y_j = 1|x_j))$

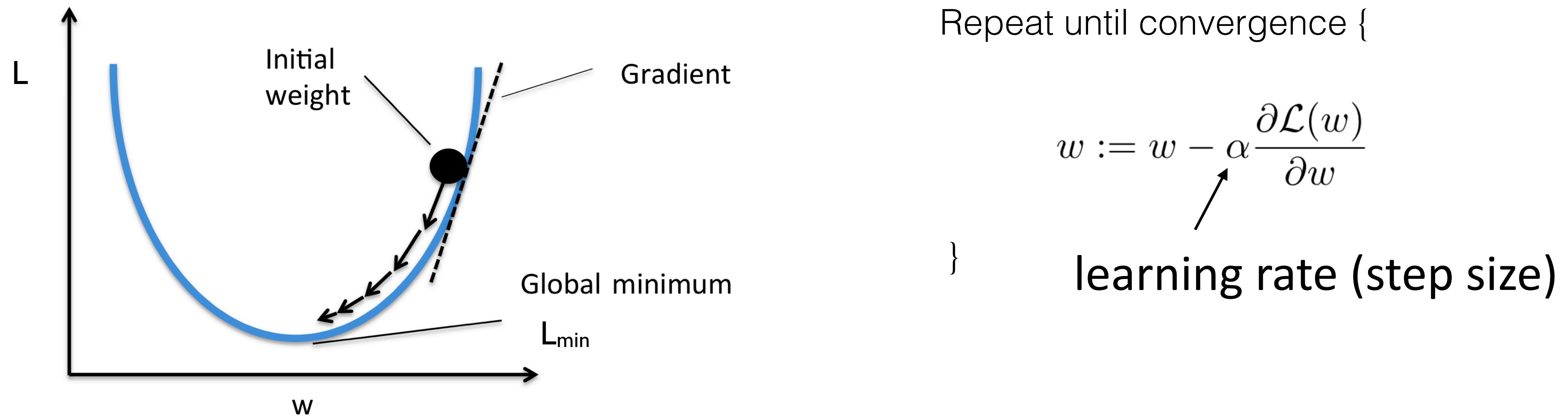
Gradient Decent

- Can combine these gradients as $\frac{\partial \mathcal{L}(x_j, y_j)}{\partial w} = x_j(y_j - P(y_j = 1|x_j))$


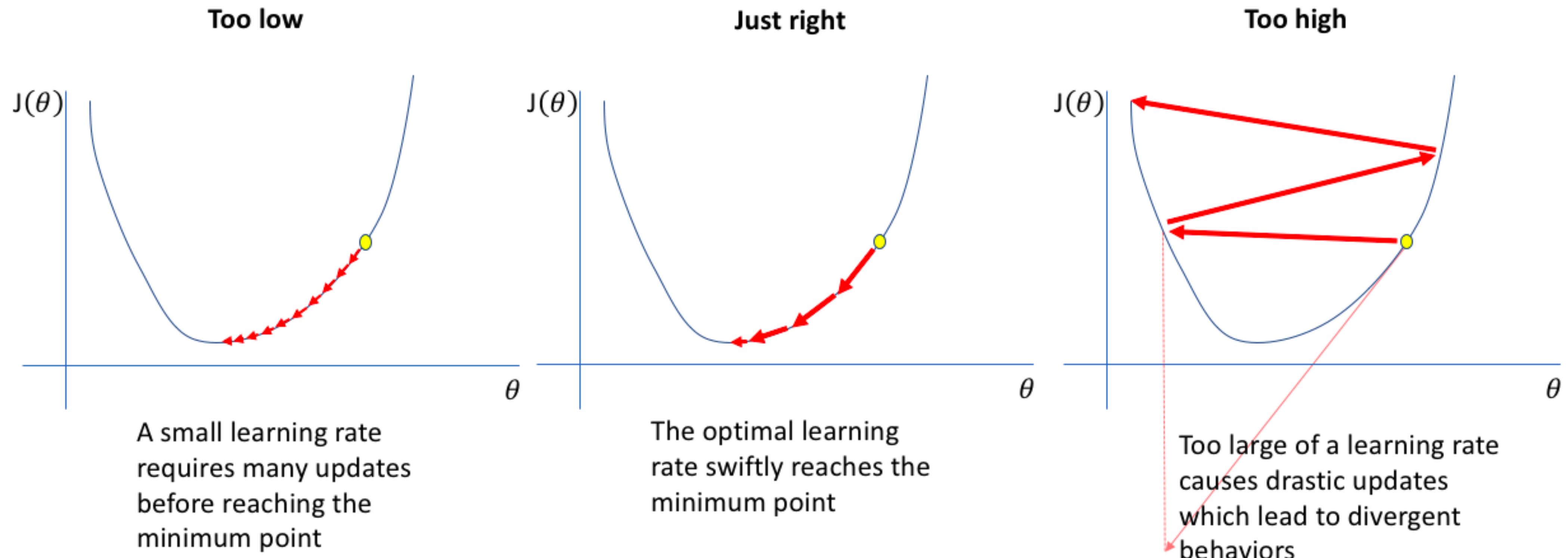
The diagram consists of two arrows. One arrow points from the text "log likelihood of data $P(y|x)$ " to the term x_j in the equation. Another arrow points from the text "data points (j)" to the term $(y_j - P(y_j = 1|x_j))$.
- Training set log-likelihood: $\mathcal{L}(w) = \frac{1}{m} \sum_{j=1}^m \mathcal{L}(x_j, y_j)$
- Gradient vector: $\frac{\partial \mathcal{L}(w)}{\partial w} = \left(\frac{\partial \mathcal{L}}{\partial w_1}, \frac{\partial \mathcal{L}}{\partial w_2}, \dots, \frac{\partial \mathcal{L}}{\partial w_n} \right)$

Gradient Decent

- ▶ Gradient decent (or ascent) is an iterative optimization algorithm for finding the minimum (or maximum) of a function.



