Instance-Based Learning

Instructor: Alan Ritter

Many Slides from Pedro Domingos

Preview

- *K*-nearest neighbor
- Other forms of IBL
- Collaborative filtering

Instance-Based Learning

Key idea: Just store all training examples $\langle x_i, f(x_i) \rangle$

Nearest neighbor:

- Given query instance x_q, first locate nearest training example x_n, then estimate $\hat{f}(x_q) \leftarrow f(x_n)$

k-Nearest neighbor:

- Given x_q, take vote among its k nearest neighbors (if discrete-valued target function)
- Take mean of f values of k nearest neighbors (if real-valued)

$$\hat{f}(x_q) \leftarrow \frac{1}{k} \sum_{i=1}^{k} f(x_i)$$

Advantages and Disadvantages

Advantages:

- Training is very fast
- Learn complex target functions easily
- Don't lose information

Disadvantages:

- Slow at query time
- Lots of storage
- Easily fooled by irrelevant attributes

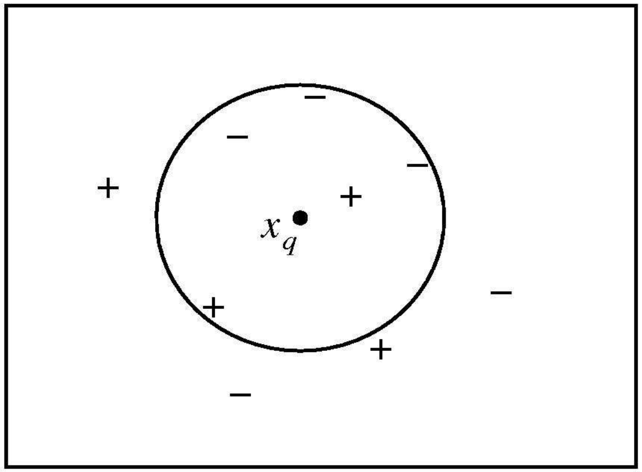
Distance Measures

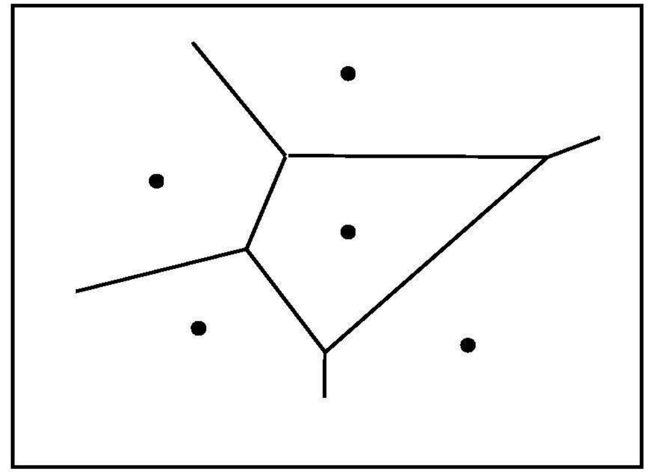
- **Numeric features:**
  - Euclidean, Manhattan, L^n-norm:

    $$L^n(\mathbf{x}_1, \mathbf{x}_2) = \sqrt[n]{\sum_{i=1}^{\#\mathrm{dim}} |\mathbf{x}_{1,i} - \mathbf{x}_{2,i}|^n}$$
  - Normalized by: range, std. deviation
- **Symbolic features:**
  - Hamming/overlap
  - Value difference measure (VDM):

    $$\delta(val_i, val_j) = \sum_{h=1}^{\#\mathrm{classes}} |P(c_h|val_i) - P(c_h|val_j)|^n$$
- **In general:** arbitrary, encode knowledge

Voronoi Diagram





S: Training set

Voronoi cell of $\mathbf{x} \in S$:
All points closer to $\mathbf{x}$ than to any other instance in S

Region of class C:
Union of Voronoi cells of instances of C in S

Behavior in the Limit

$\epsilon^*(\mathbf{x})$: Error of optimal prediction
$\epsilon_{NN}(\mathbf{x})$: Error of nearest neighbor

Theorem: $\lim_{n\rightarrow\infty} \epsilon_{NN} \leq 2\epsilon^*$

Proof sketch (2-class case):

$$\begin{aligned}\epsilon_{NN} &= p_+ p_{NN\in -} + p_- p_{NN\in +}\\ &= p_+(1 - p_{NN\in +}) + (1 - p_+) p_{NN\in +}\end{aligned}$$

$$\lim_{n\rightarrow\infty} p_{NN\in +} = p_+, \quad \lim_{n\rightarrow\infty} p_{NN\in -} = p_-$$

$$\lim_{n\rightarrow\infty} \epsilon_{NN} = p_+(1-p_+)+(1-p_+)p_+ = 2\epsilon^*(1-\epsilon^*) \leq 2\epsilon^*$$

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$$\lim_{n\to\infty} \epsilon_{NN} = p_+(1-p_+)+(1-p_+)p_+ = 2\epsilon^*(1-\epsilon^*) \leq 2\epsilon^*$$

Theorem: $\lim_{n\to\infty,\, k\to\infty,\, k/n\to 0} \epsilon_{kNN} = \epsilon^*$

