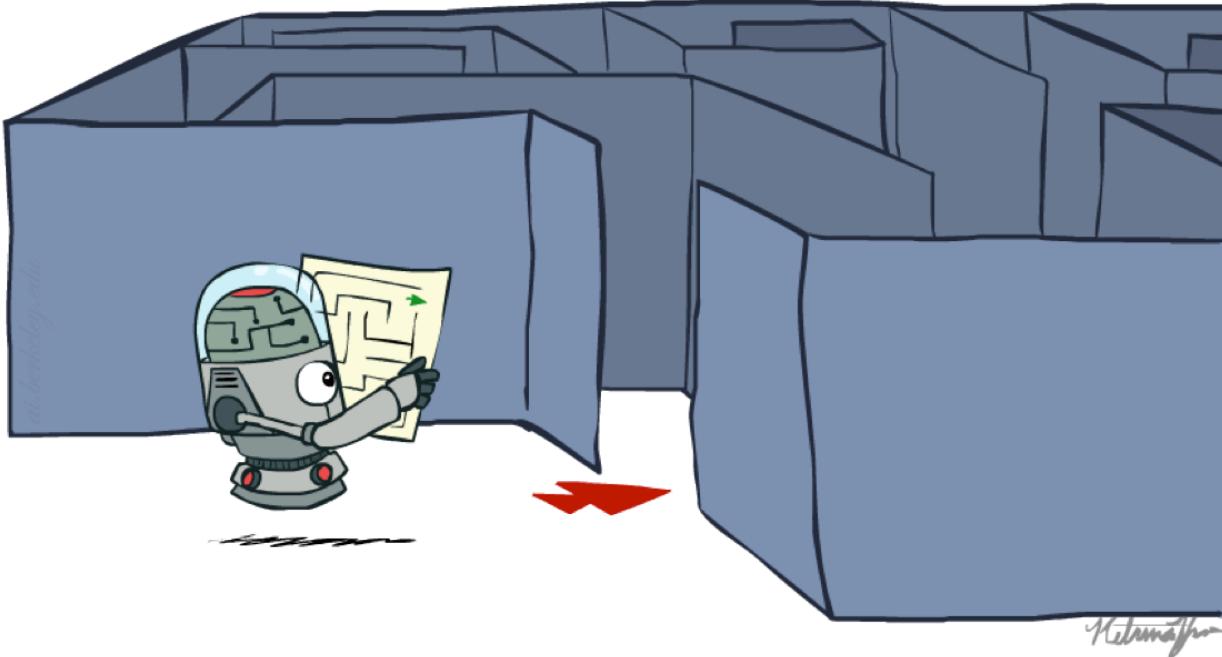


# CS 5522: Artificial Intelligence II

## Search



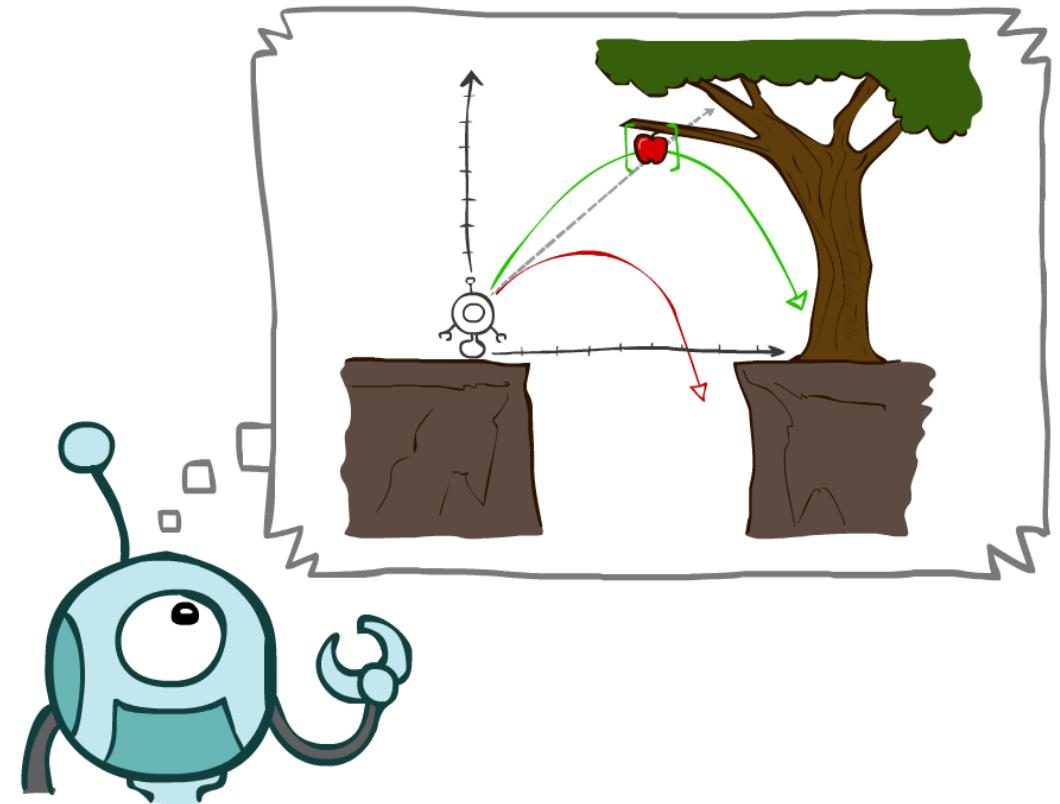
Instructor: Alan Ritter

Ohio State University

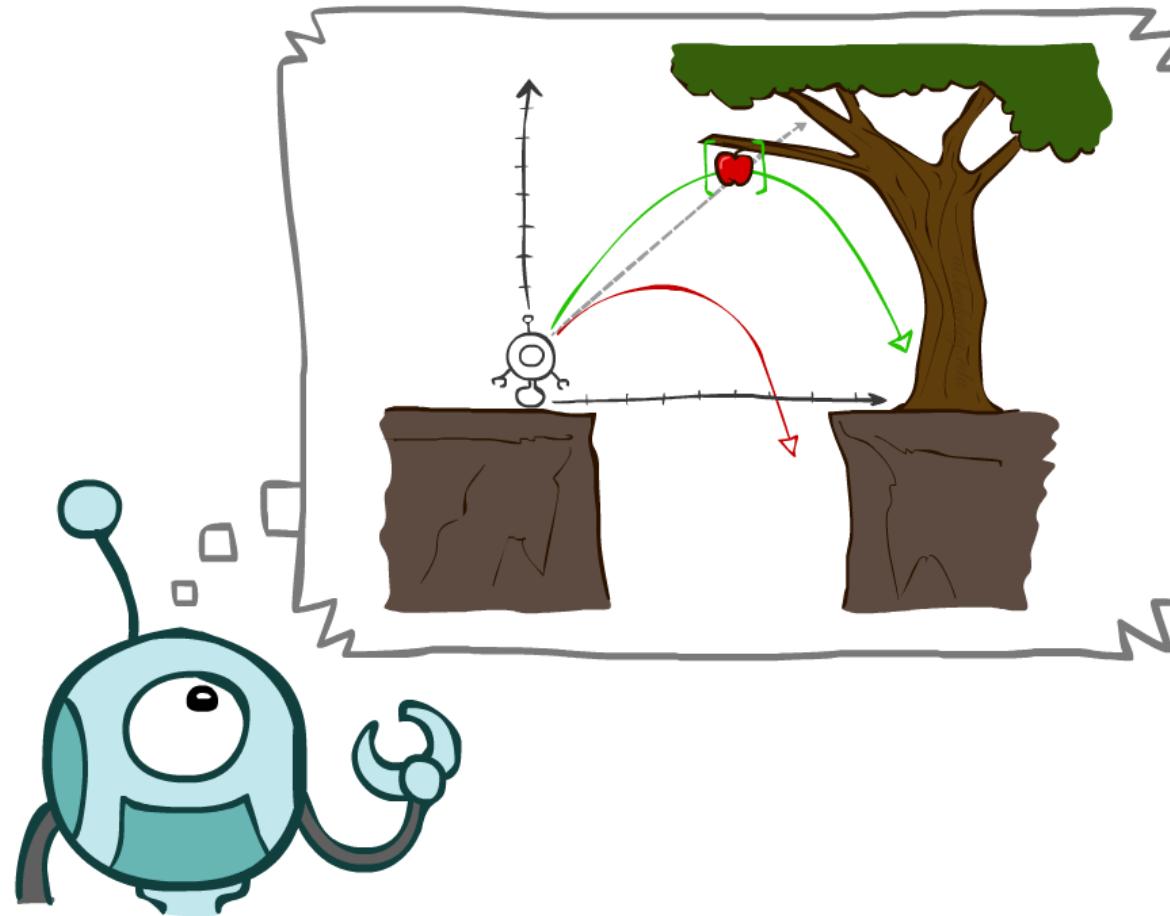
[These slides were adapted from CS188 Intro to AI at UC Berkeley. All materials available at <http://ai.berkeley.edu>.]

# Today

- Agents that Plan Ahead
- Search Problems
- Uninformed Search Methods
  - Depth-First Search
  - Breadth-First Search
  - Uniform-Cost Search

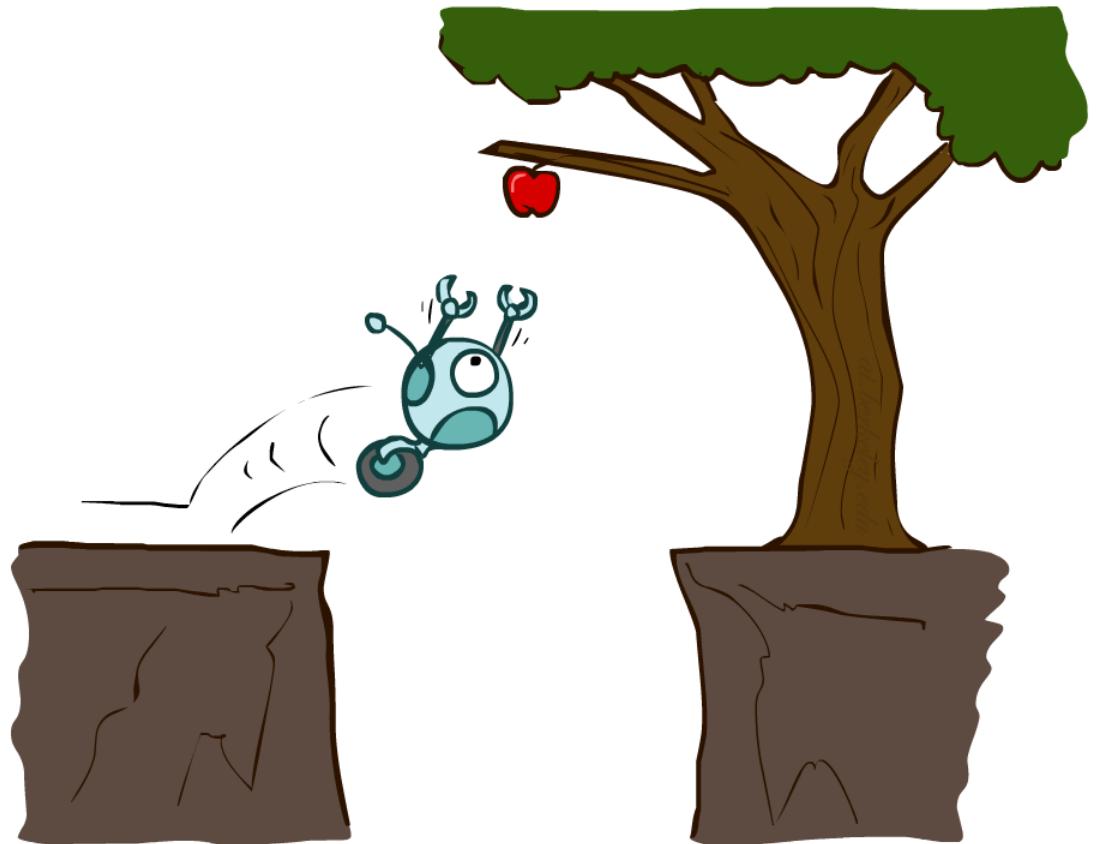


# Agents that Plan



# Reflex Agents

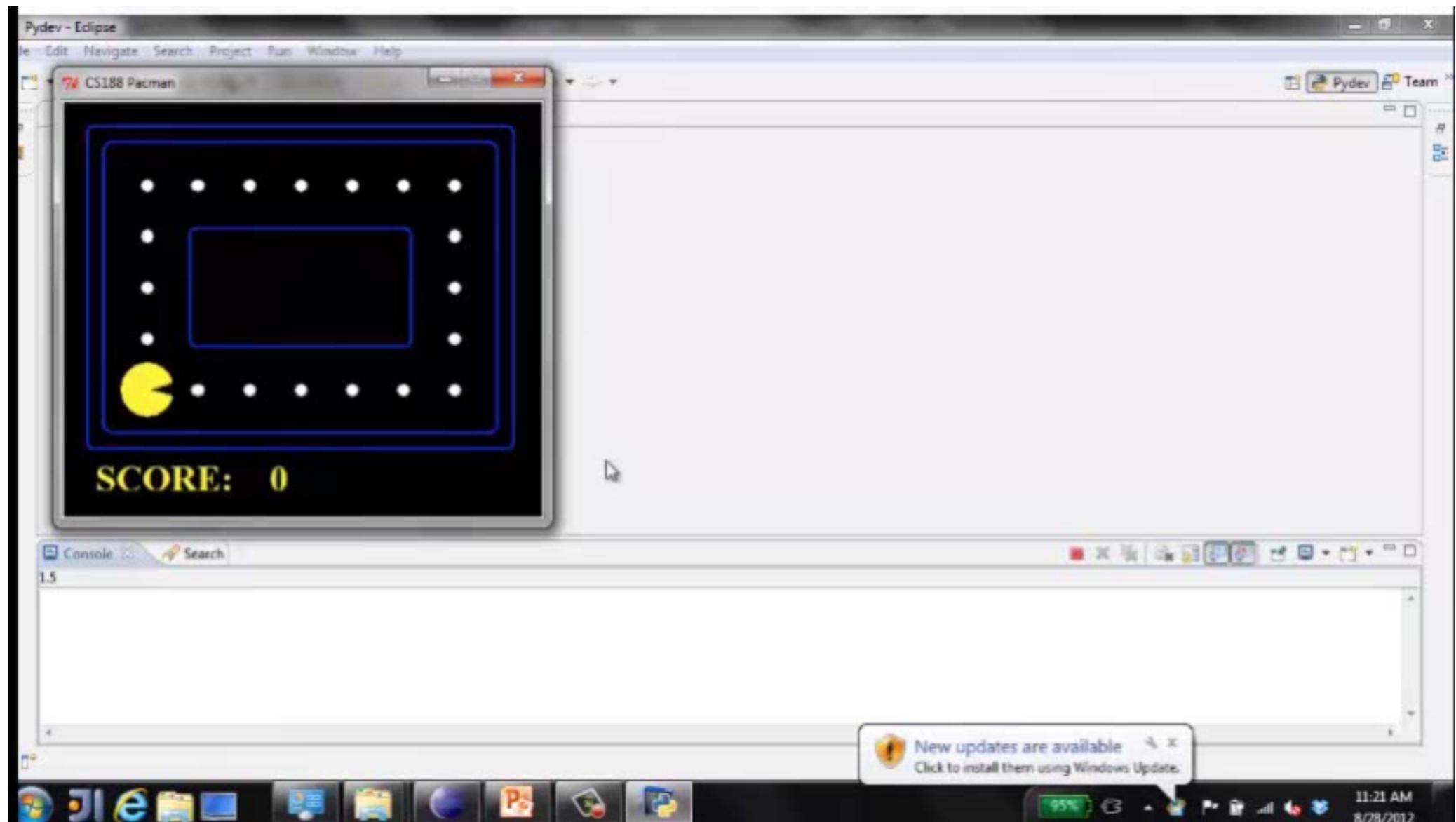
- Reflex agents:
  - Choose action based on current percept (and maybe memory)
  - May have memory or a model of the world's current state
  - Do not consider the future consequences of their actions
  - Consider how the world IS
- Can a reflex agent be rational?



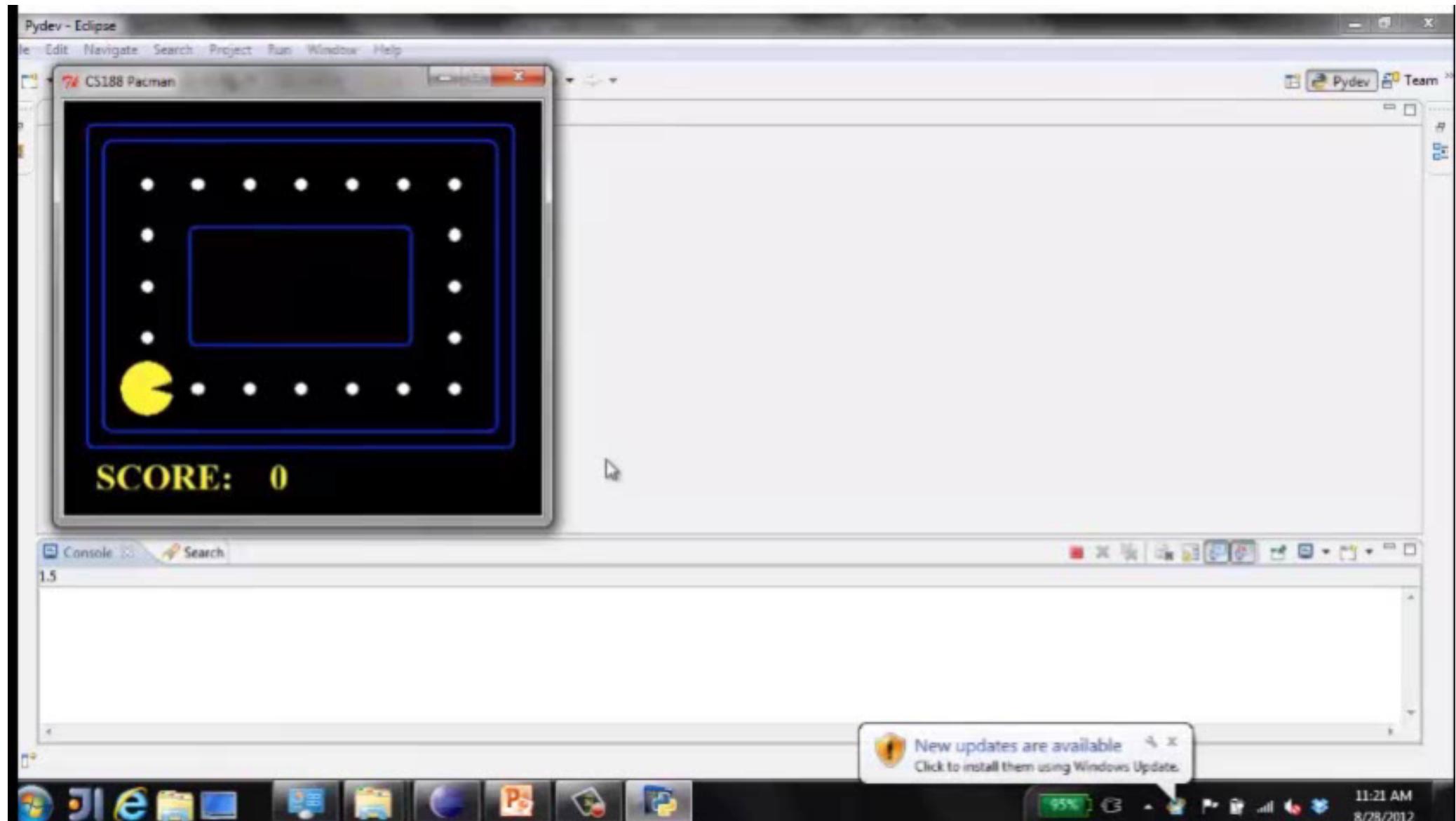
[Demo: reflex optimal (L2D1)]

[Demo: reflex optimal (L2D2)]

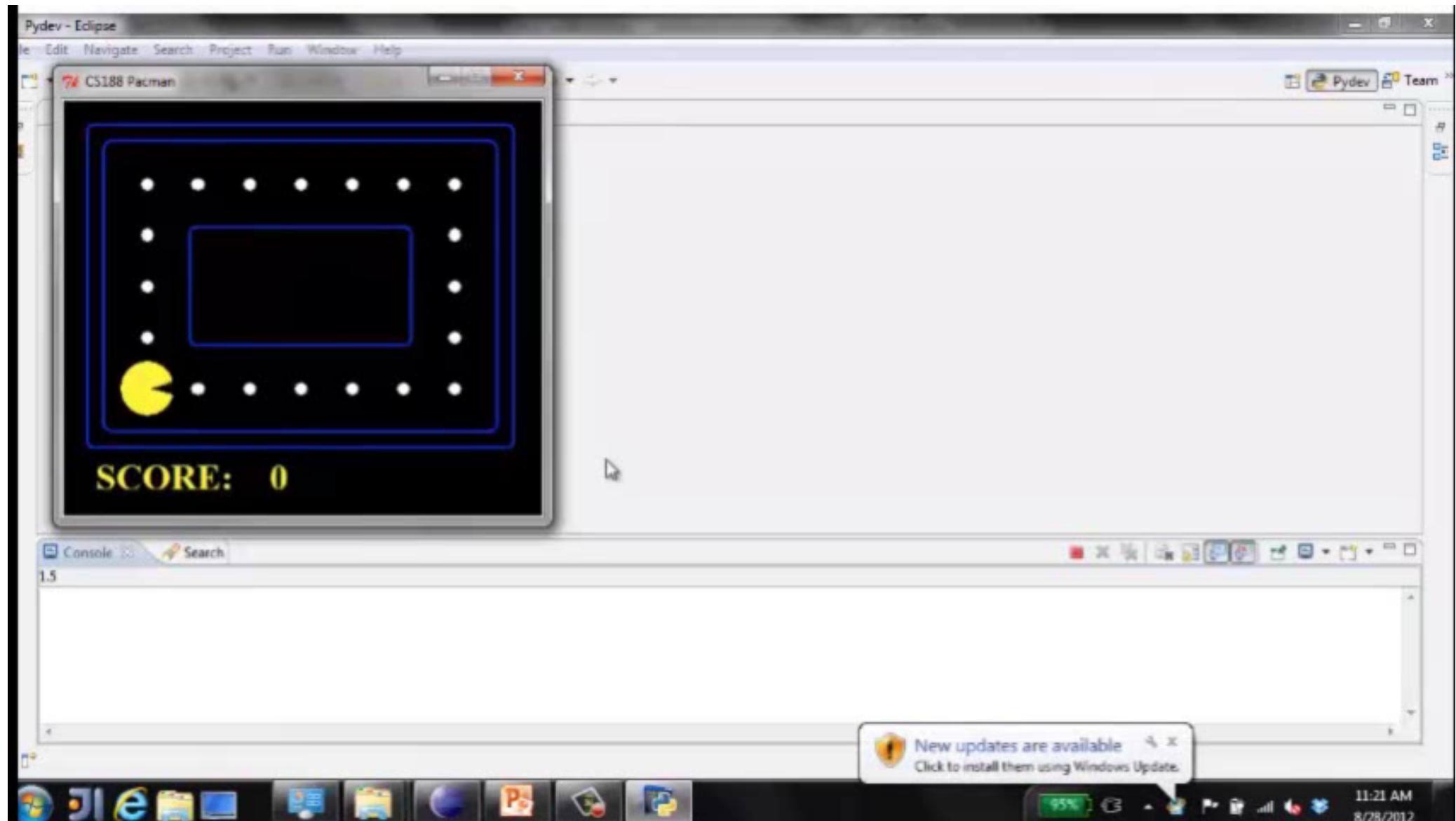
# Video of Demo Reflex Optimal



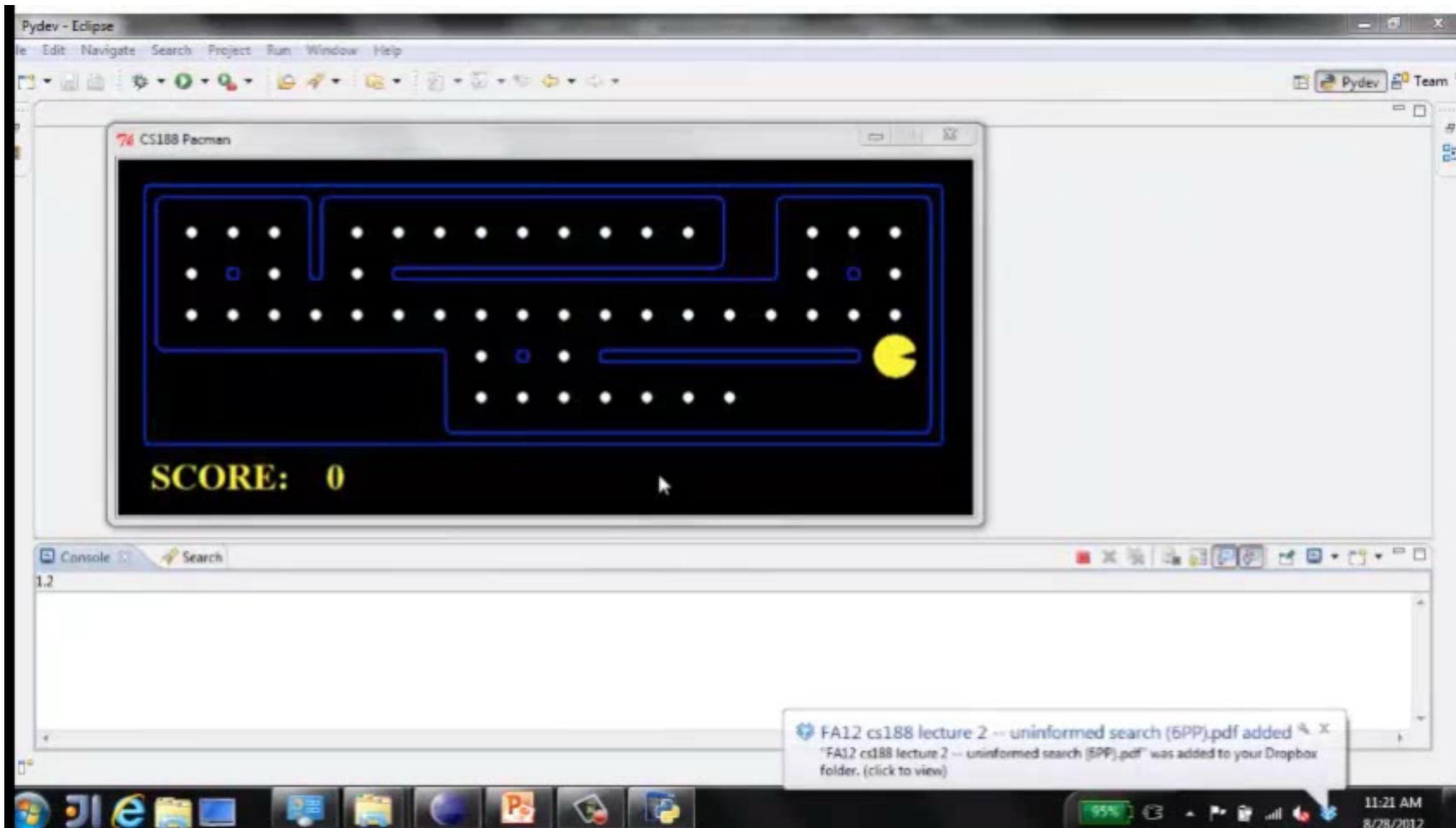
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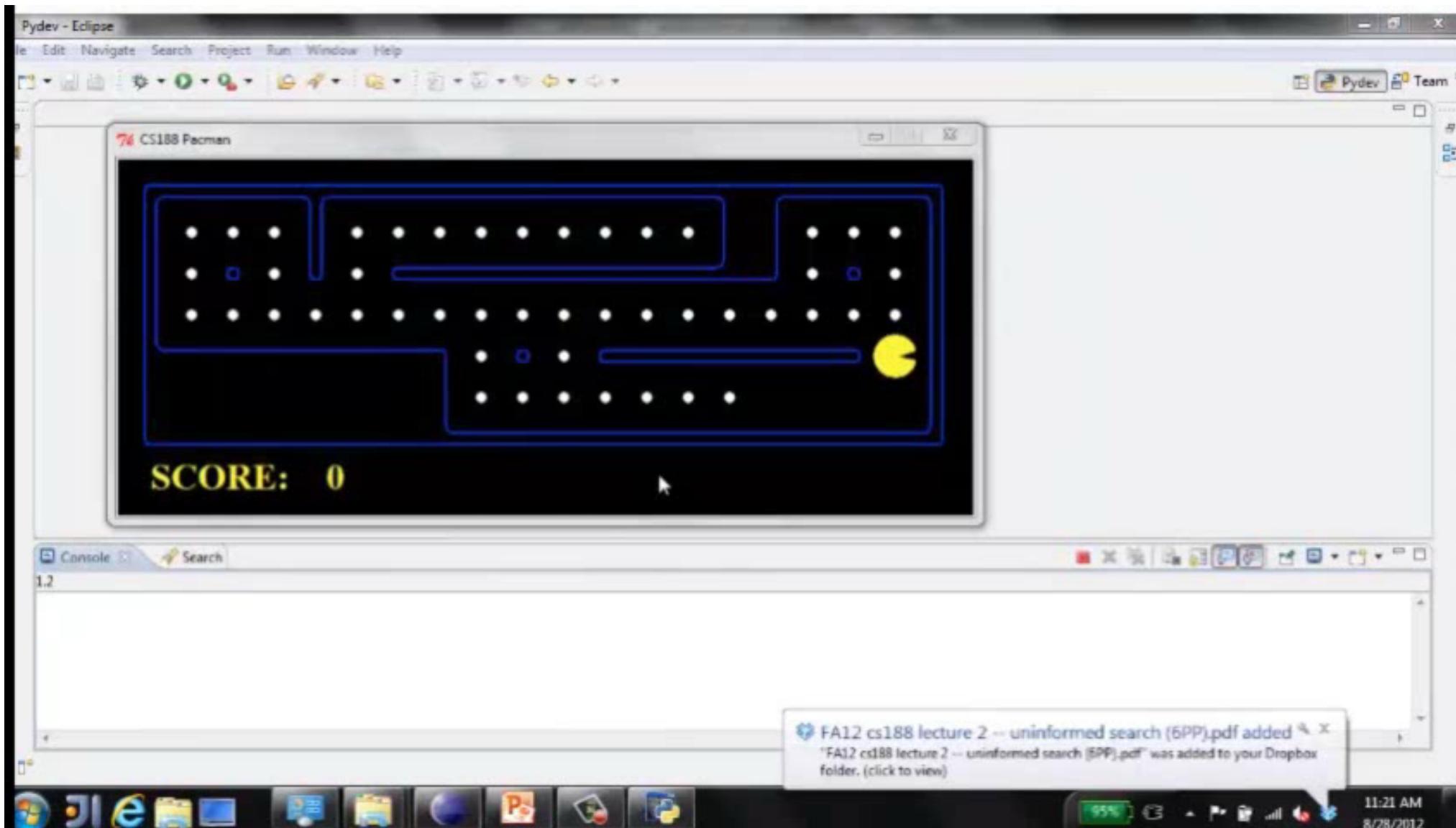
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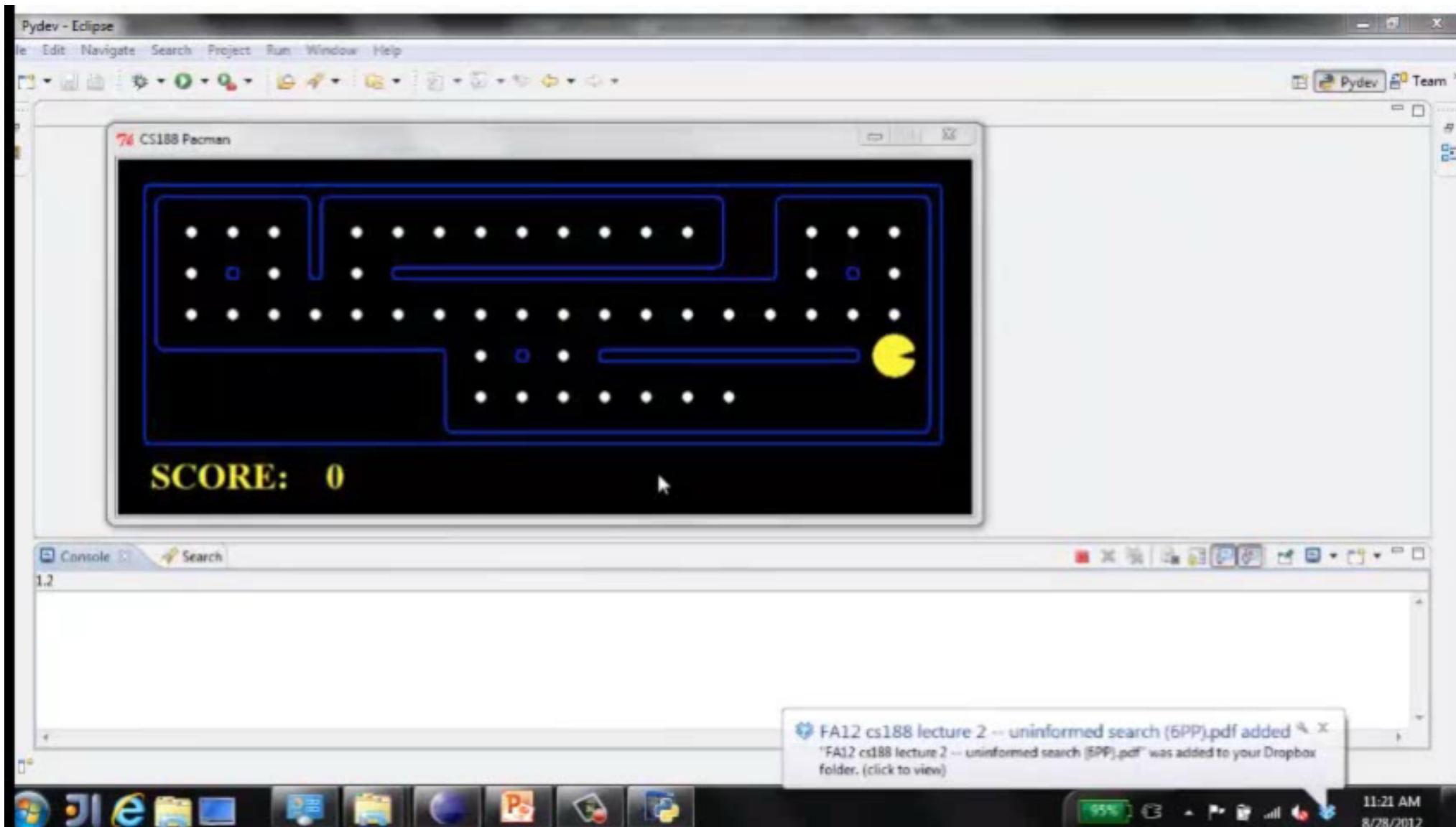
# Video of Demo Reflex Odd



# Video of Demo Reflex Odd

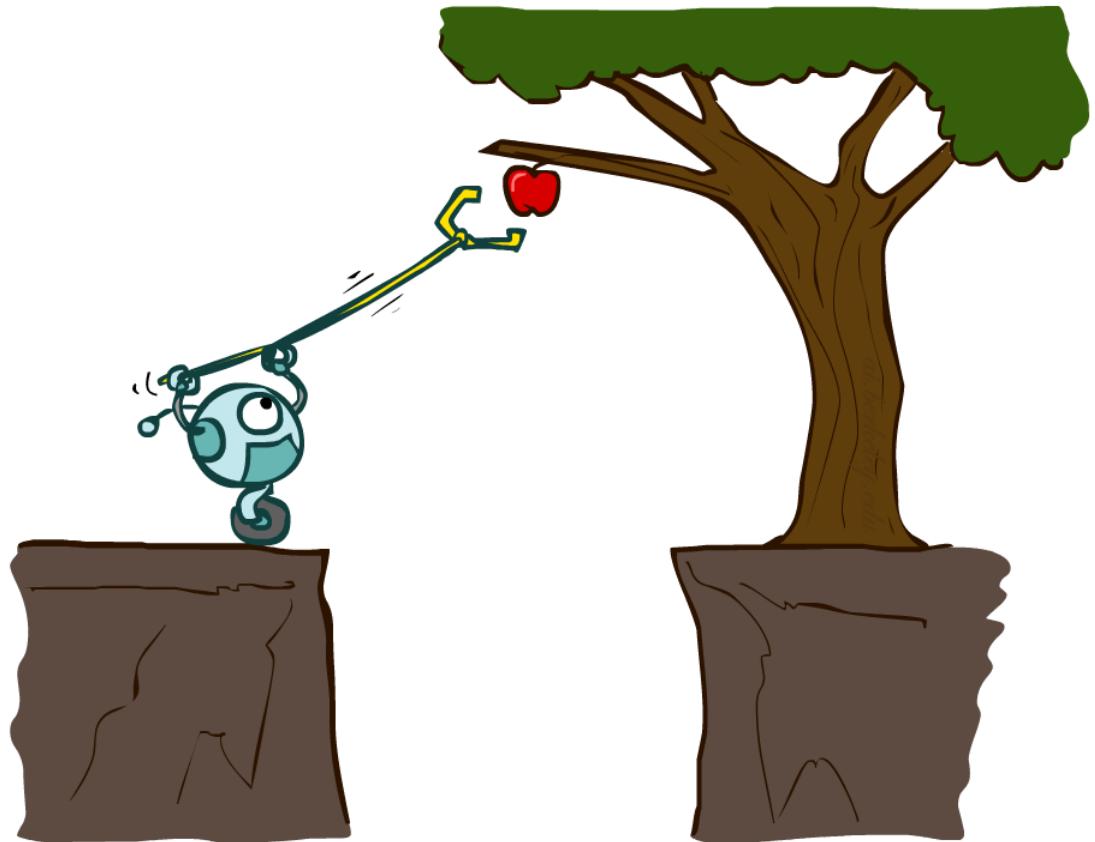


# Video of Demo Reflex Odd



# Planning Agents

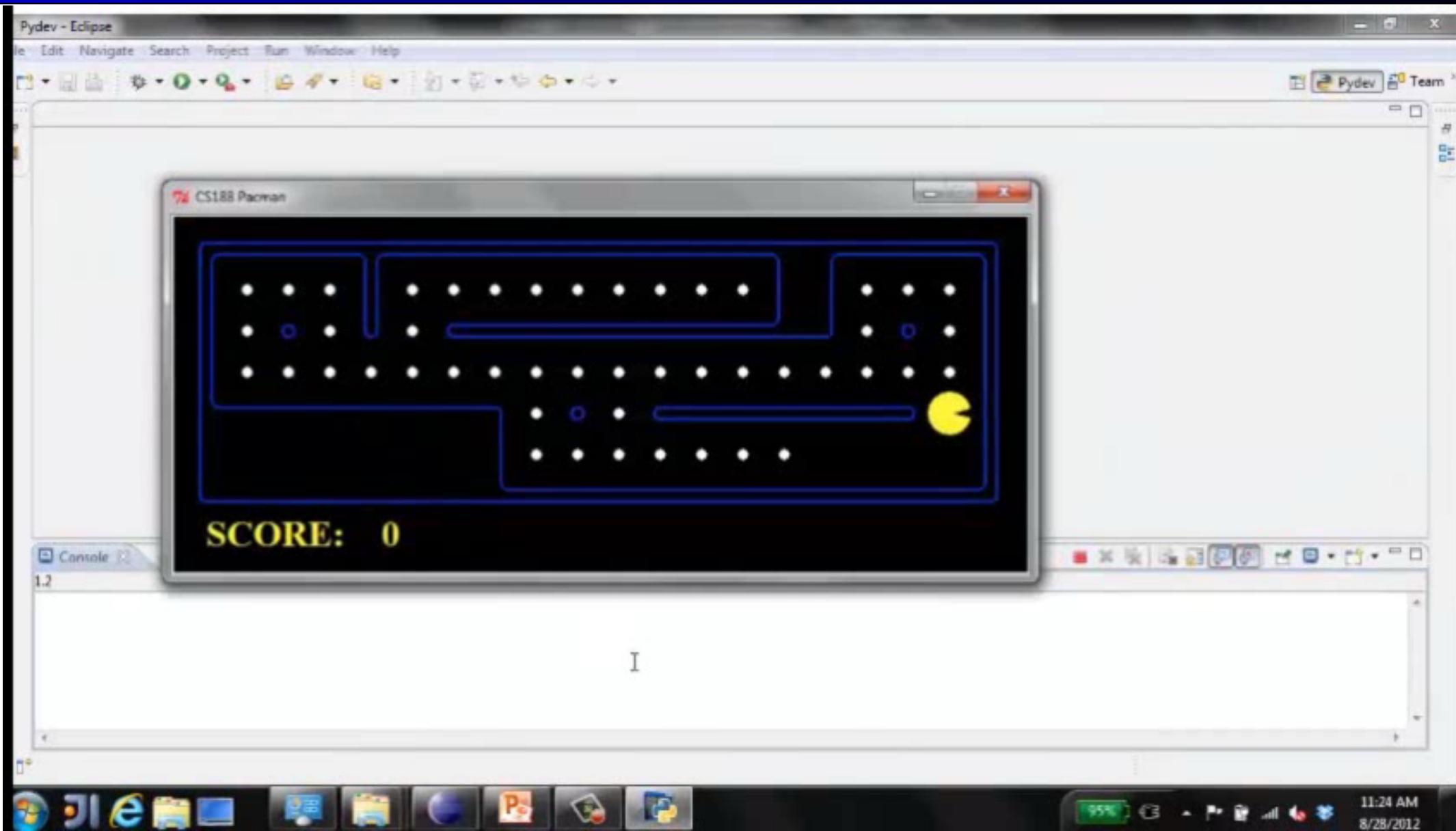
- Planning agents:
  - Ask “what if”
  - Decisions based on (hypothesized) consequences of actions
  - Must have a model of how the world evolves in response to actions
  - Must formulate a goal (test)
  - Consider how the world **WOULD BE**
- Optimal vs. complete planning
- Planning vs. replanning



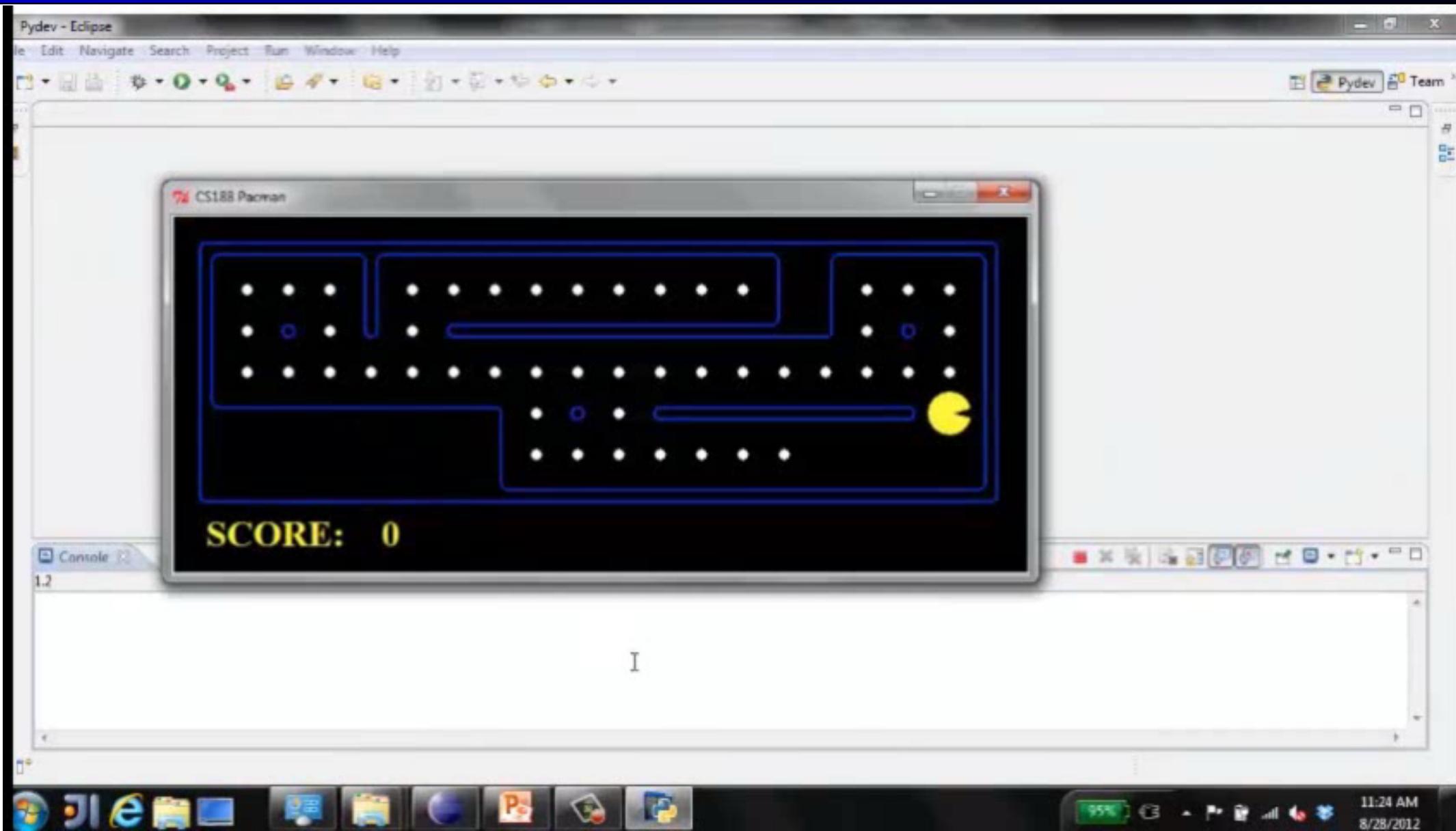
[Demo: replanning (L2D3)]

[Demo: mastermind (L2D4)]