Learning Rate



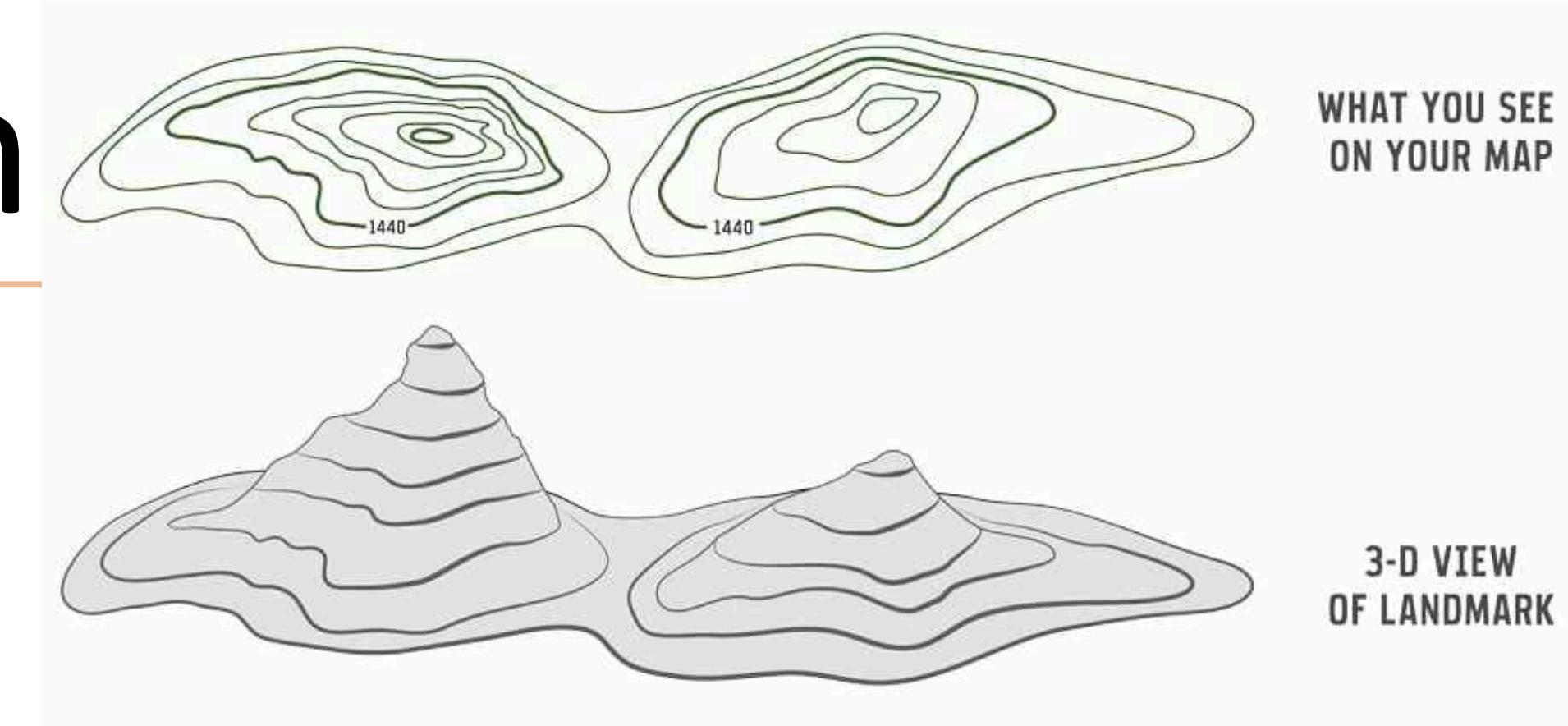
Credit: Jeremy Jordan

Optimization

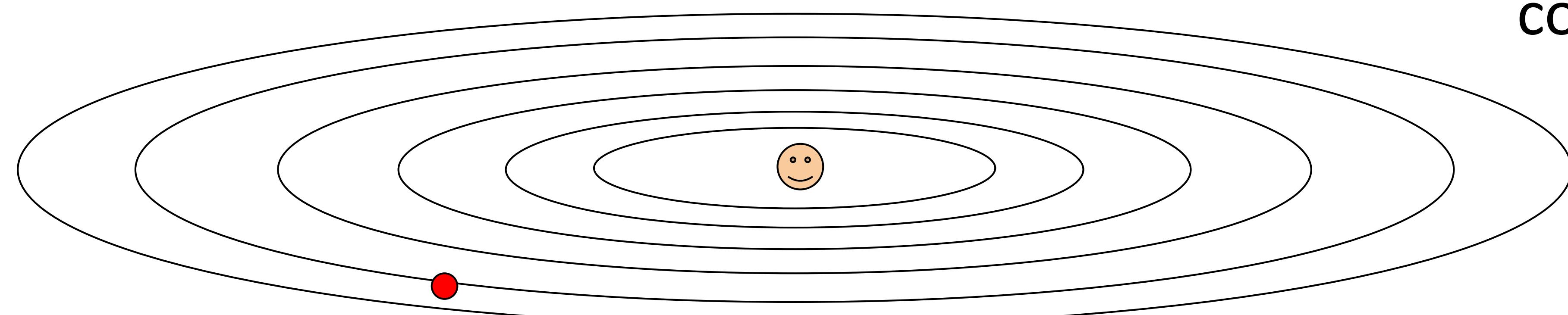
WHAT YOU SEE
ON YOUR MAP

- Gradient descent

$$w := w - \alpha \frac{\partial \mathcal{L}(w)}{\partial w}$$



Q: What if loss changes quickly in one direction and slowly in another direction?