Distance-Weighted k-NN

Might want to weight nearer neighbors more heavily ...

$$\hat{f}(x_q) \leftarrow \frac{\sum_{i=1}^{k} w_i f(x_i)}{\sum_{i=1}^{k} w_i}$$

where

$$w_i \equiv \frac{1}{d(x_q, x_i)^2}$$

and $d(x_q, x_i)$ is distance between x_q and x_i

Notice that now it makes sense to use *all* training examples instead of just k

Curse of Dimensionality

- Imagine instances described by 20 attributes, but only 2 are relevant to target function
- **Curse of dimensionality**:
  - Nearest neighbor is easily misled when hi-dim X
  - Easy problems in low-dim are hard in hi-dim
  - Low-dim intuitions don't apply in hi-dim
- **Examples:**
  - Normal distribution
  - Uniform distribution on hypercube
  - Points on hypergrid
  - Approximation of sphere by cube
  - Volume of hypersphere

Feature Selection

- **Filter approach:**
  Pre-select features individually
  - E.g., by info gain
- **Wrapper approach:**
  Run learner with different combinations of features
  - Forward selection
  - Backward elimination
  - Etc.

FORWARD_SELECTION(FS)
    FS: Set of features used to describe examples
Let $SS = \emptyset$
Let $BestEval = 0$
Repeat
    Let $BestF = None$
    For each feature F in FS and not in SS
        Let $SS' = SS \cup \{F\}$
        If $Eval(SS') > BestEval$
            Then Let $BestF = F$
                    Let $BestEval = Eval(SS')$
    If $BestF \neq None$
        Then Let $SS = SS \cup \{BestF\}$
Until $BestF = None$ or $SS = FS$
Return SS

```
BACKWARD_ELIMINATION(FS)
    FS: Set of features used to describe examples
Let SS = FS
Let BestEval = Eval(SS)
Repeat
    Let WorstF = None.
    For each feature F in SS
        Let SS' = SS − {F}
        If Eval(SS') ≥ BestEval
            Then Let WorstF = F
                    Let BestEval = Eval(SS')
    If WorstF ≠ None
        Then Let SS = SS − {WorstF}
Until WorstF = None or SS = ∅
Return SS
```

Feature Weighting

- Stretch jth axis by weight z_j, where $z_1, \ldots, z_n$ chosen to minimize prediction error
- Use gradient descent to find weights $z_1, \ldots, z_n$
- Setting z_j to zero eliminates this dimension altogether

Reducing Computational Cost

- Efficient retrieval: k-D trees
  (only work in low dimensions)
- Efficient similarity comparison:
  - Use cheap approx. to weed out most instances
  - Use expensive measure on remainder
- Form prototypes
- Edited k-NN:
  Remove instances that don't affect frontier

Edited k-Nearest Neighbor

EDITED_k-NN(S)
  S: Set of instances
For each instance $\mathbf{x}$ in S
  If $\mathbf{x}$ is correctly classified by $S - \{\mathbf{x}\}$
    Remove $\mathbf{x}$ from S
Return S

EDITED_k-NN(S)
  S: Set of instances
$T = \emptyset$
For each instance $\mathbf{x}$ in S
  If $\mathbf{x}$ is **not** correctly classified by T
    Add $\mathbf{x}$ to T
Return T

Overfitting Avoidance

- Set k by cross-validation
- Form prototypes
- Remove noisy instances
  - E.g., remove $\mathbf{x}$ if all of $\mathbf{x}$'s k nearest neighbors are of another class

Collaborative Filtering

(AKA Recommender Systems)

- **Problem:**
  Predict whether someone will like a Web page, newsgroup posting, movie, book, CD, etc.

- **Previous approach:**
  Look at content

- **Collaborative filtering:**
  - Look at what similar users liked
  - Similar users = Similar likes & dislikes

Collaborative Filtering

- Represent each user by vector of ratings
- Two types:
  - Yes/No
  - Explicit ratings (e.g., $0 - *****$)
- Predict rating:

$$\hat{R}_{ik} = \overline{R}_i + \alpha \sum_{X_j \in \mathbf{N}_i} W_{ij}(R_{jk} - \overline{R}_j)$$

- Similarity (Pearson coefficient):

$$W_{ij} = \frac{\sum_k (R_{ik} - \overline{R}_i)(R_{jk} - \overline{R}_j)}{\sqrt{\sum_k (R_{ik} - \overline{R}_i)^2 (R_{jk} - \overline{R}_j)^2}}$$

Fine Points

- Primitive version:

$$\hat{R}_{ik} = \alpha \sum_{X_j \in \mathbf{N}_i} W_{ij} R_{jk}$$

- $\alpha = (\sum |W_{ij}|)^{-1}$
- $\mathbf{N}_i$ can be whole database, or only k nearest neighbors
- R_{jk} = Rating of user j on item k
- $\overline{R}_j$ = Average of all of user j's ratings
- Summation in Pearson coefficient is over all items rated by *both* users
- In principle, any prediction method can be used for collaborative filtering

Example

	R_1	R_2	R_3	R_4	R_5	R_6
Alice	2	-	4	4	-	5
Bob	1	5	4	-	3	4
Chris	5	2	-	2	1	-
Diana	3	-	2	2	-	4