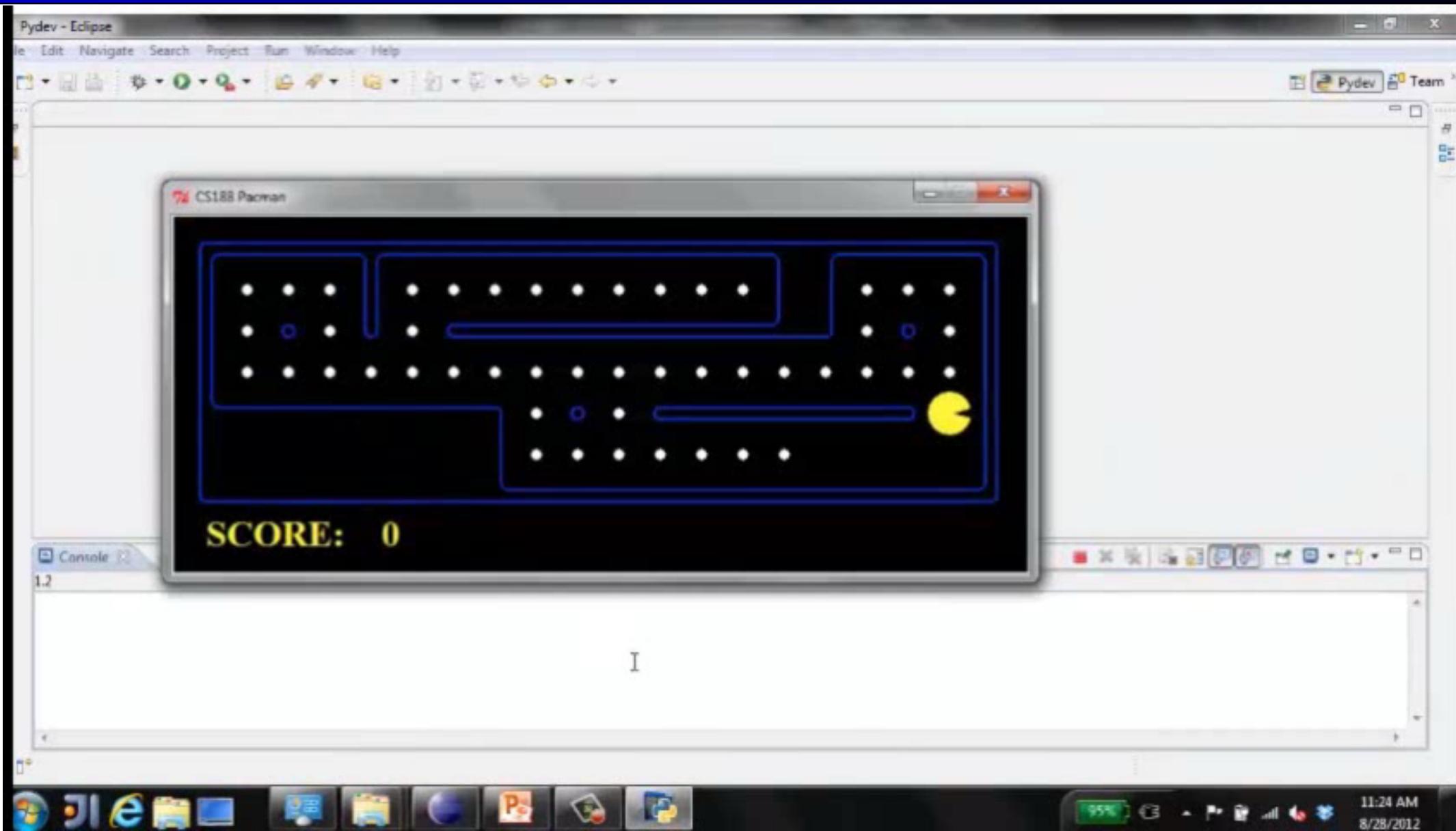
# Video of Demo Replanning



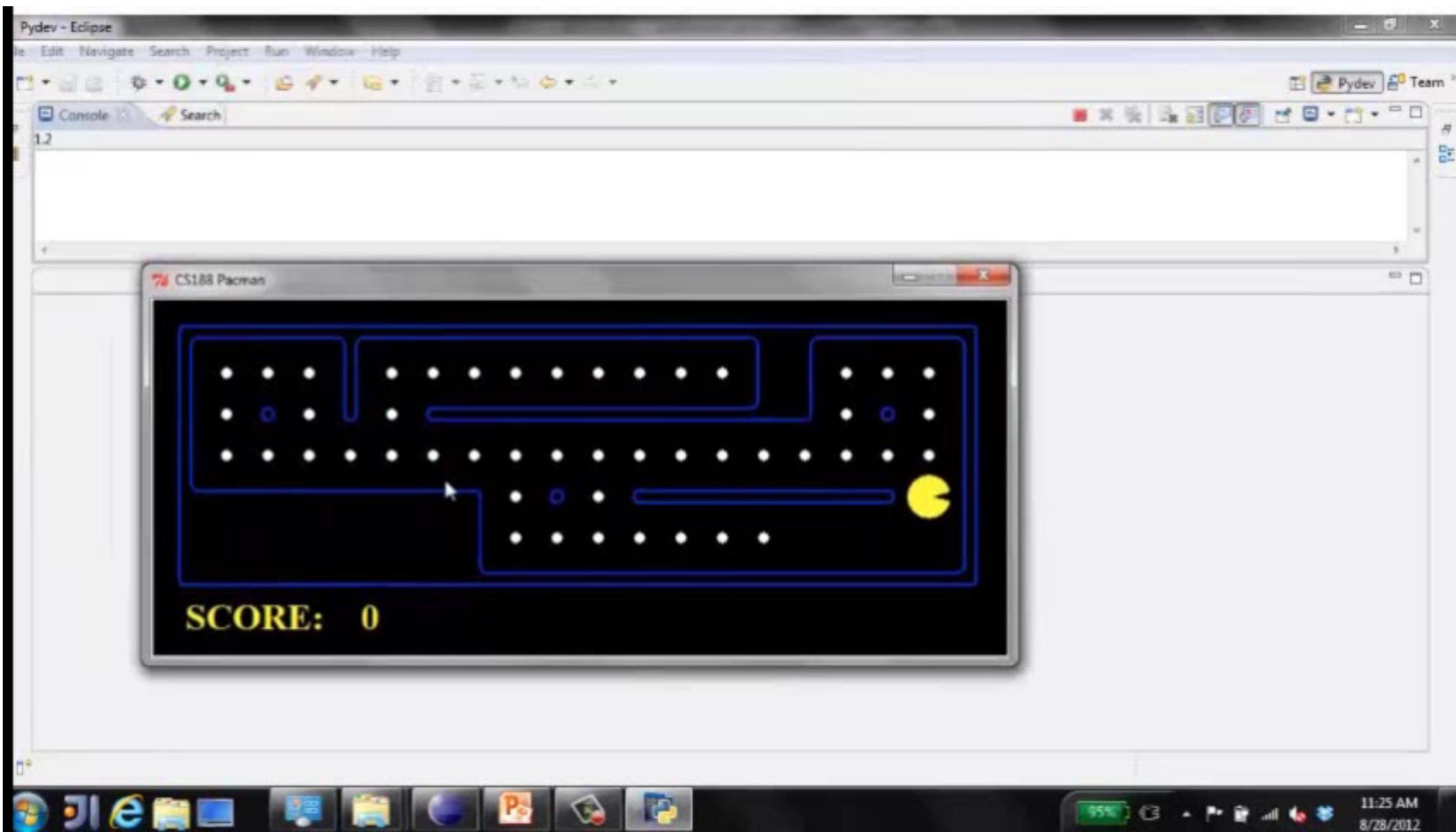
# Video of Demo Replanning



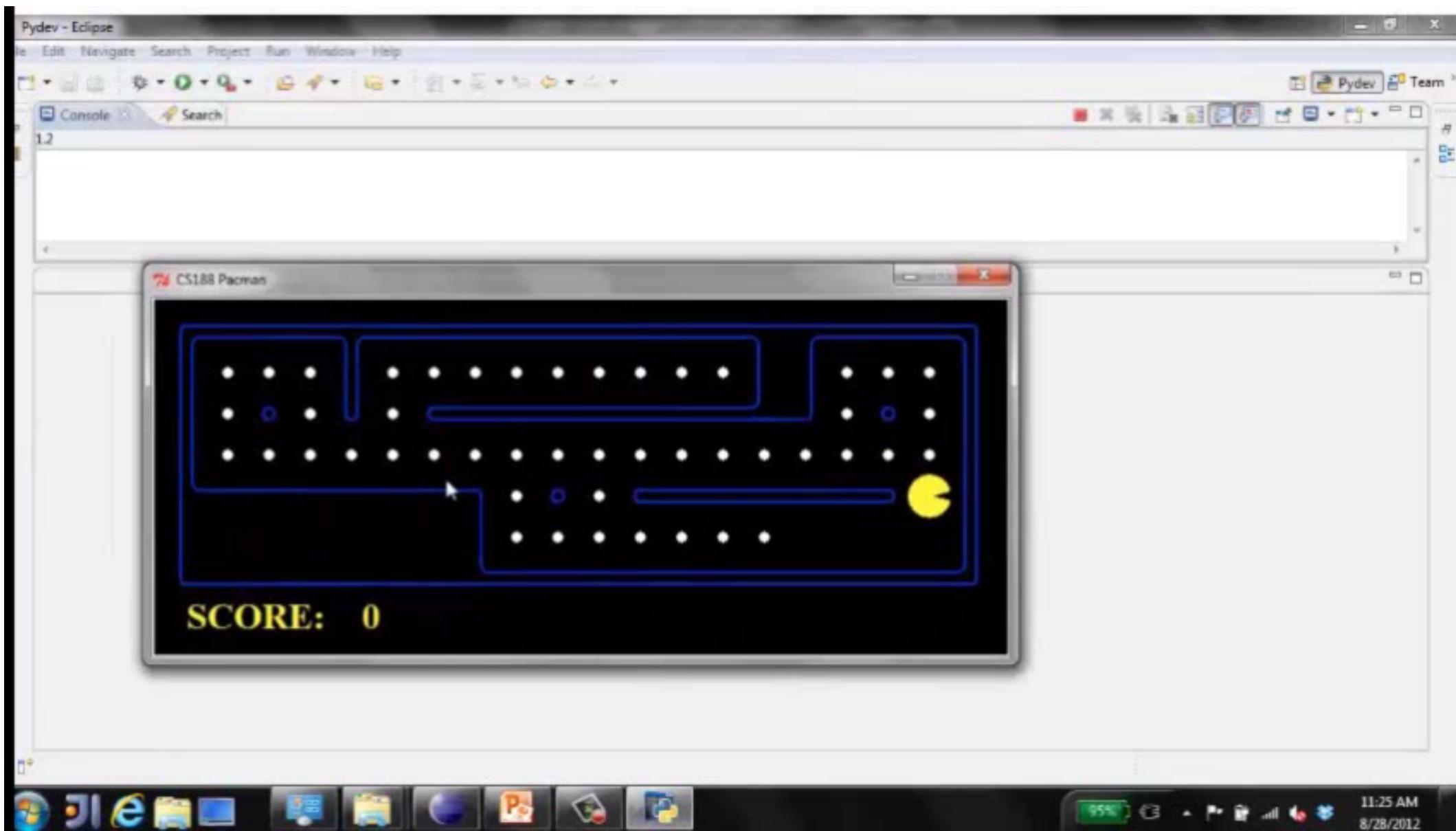
# Video of Demo Replanning



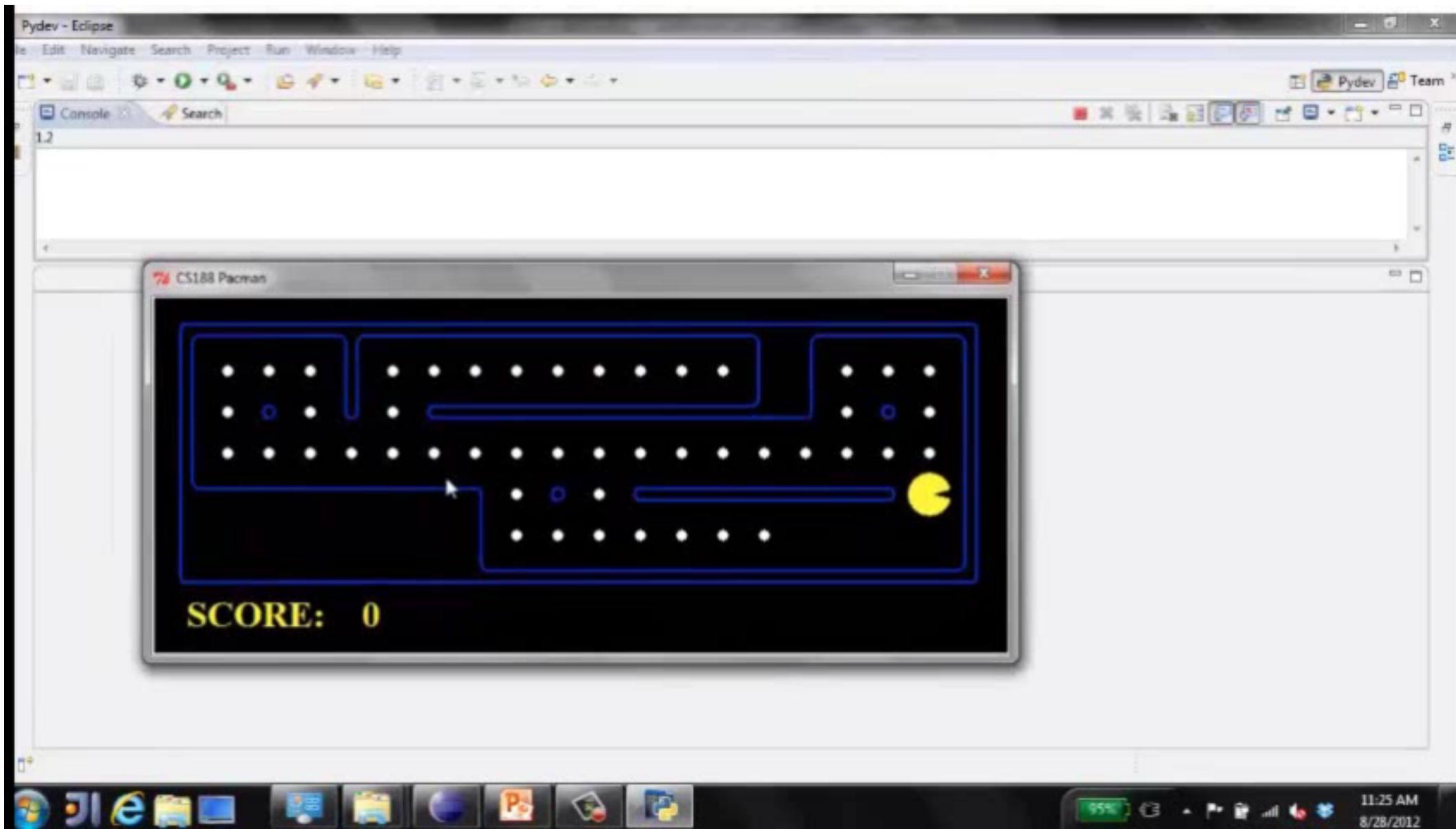
# Video of Demo Mastermind



# Video of Demo Mastermind



# Video of Demo Mastermind



# Search Problems

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# Search Problems

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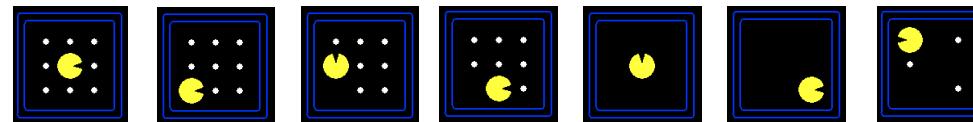
- A search problem consists of:

# Search Problems

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- A search problem consists of:

- A state space

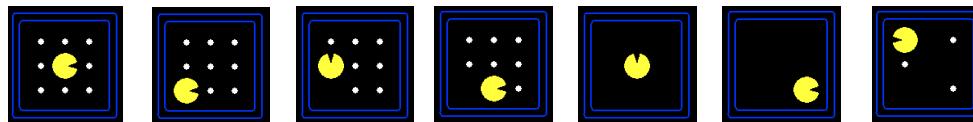


# Search Problems

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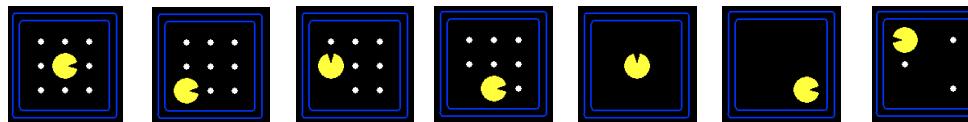


- A successor function  
(with actions, costs)

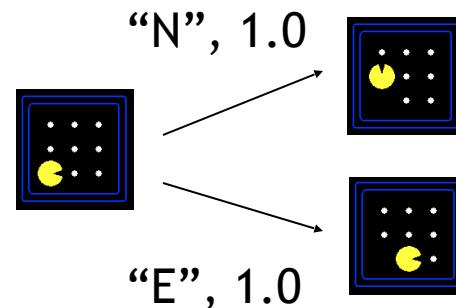
# Search Problems

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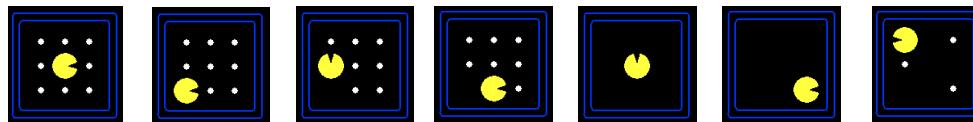
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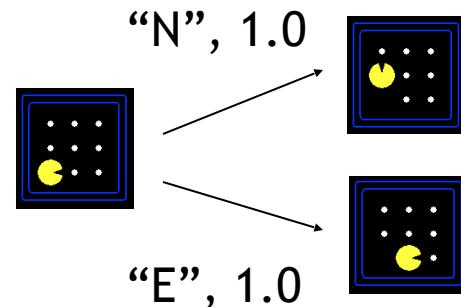
# Search Problems

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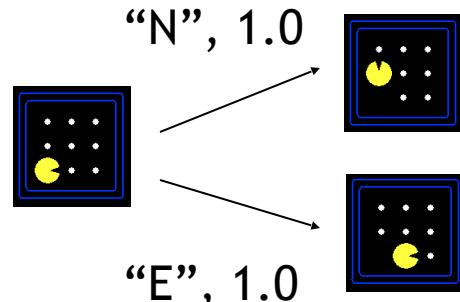


- A successor function  
(with actions, costs)



- A start state and a goal test

# Search Problems

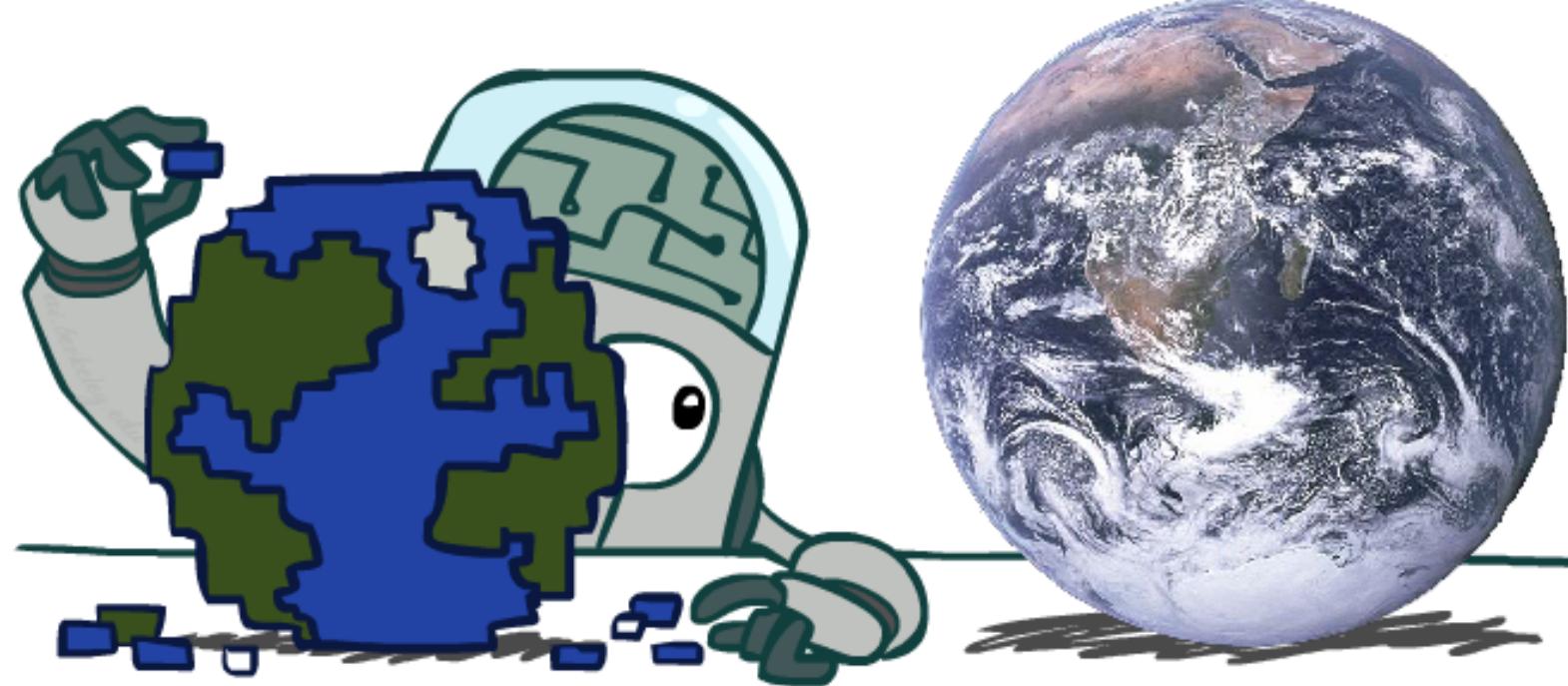
- A search problem consists of:
  - A state space
  - A successor function (with actions, costs)
  - A start state and a goal test
  - A solution is a sequence of actions (a plan) which transforms the start state to a goal state

# Search Problems Are Models

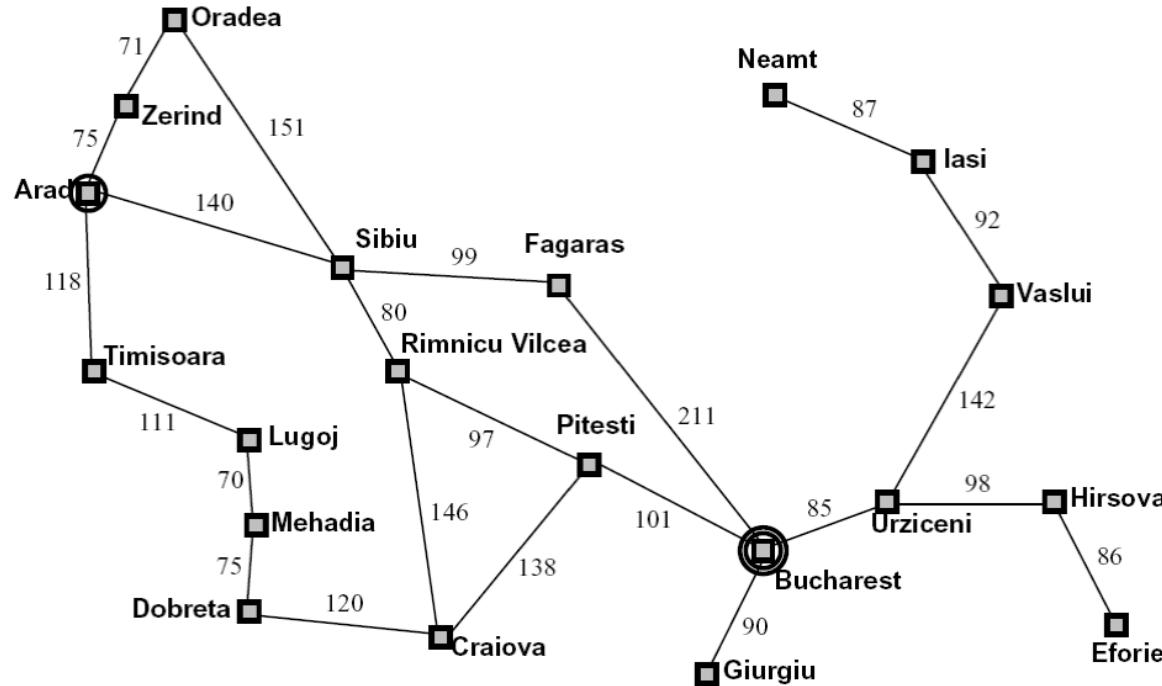
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# Search Problems Are Models

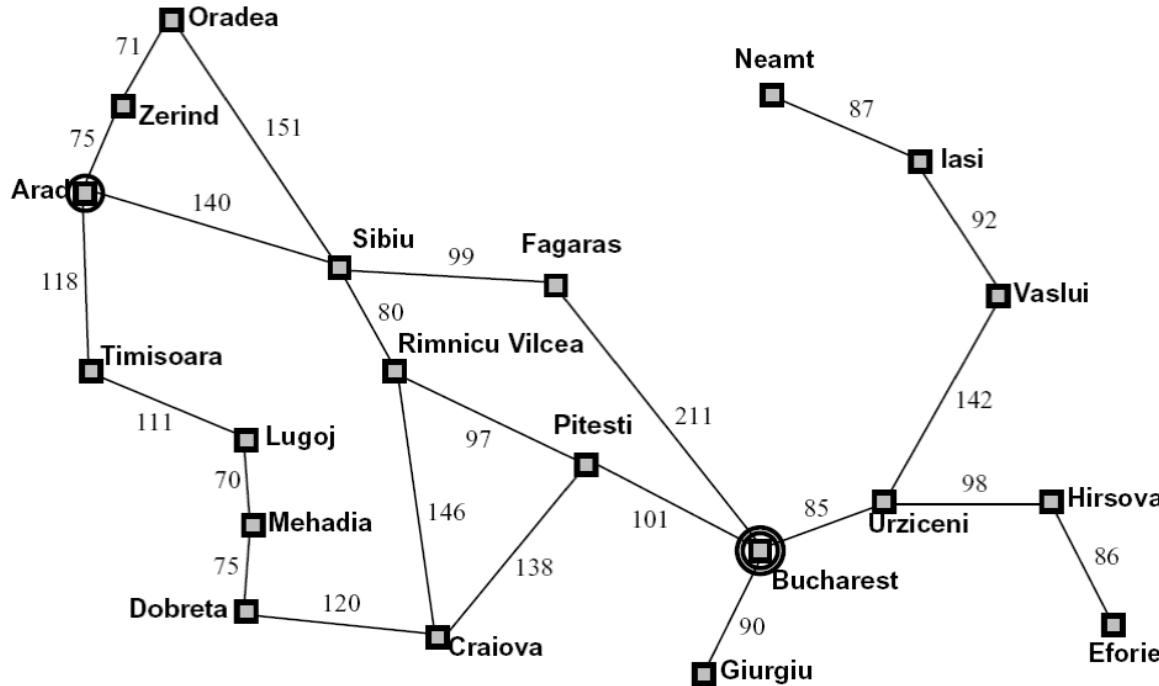
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# Example: Traveling in Romania

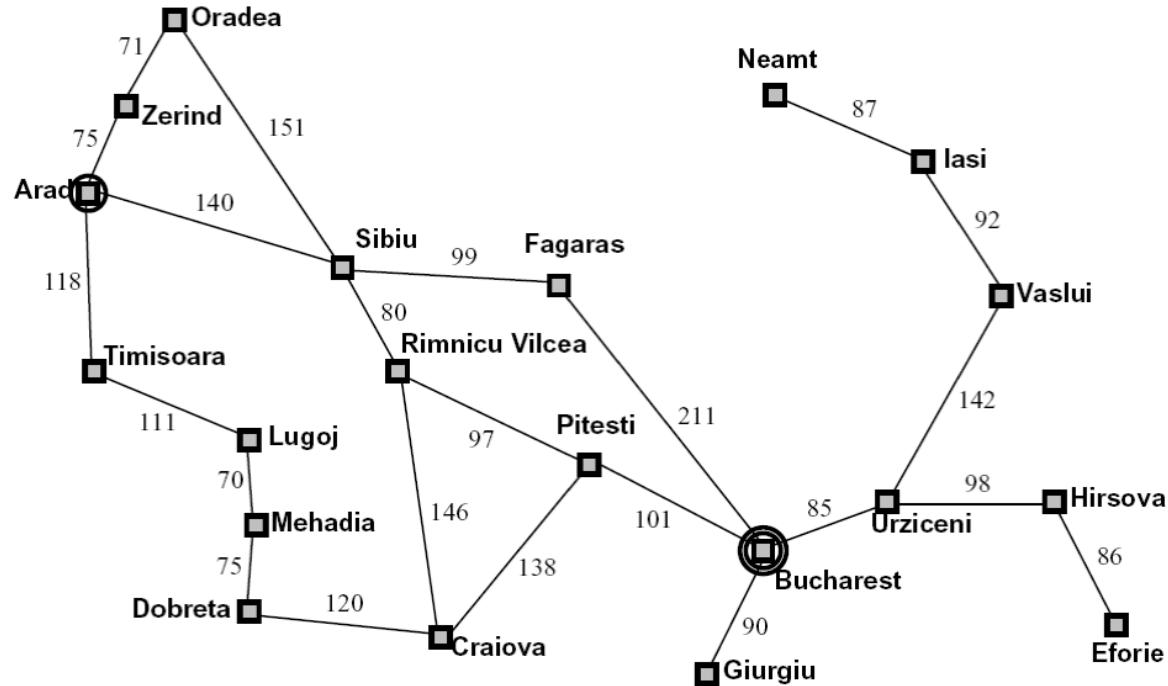


# Example: Traveling in Romania



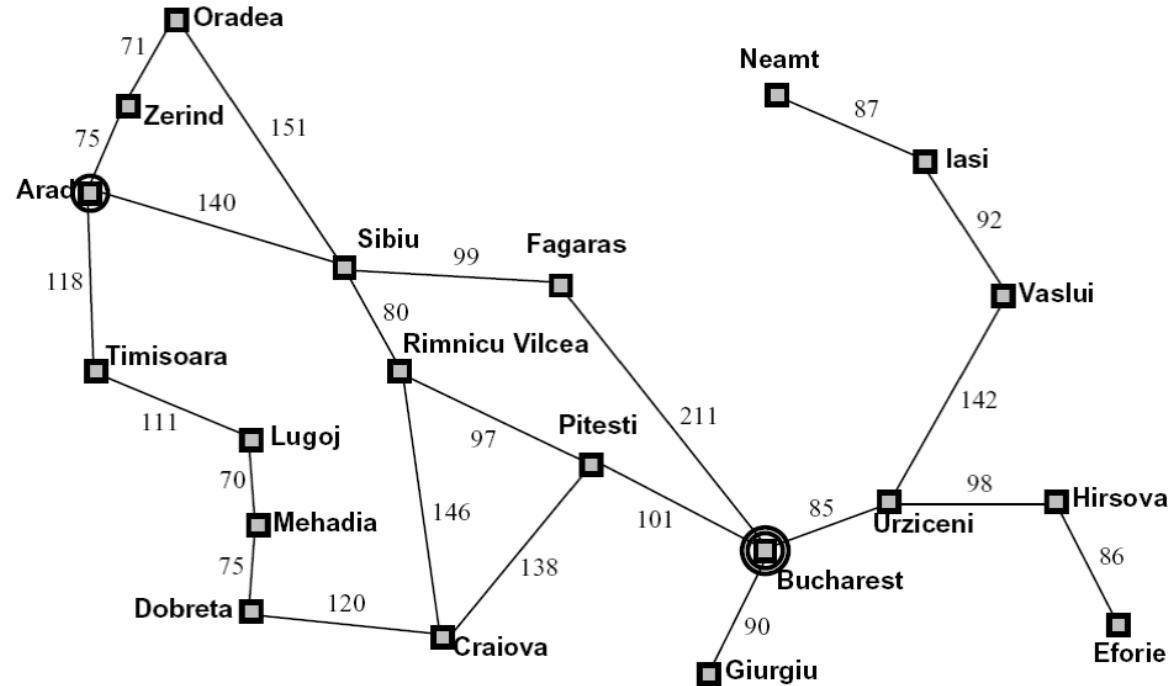
- State space:
  - Cities

# Example: Traveling in Romania



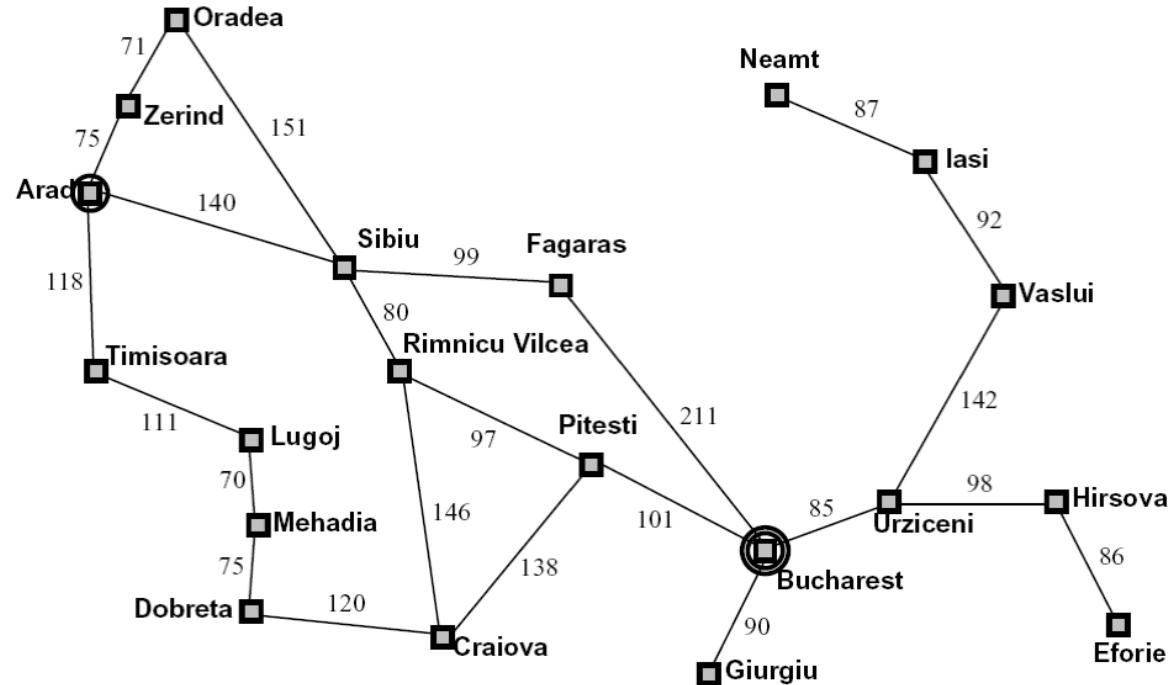
- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance

# Example: Traveling in Romania



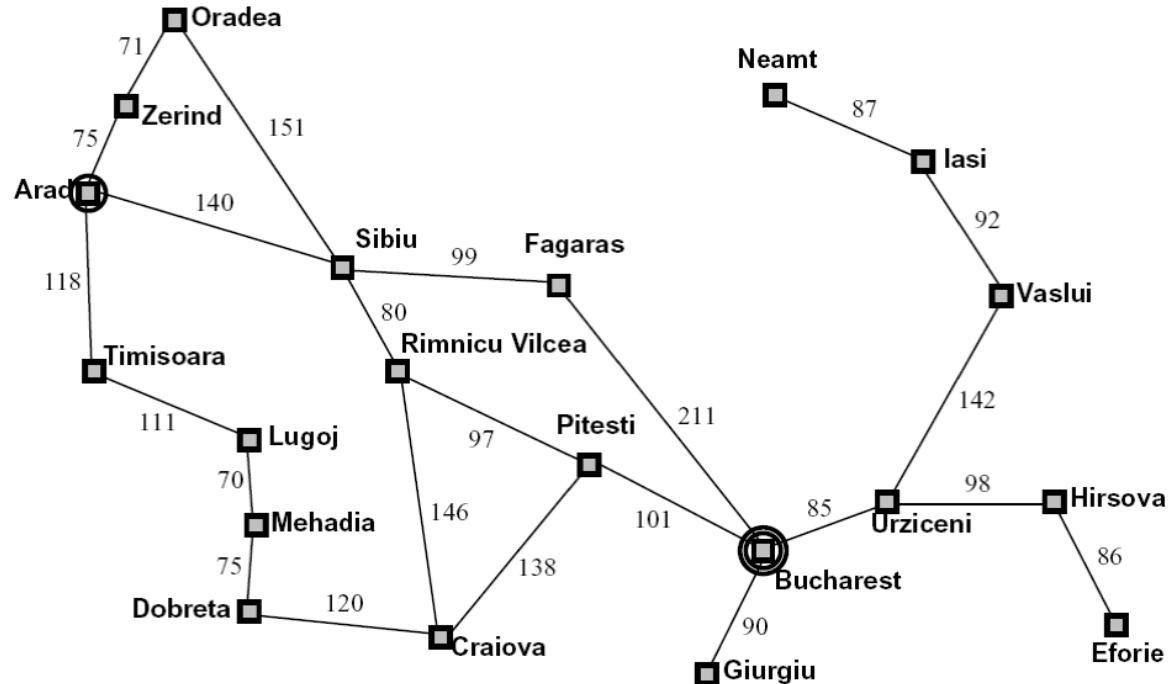
- State space:
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  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad

# Example: Traveling in Romania



- State space:
  - Cities
- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?

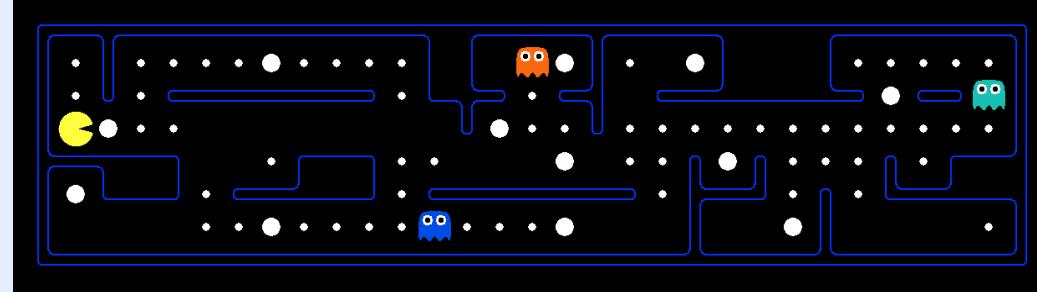
# Example: Traveling in Romania



- State space:
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- Successor function:
  - Roads: Go to adjacent city with cost = distance
- Start state:
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- Goal test:
  - Is state == Bucharest?
- Solution?

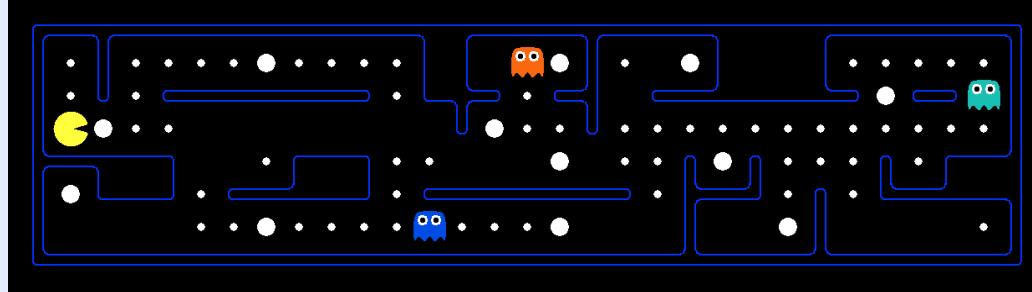
# What's in a State Space?

The **world state** includes every last detail of the environment



# What's in a State Space?

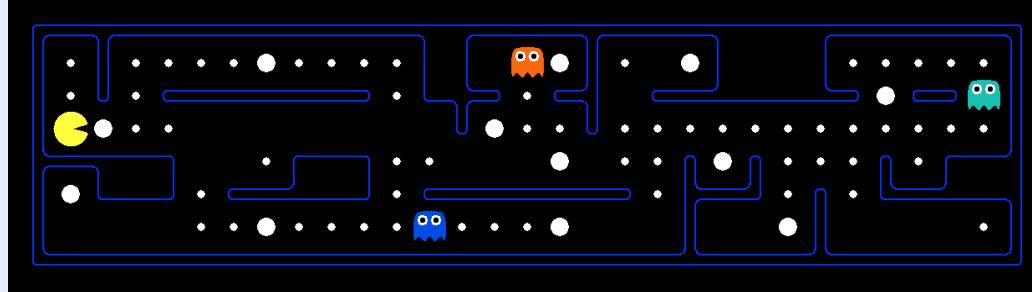
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A **search state** keeps only the details needed for planning (abstraction)

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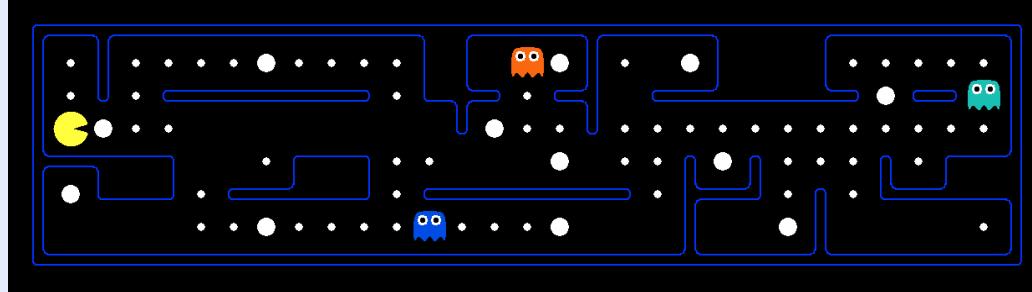


A **search state** keeps only the details needed for planning (abstraction)

- **Problem: Pathing**
  - States: (x,y) location
  - Actions: NSEW
  - Successor: update location only
  - Goal test: is (x,y)=END

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The **world state** includes every last detail of the environment



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- **Problem: Pathing**

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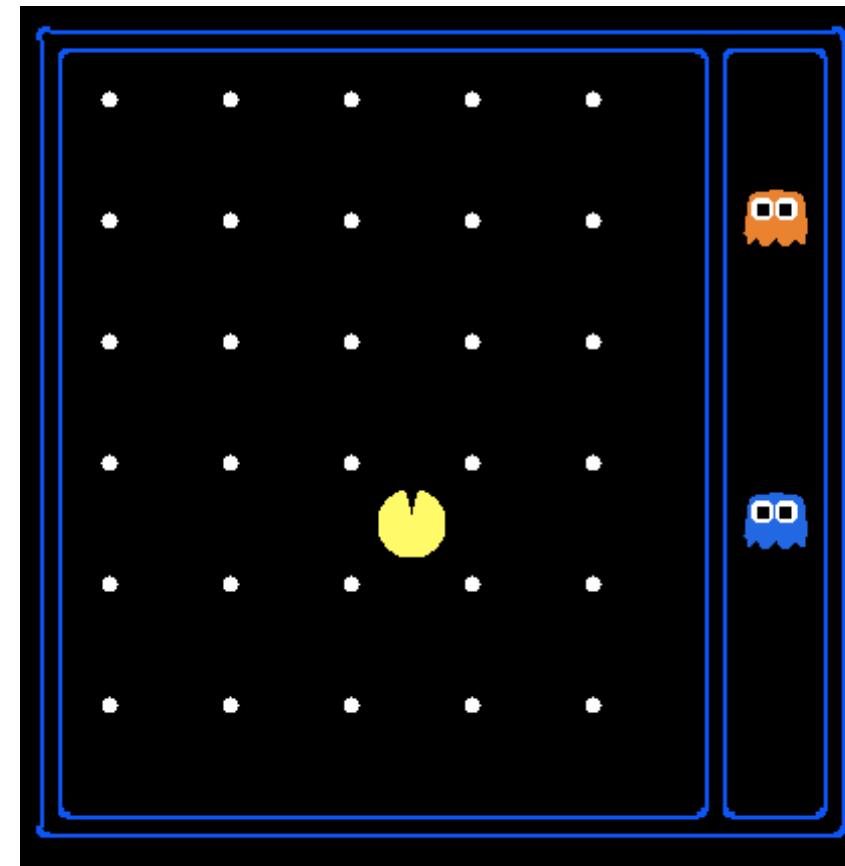
- **Problem: Eat-All-Dots**

- States:  $\{(x,y), \text{dot booleans}\}$
- Actions: NSEW
- Successor: update location and possibly a dot boolean
- Goal test: dots all false

# State Space Sizes?

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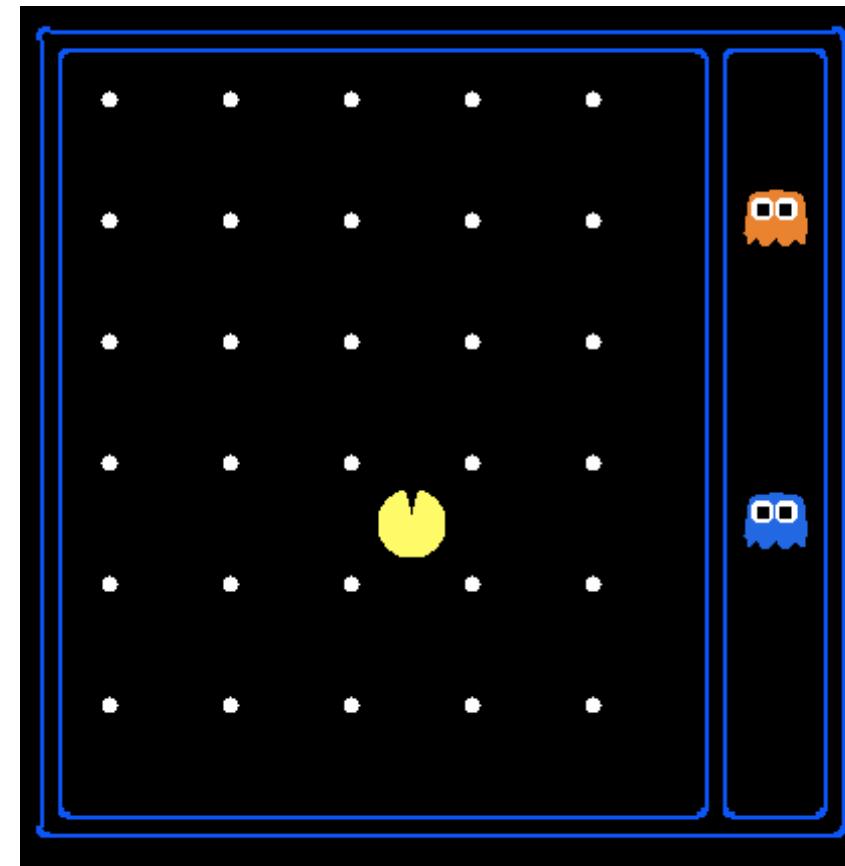
- World state:
  - Agent positions: 120
  - Food count: 30
  - Ghost positions: 12
  - Agent facing: NSEW