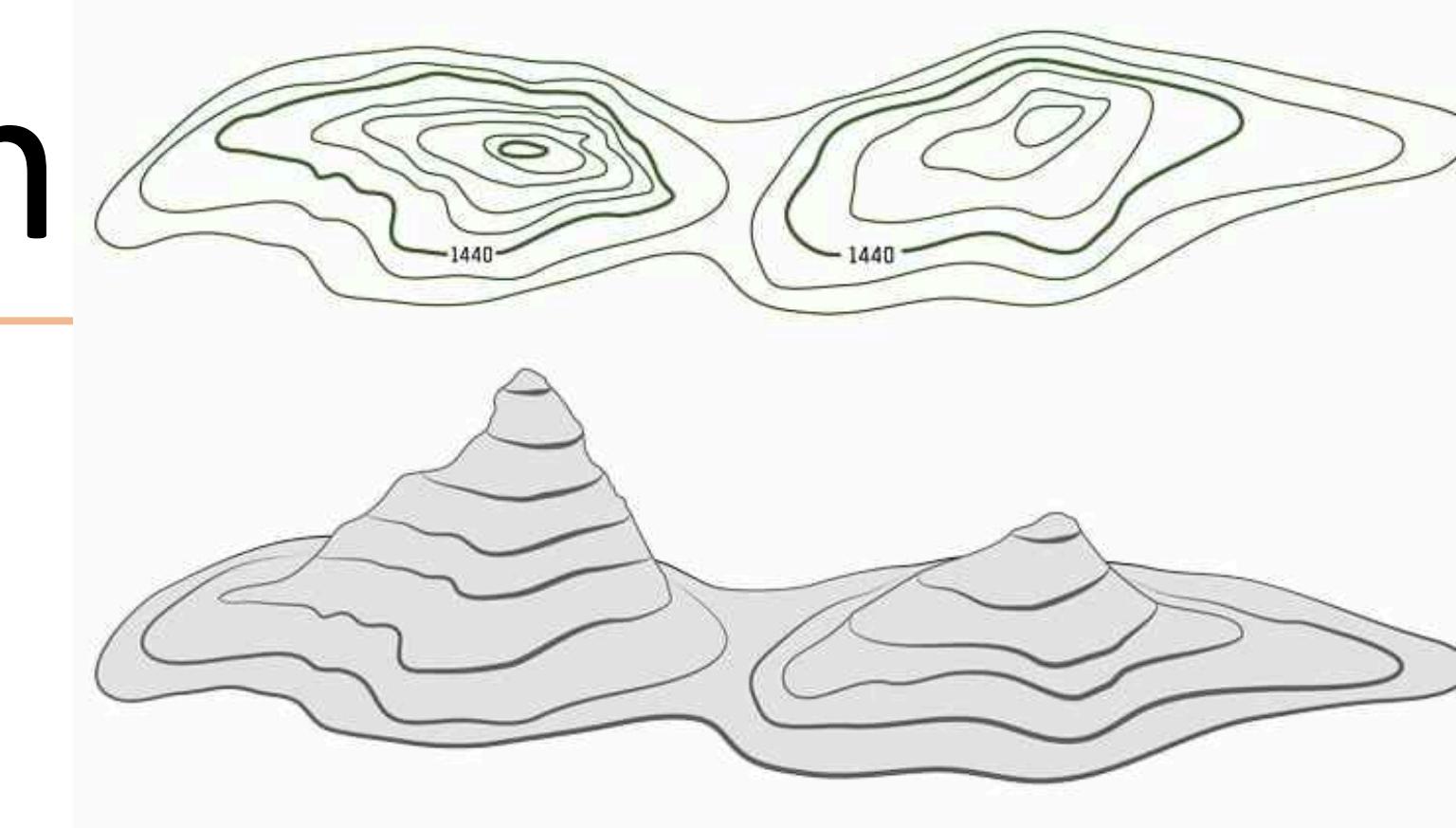
contour plot

Credit: Stanford CS231n

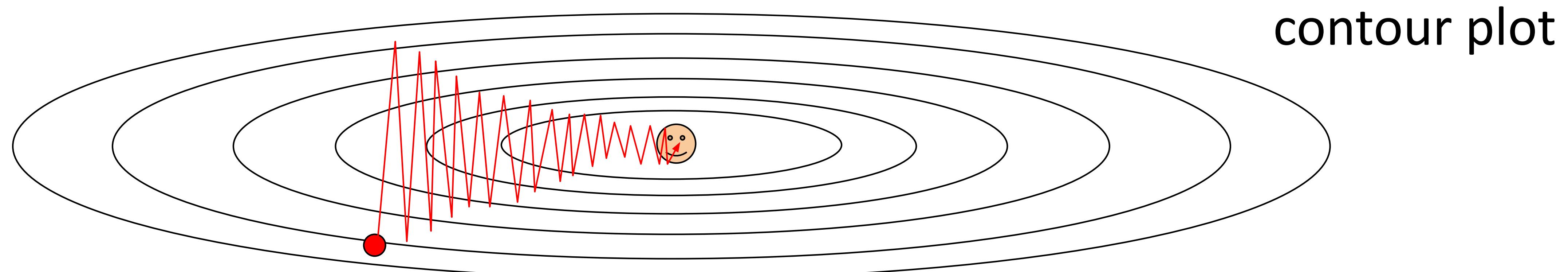
Optimization

► Gradient descent

$$w := w - \alpha \frac{\partial \mathcal{L}(w)}{\partial w}$$



Q: What if loss changes quickly in one direction and slowly in another direction?



Solution: feature scaling!

Credit: Stanford CS231n

Regularization

Regularization

- ▶ Regularizing an objective can mean many things, including an L2-norm penalty to the weights:

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 - ▶ For neural networks: dropout and gradient clipping

Logistic Regression: Summary

- ▶ Model

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- ▶ Learning: gradient ascent on the (regularized) discriminative log-likelihood

Perceptron/SVM

Perceptron

- Invented in 1958
 - By [Frank Rosenblatt](#)
 - At the [Cornell Aeronautical Laboratory](#)
- Implemented in custom-built hardware
- Connected to a camera with 20×20 cadmium sulfide photocells to make a 400-pixel image.
- Weights were encoded in [potentiometers](#), and weight updates during learning were performed by electric motors.



Source: Wikipedia

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 - ▶ If incorrect: if positive, $w \leftarrow w + x$
 - if negative, $w \leftarrow w - x$
- ▶ Guaranteed to eventually separate the data if the data are separable

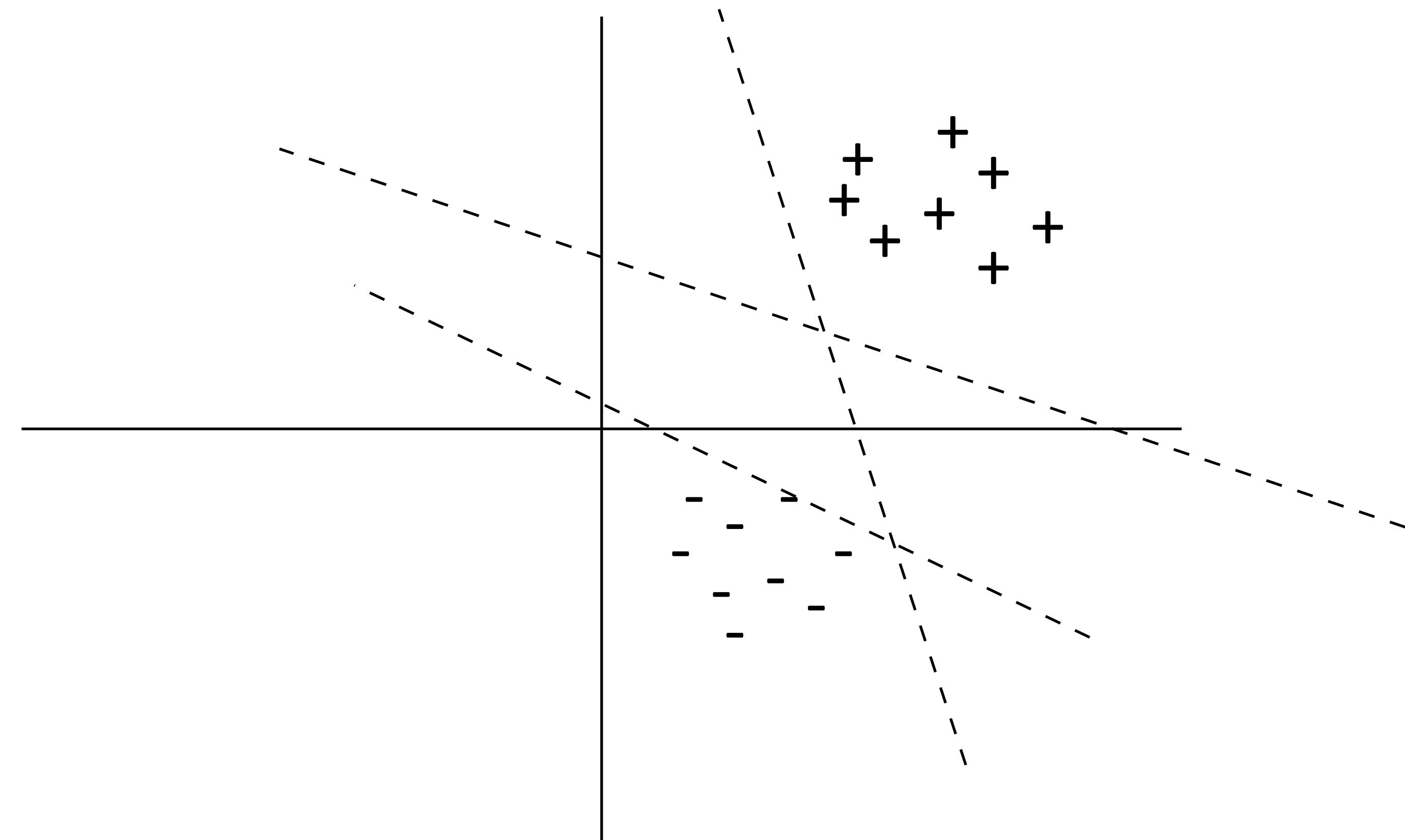
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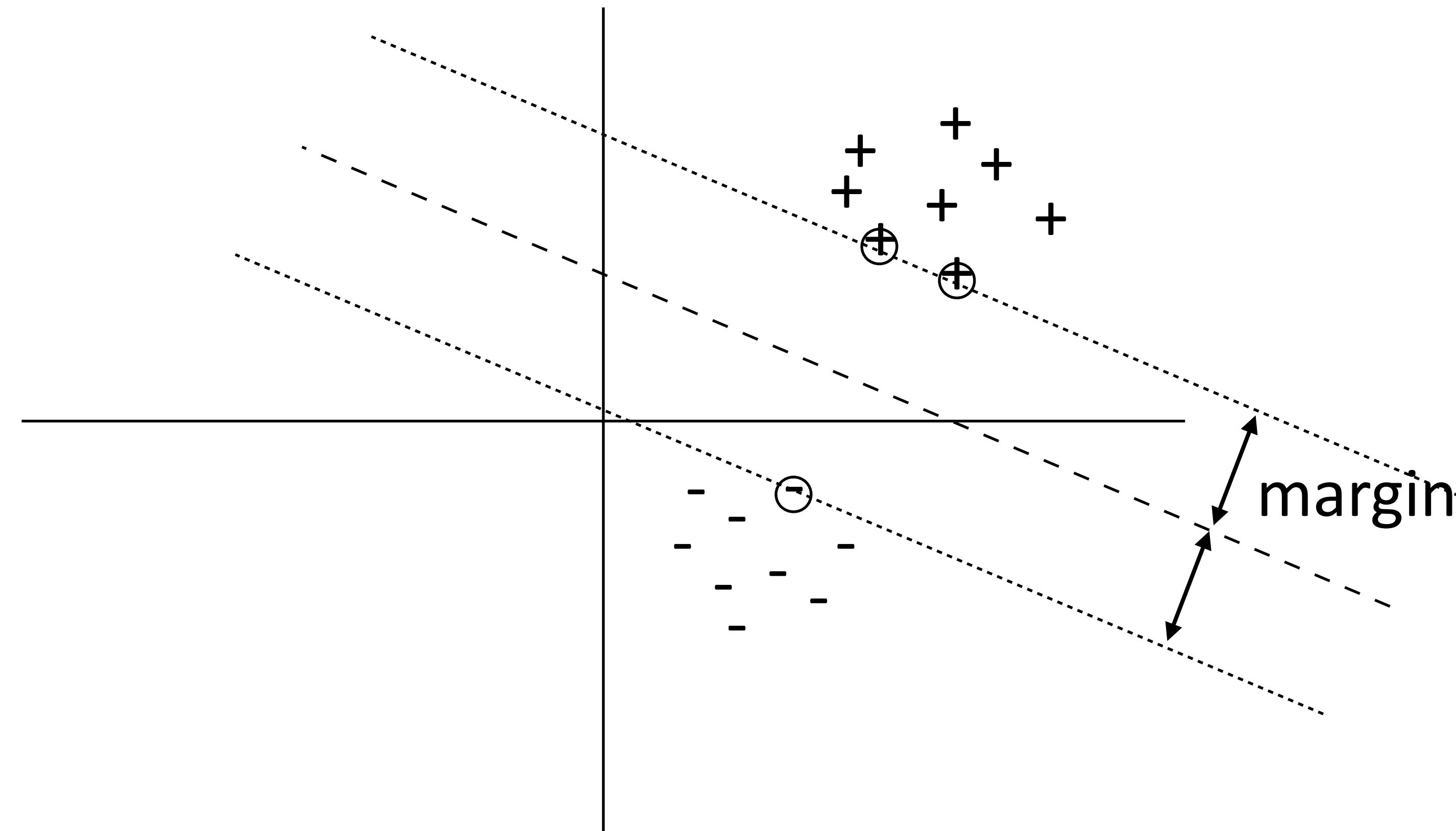
Support Vector Machines

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- ▶ Generally no solution (data is generally non-separable) – need slack!