# State Space Sizes?

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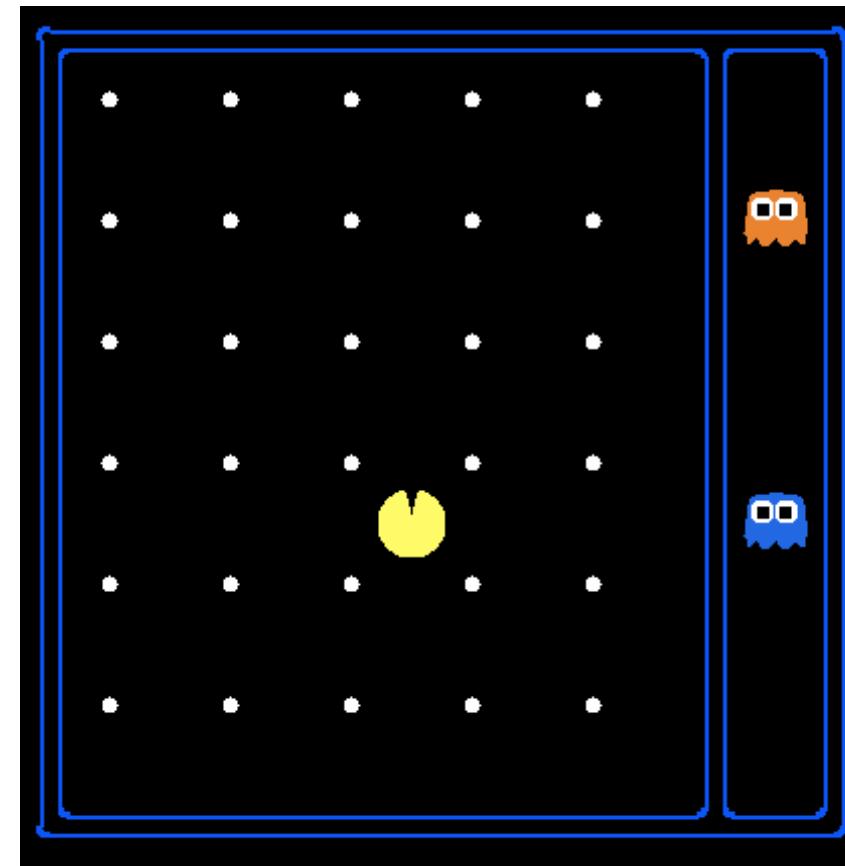
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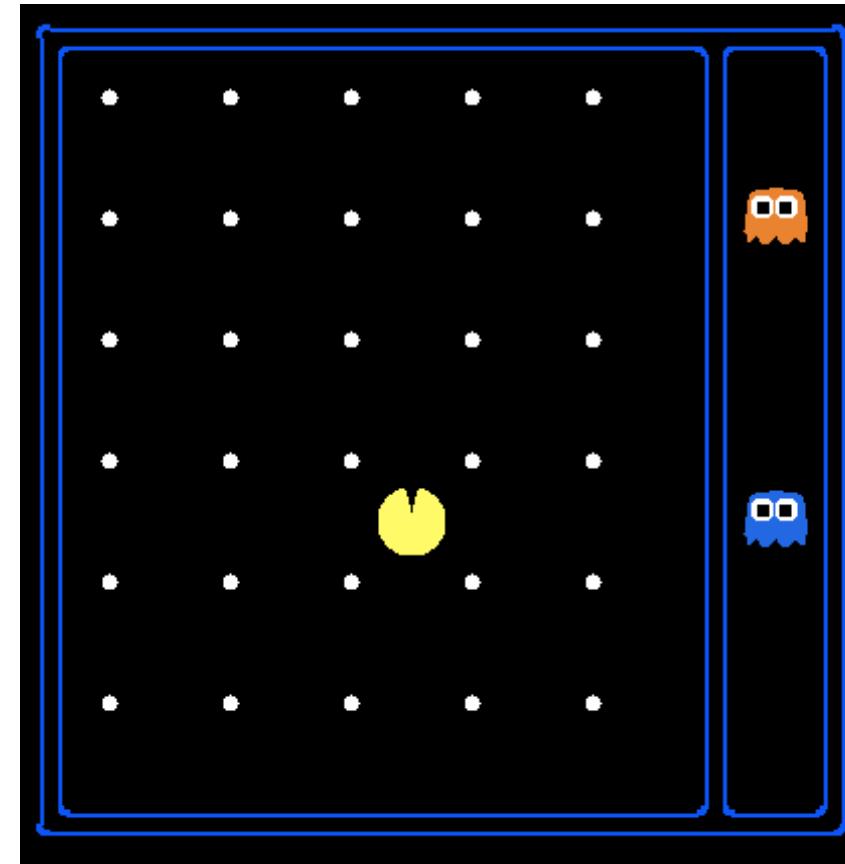
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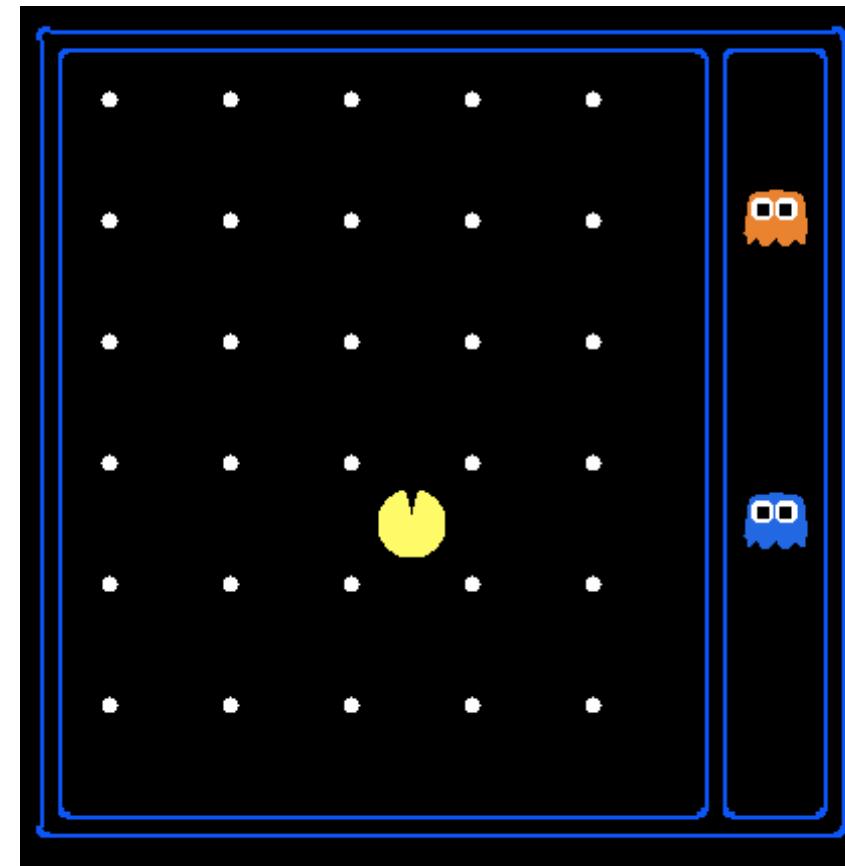
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$$120 \times (2^{30}) \times (12^2) \times 4$$



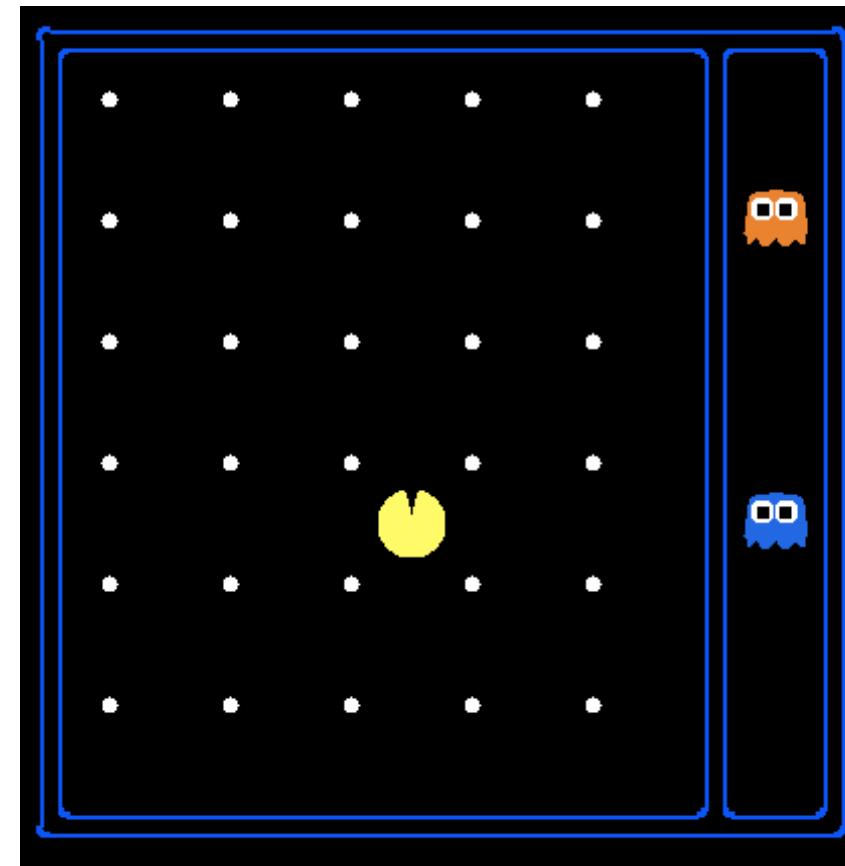
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  - States for pathing?



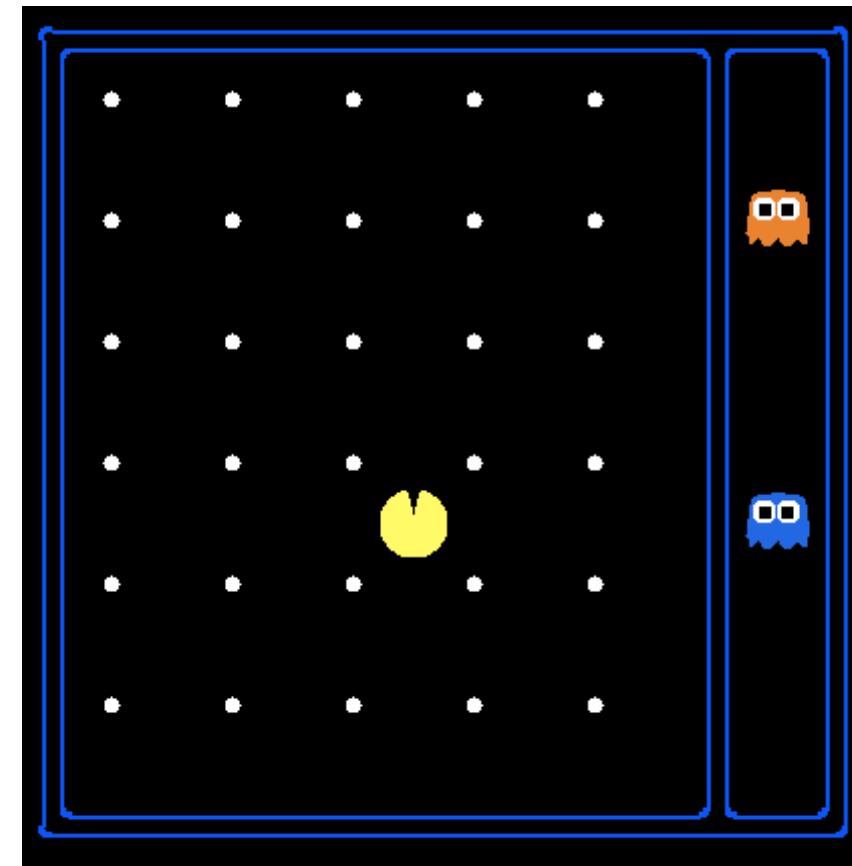
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120



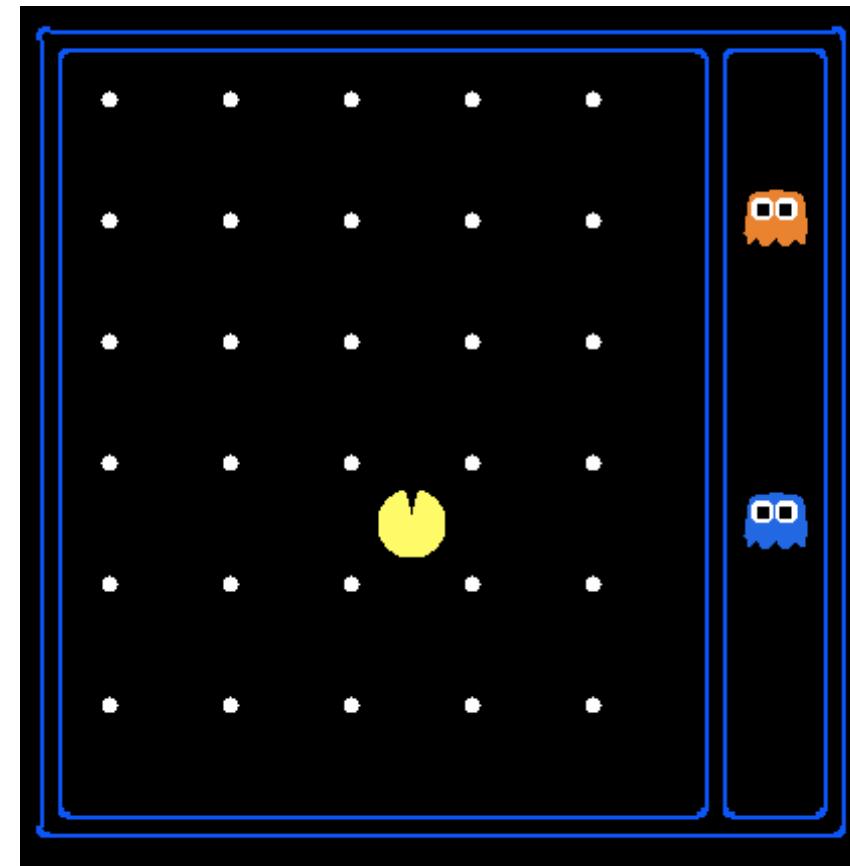
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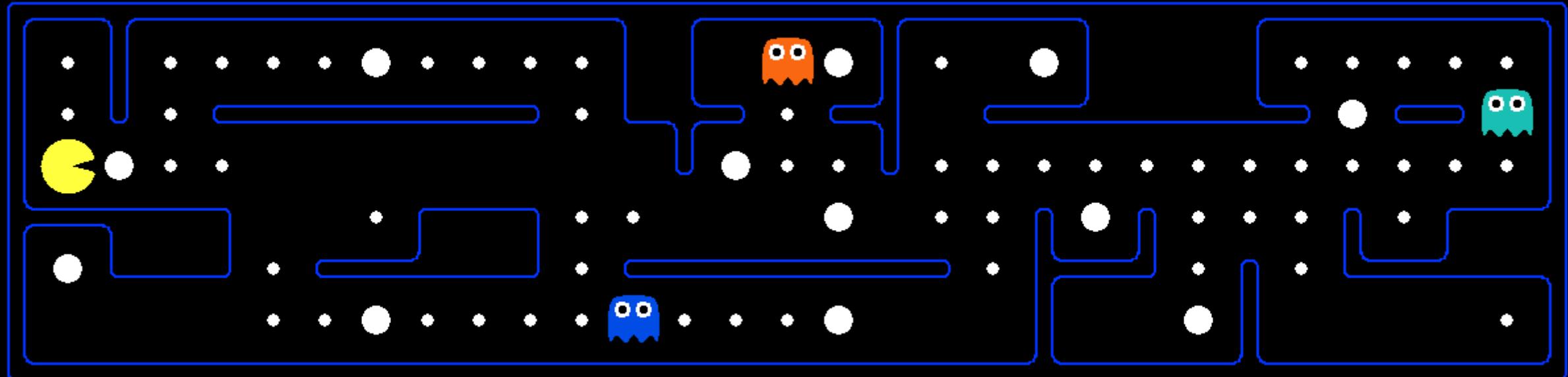


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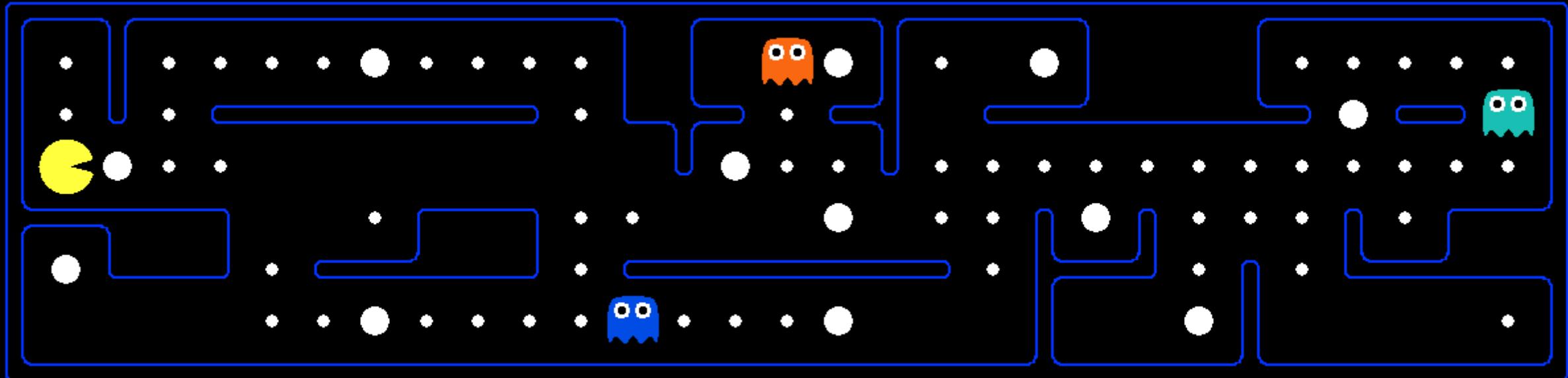


# Quiz: Safe Passage



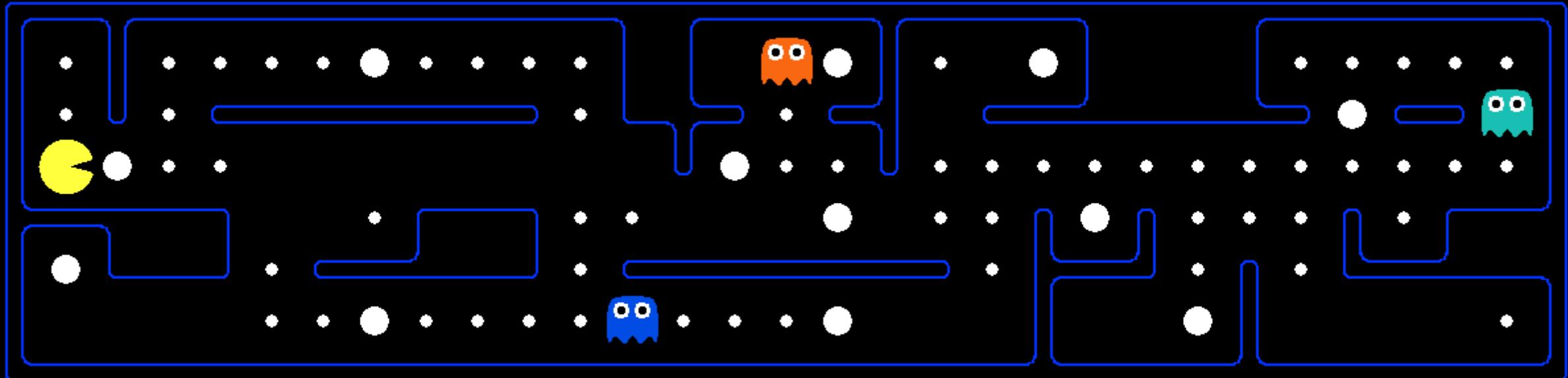
- Problem: eat all dots while keeping the ghosts perma-scared

# Quiz: Safe Passage



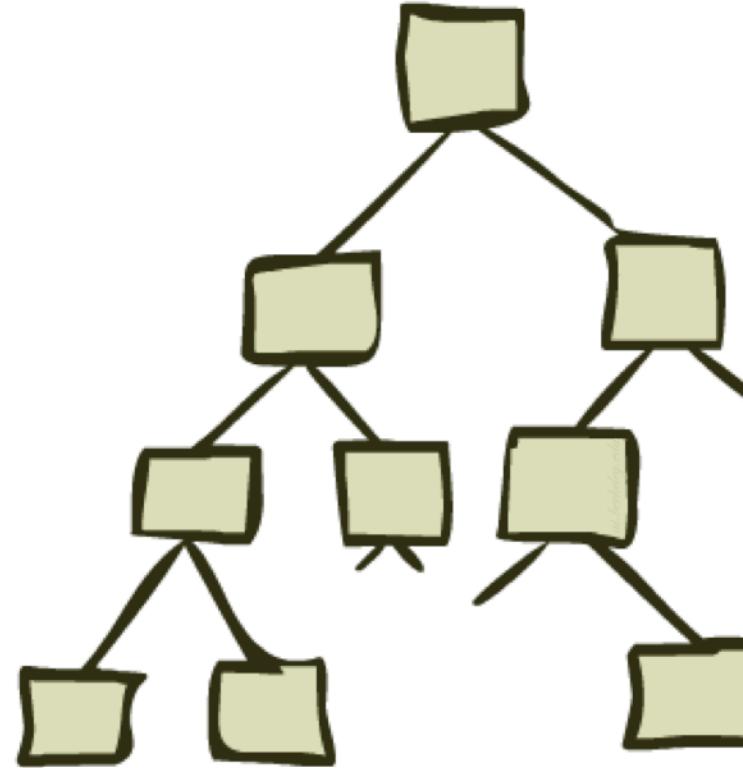
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- What does the state space have to specify?

# Quiz: Safe Passage



- Problem: eat all dots while keeping the ghosts perma-scared
- What does the state space have to specify?
  - (agent position, dot booleans, power pellet booleans, remaining scared time)

# State Space Graphs and Search Trees



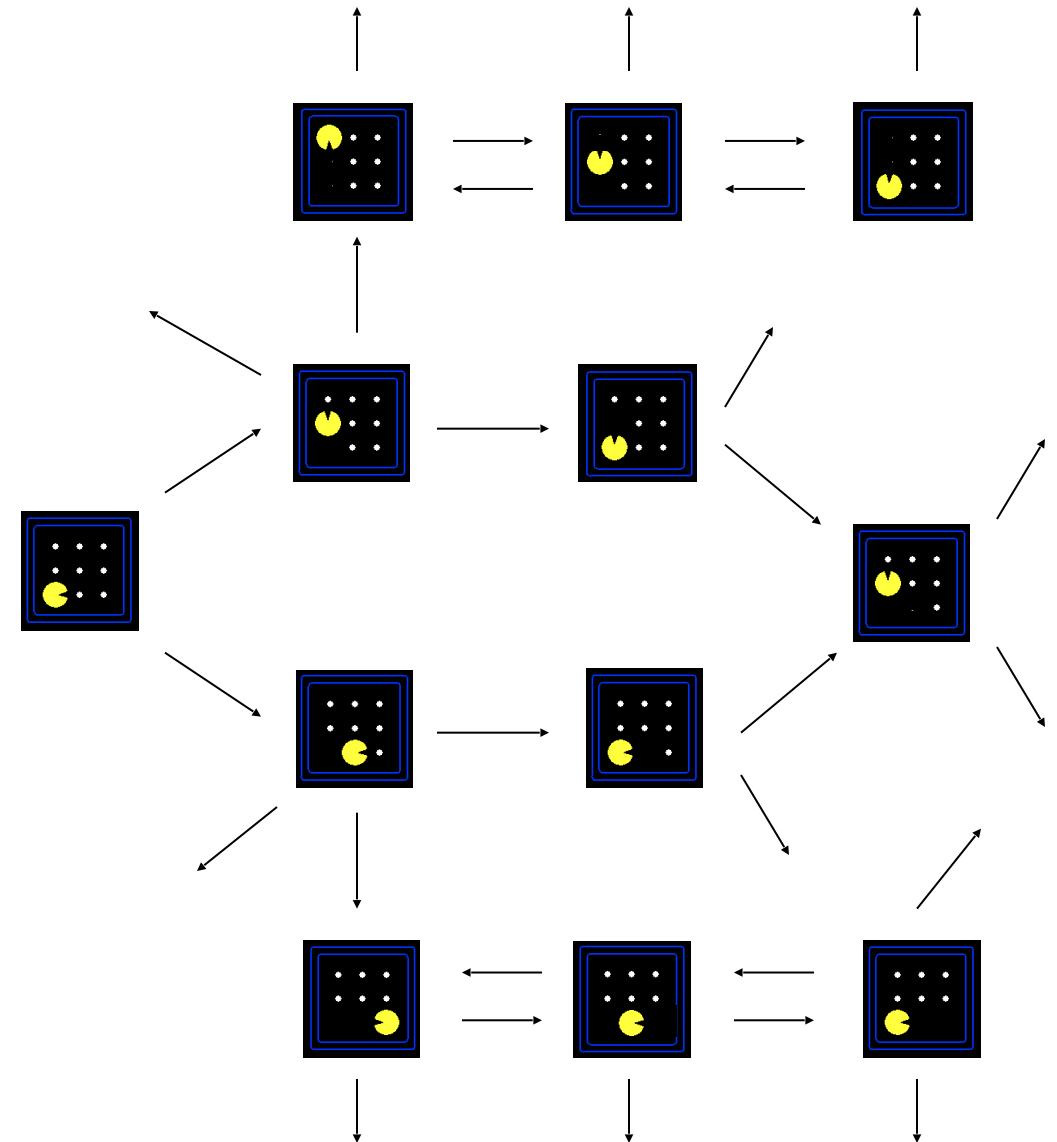
# State Space Graphs

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- State space graph: A mathematical representation of a search problem

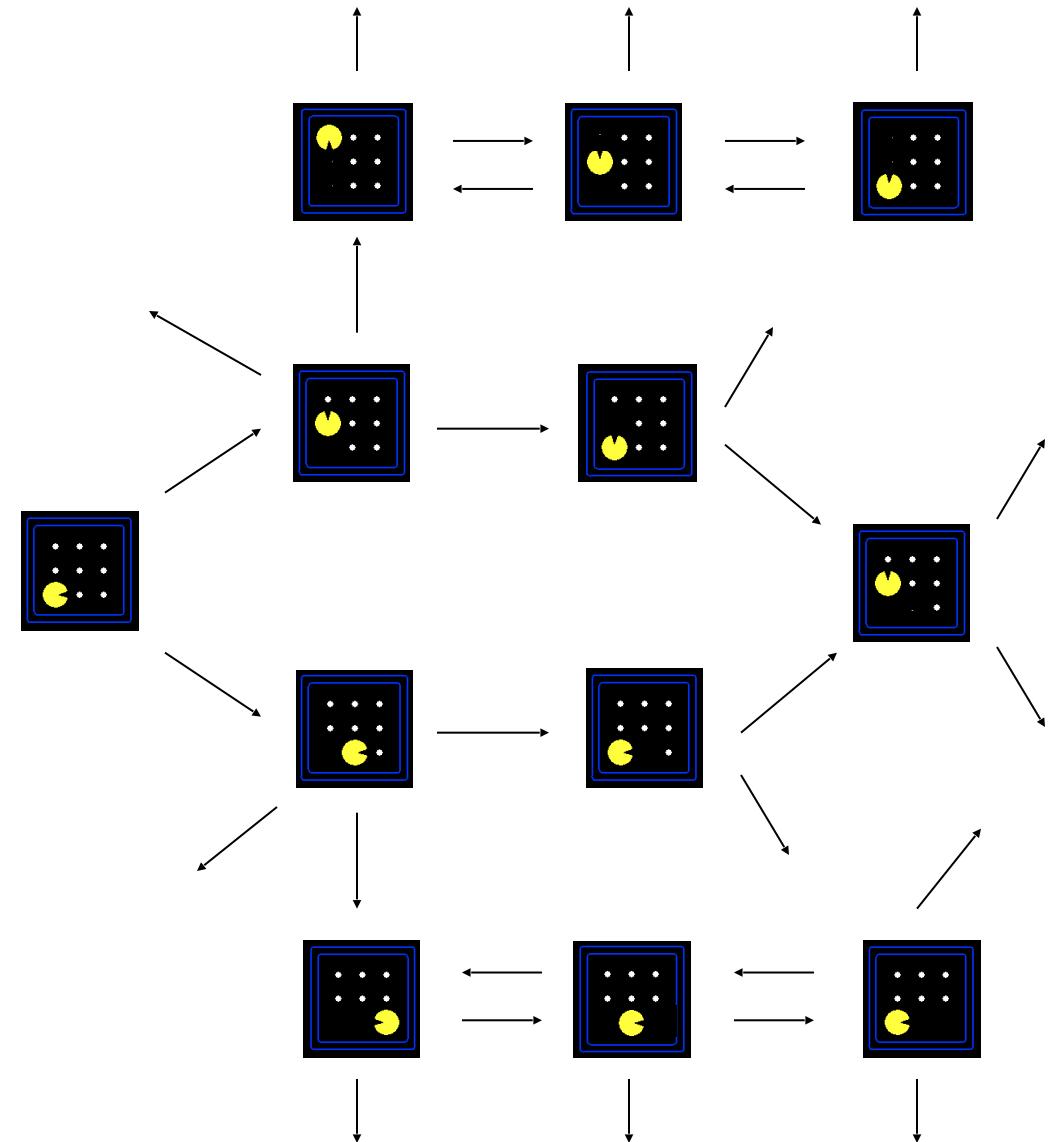
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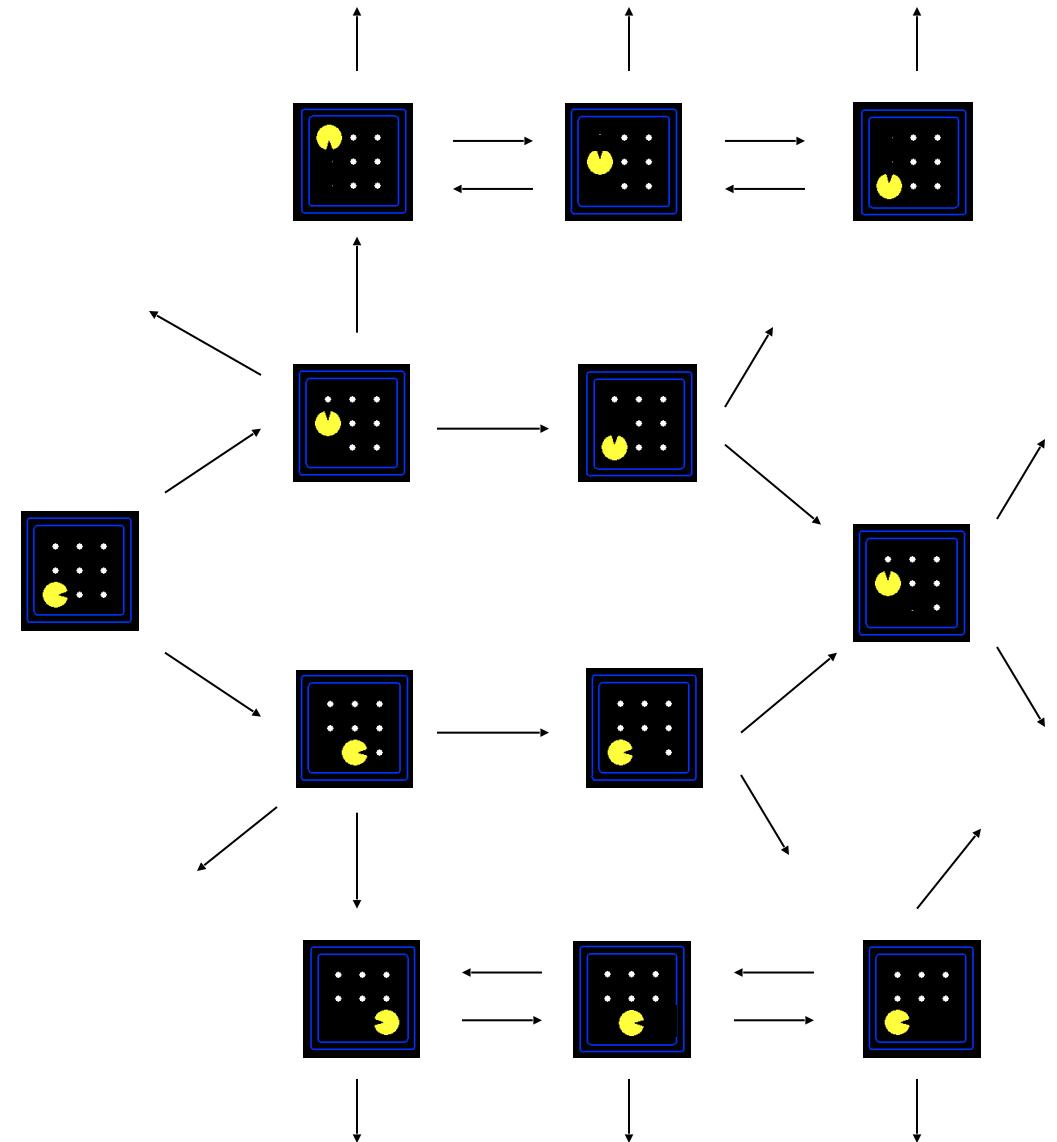
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- State space graph: A mathematical representation of a search problem
  - Nodes are (abstracted) world configurations



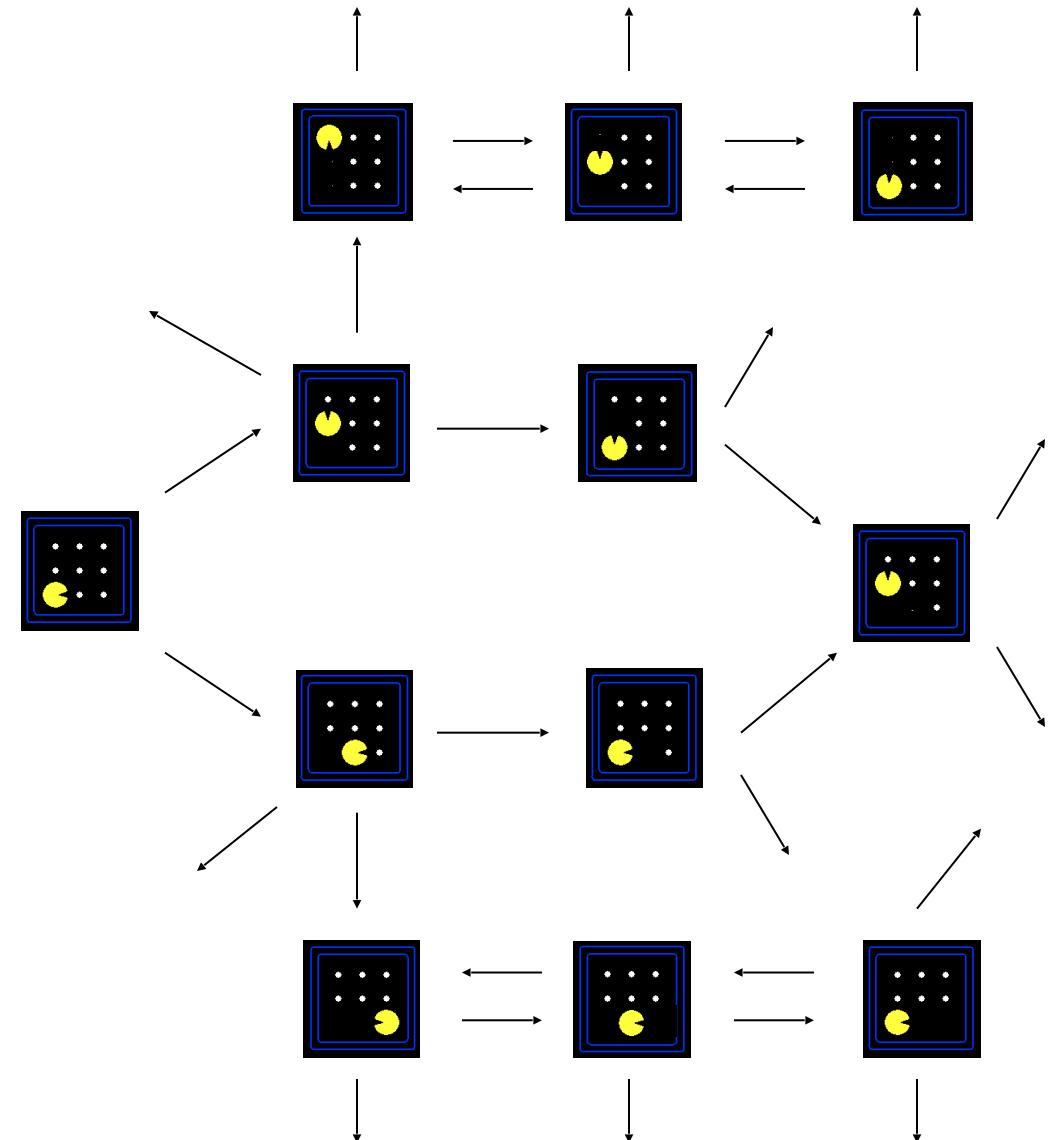
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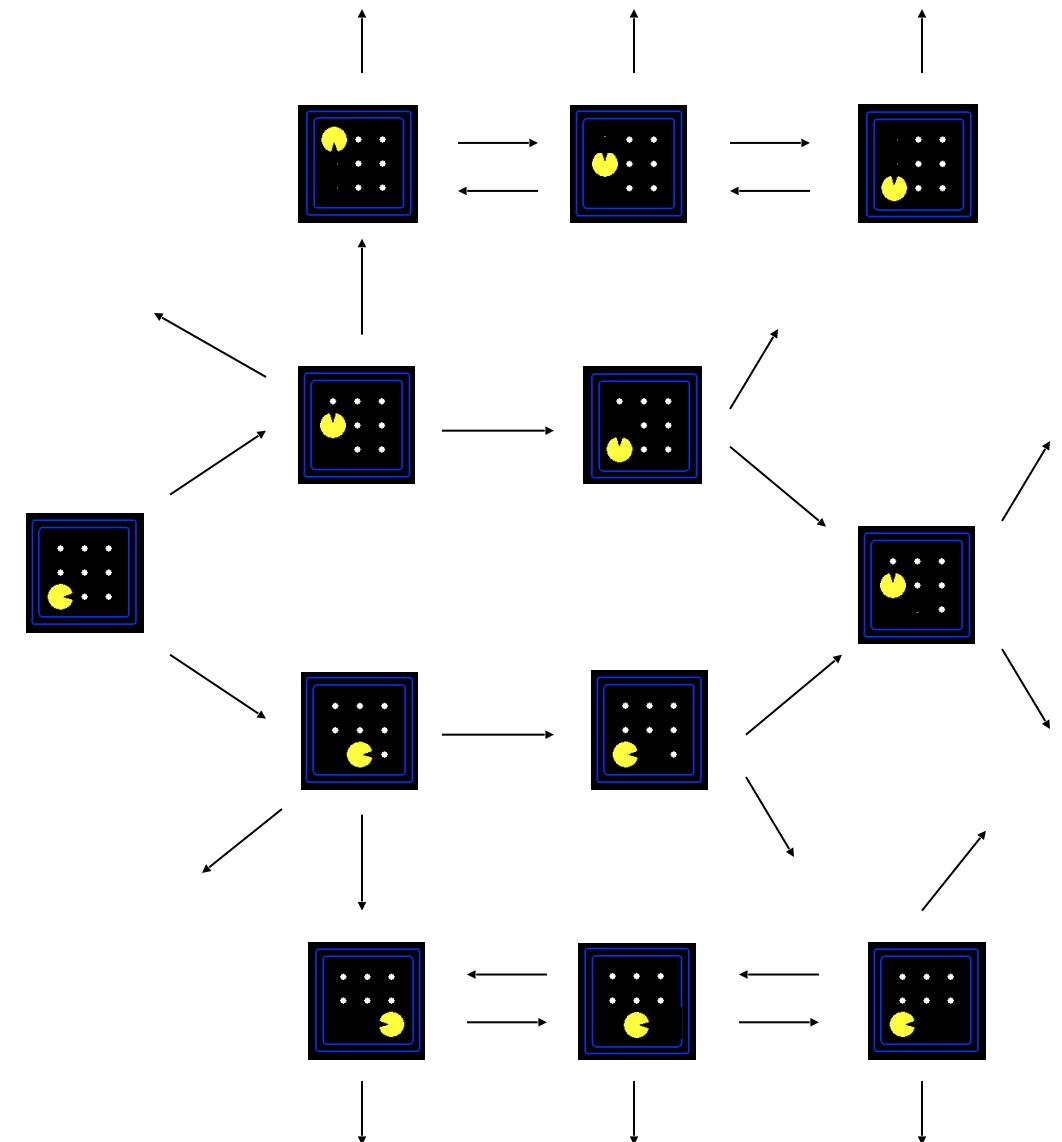
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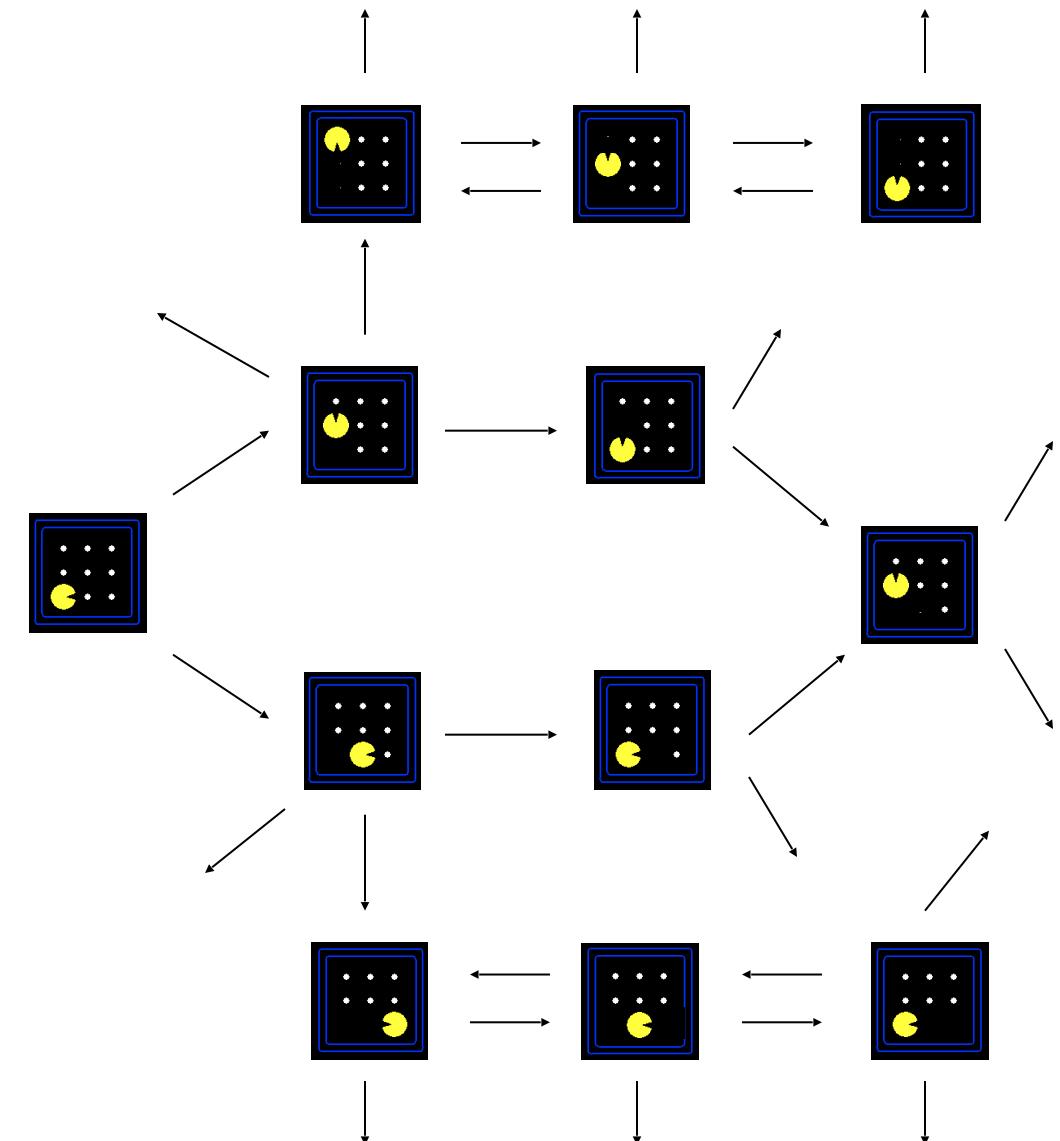
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  - In a state space graph, each state occurs only once!