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- ▶ Looks like the perceptron! But updates more frequently

Gradients on Positive Examples

Logistic regression

$$x(1 - \text{logistic}(w^\top x))$$

Perceptron

$$x \text{ if } w^\top x < 0, \text{ else } 0$$

SVM (ignoring regularizer)

$$x \text{ if } w^\top x < 1, \text{ else } 0$$

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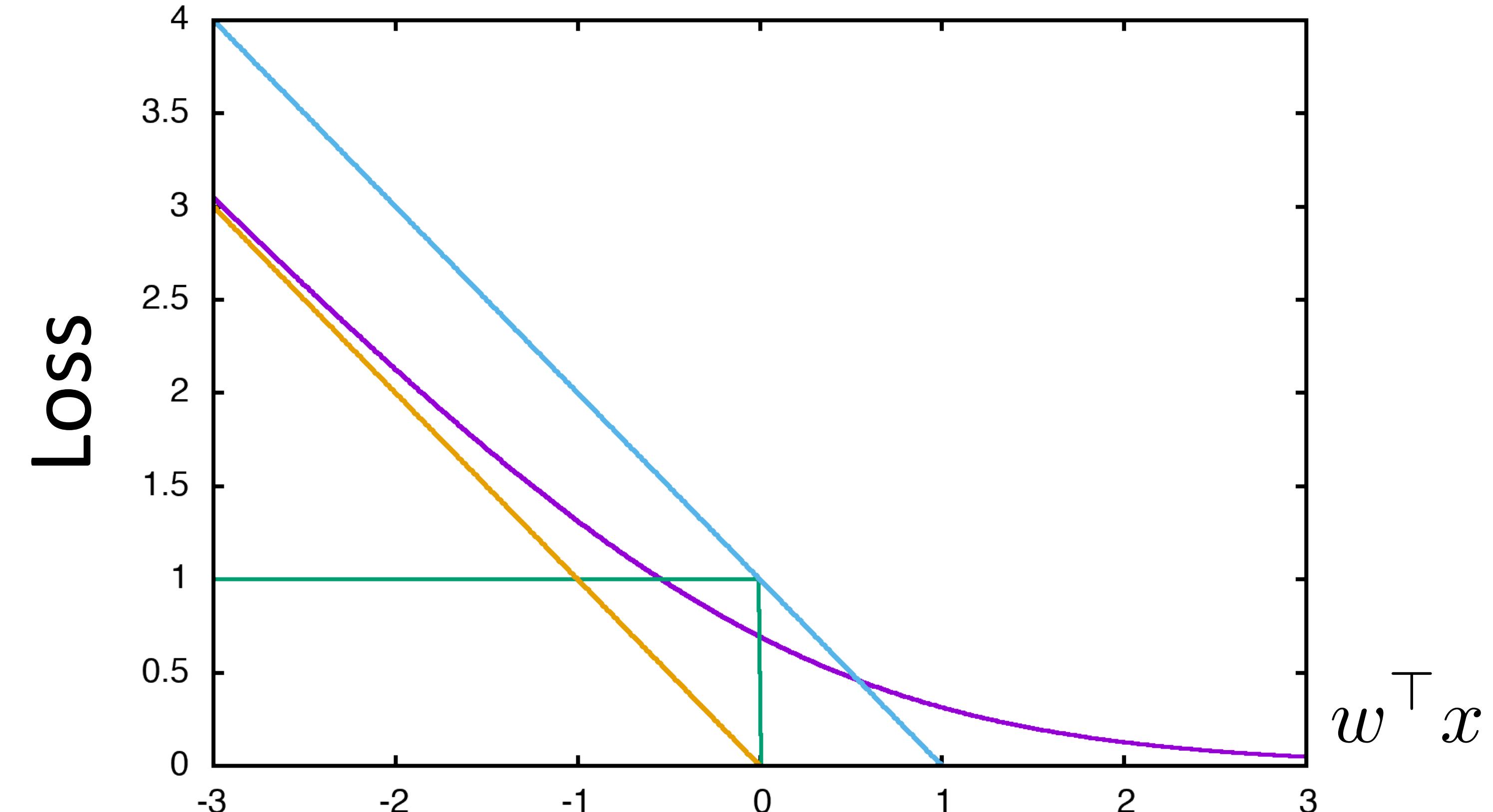
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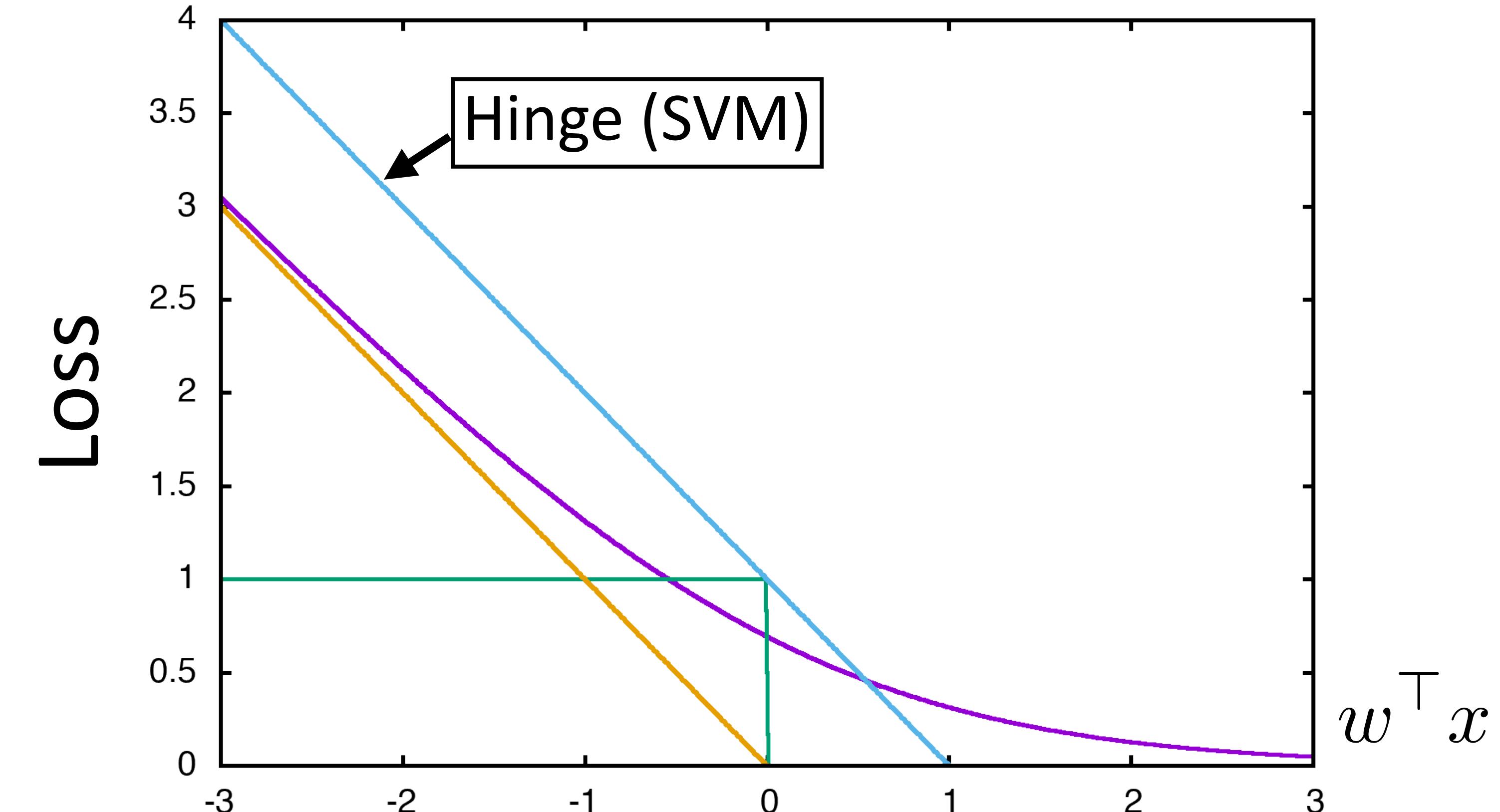
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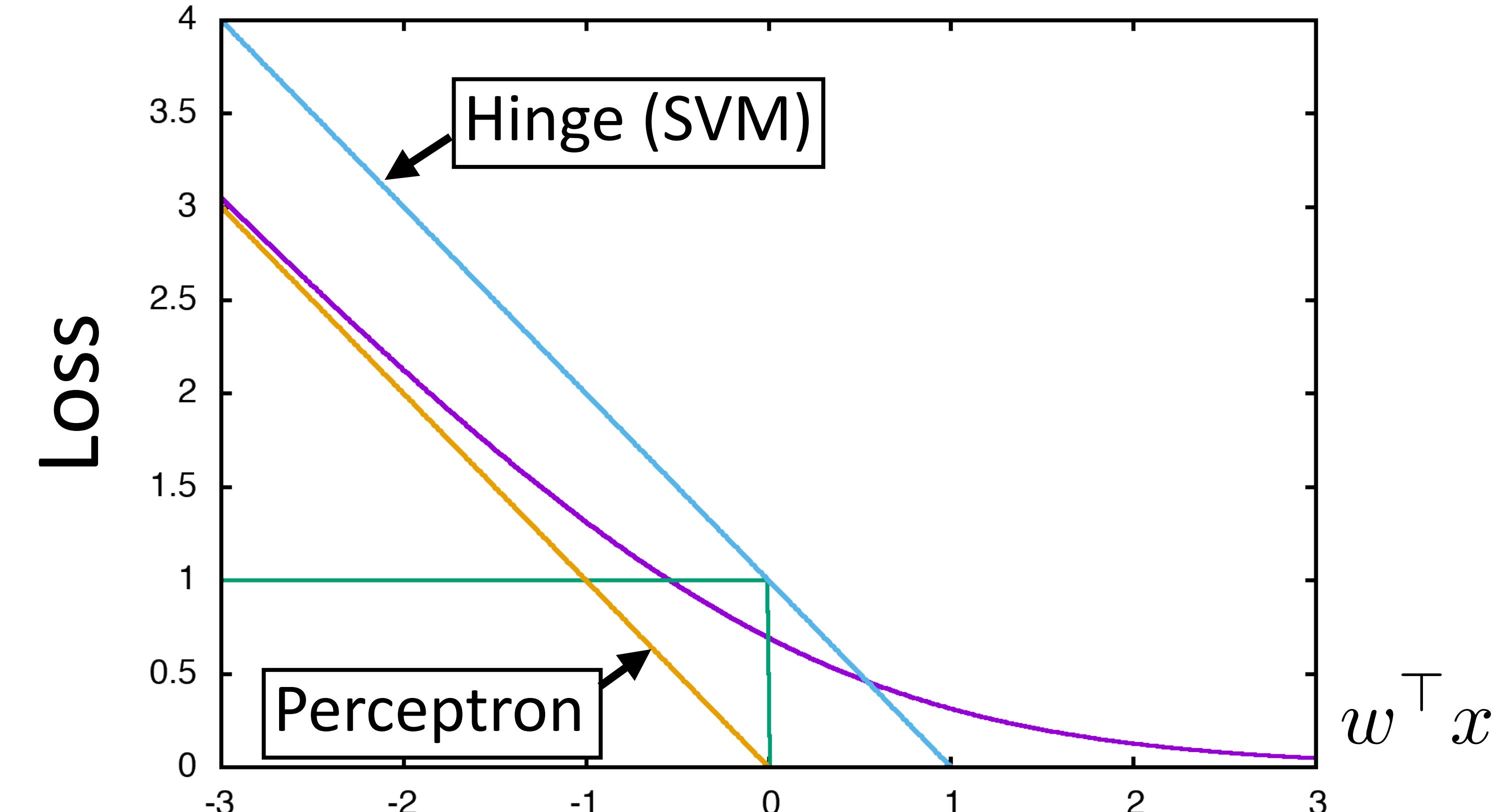
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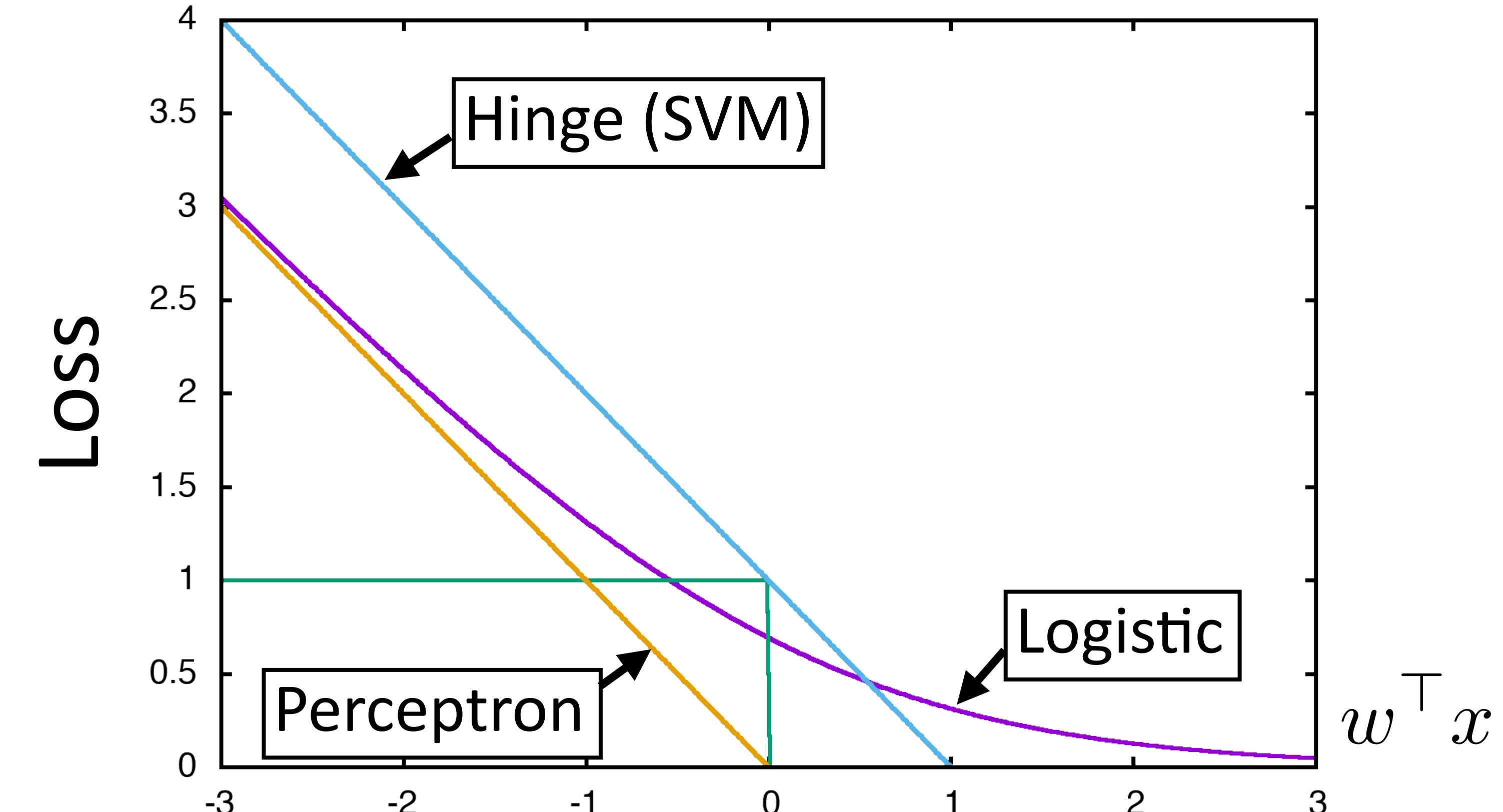