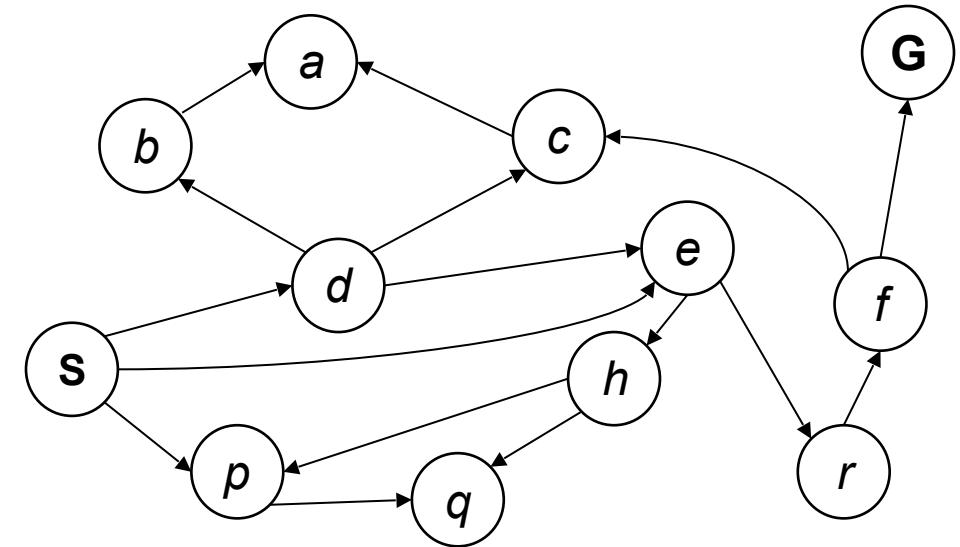
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- We can rarely build this full graph in memory (it's too big), but it's a useful idea



# State Space Graphs

- State space graph: A mathematical representation of a search problem
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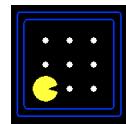
*Tiny search graph for a tiny search problem*

# Search Trees

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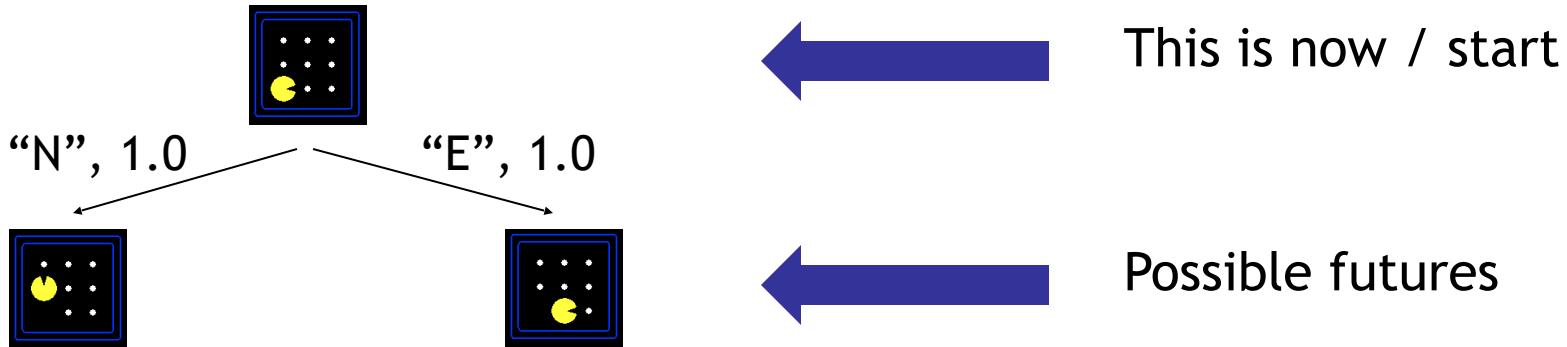
# Search Trees

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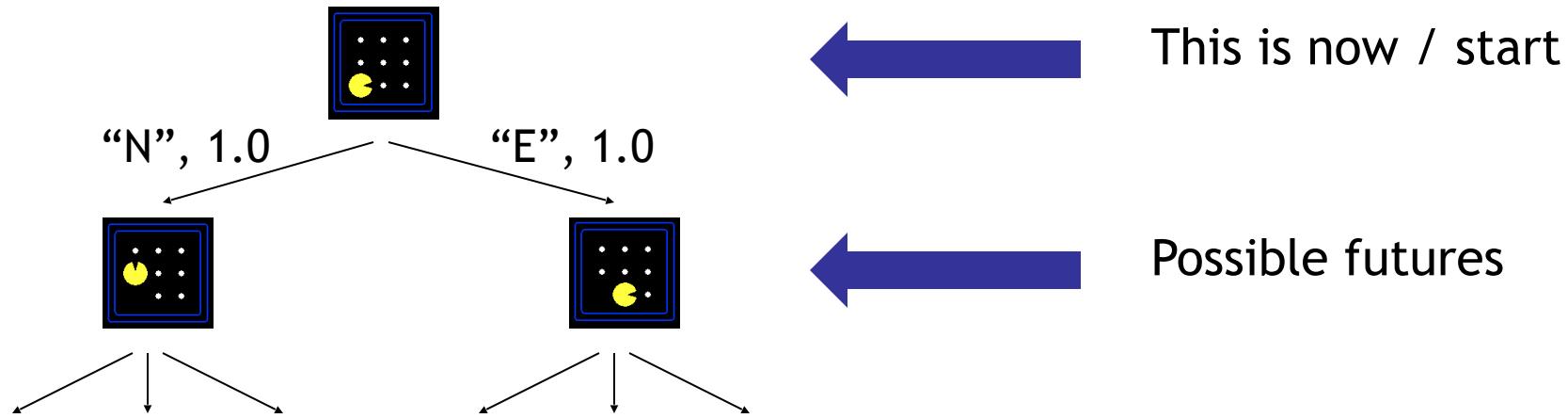


This is now / start

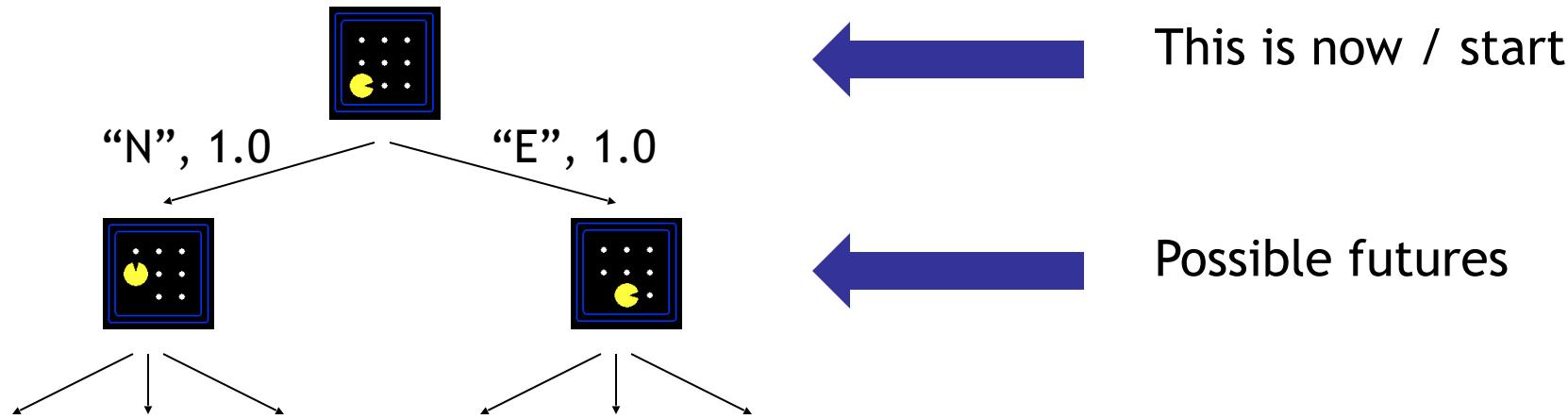
# Search Trees



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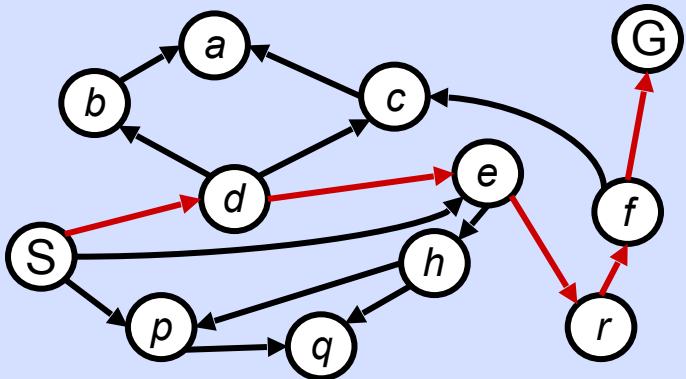
# Search Trees



- A **search tree**:
  - A “what if” tree of plans and their outcomes
  - The start state is the root node
  - Children correspond to successors
  - Nodes show states, but correspond to PLANS that achieve those states
  - **For most problems, we can never actually build the whole tree**

# State Space Graphs vs. Search Trees

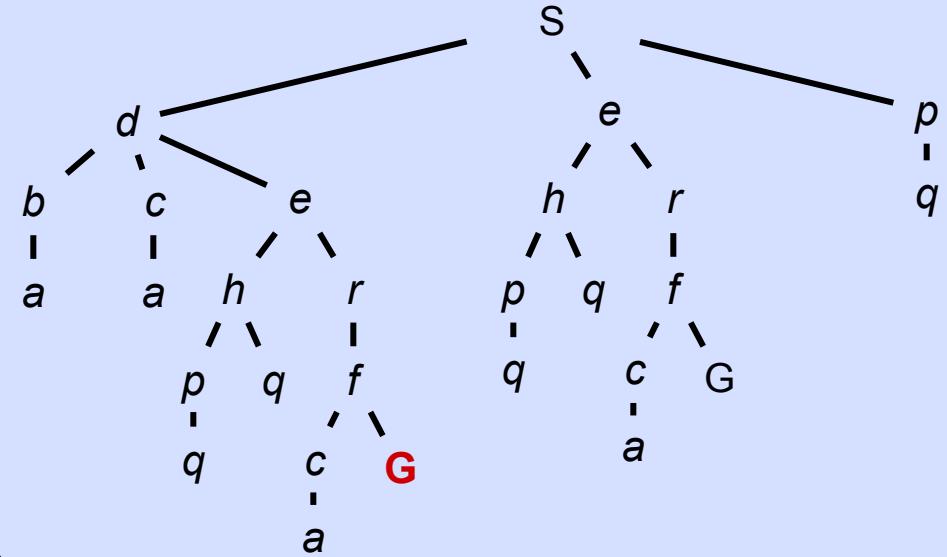
State Space Graph



*Each NODE in in  
the search tree is  
an entire PATH in  
the state space  
graph.*

*We construct both  
on demand - and  
we construct as  
little as possible.*

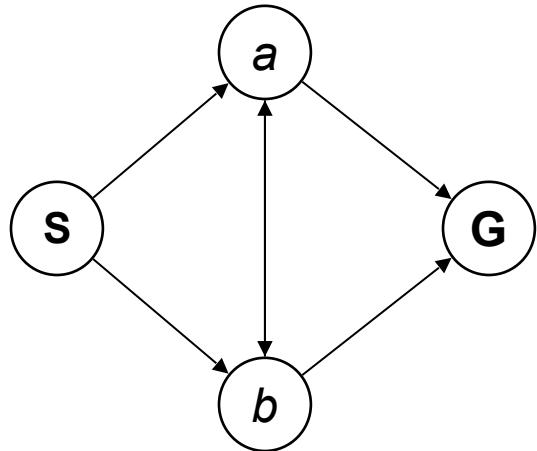
Search Tree



# Quiz: State Space Graphs vs. Search Trees

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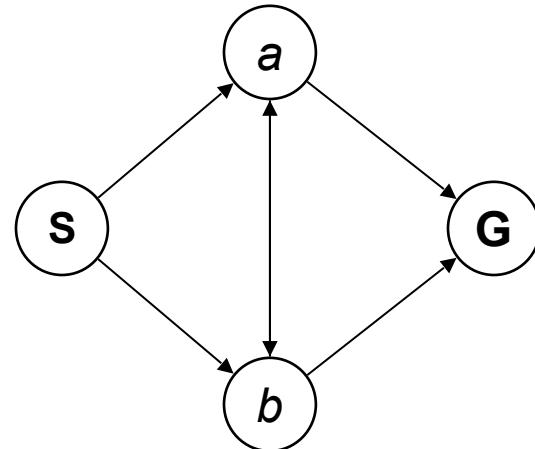
Consider this 4-state graph:



# Quiz: State Space Graphs vs. Search Trees

---

Consider this 4-state graph:

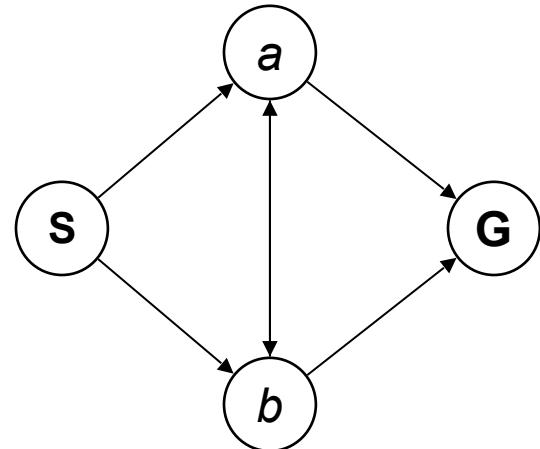


How big is its search tree (from S)?

# Quiz: State Space Graphs vs. Search Trees

---

Consider this 4-state graph:



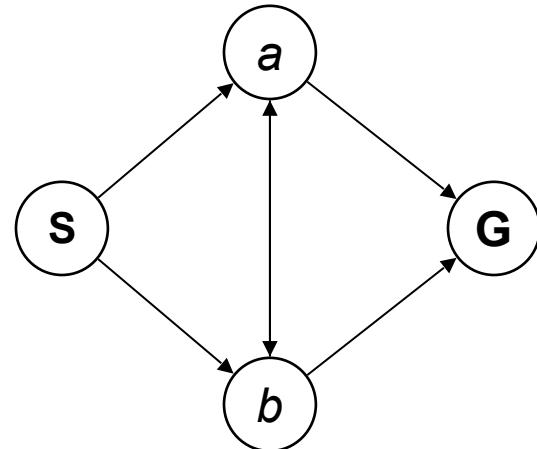
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# Quiz: State Space Graphs vs. Search Trees

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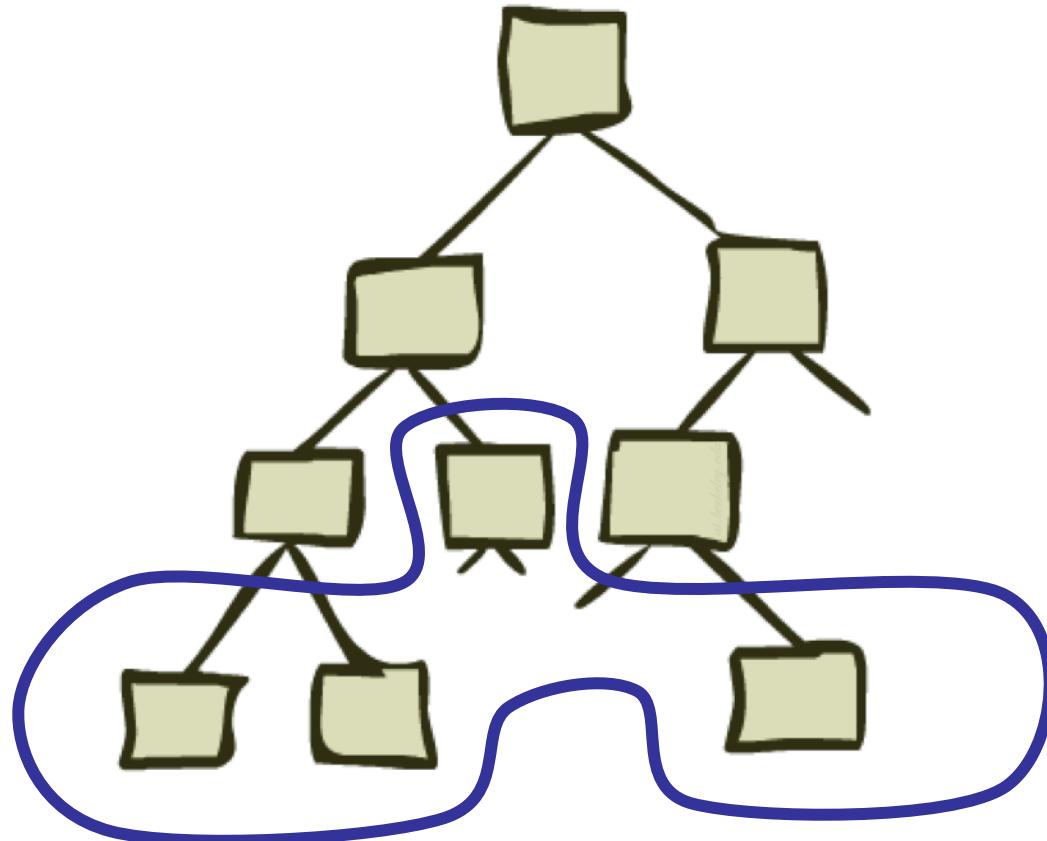
How big is its search tree (from S)?



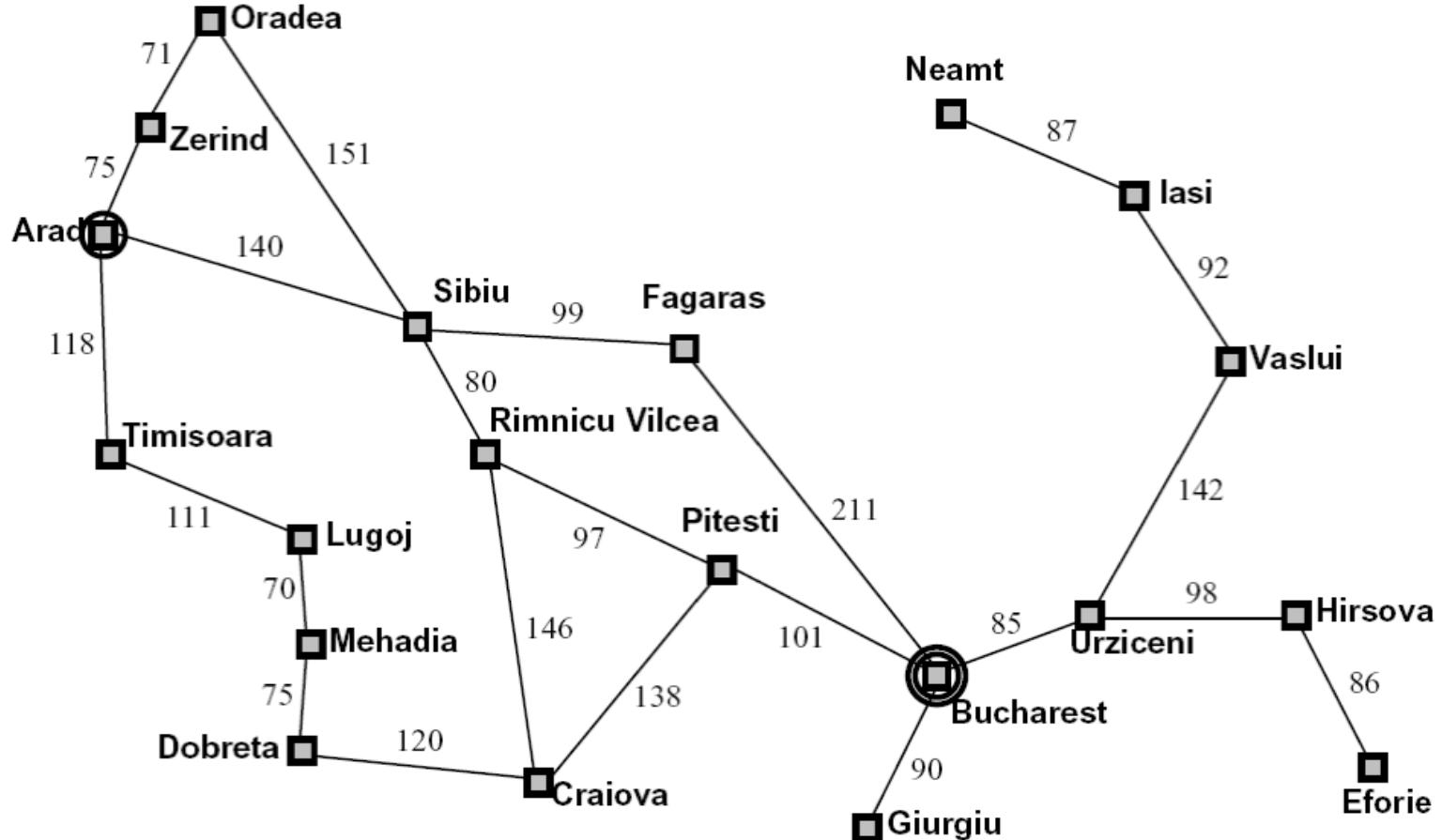
Important: Lots of repeated structure in the search tree!

# Tree Search

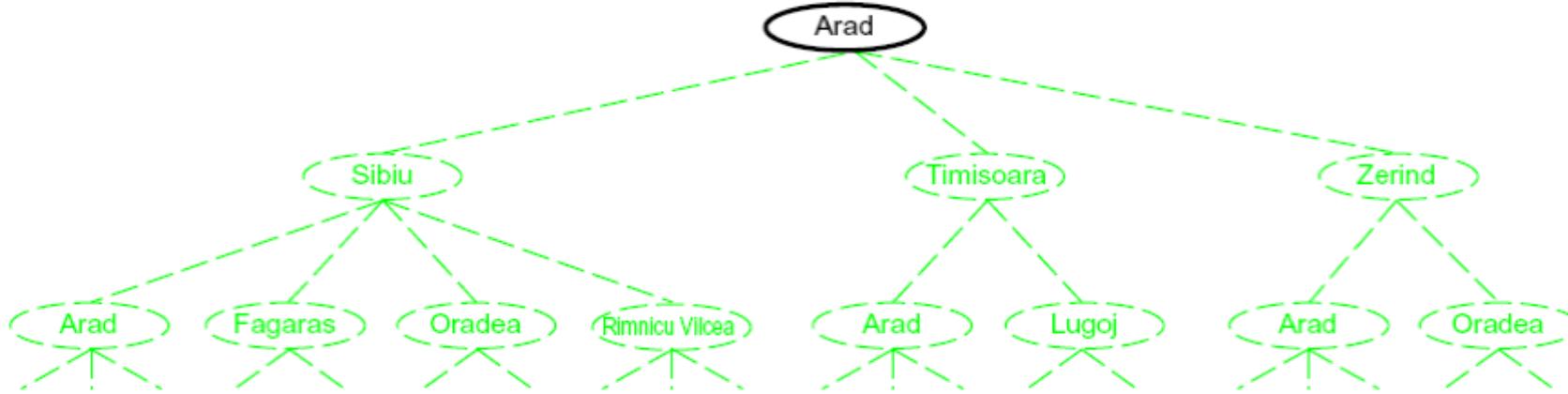
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# Search Example: Romania

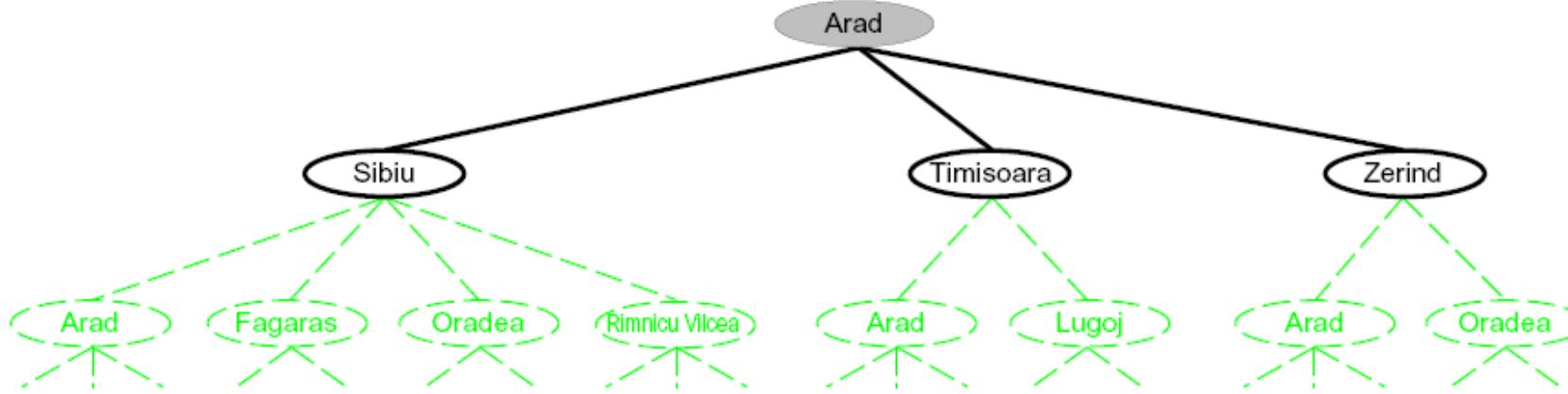


# Searching with a Search Tree



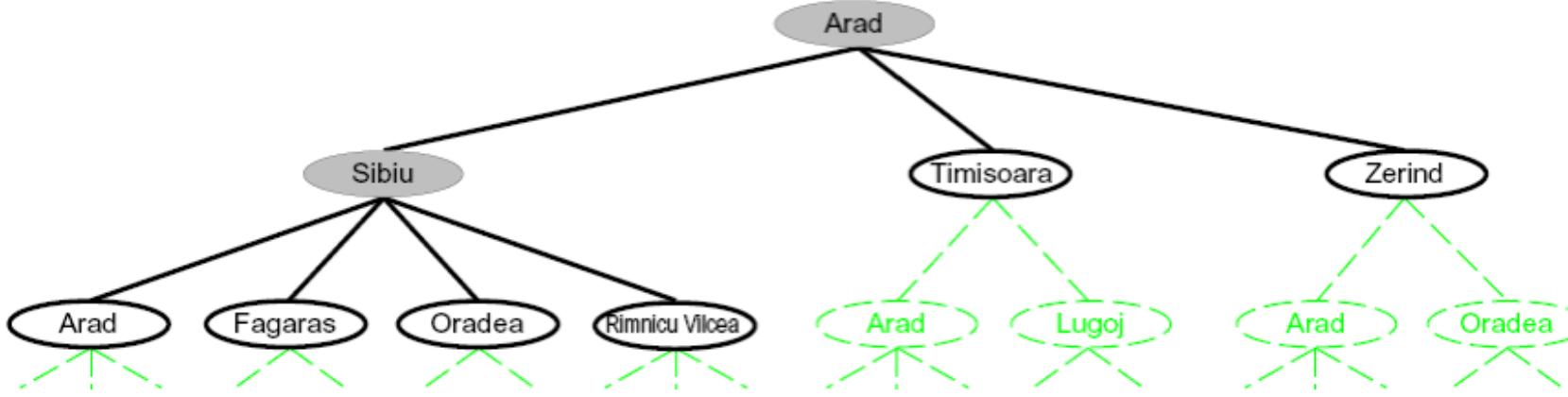
- Search:
  - Expand out potential plans (tree nodes)
  - Maintain a **fringe** of partial plans under consideration
  - Try to expand as few tree nodes as possible

# Searching with a Search Tree



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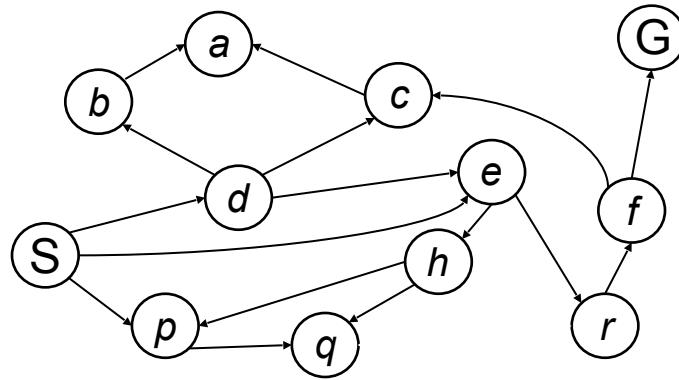
- **Search:**
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  - Maintain a **fringe** of partial plans under consideration
  - Try to expand as few tree nodes as possible

# General Tree Search

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy
- Main question: which fringe nodes to explore?

# Example: Tree Search



# Depth-First Search

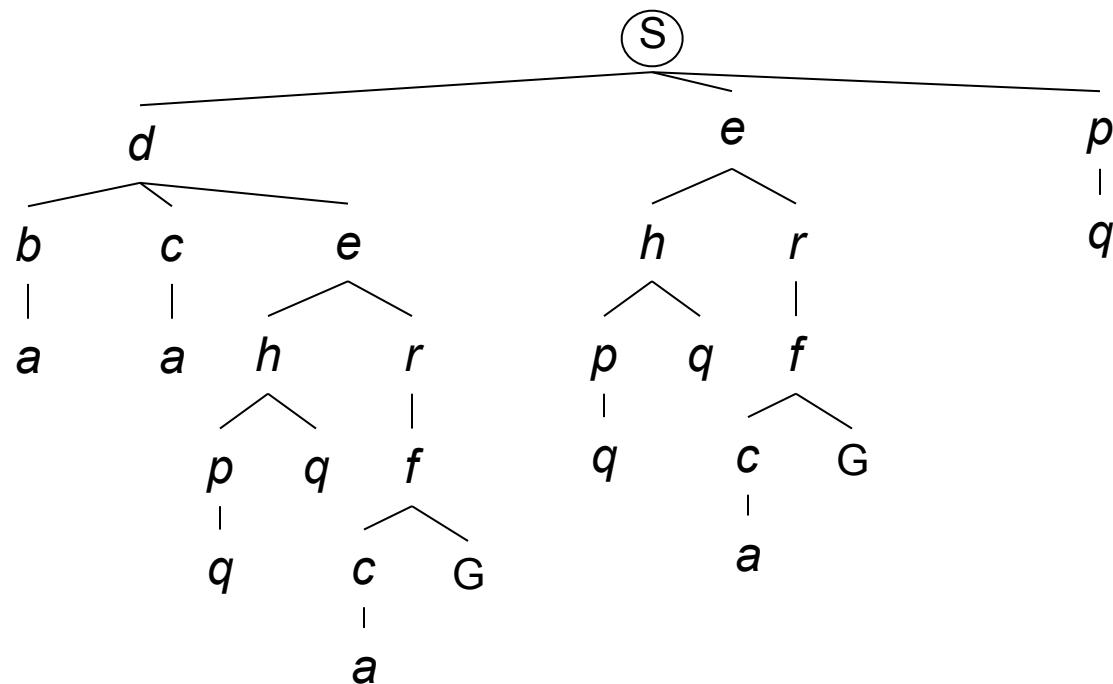
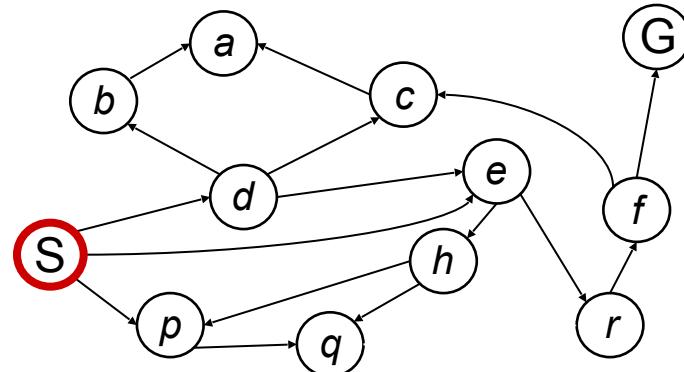
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# Depth-First Search

*Strategy: expand a deepest node first*

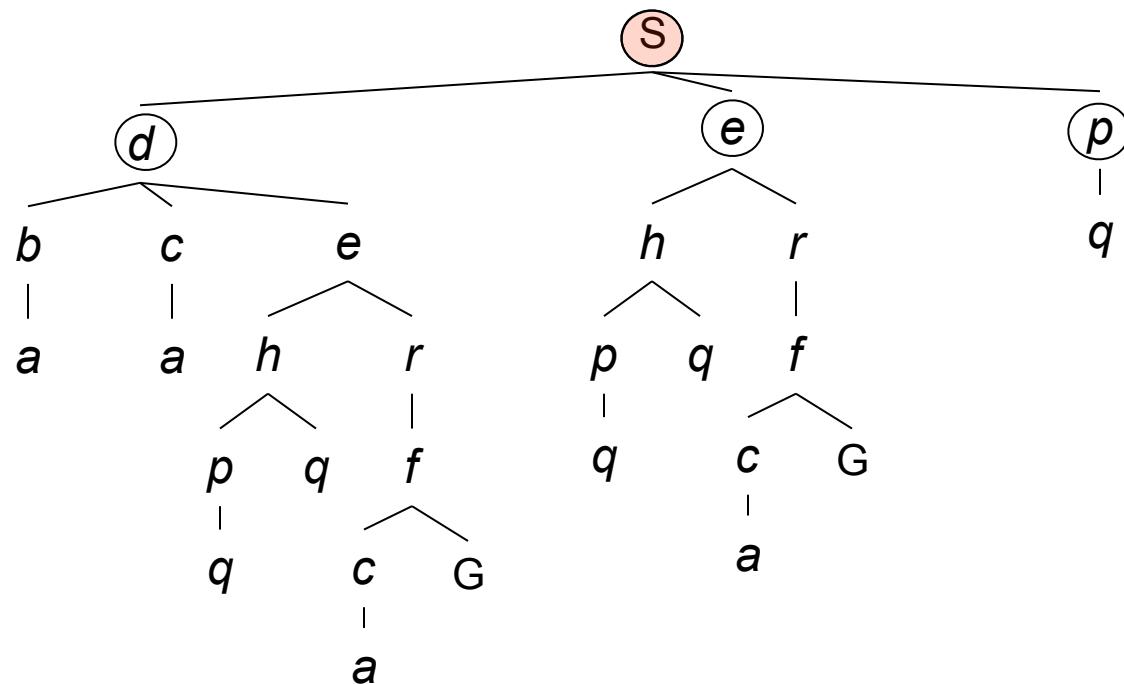
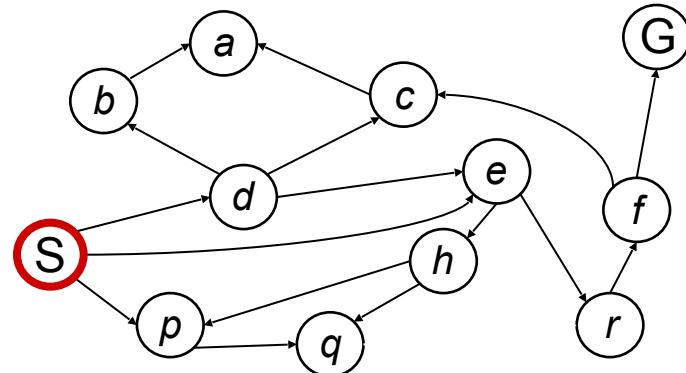
*Implementation:  
Fringe is a LIFO stack*



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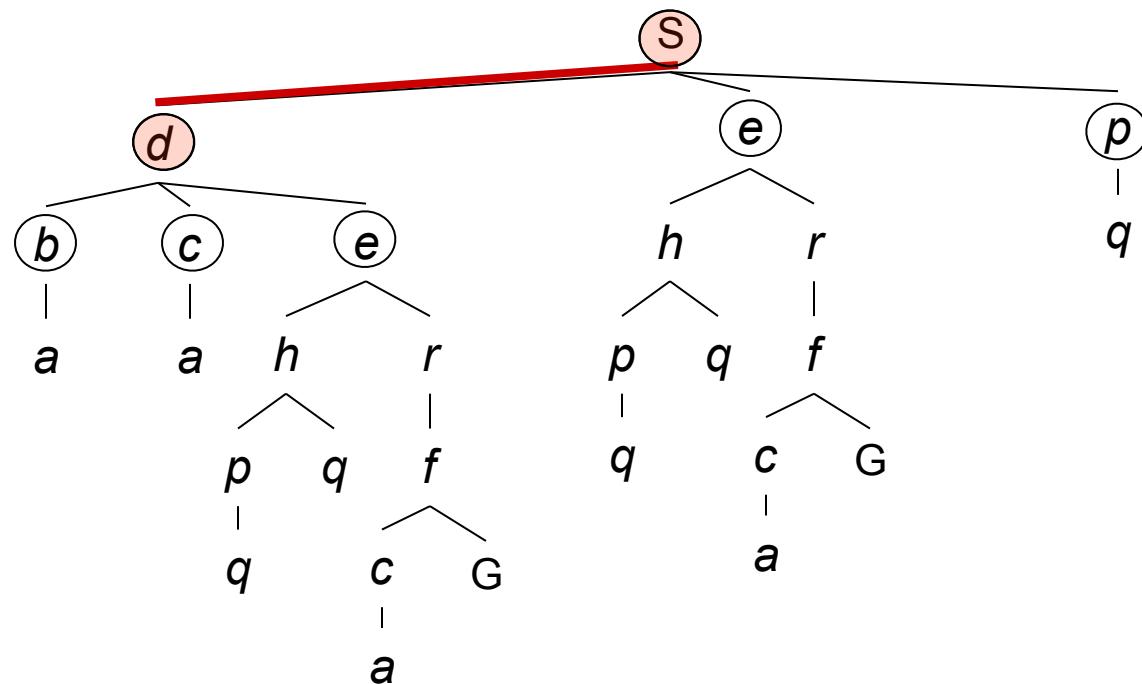
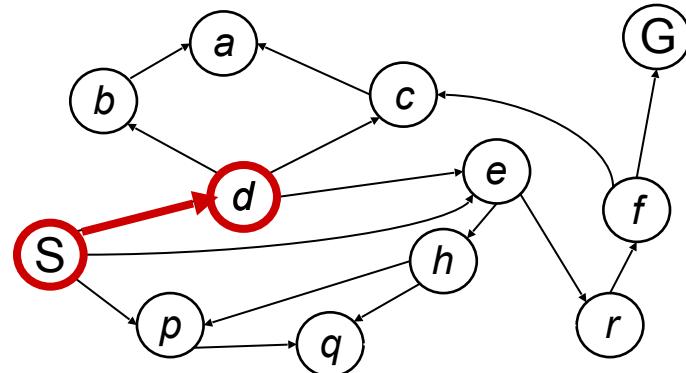
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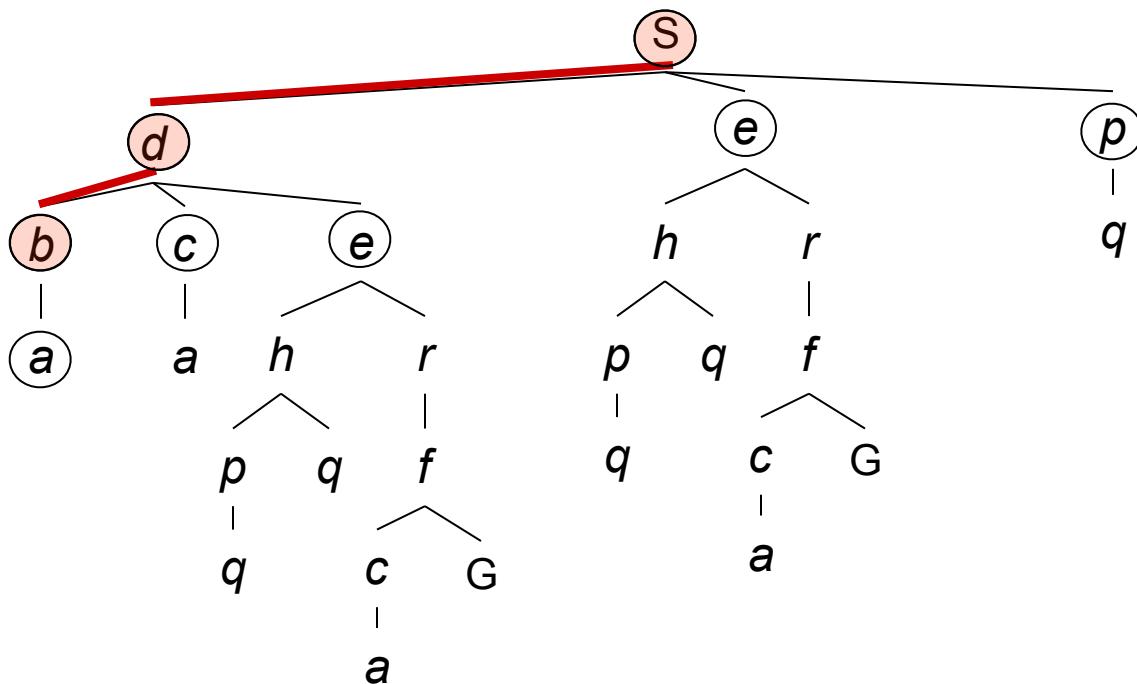
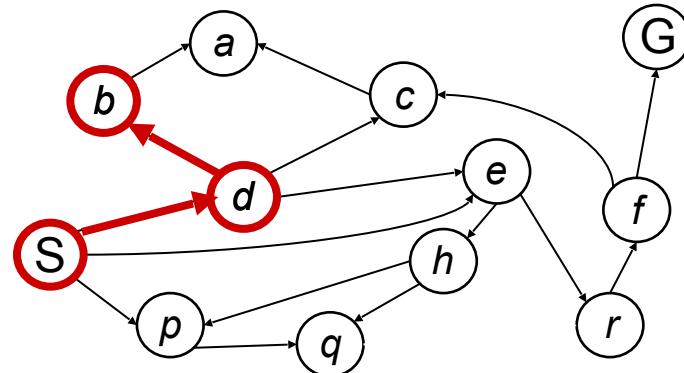
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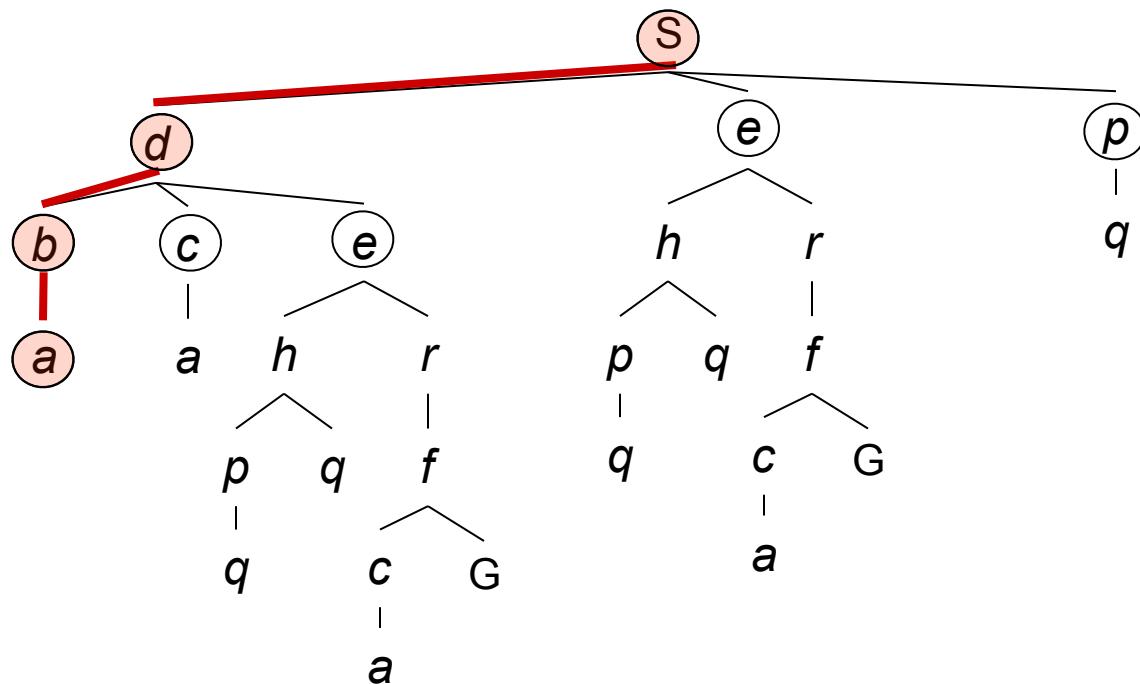
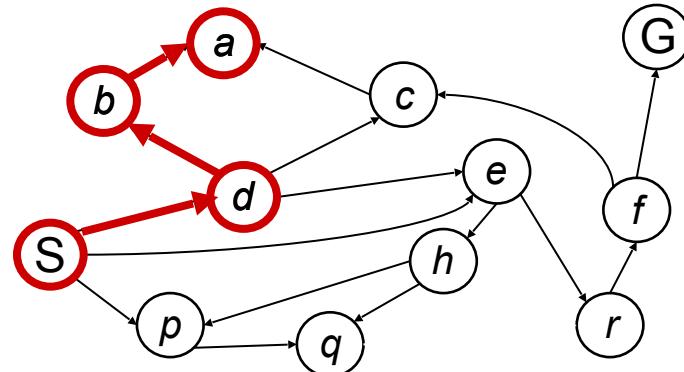
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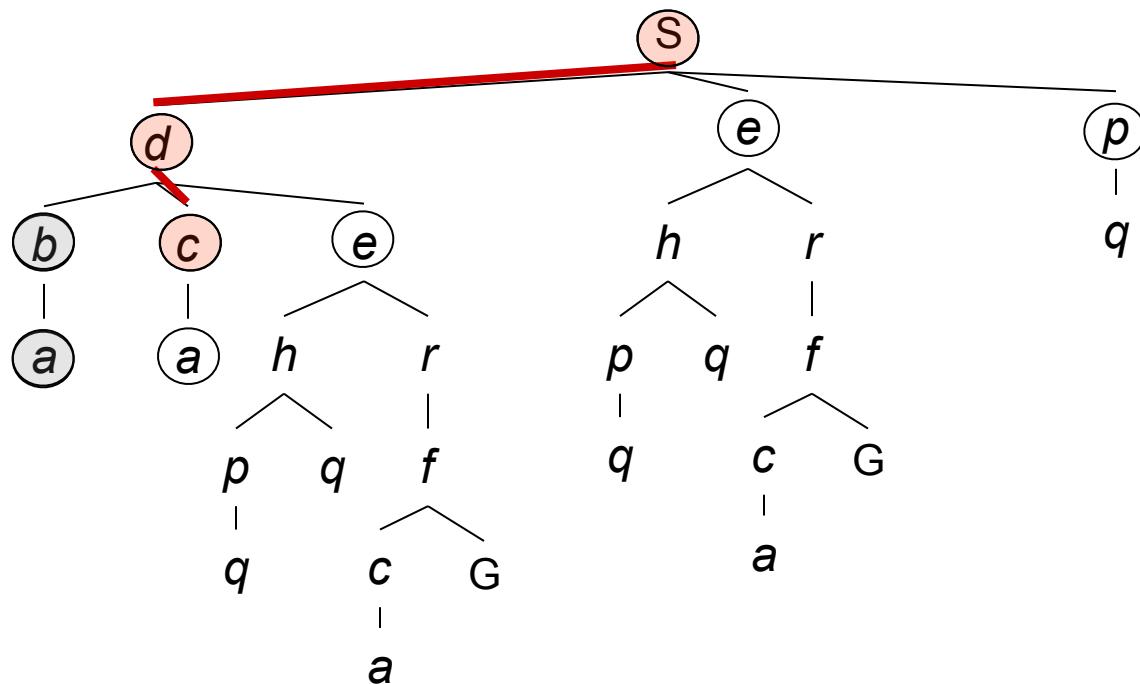
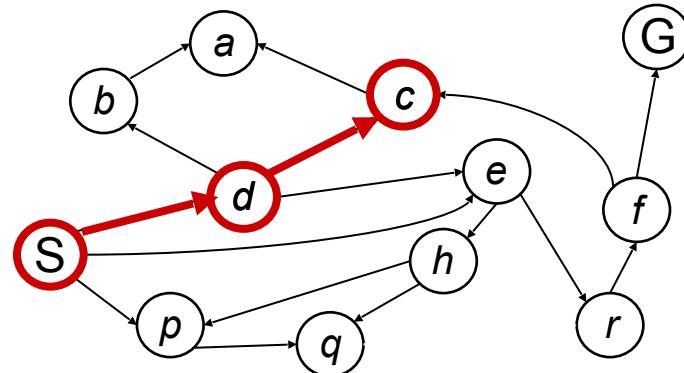
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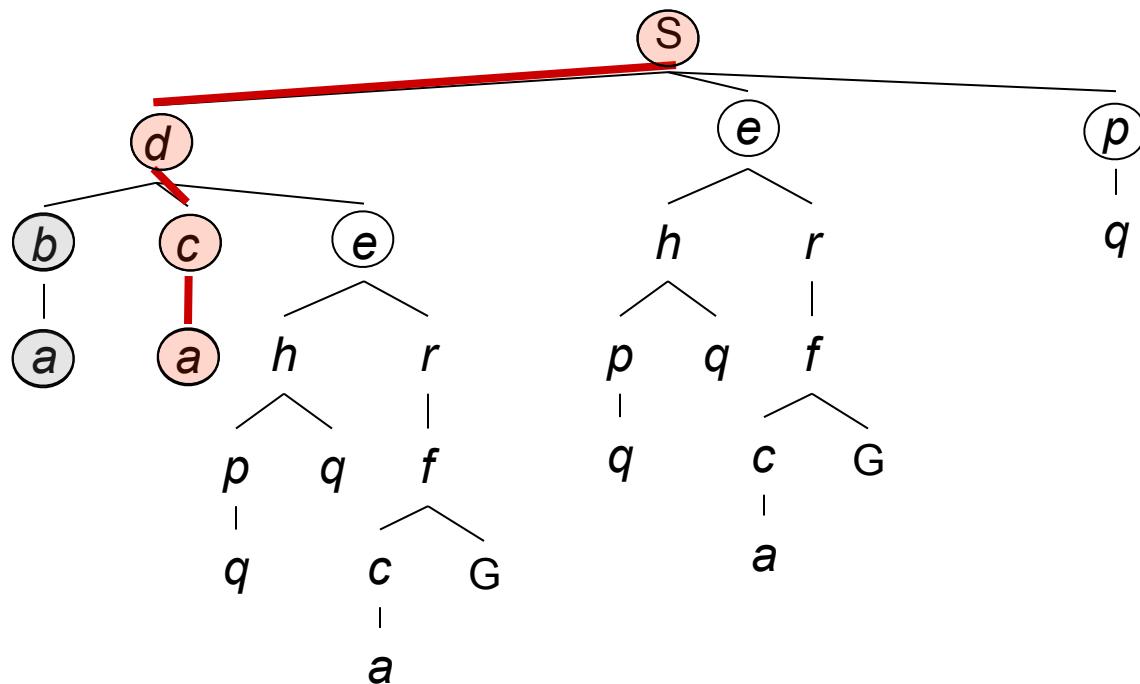
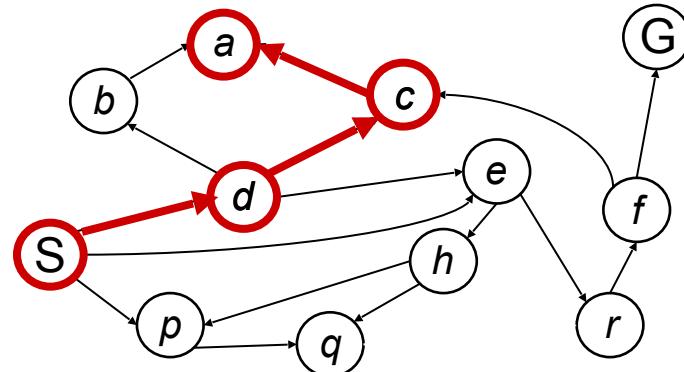
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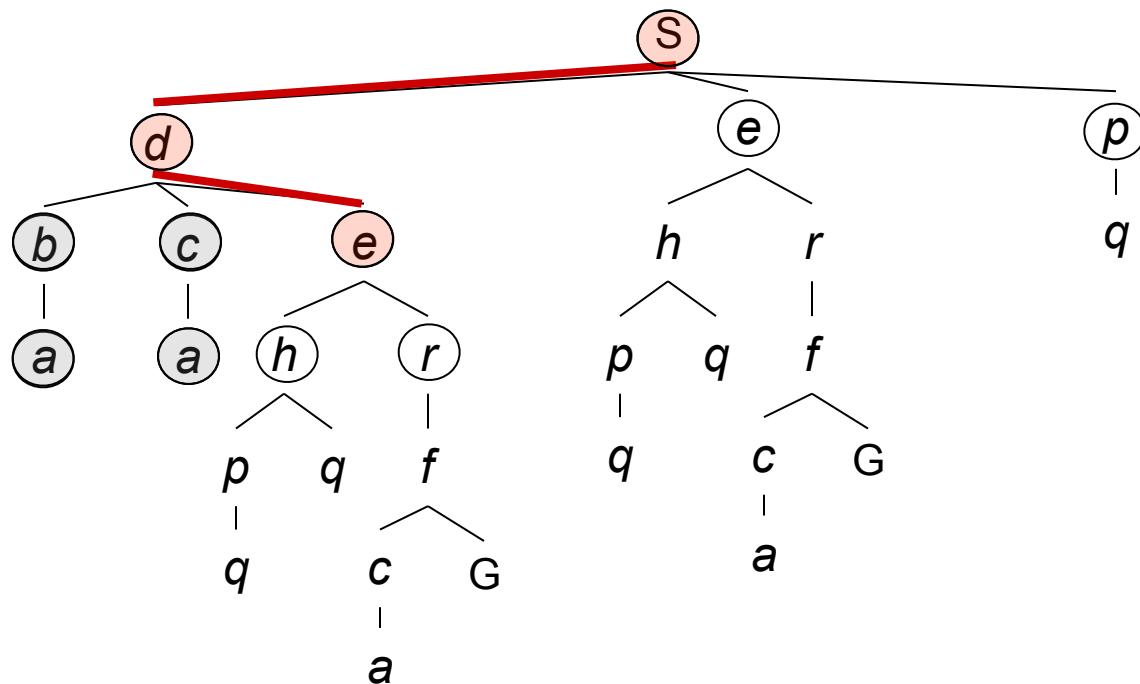
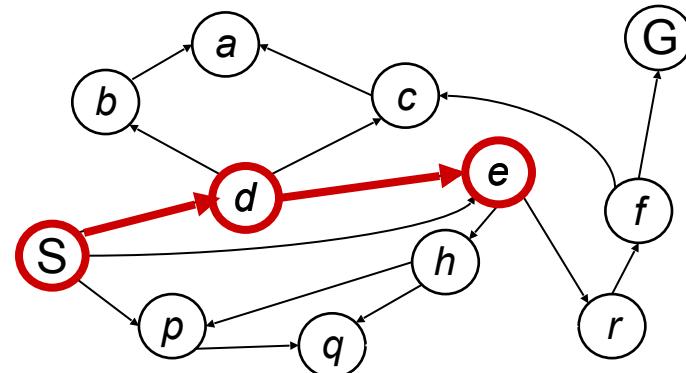
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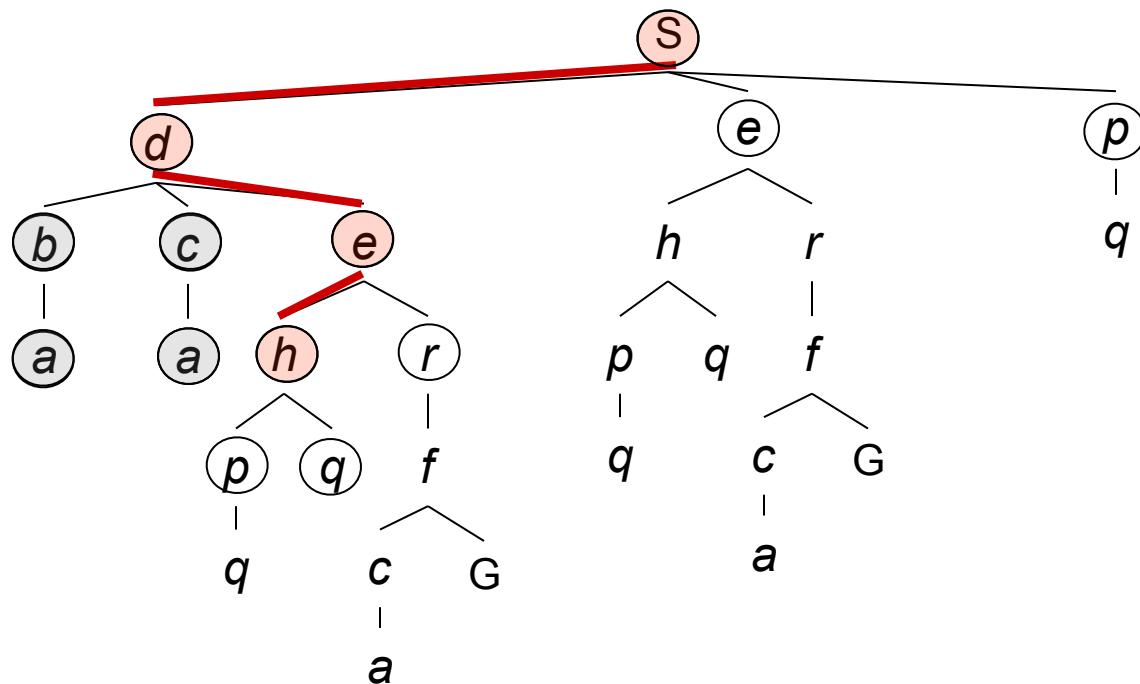
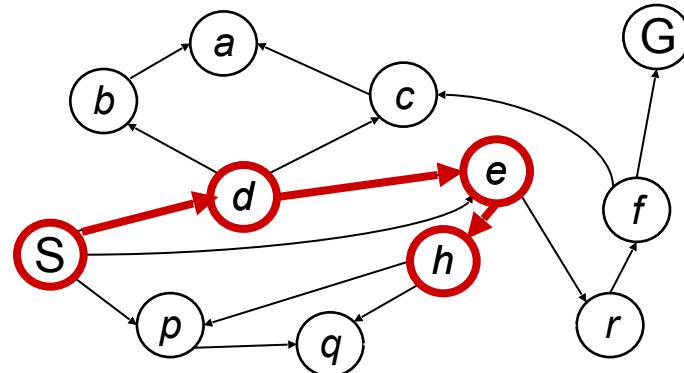
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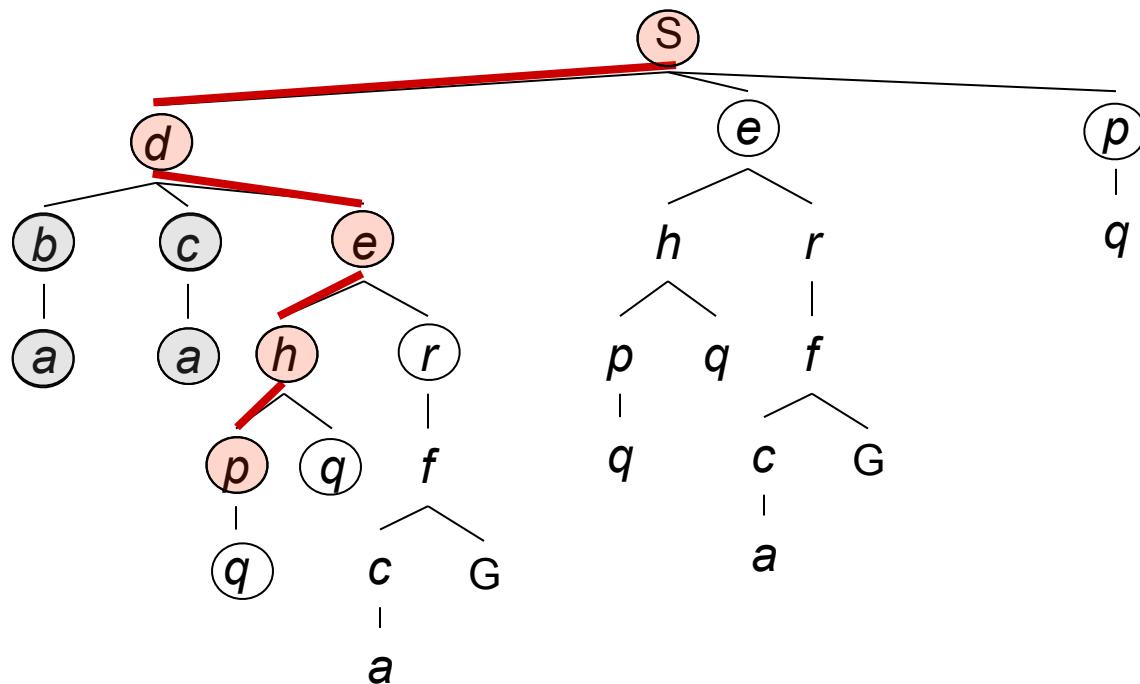
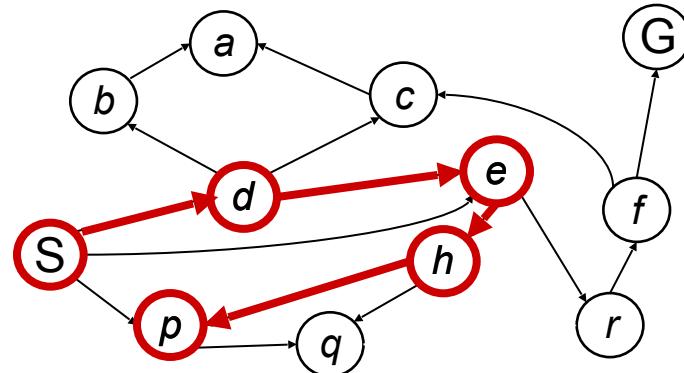
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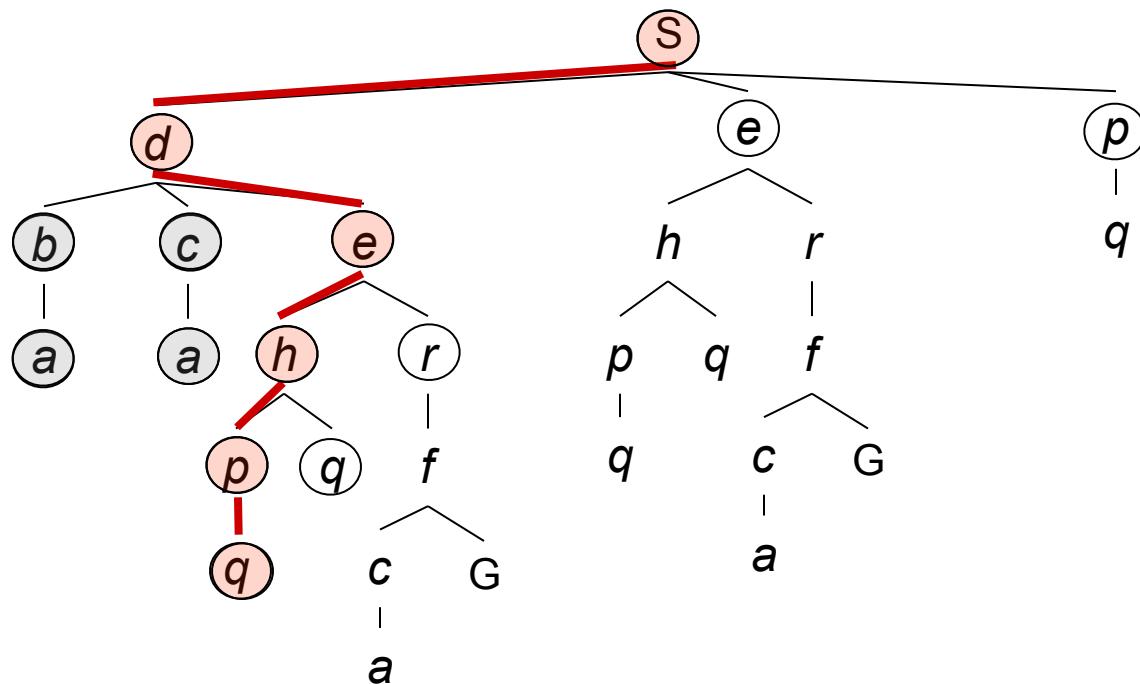
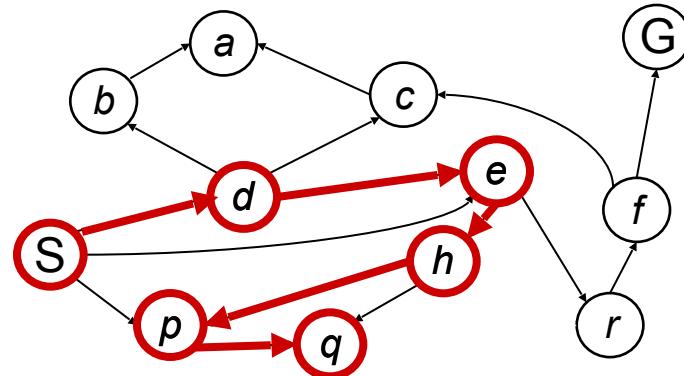
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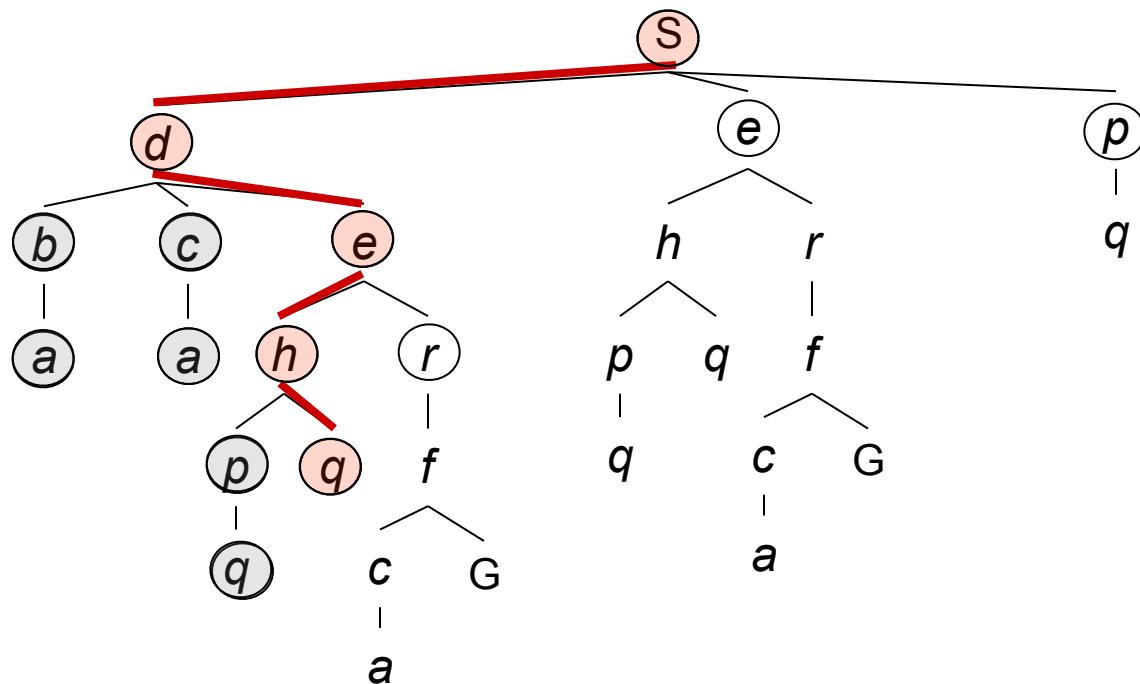
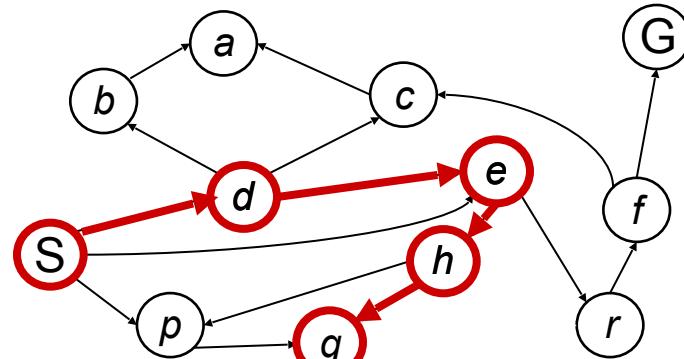
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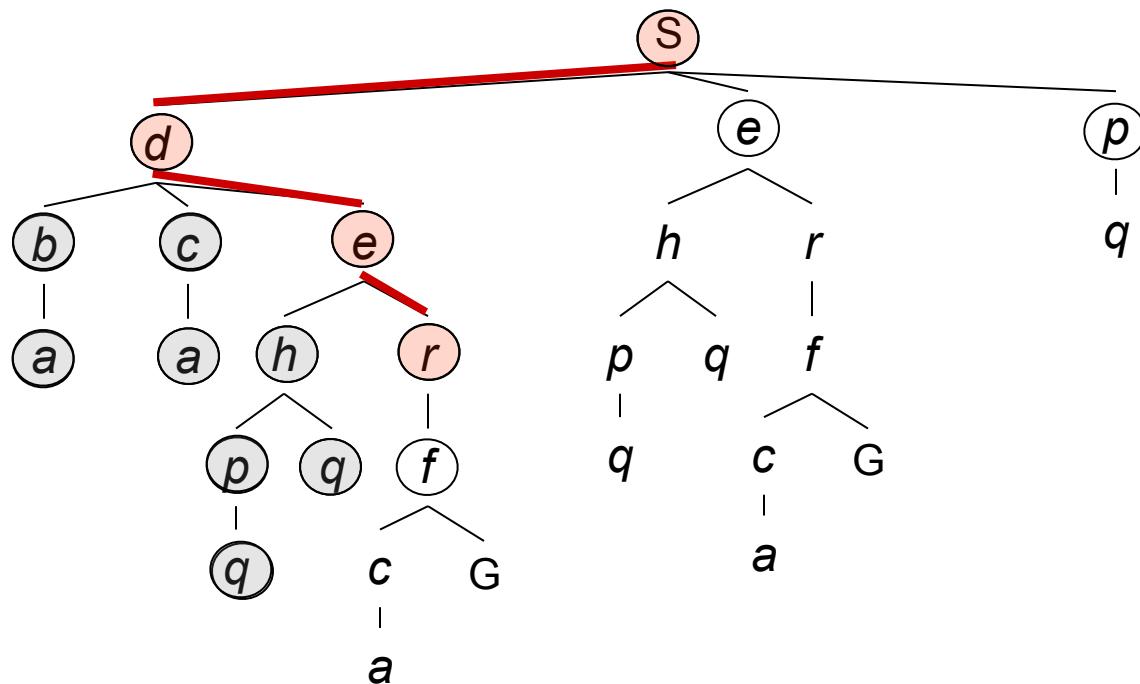
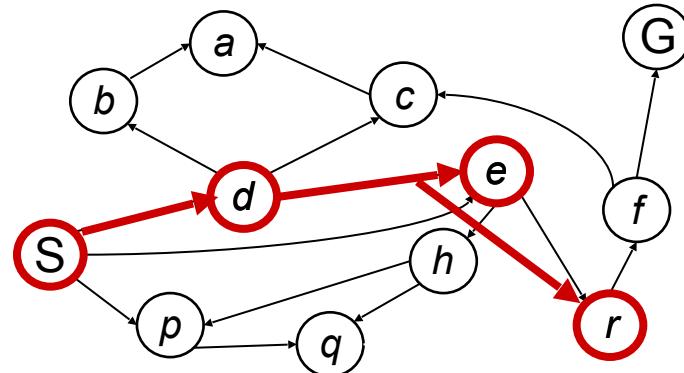
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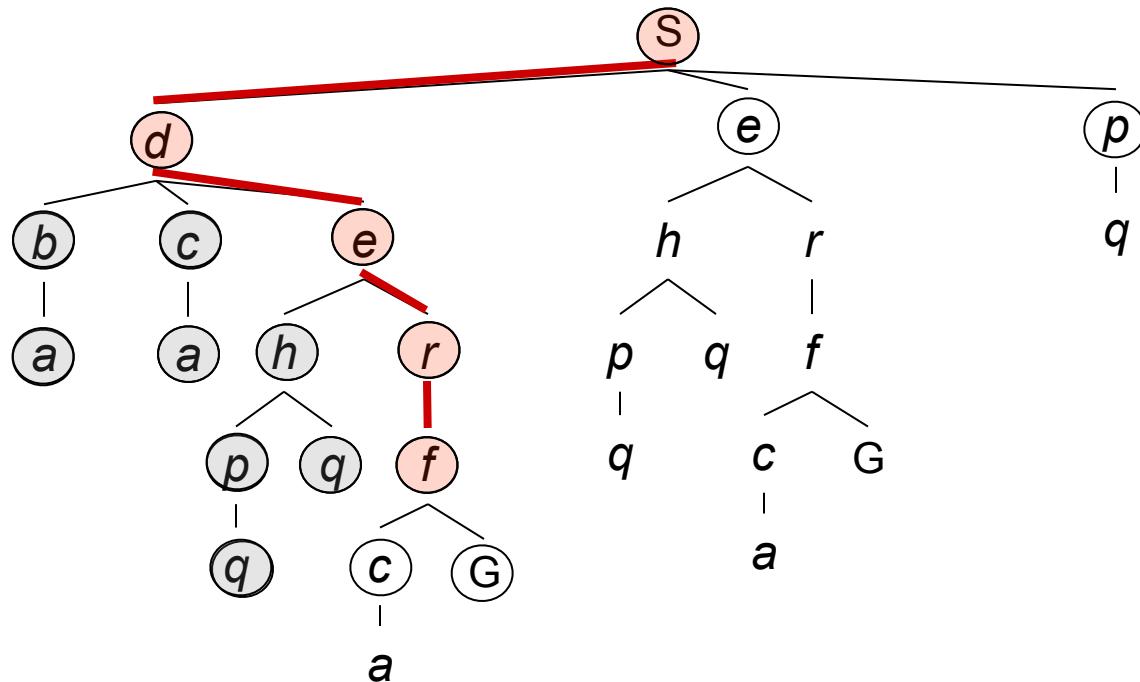
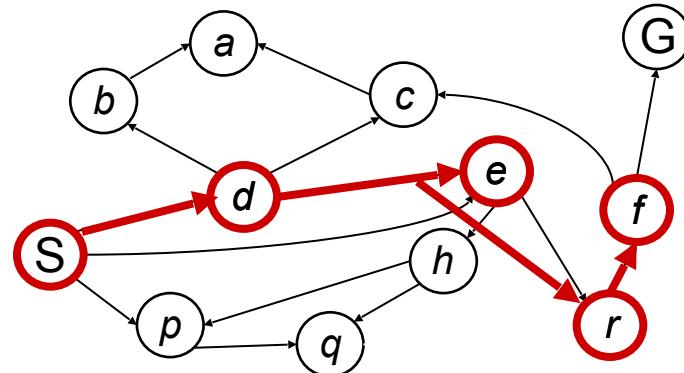
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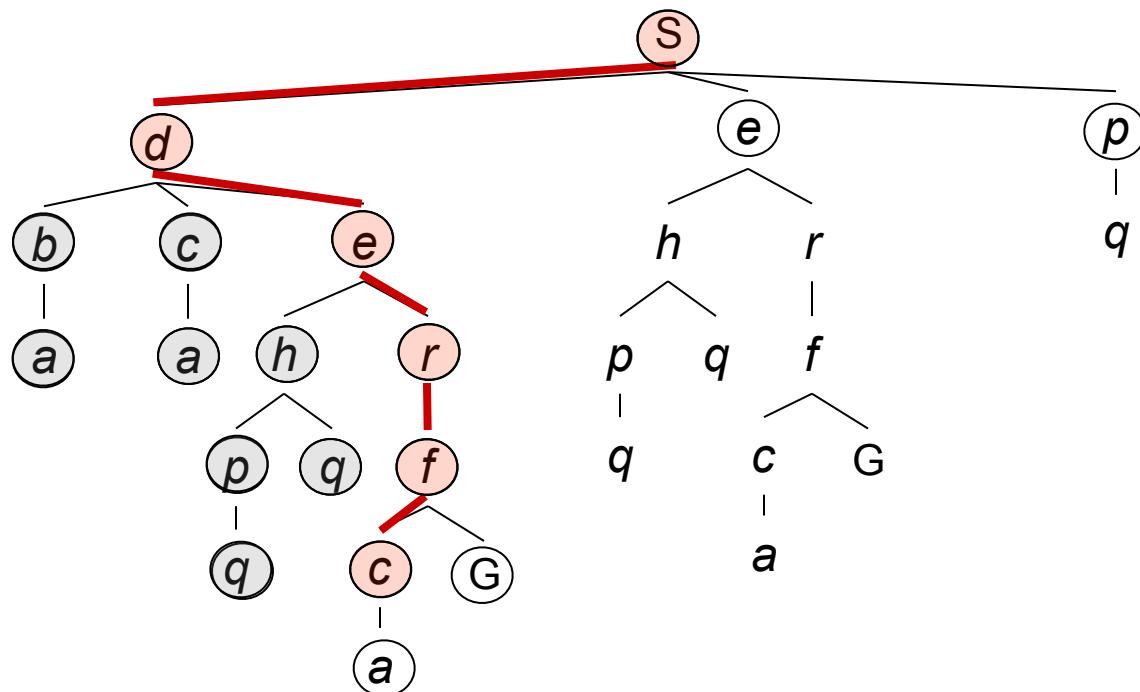
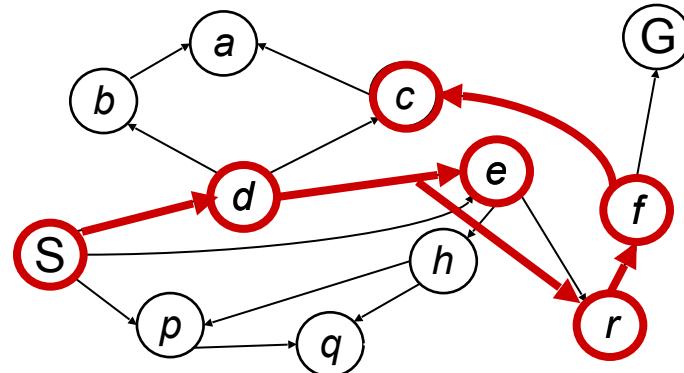
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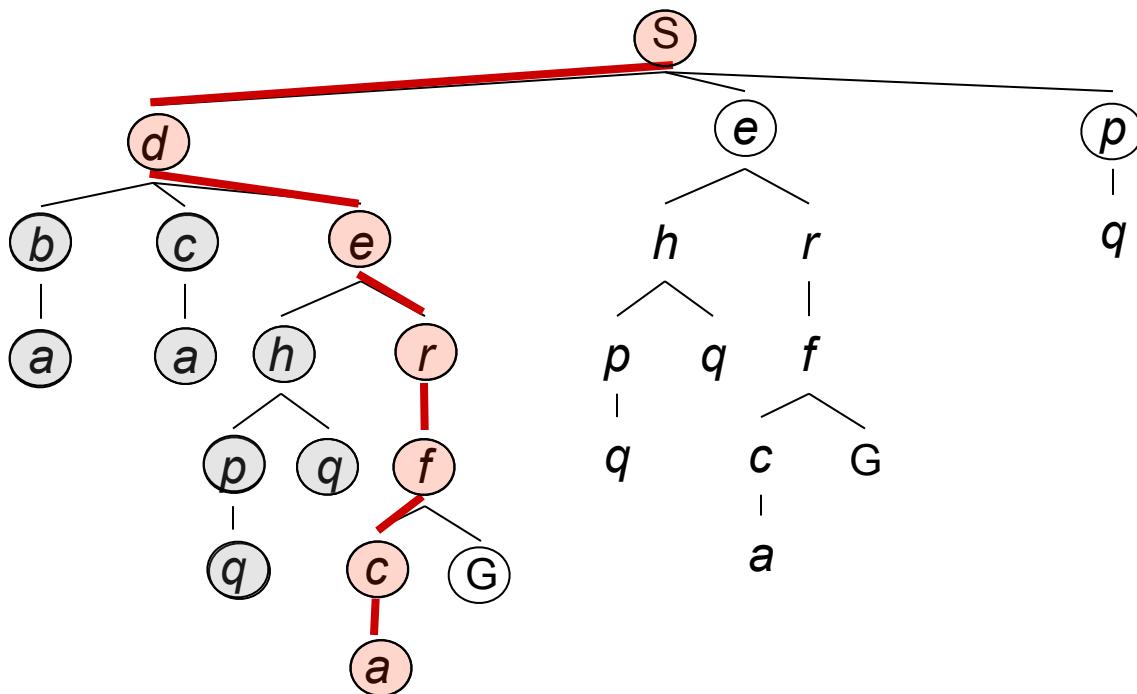
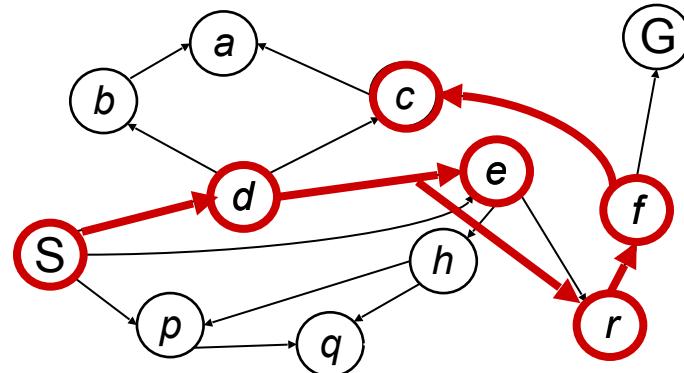
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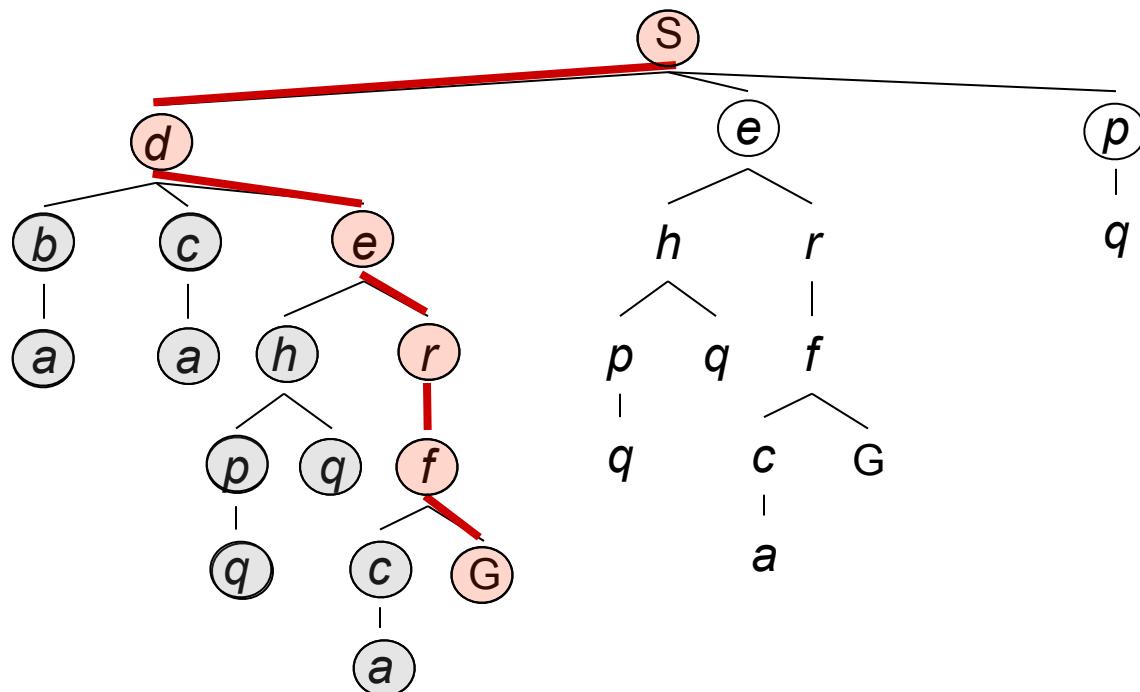
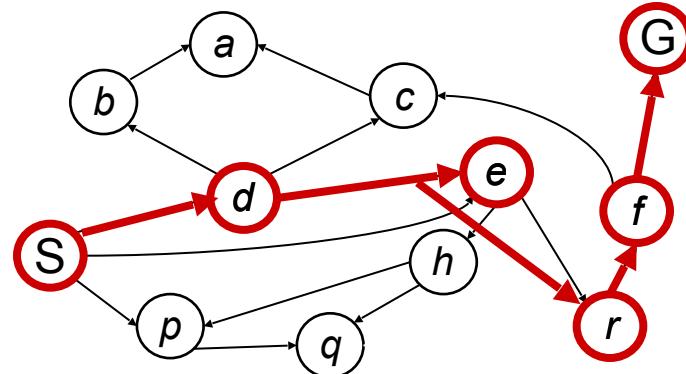
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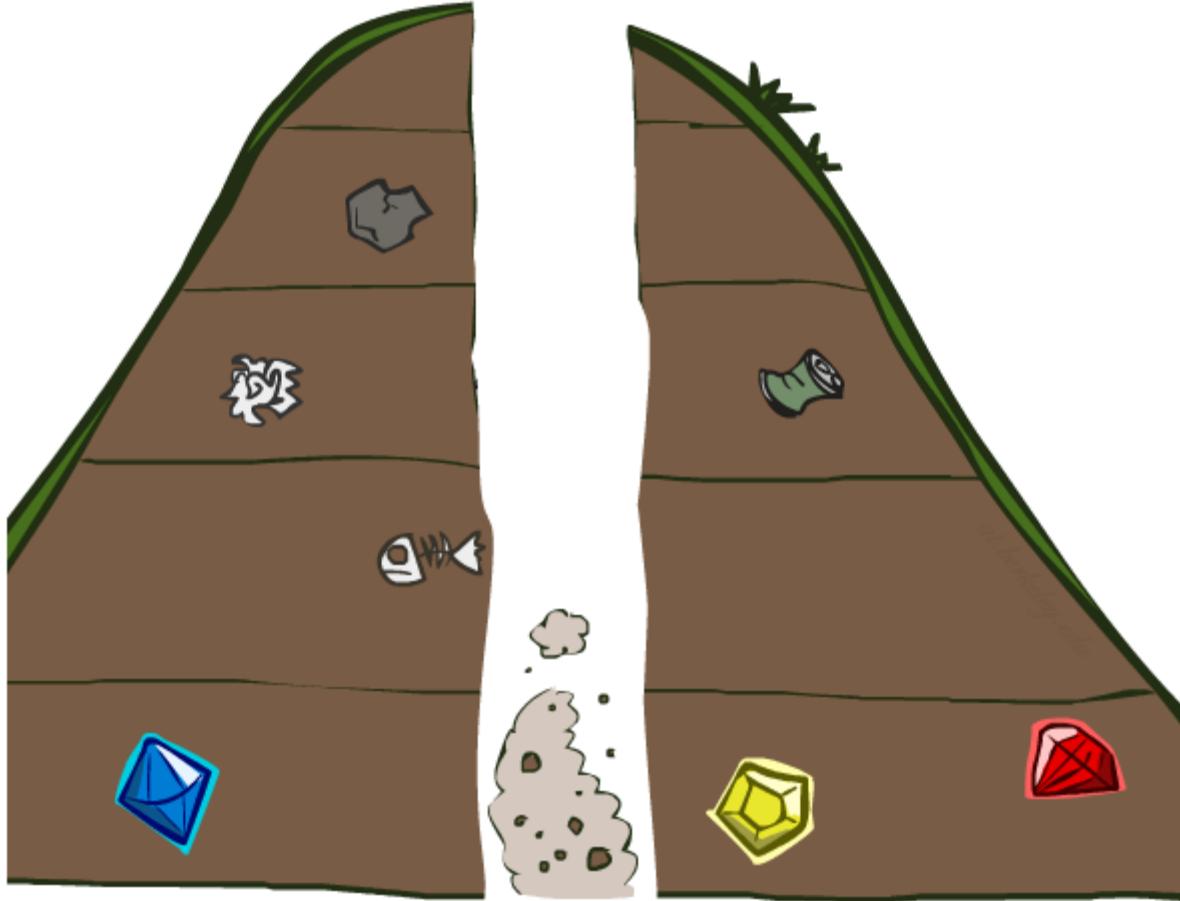
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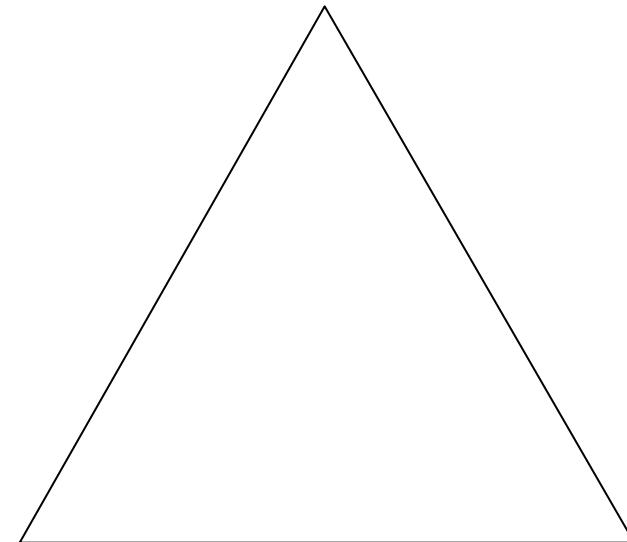
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- Cartoon of search tree:

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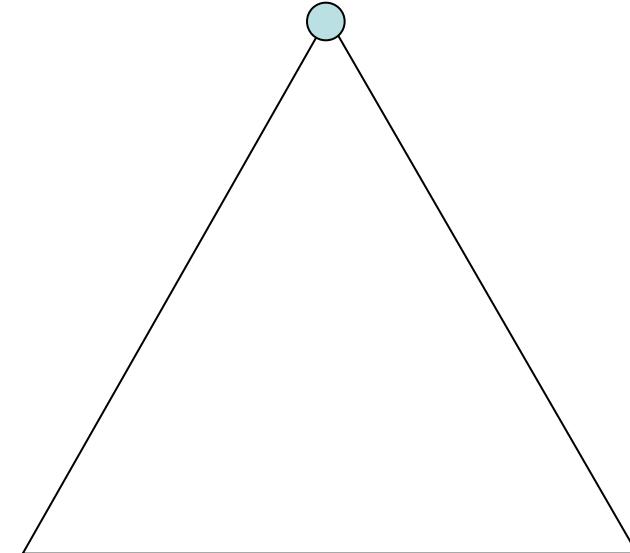
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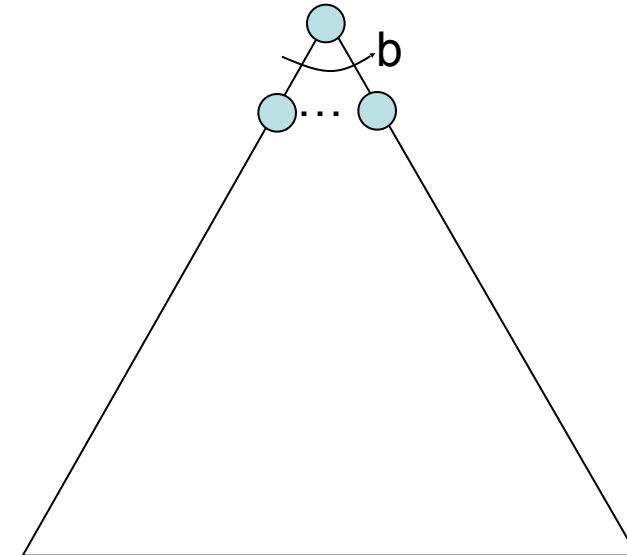
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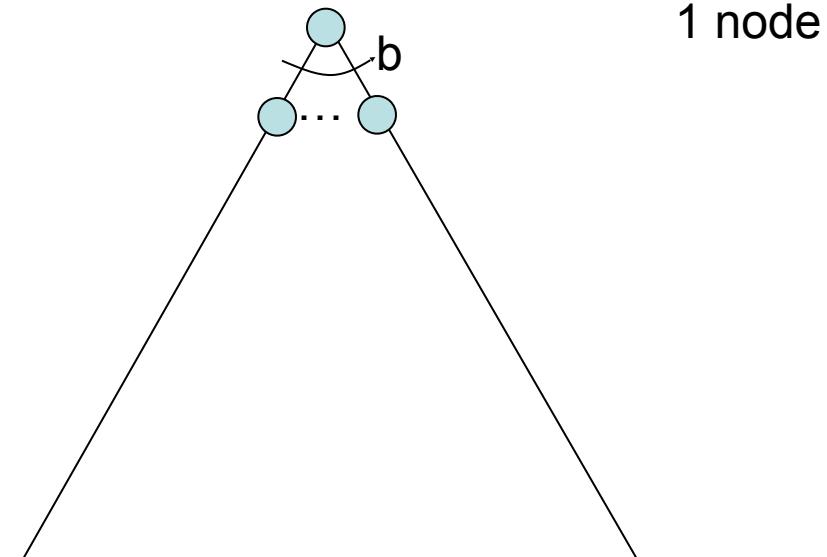
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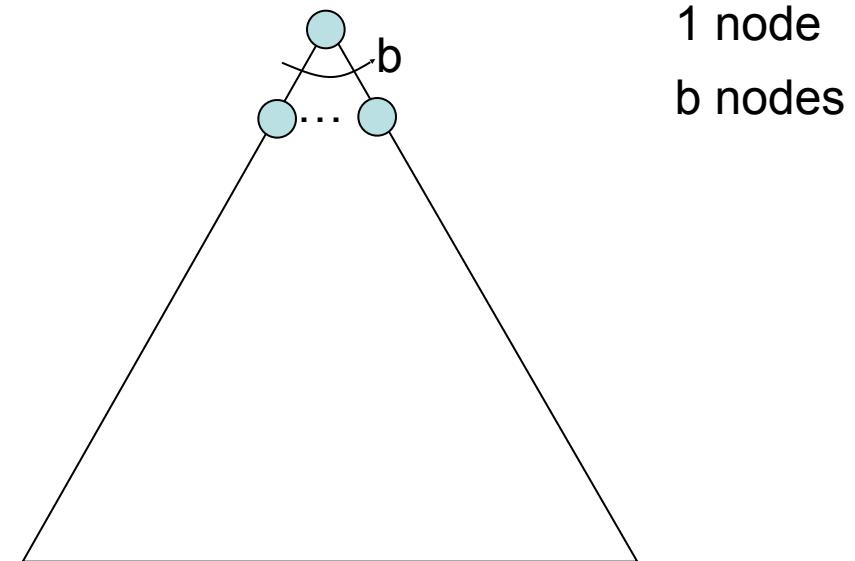
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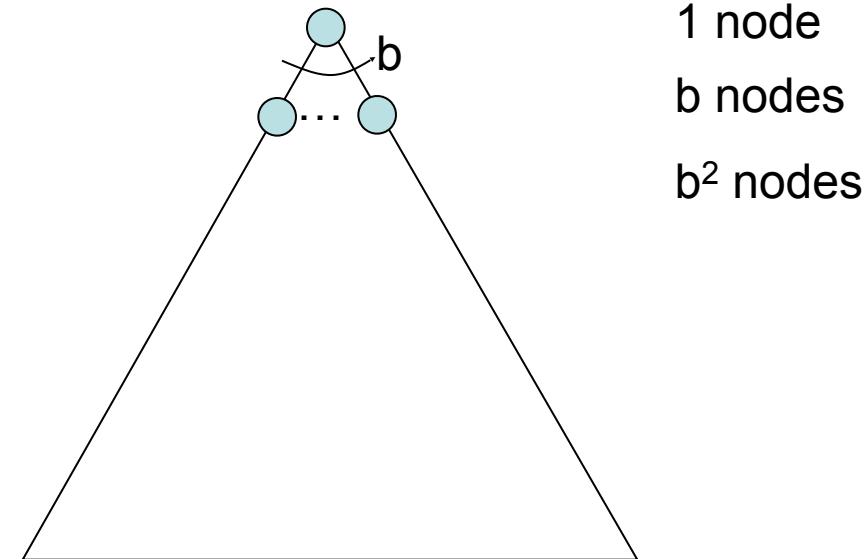
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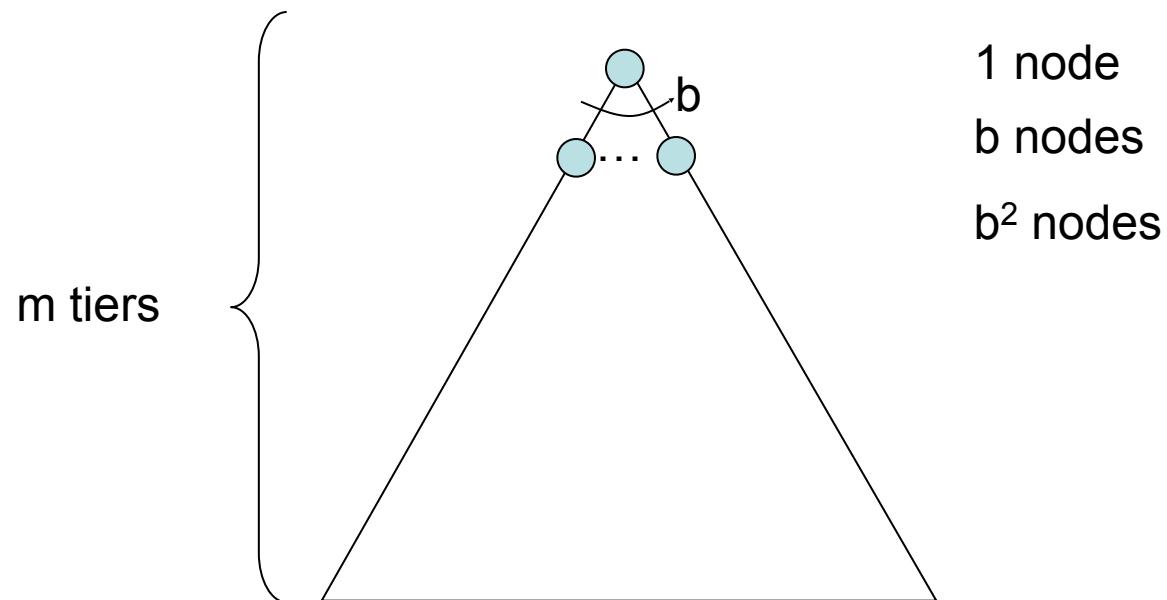
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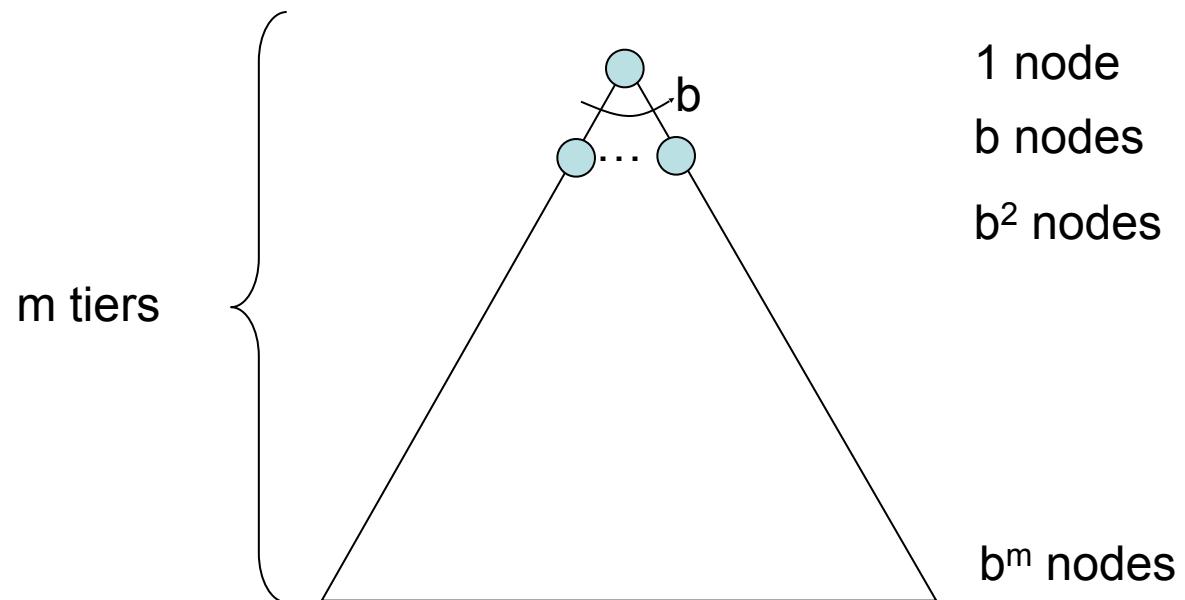
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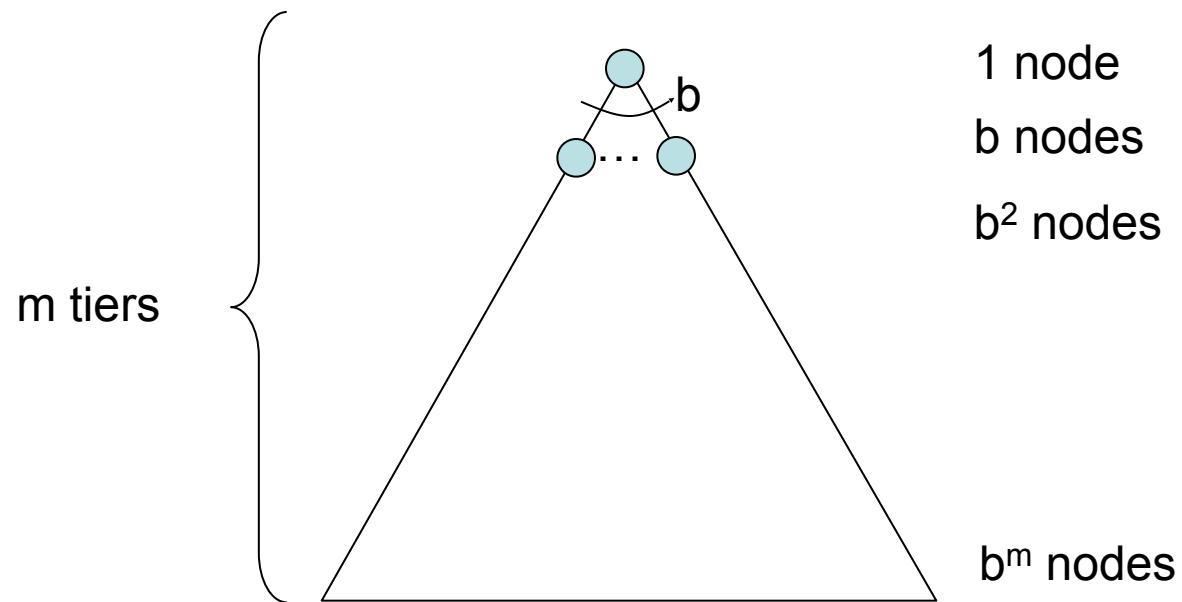
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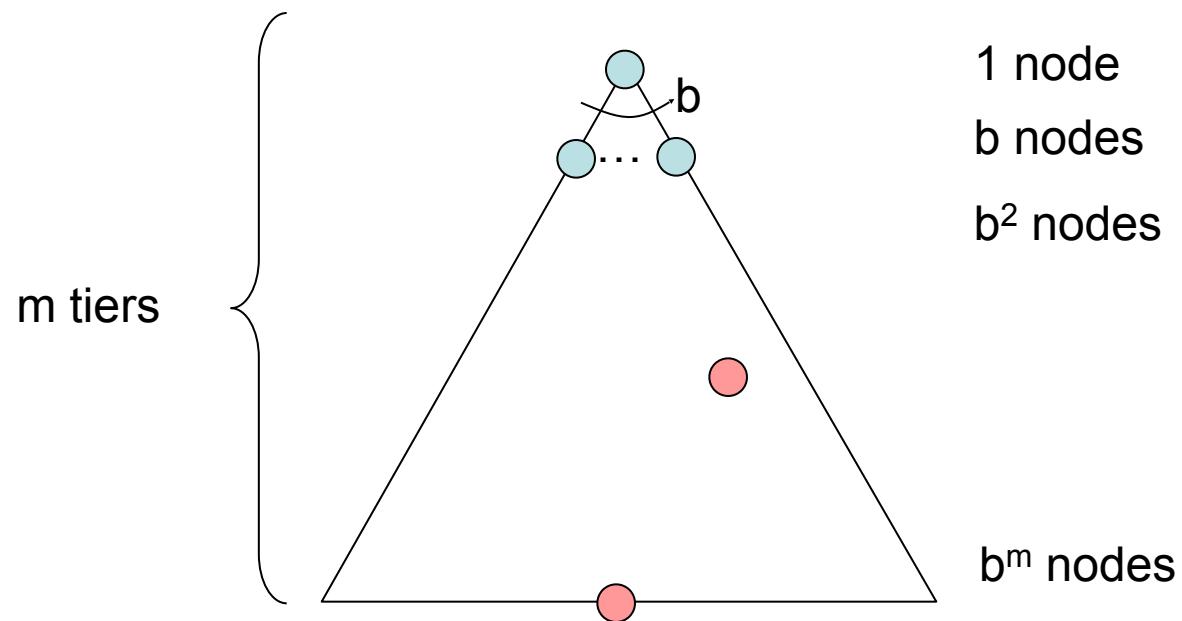
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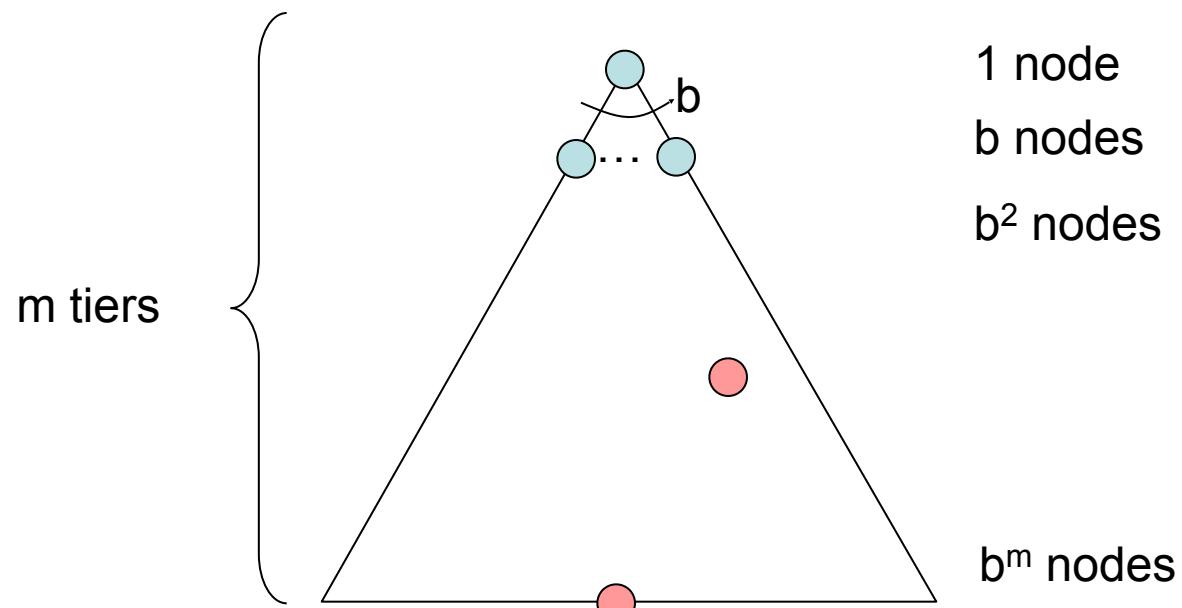
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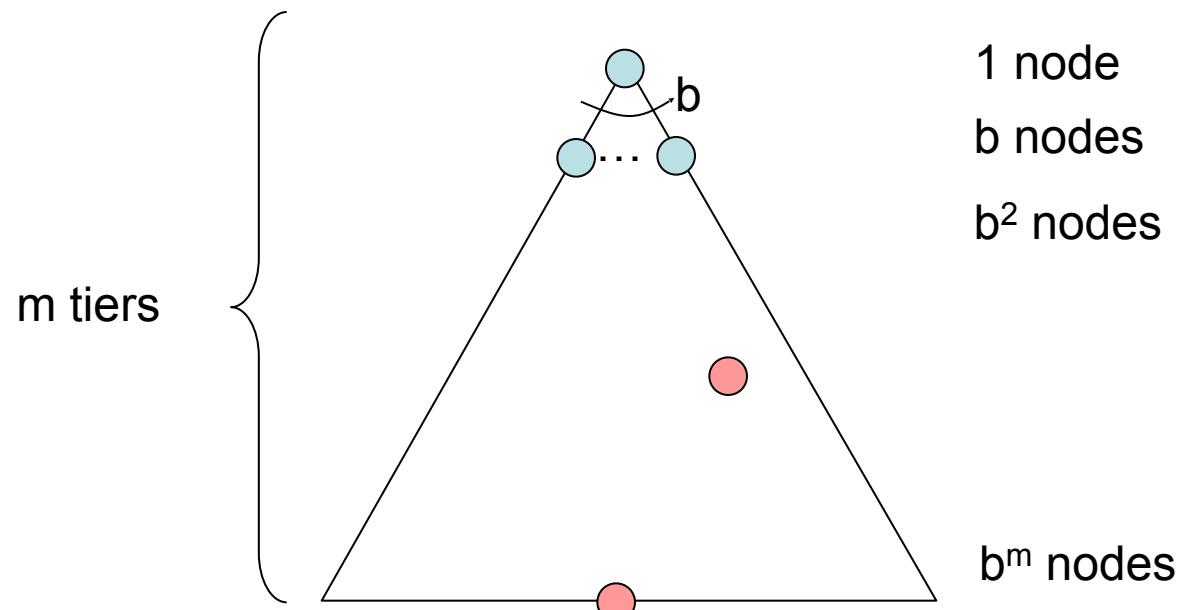
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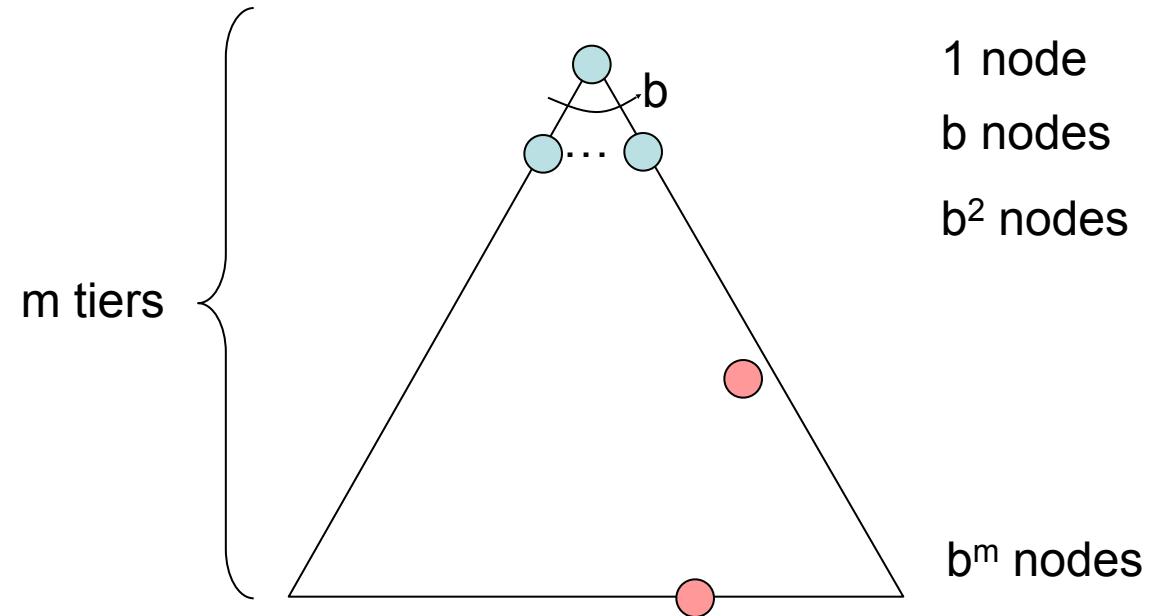
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  - $1 + b + b^2 + \dots + b^m = O(b^m)$



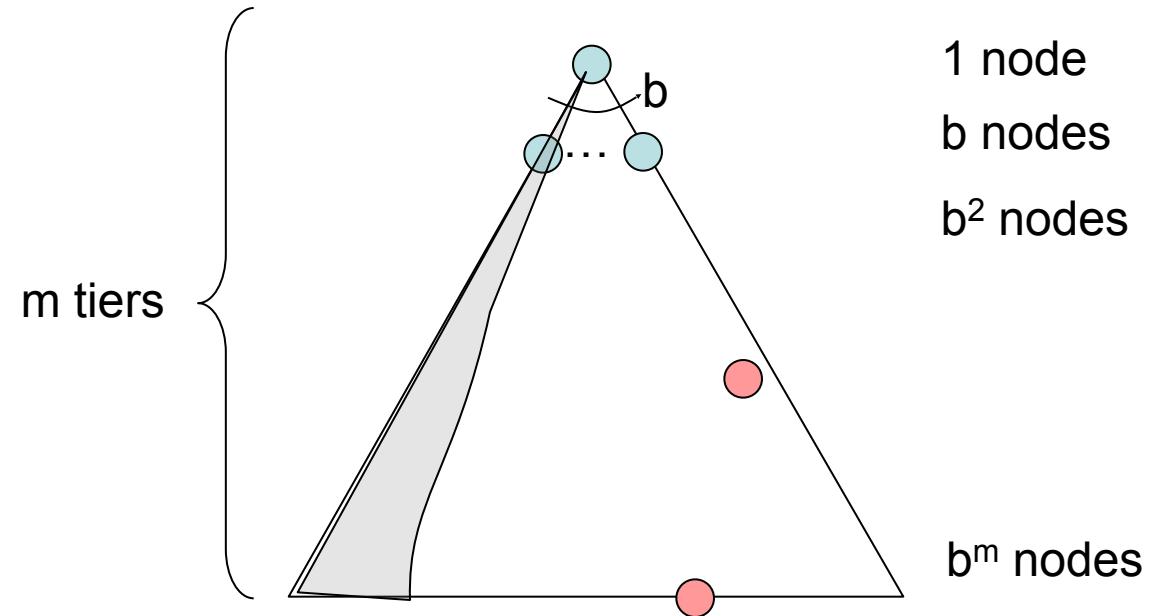
# Depth-First Search (DFS) Properties

- What nodes DFS expand?



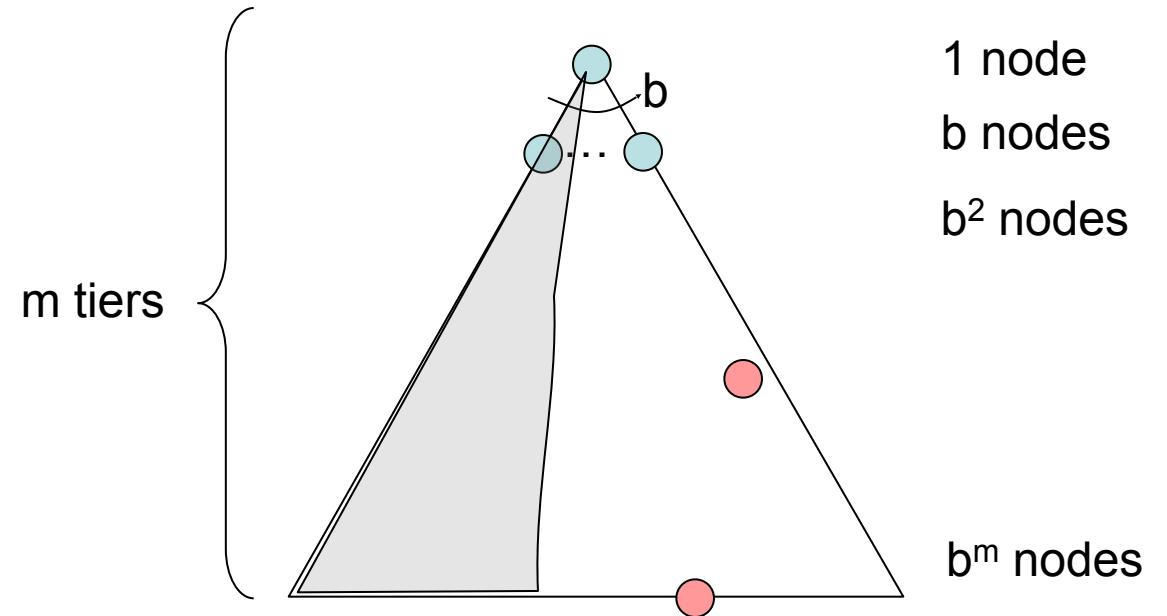
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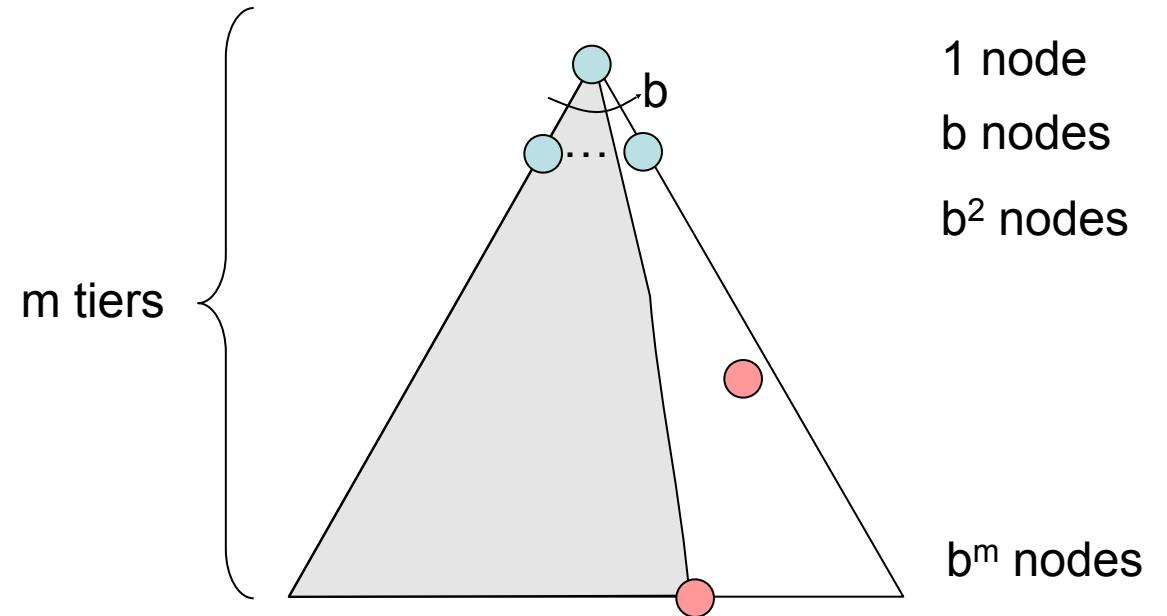
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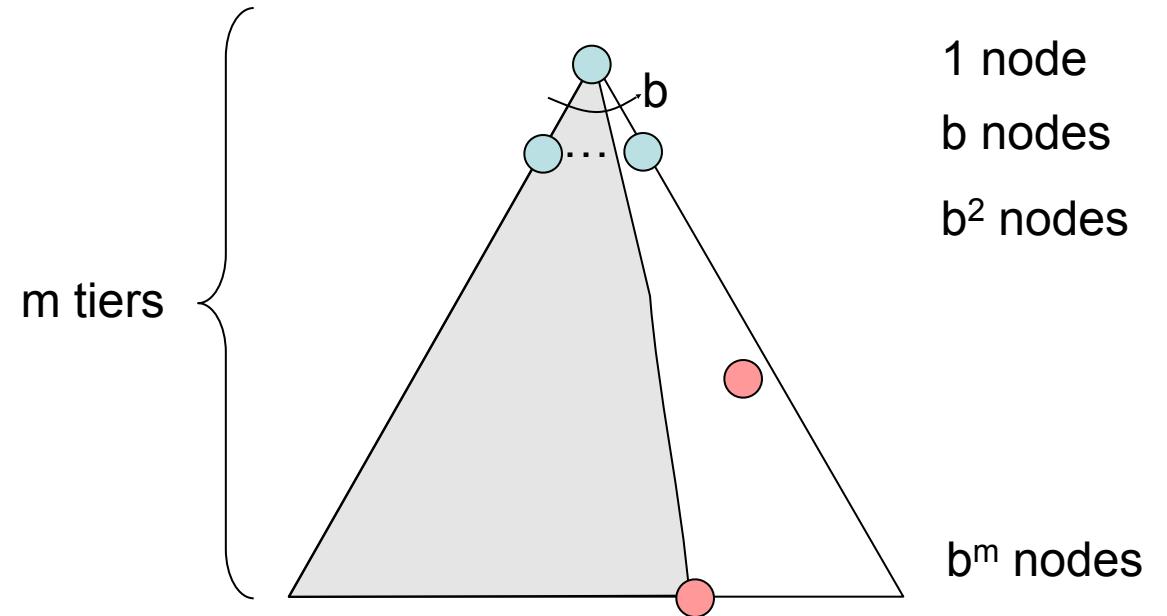
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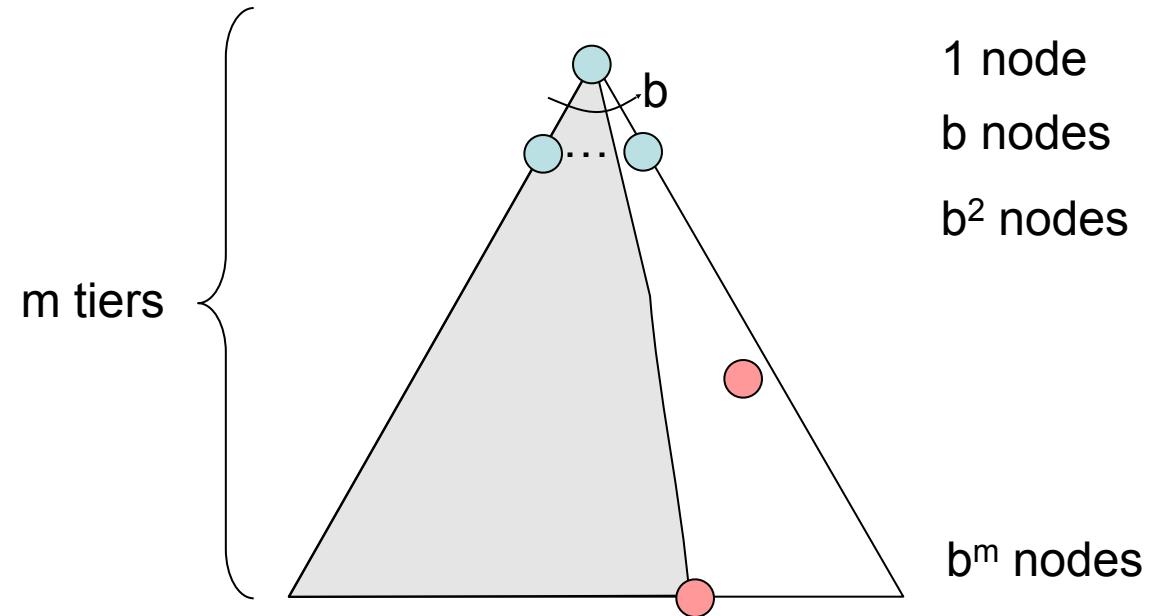
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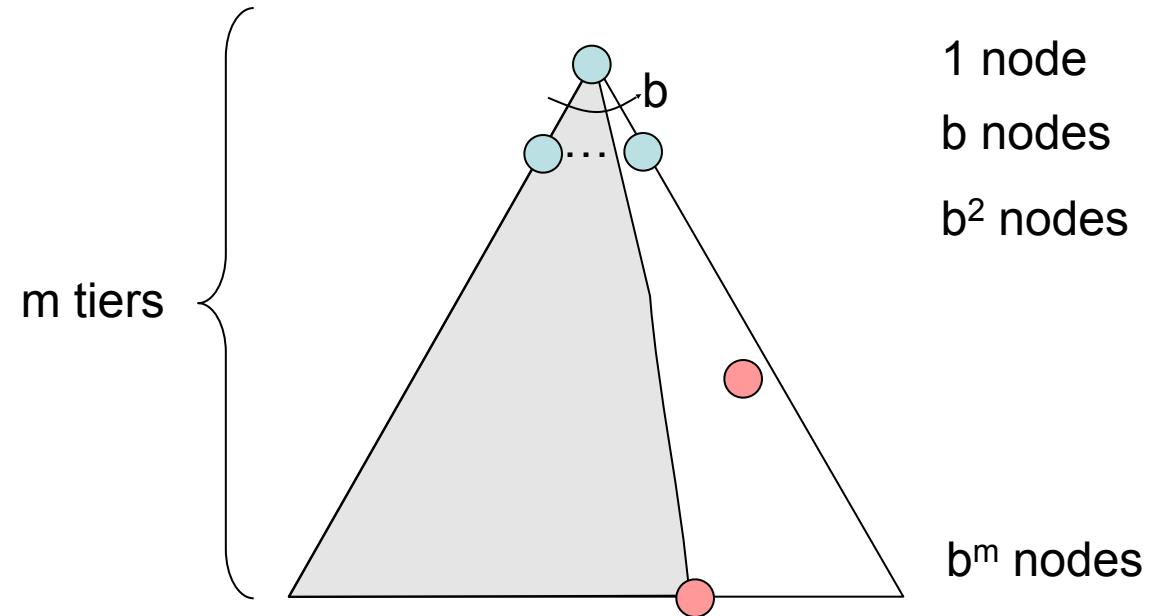
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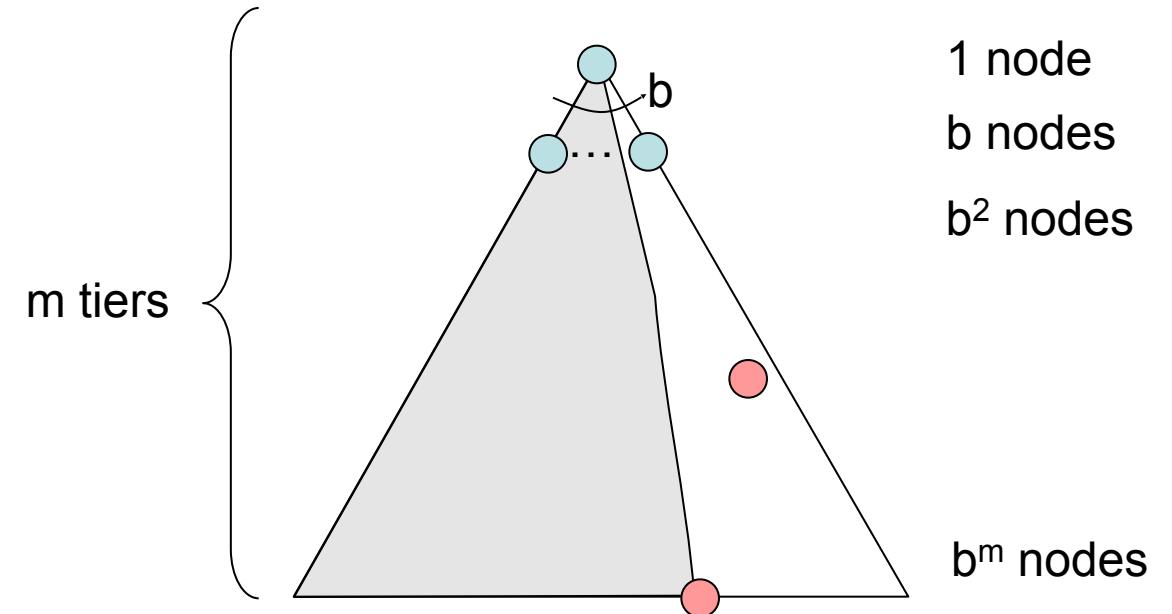


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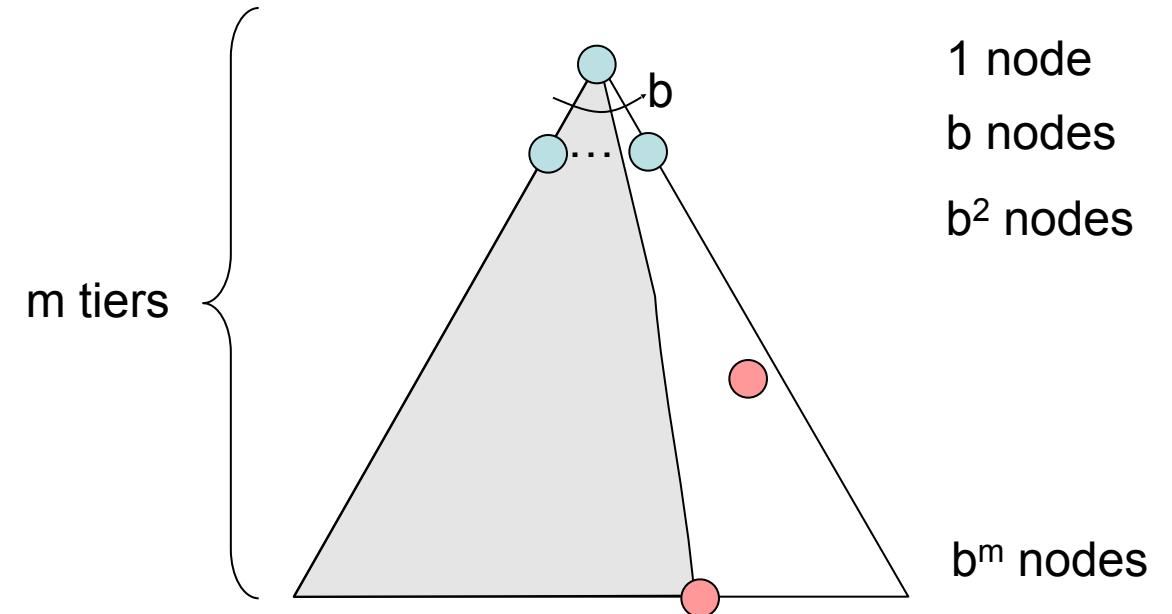
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- What nodes DFS expand?

- Some left prefix of the tree.
- Could process the whole tree!
- If  $m$  is finite, takes time  $O(b^m)$

- How much space does the fringe take?

- Only has siblings on path to root, so  $O(bm)$



# Depth-First Search (DFS) Properties

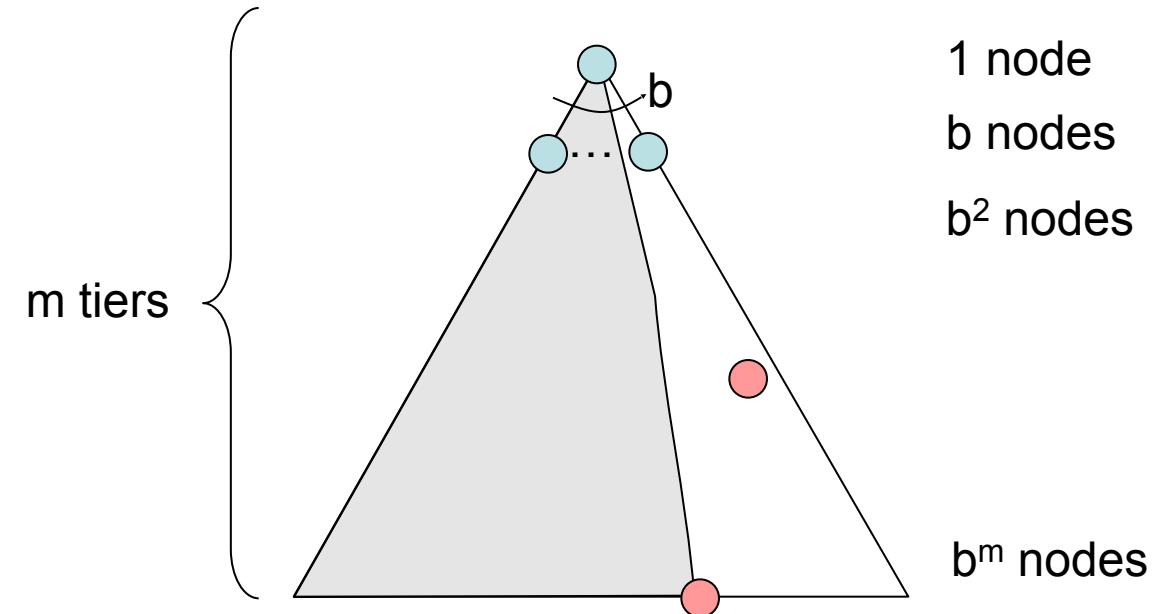
- What nodes DFS expand?

- Some left prefix of the tree.
- Could process the whole tree!
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- Only has siblings on path to root, so  $O(bm)$

- Is it complete?



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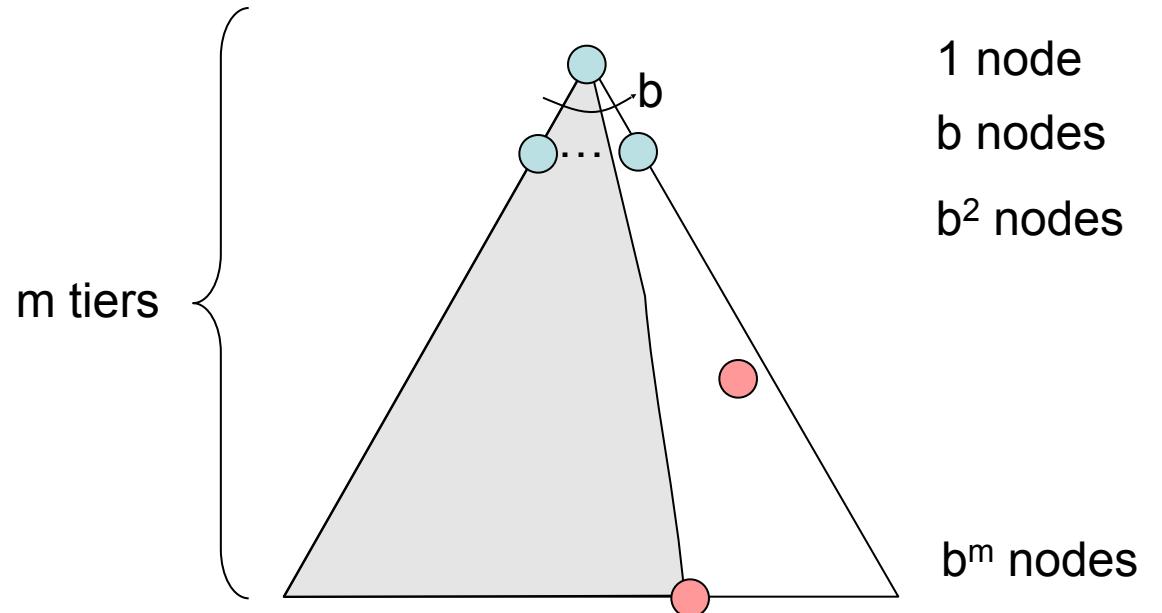
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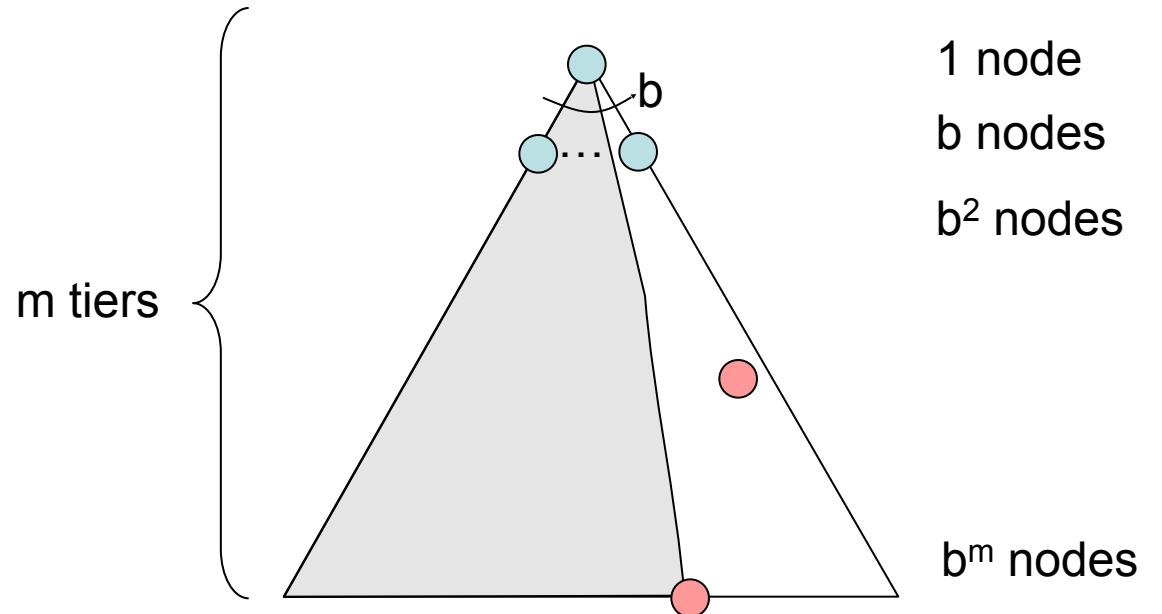
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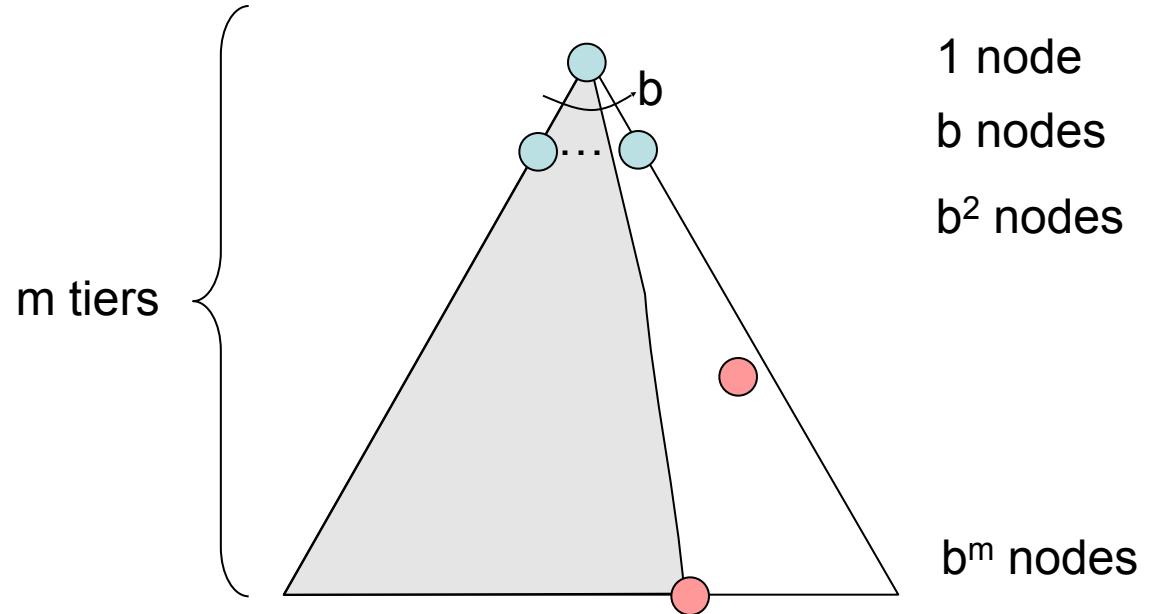
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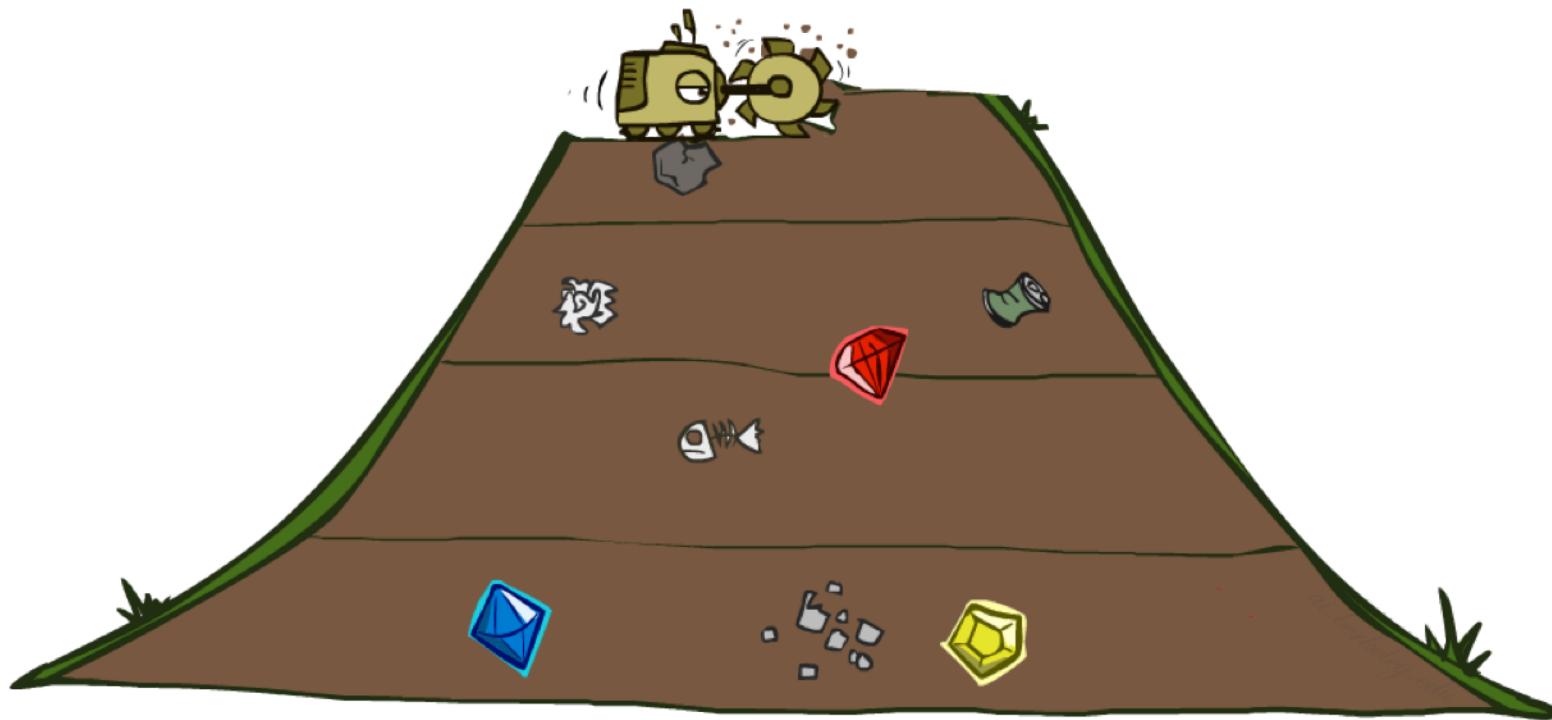
- Is it optimal?