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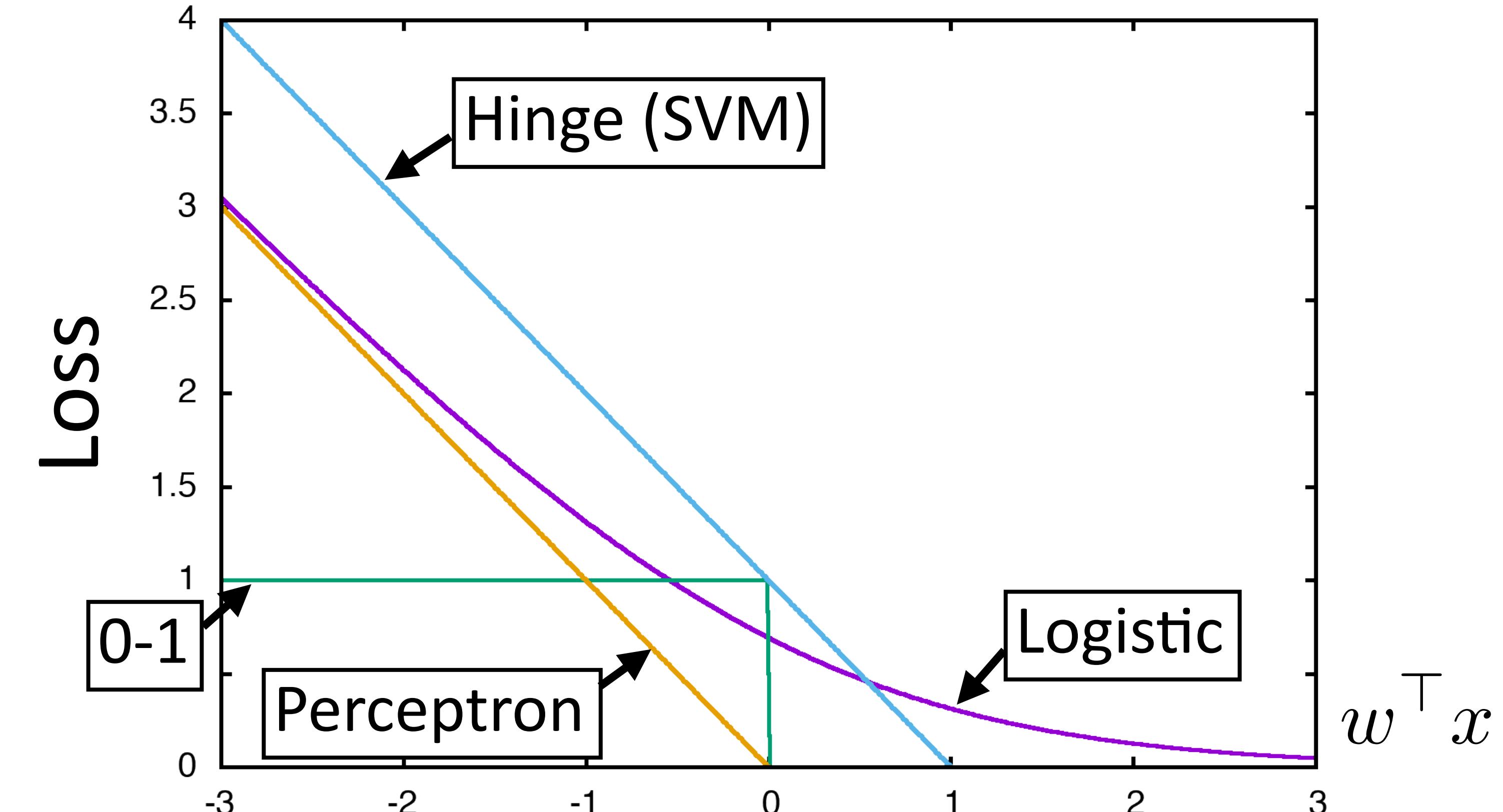
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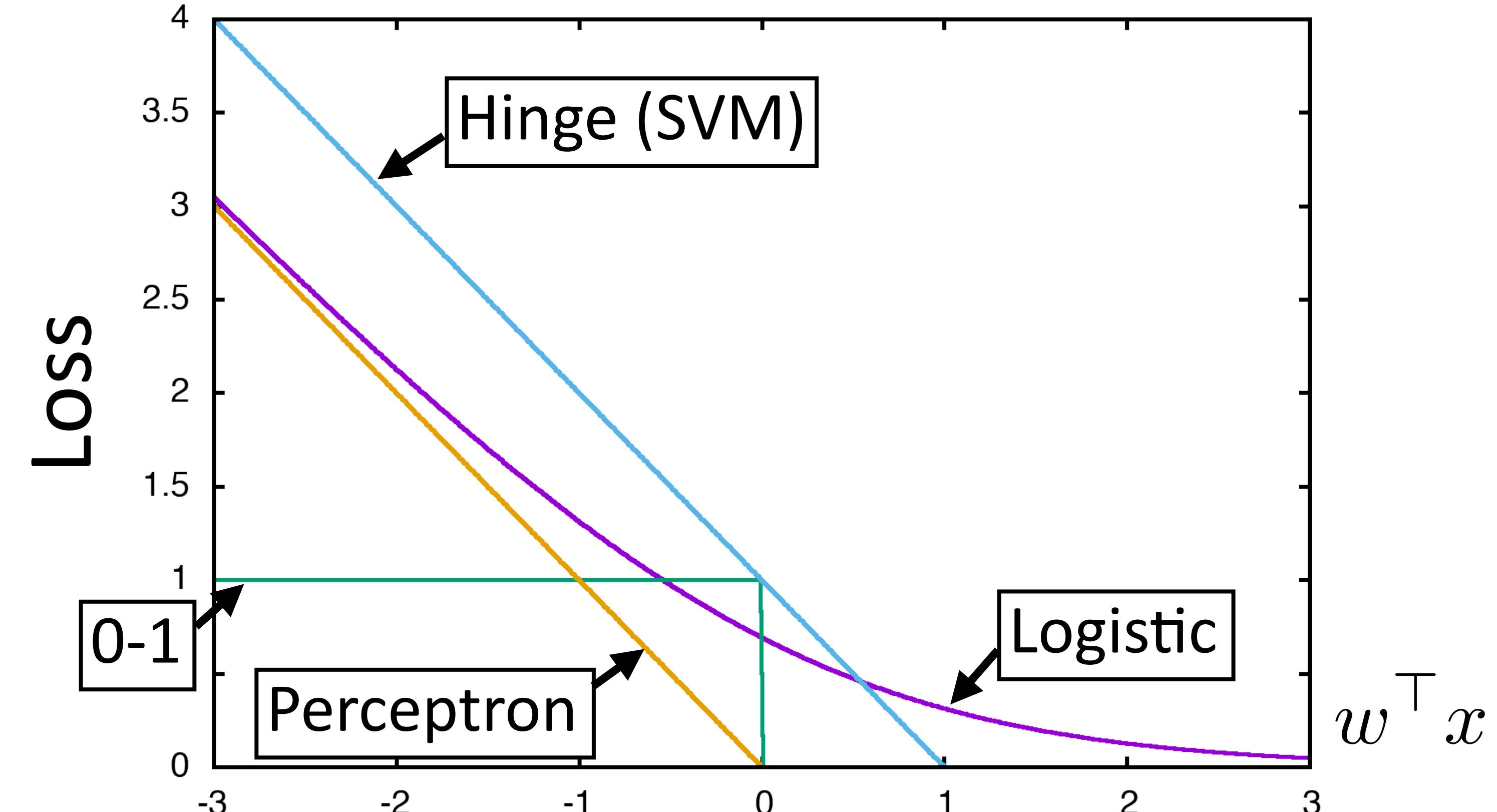
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*gradients are for maximizing things,
which is why they are flipped