- No, it finds the “leftmost” solution, regardless of depth or cost



# Breadth-First Search

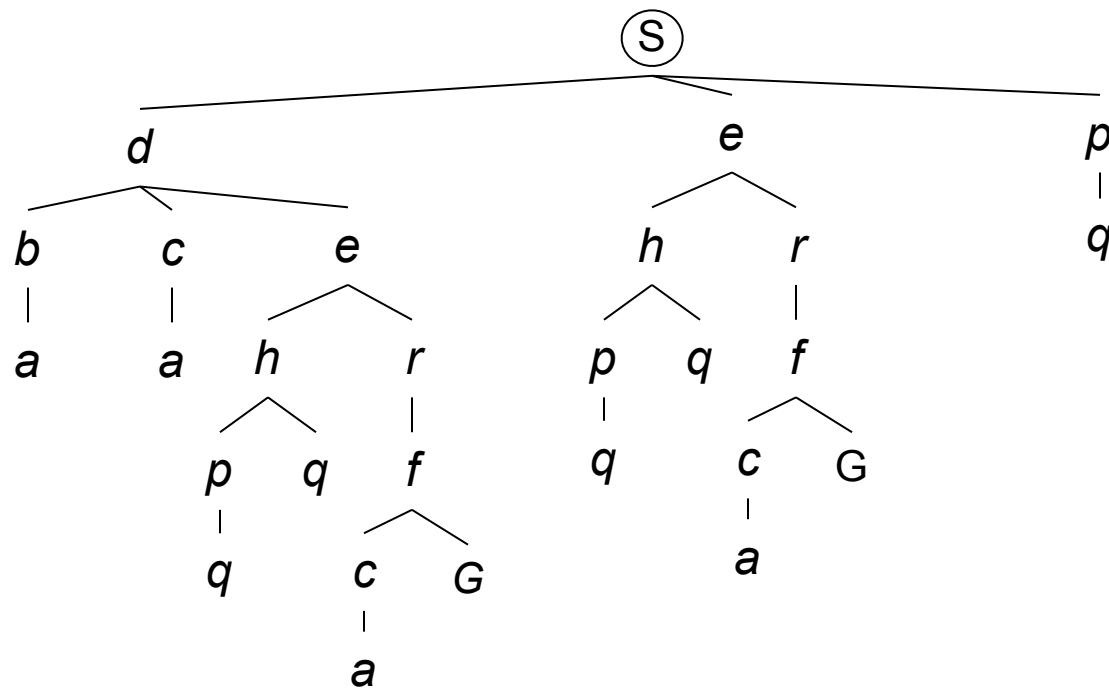
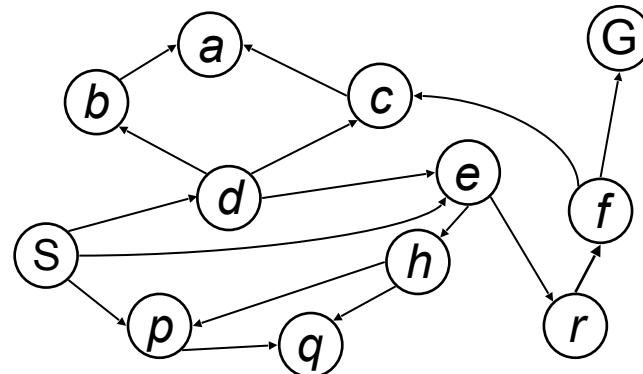
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# Breadth-First Search

*Strategy: expand a shallowest node first*

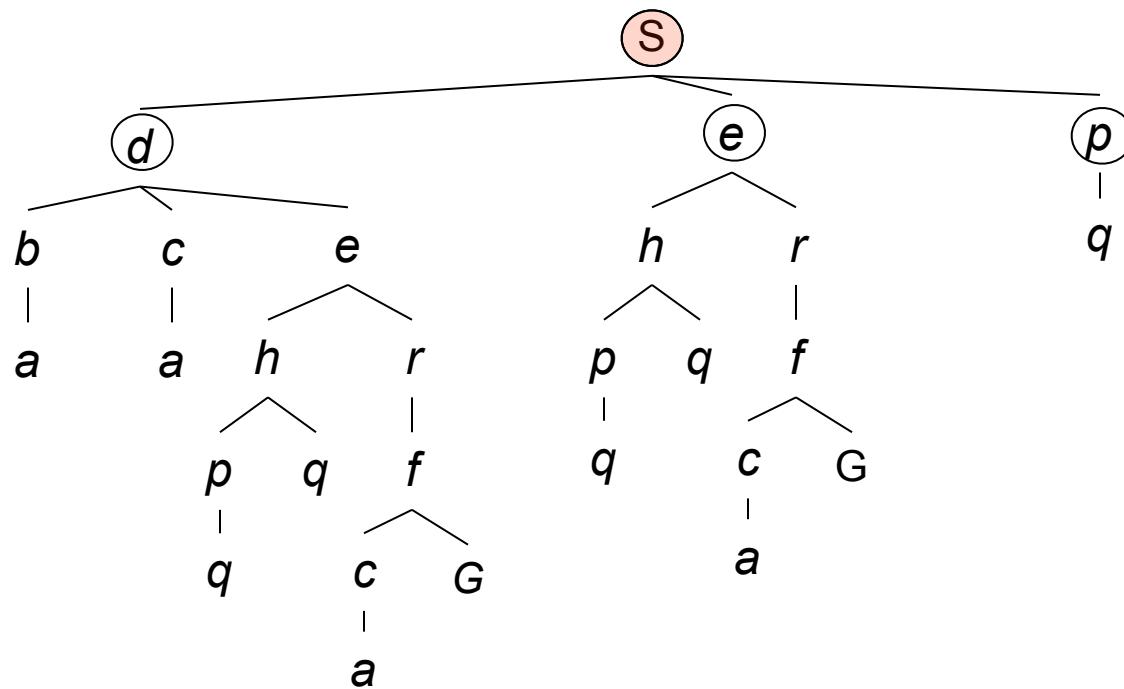
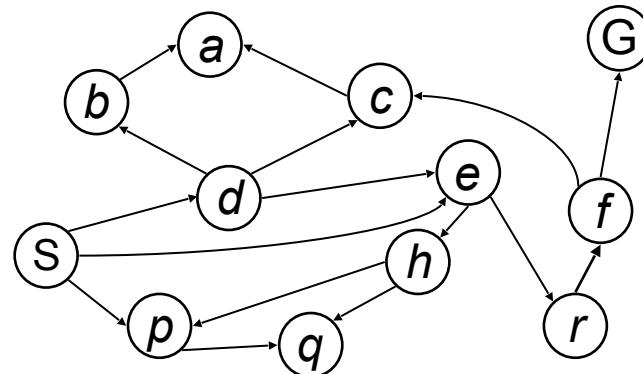
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Fringe is a FIFO  
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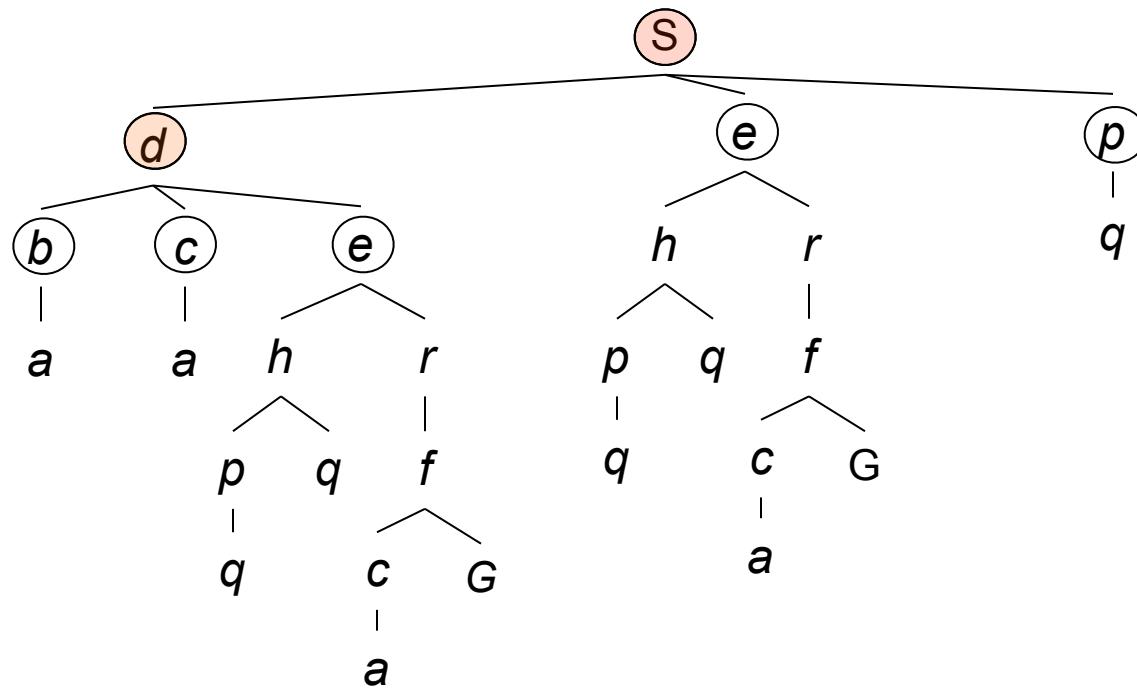
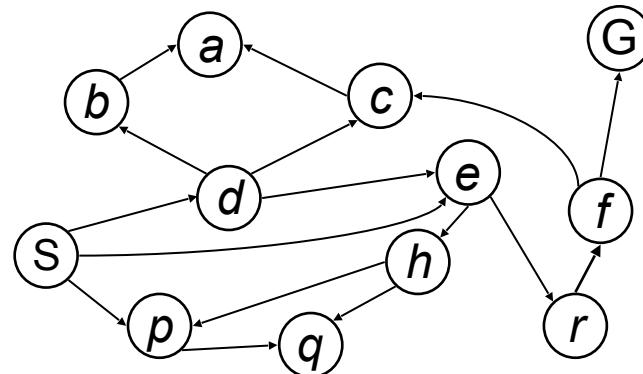
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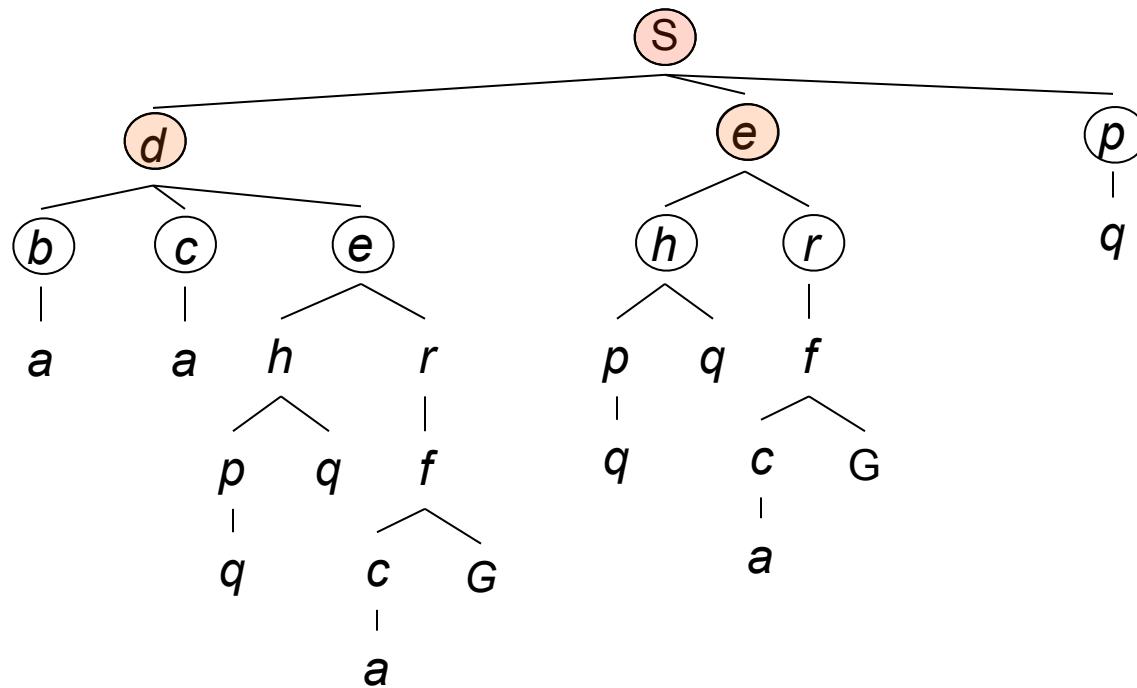
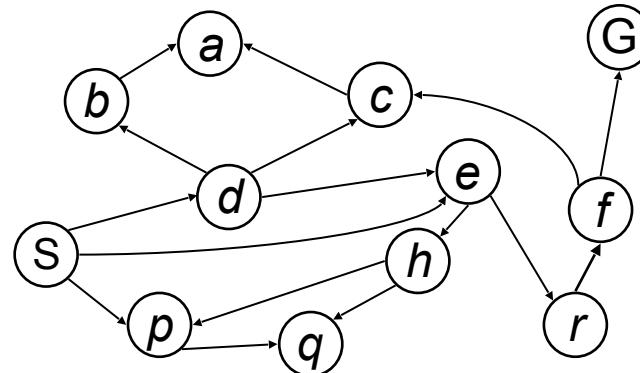
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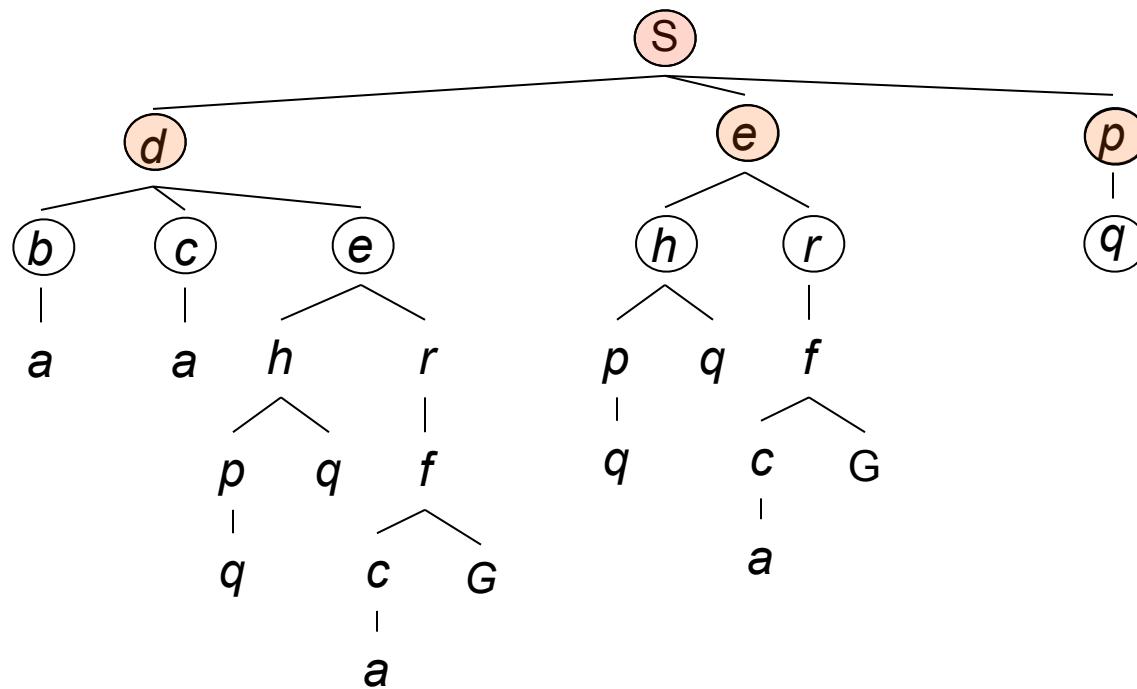
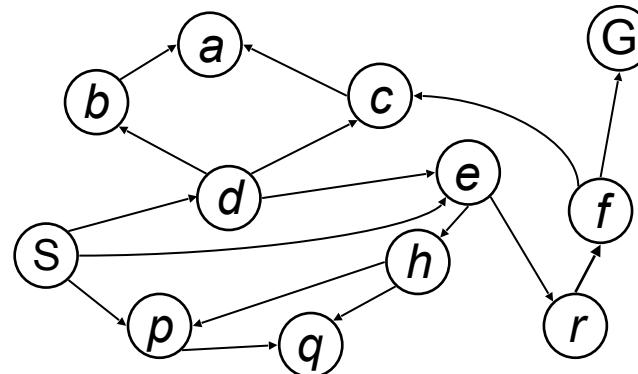
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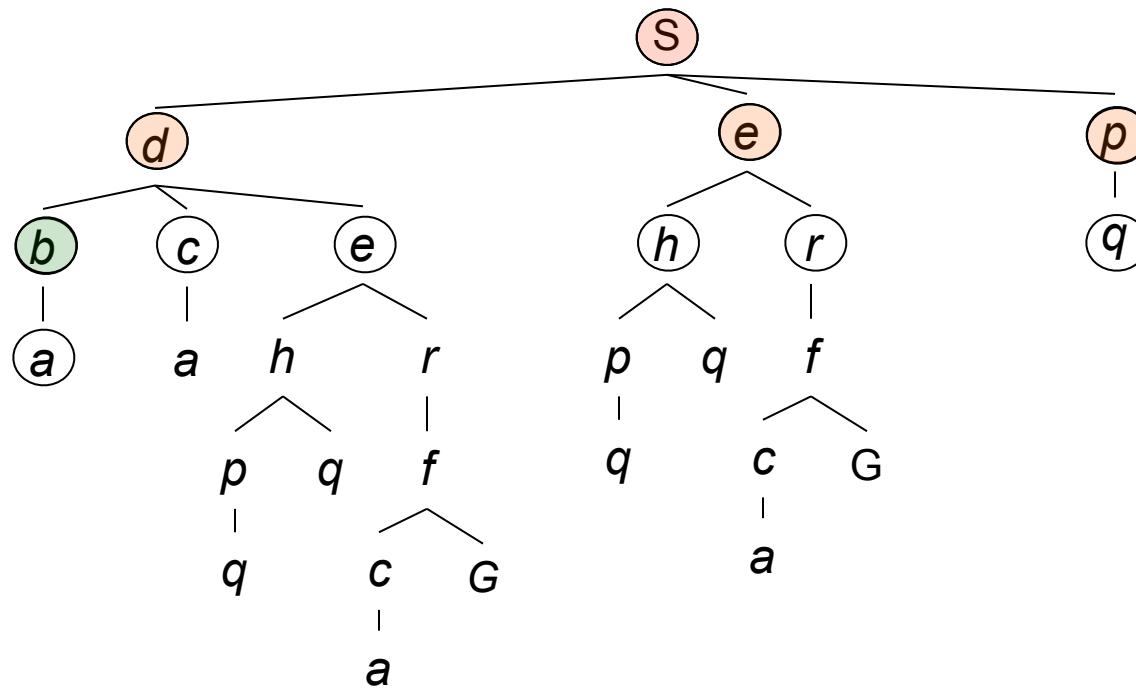
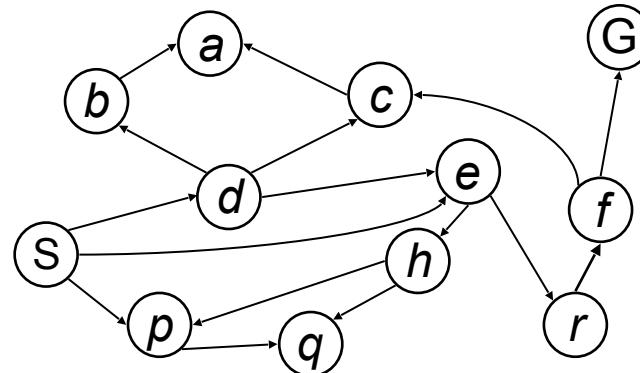
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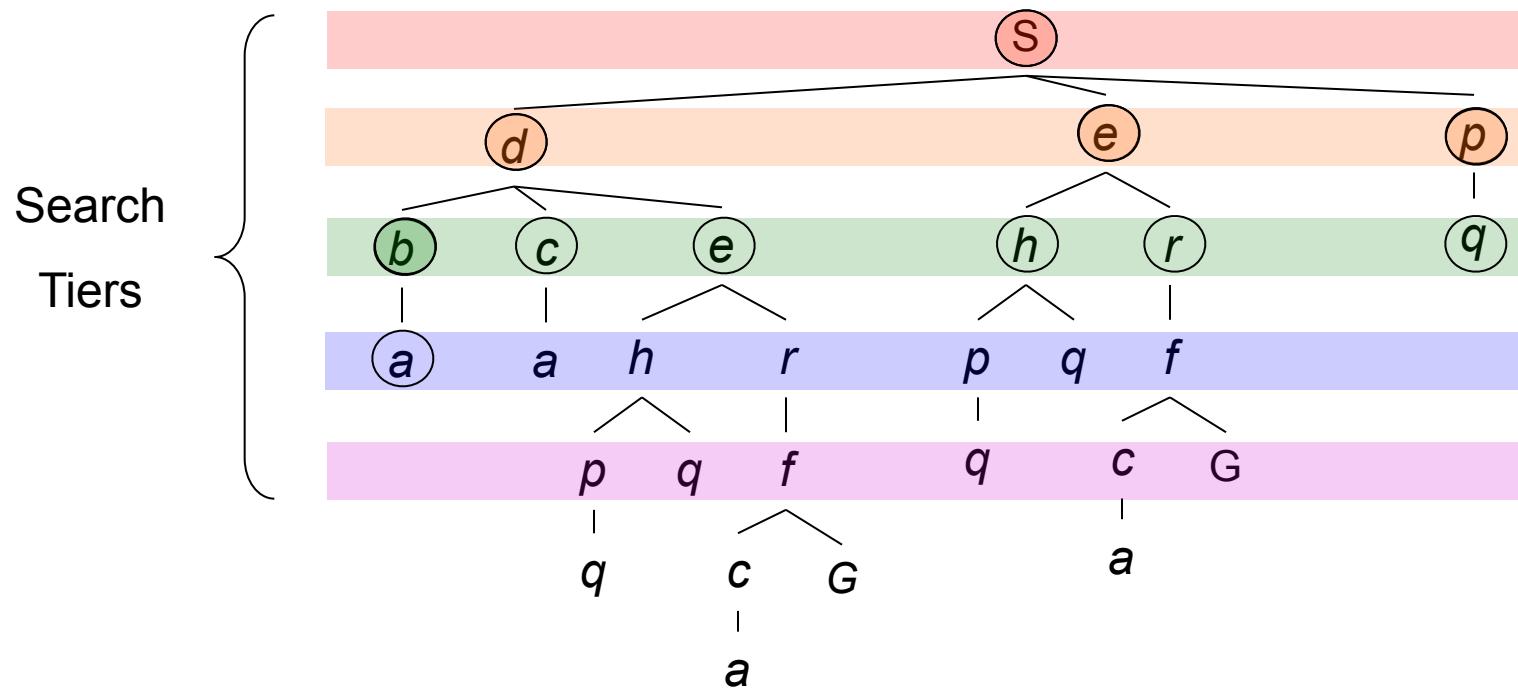
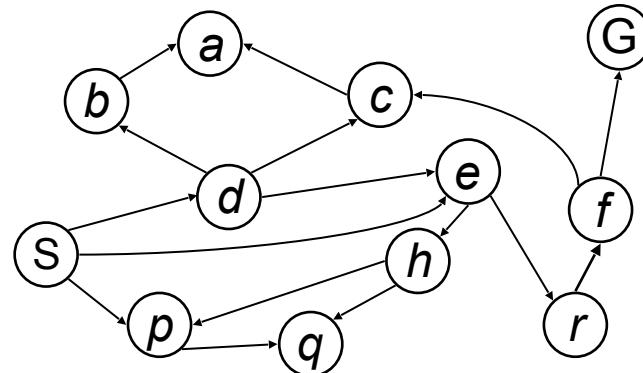
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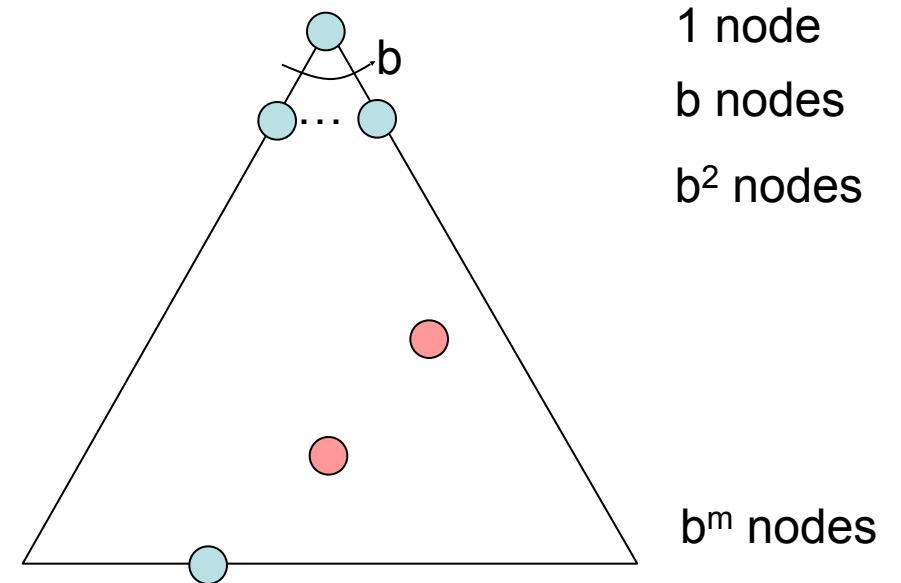
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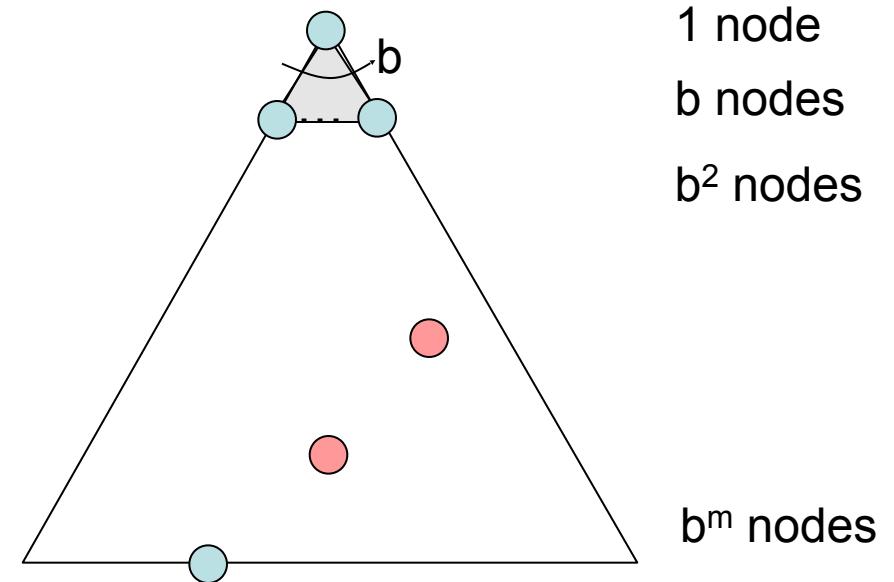
# Breadth-First Search (BFS) Properties

- What nodes does BFS expand?



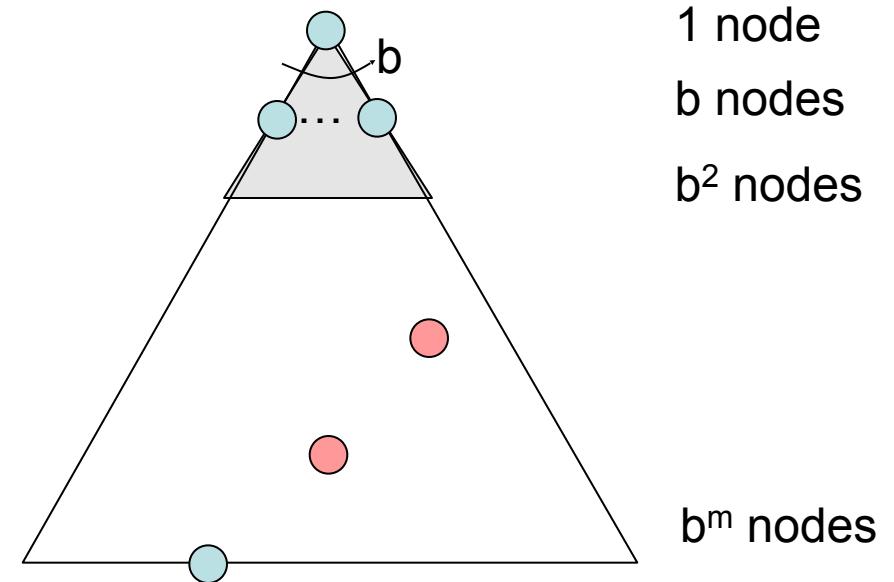
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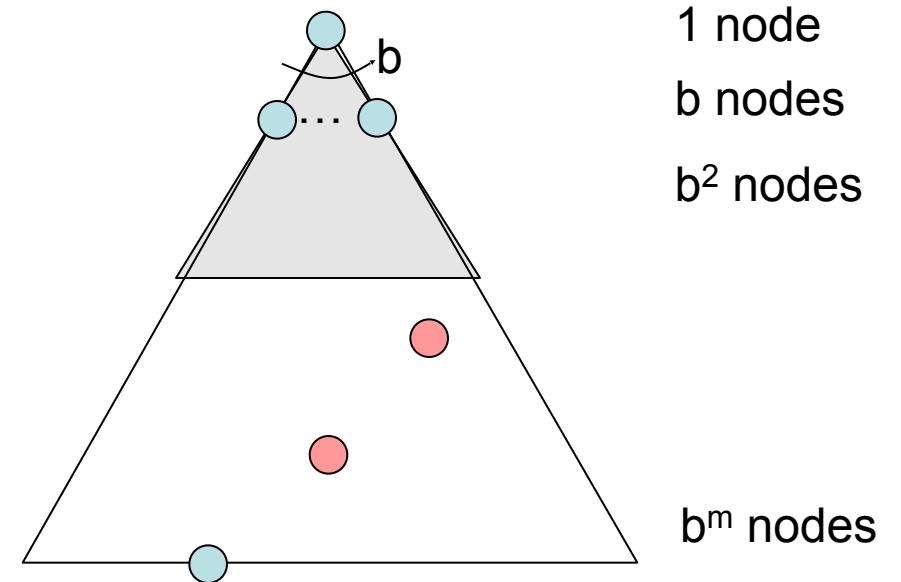
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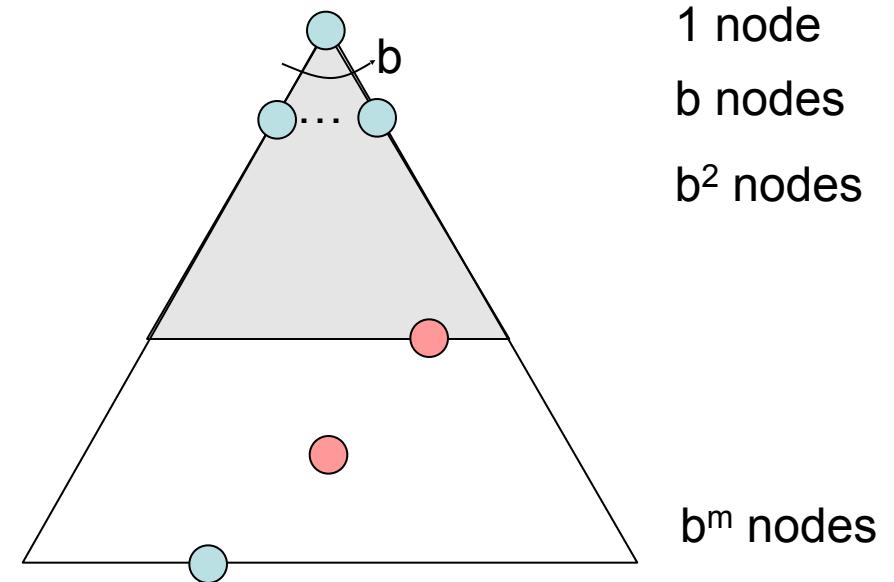
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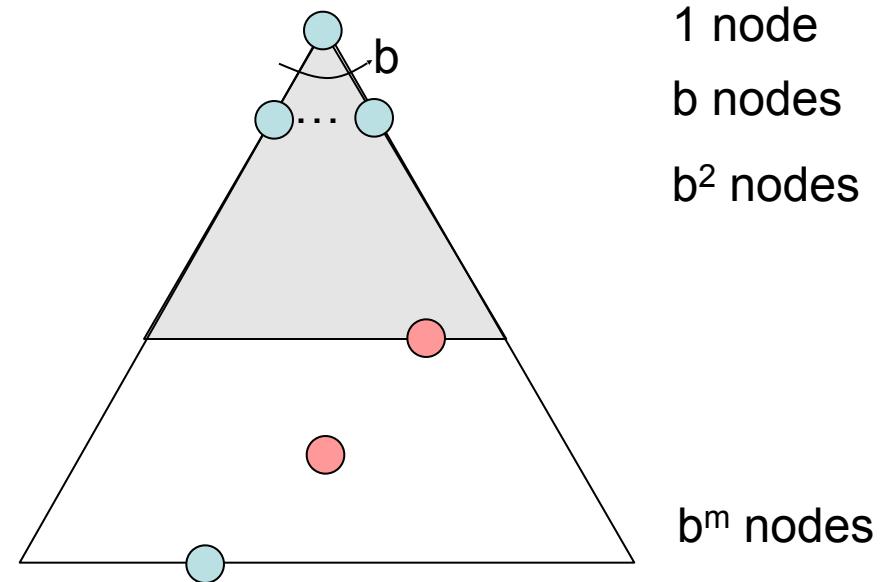
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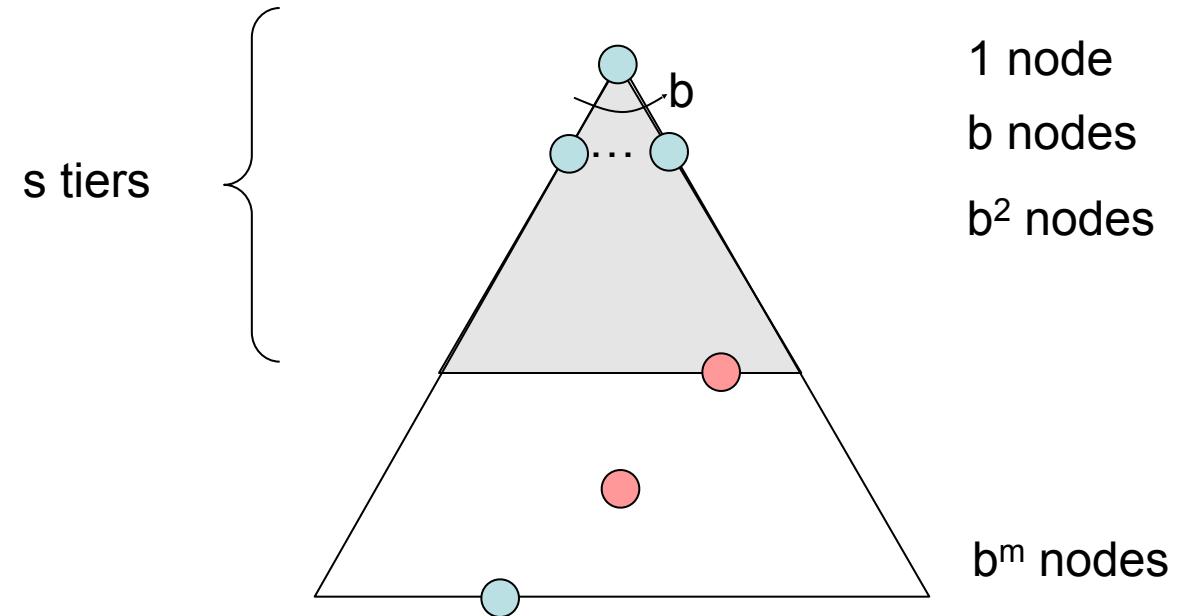
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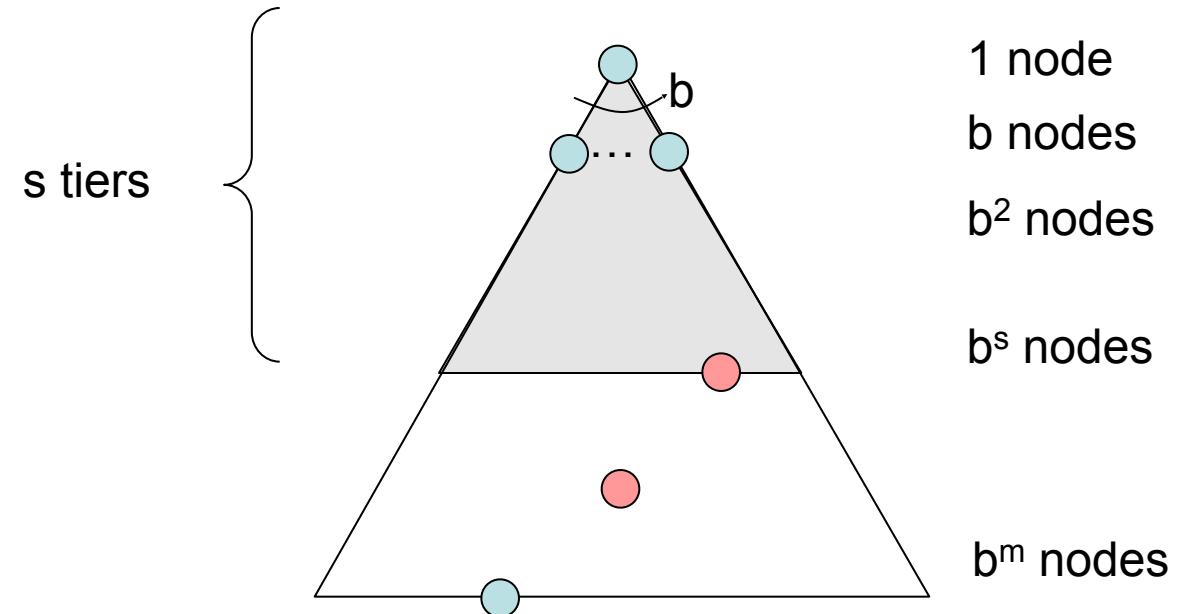
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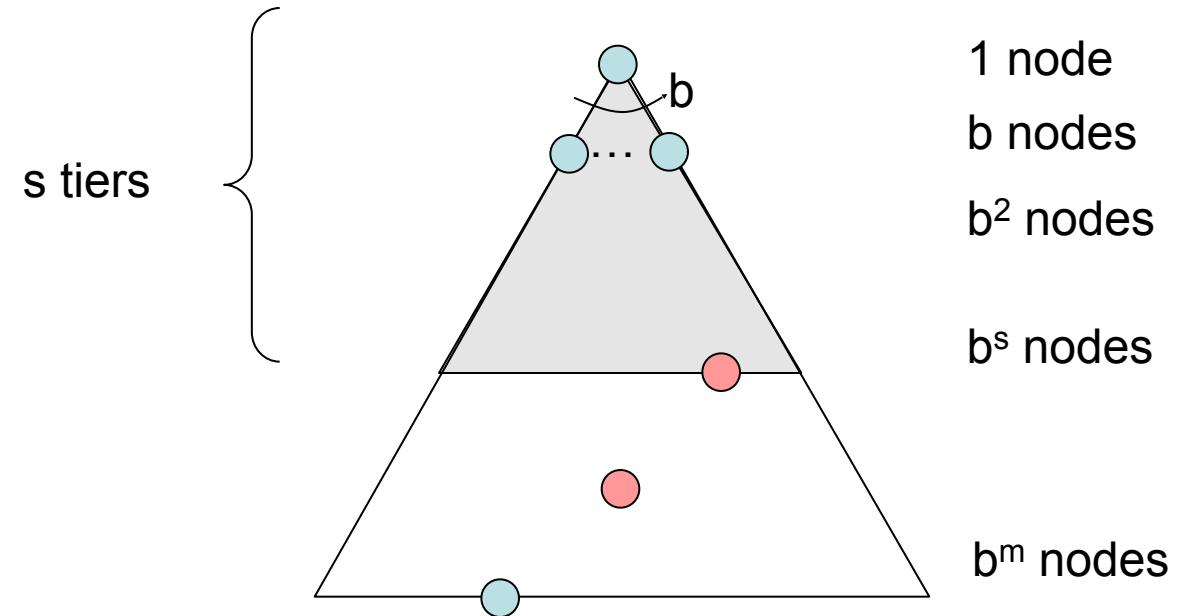
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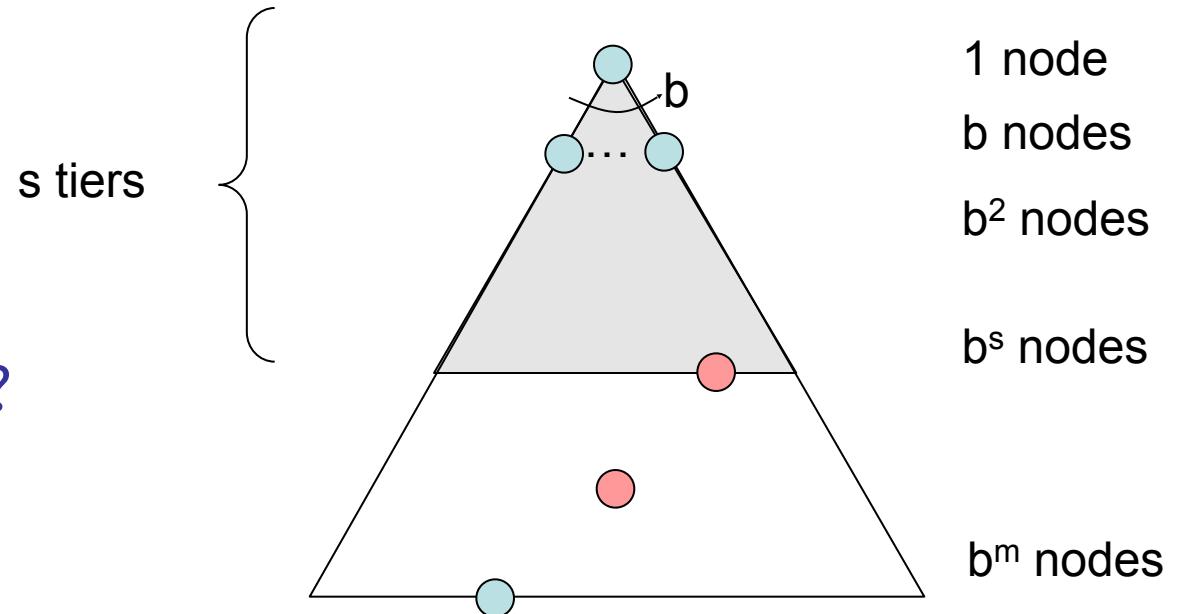