Comparing Gradient Updates (Reference)

Logistic regression (unregularized)

$$x(y - P(y = 1|x)) = x(y - \text{logistic}(w^\top x))$$

$y = 1$ for pos,

0 for neg

Perceptron

$$(2y - 1)x \text{ if classified incorrectly}$$

0 else

SVM

$$(2y - 1)x \text{ if not classified correctly with margin of 1}$$

0 else

Optimization — next time...

- ▶ Range of techniques from simple gradient descent (works pretty well) to more complex methods (can work better)
- ▶ Most methods boil down to: take a gradient and a step size, apply the gradient update times step size, incorporate estimated curvature information to make the update more effective

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- ▶ There are some ways around this: extract bigram feature for “*not X*” for all *X* following the *not*

Sentiment Analysis

Sentiment Analysis

	Features	# of features	frequency or presence?	NB	ME	SVM
(1)	unigrams	16165	freq.	78.7	N/A	72.8
(2)	unigrams	"	pres.	81.0	80.4	82.9
(3)	unigrams+bigrams	32330	pres.	80.6	80.8	82.7
(4)	bigrams	16165	pres.	77.3	77.4	77.1
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- ▶ Simple feature sets can do pretty well!

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NBSVM-uni	78.1	85.3
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RAE	76.8	85.7
RAE-pretrain	77.7	86.4
Voting-w/Rev.	63.1	81.7
Rule	62.9	81.8
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Kim (2014) CNNs

81.5 89.5

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Wang and Manning (2012)

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(Sub)gradient (unregularized): 0 if correct with margin of 1, else $x(2y - 1)$

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- ▶ SVM and perceptron inference require taking maxes, logistic regression has a similar update but is “softer” due to its probabilistic nature
- ▶ All gradient updates: “make it look more like the right thing and less like the wrong thing”