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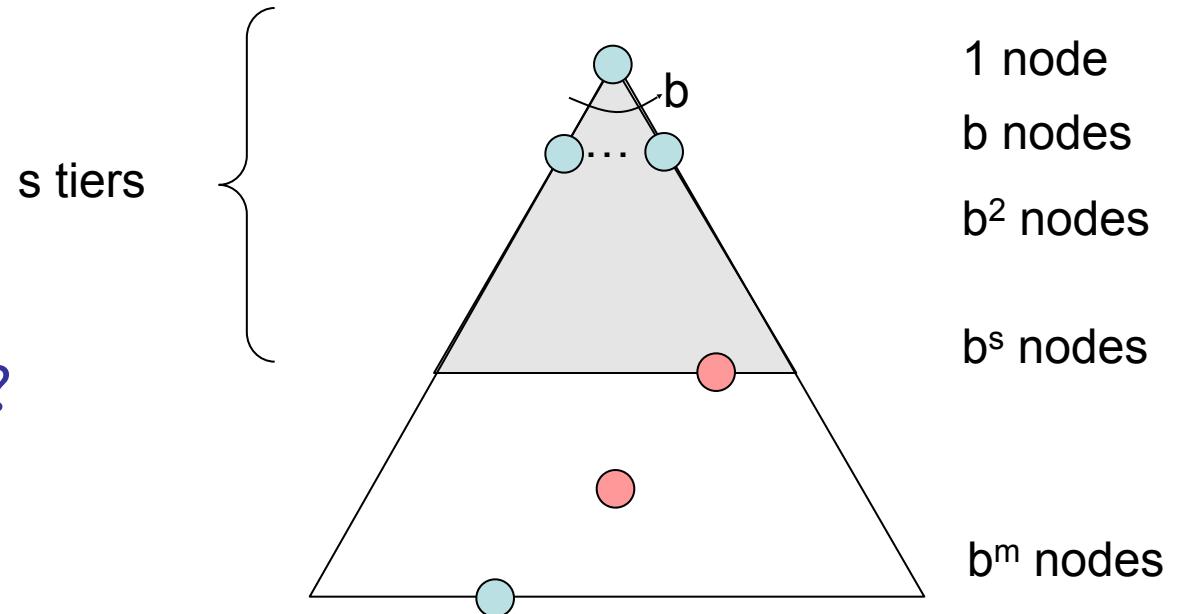
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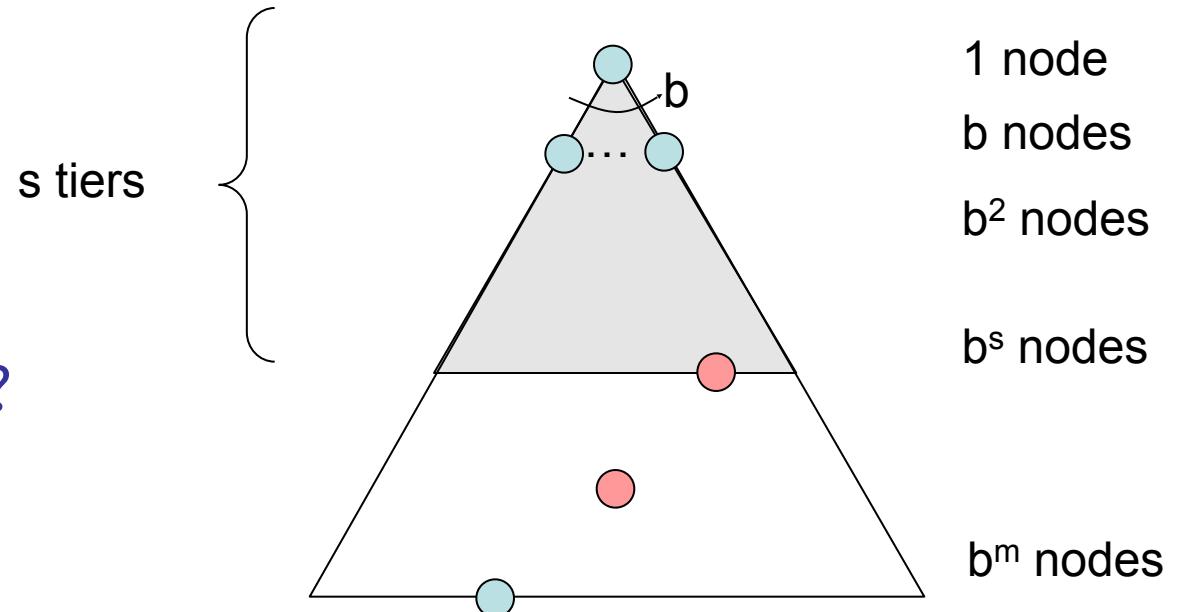
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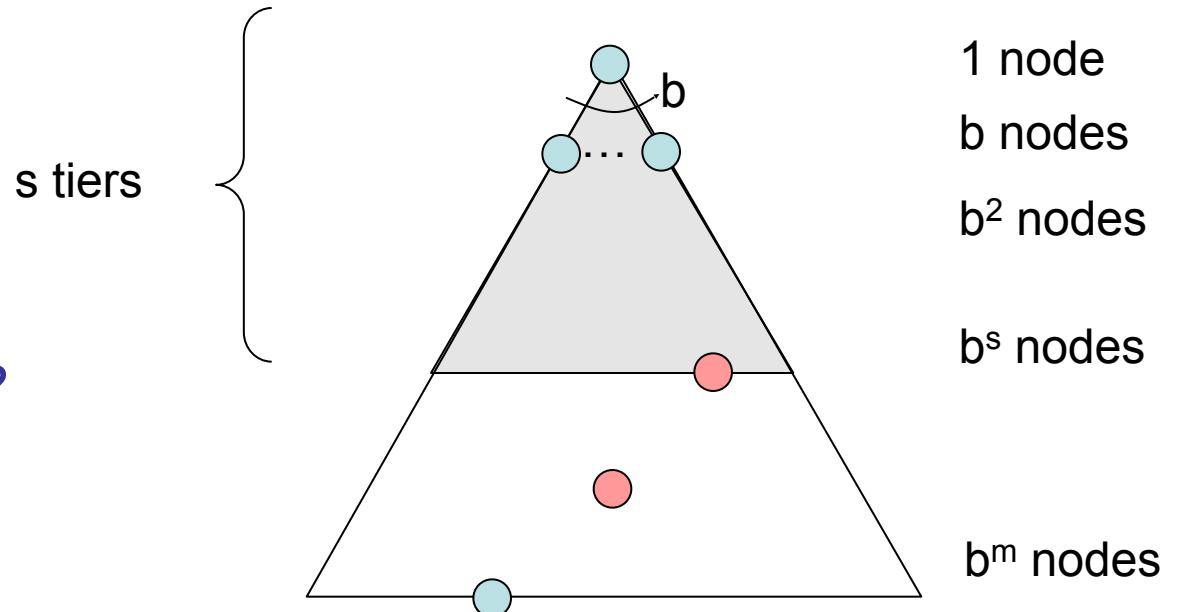
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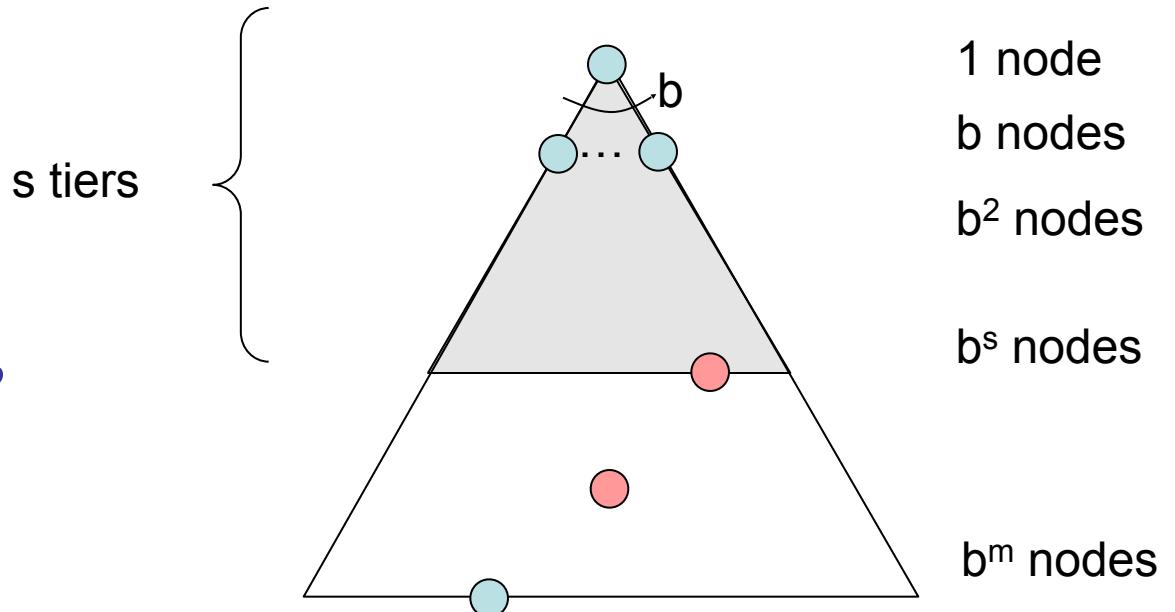
- Is it complete?

- $s$  must be finite if a solution exists, so yes!



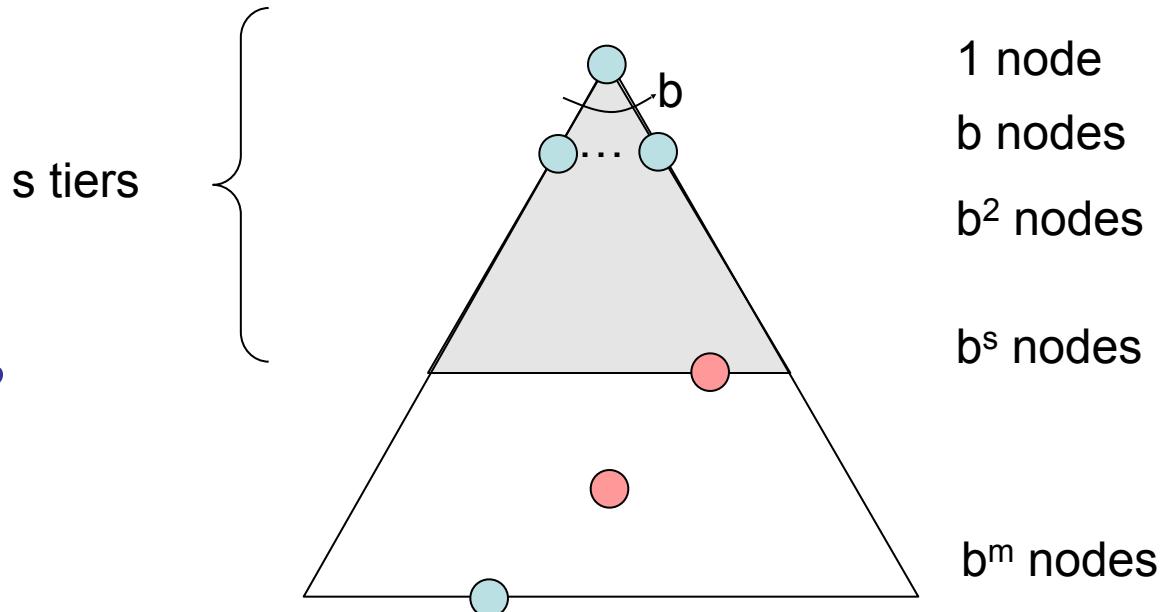
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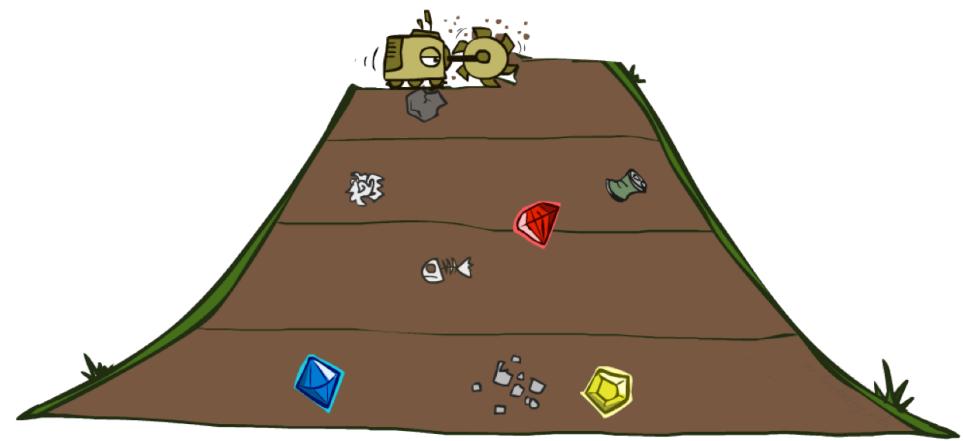
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  - Is it optimal?
    - Only if costs are all 1 (more on costs later)



# Quiz: DFS vs BFS

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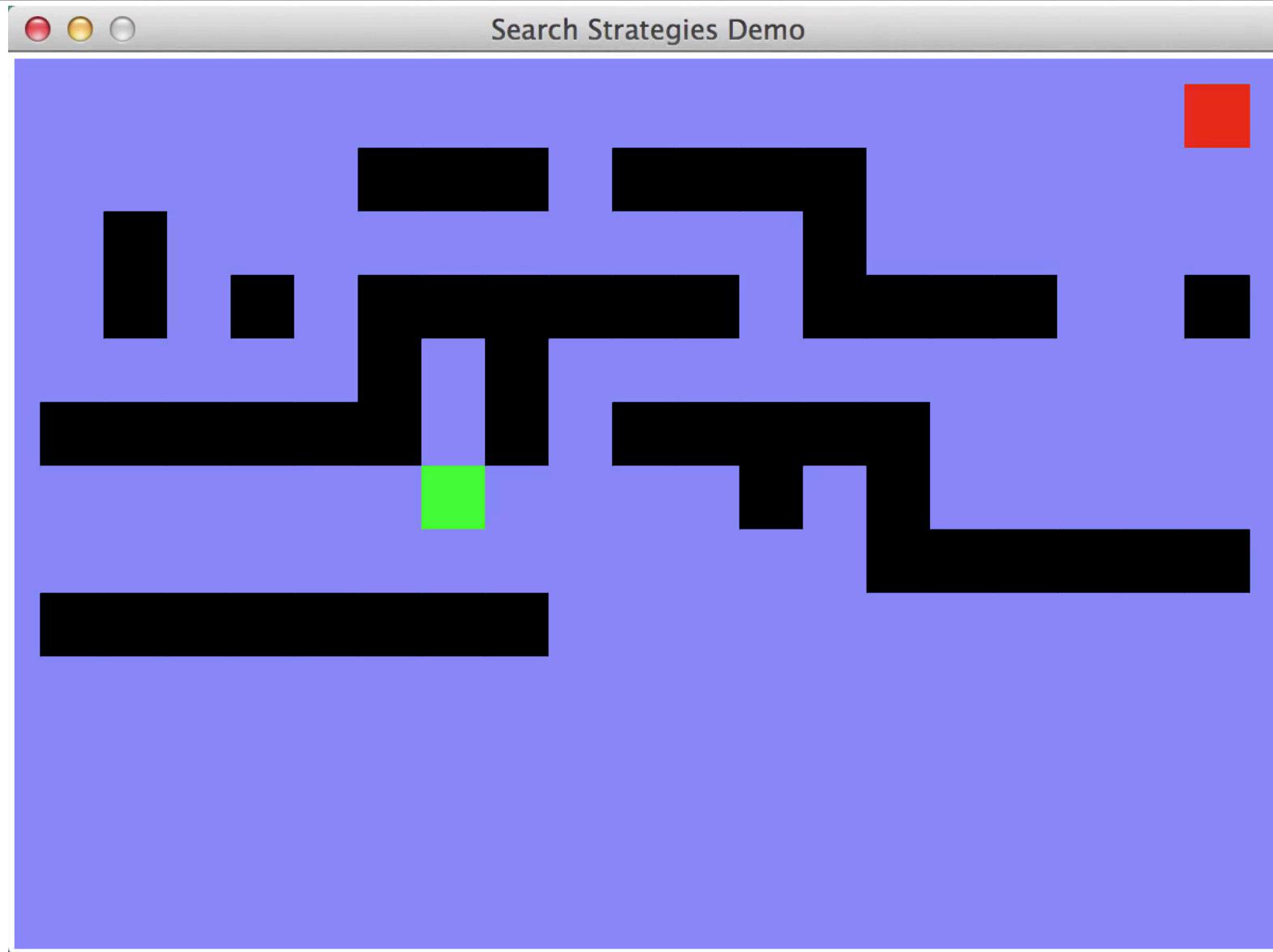


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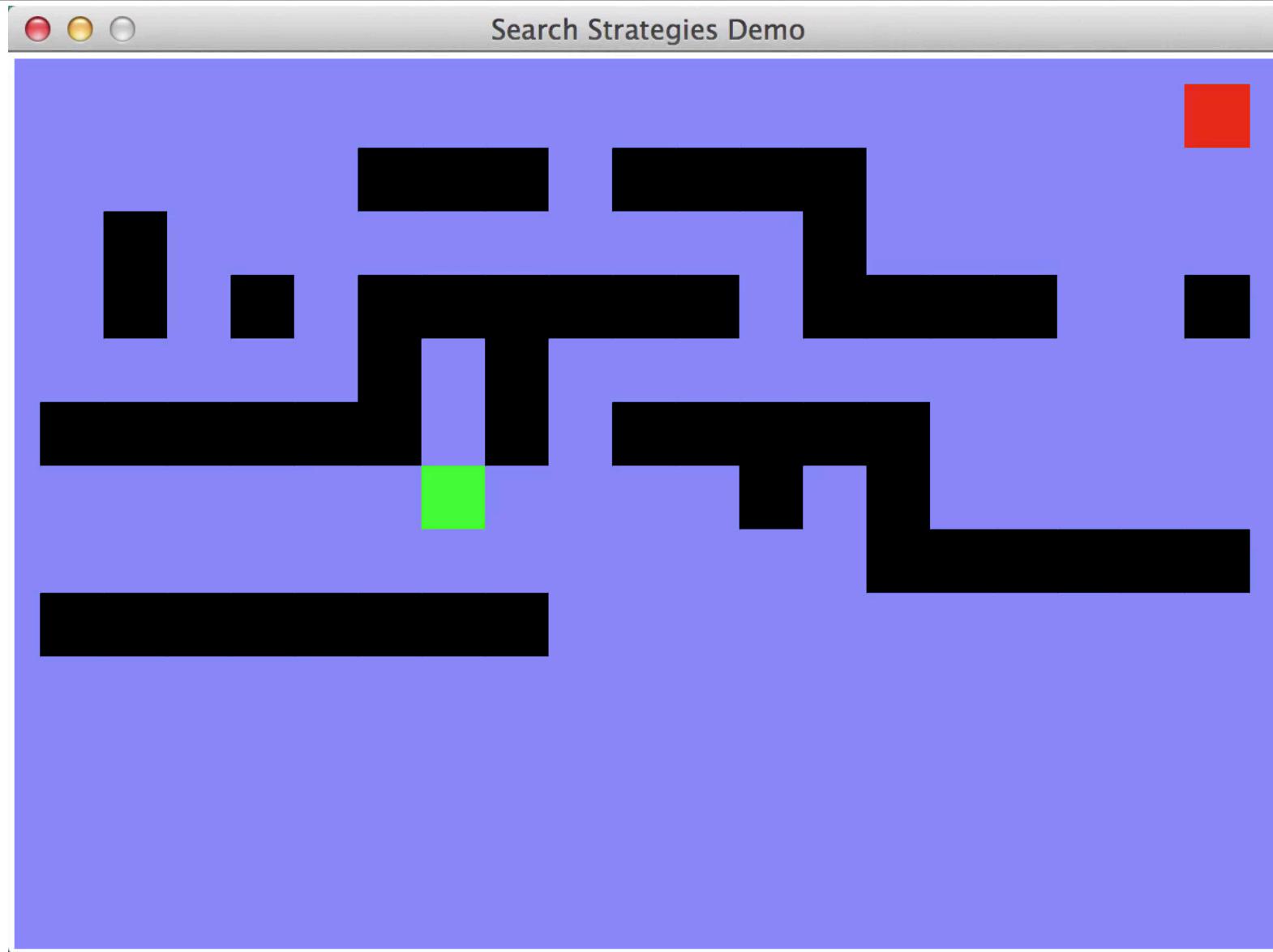
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- When will BFS outperform DFS?
- When will DFS outperform BFS?

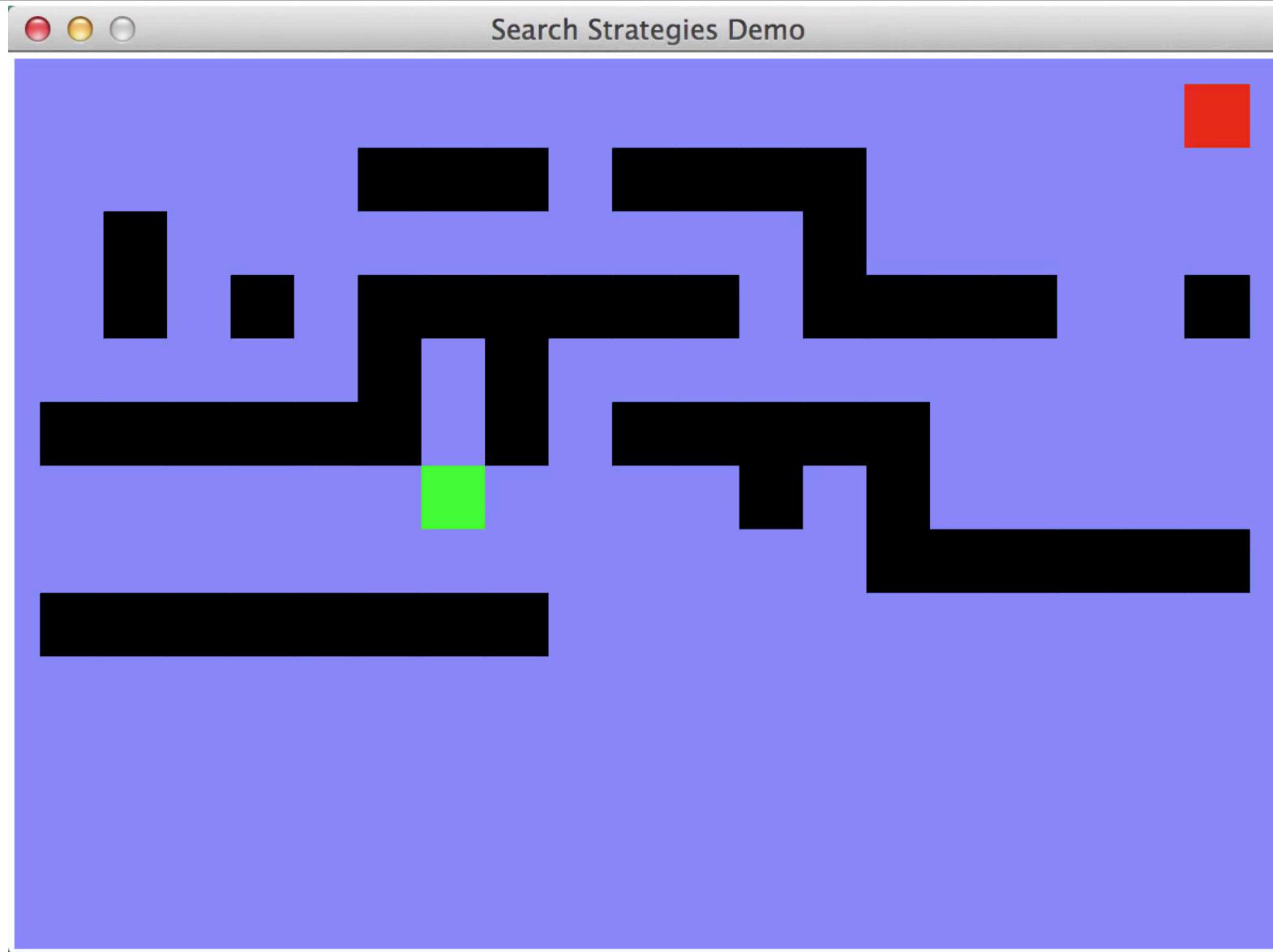
# Video of Demo Maze Water DFS/BFS (part 1)



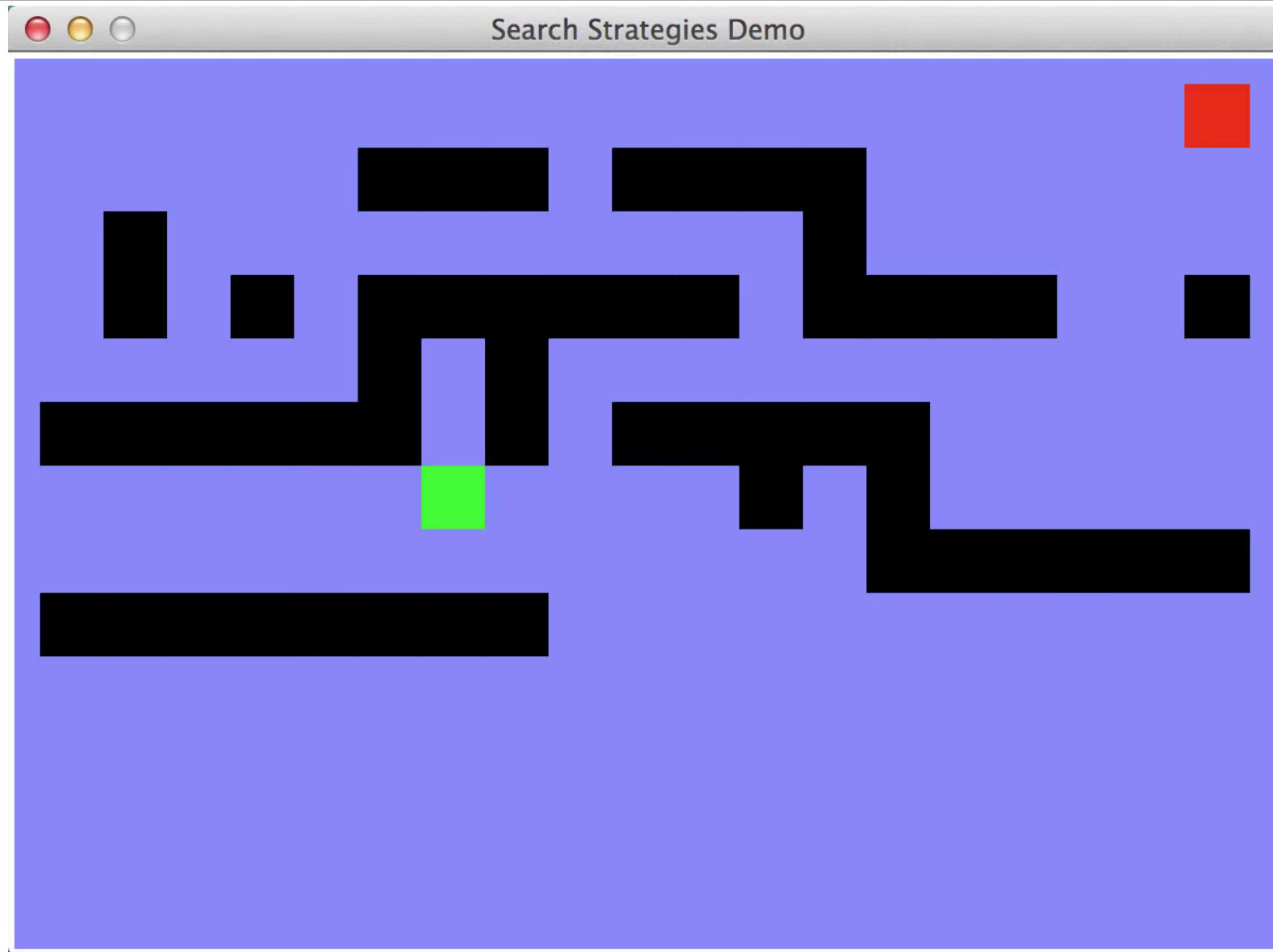
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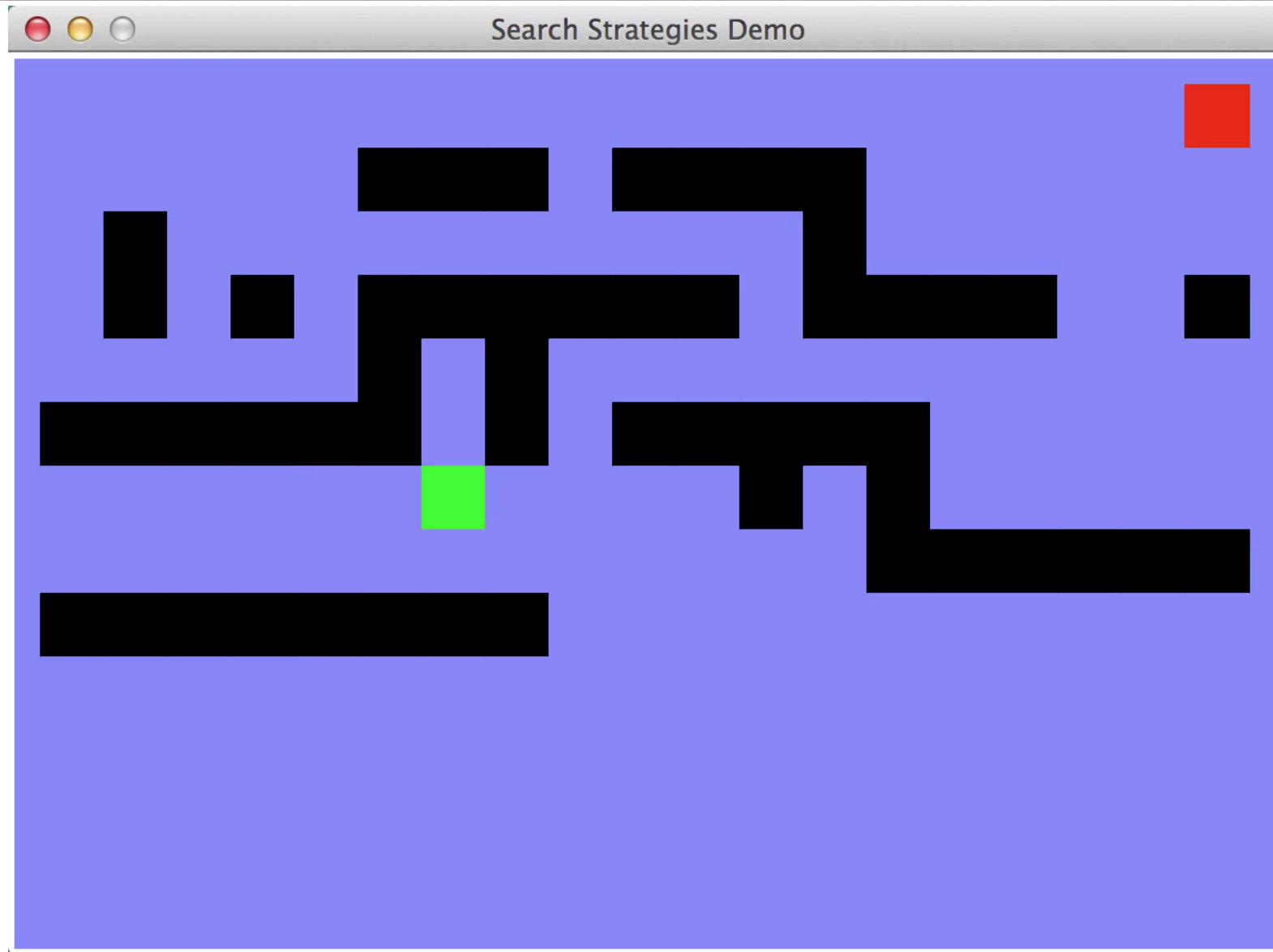
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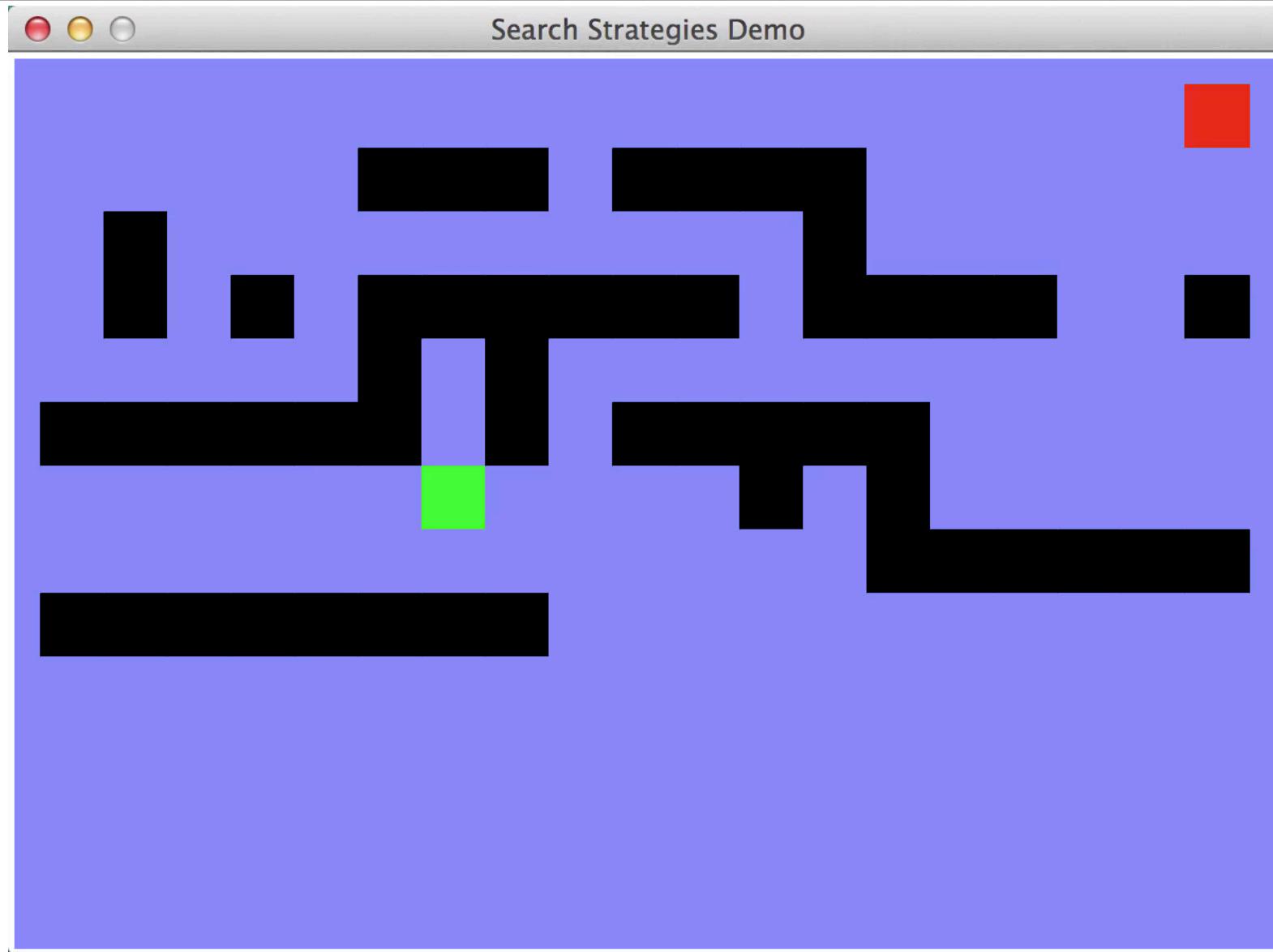
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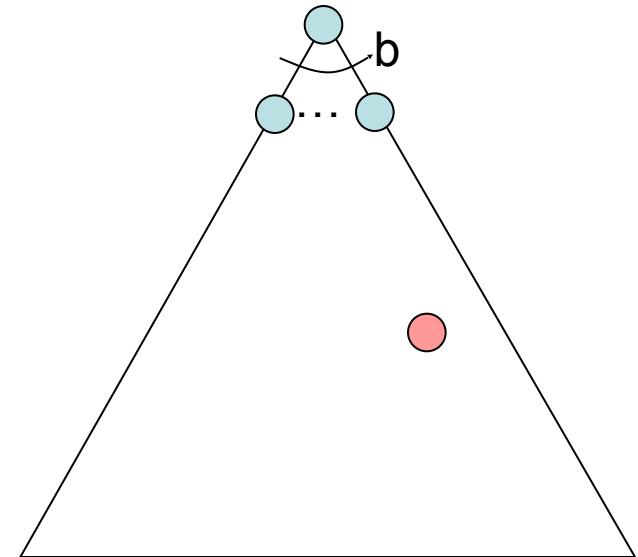
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# Iterative Deepening

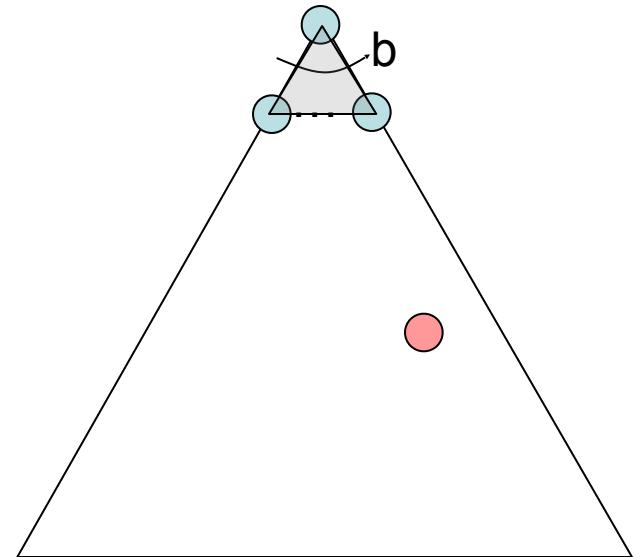
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- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages



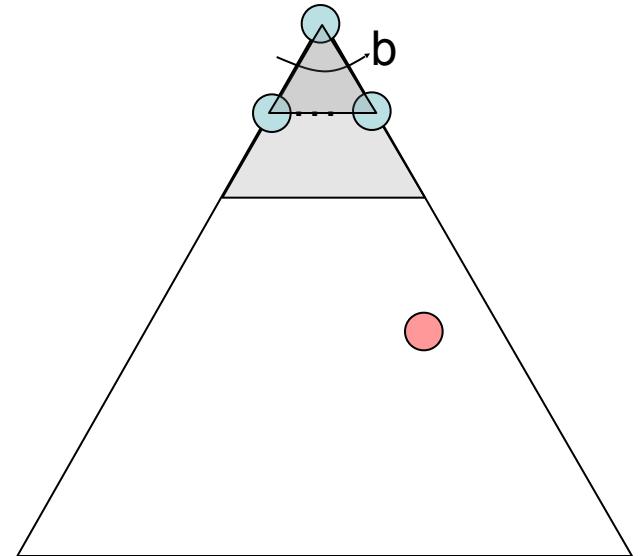
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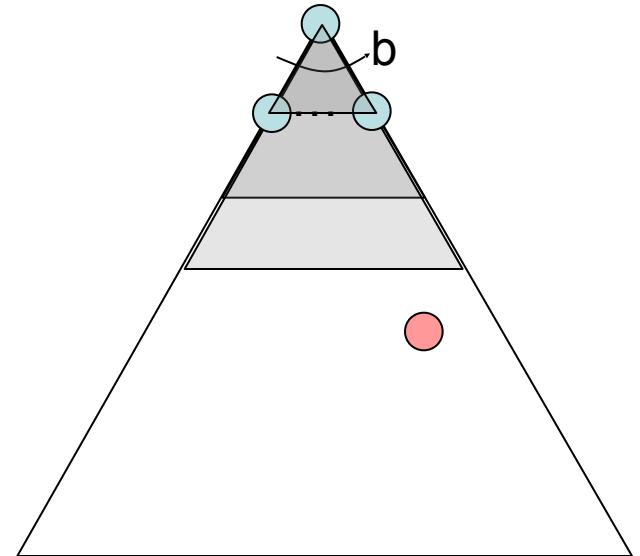
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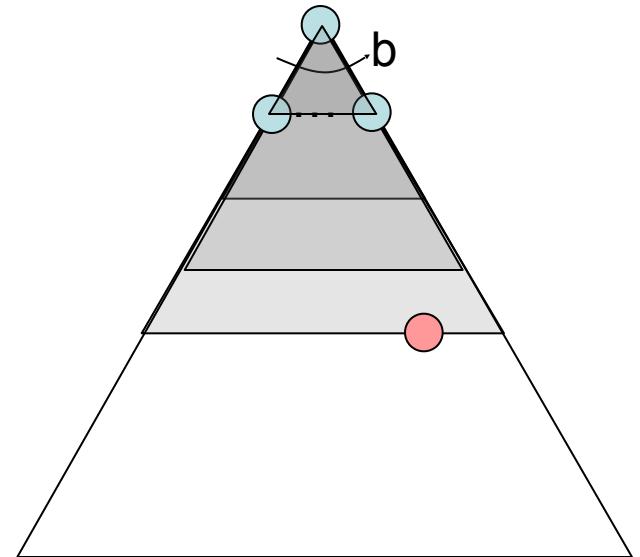
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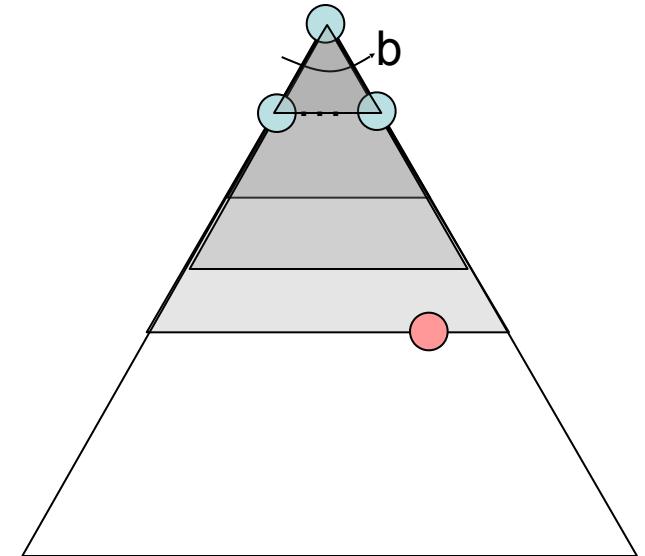


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- Isn't that wastefully redundant?

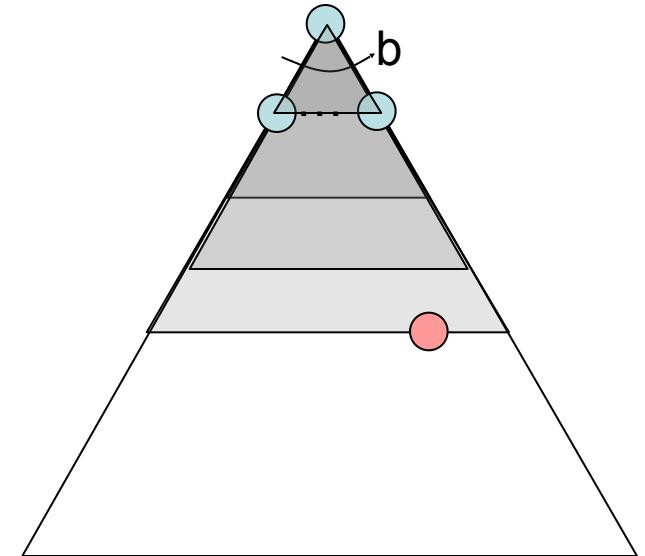


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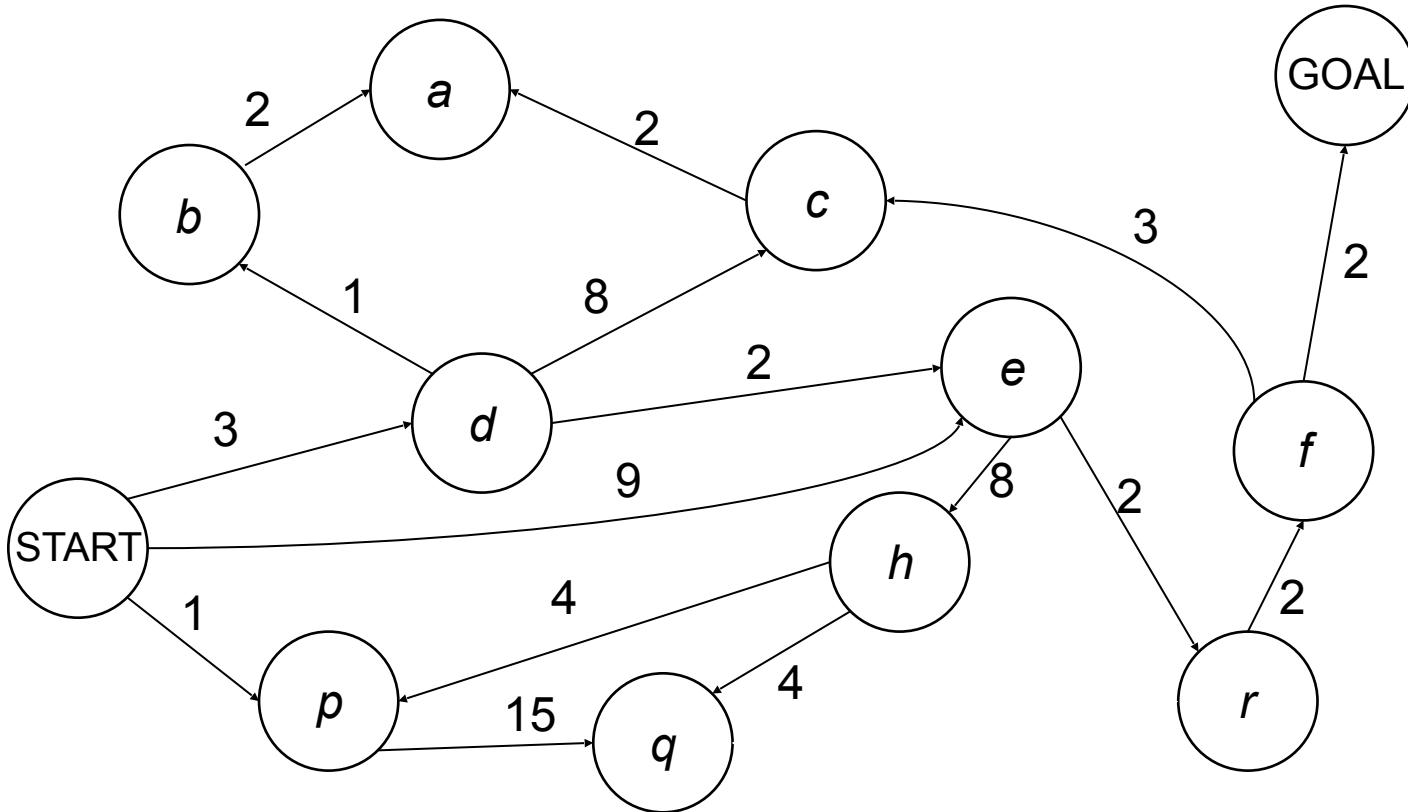
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- Run a DFS with depth limit 3. ....

- Isn't that wastefully redundant?
  - Generally most work happens in the lowest level searched, so not so bad!



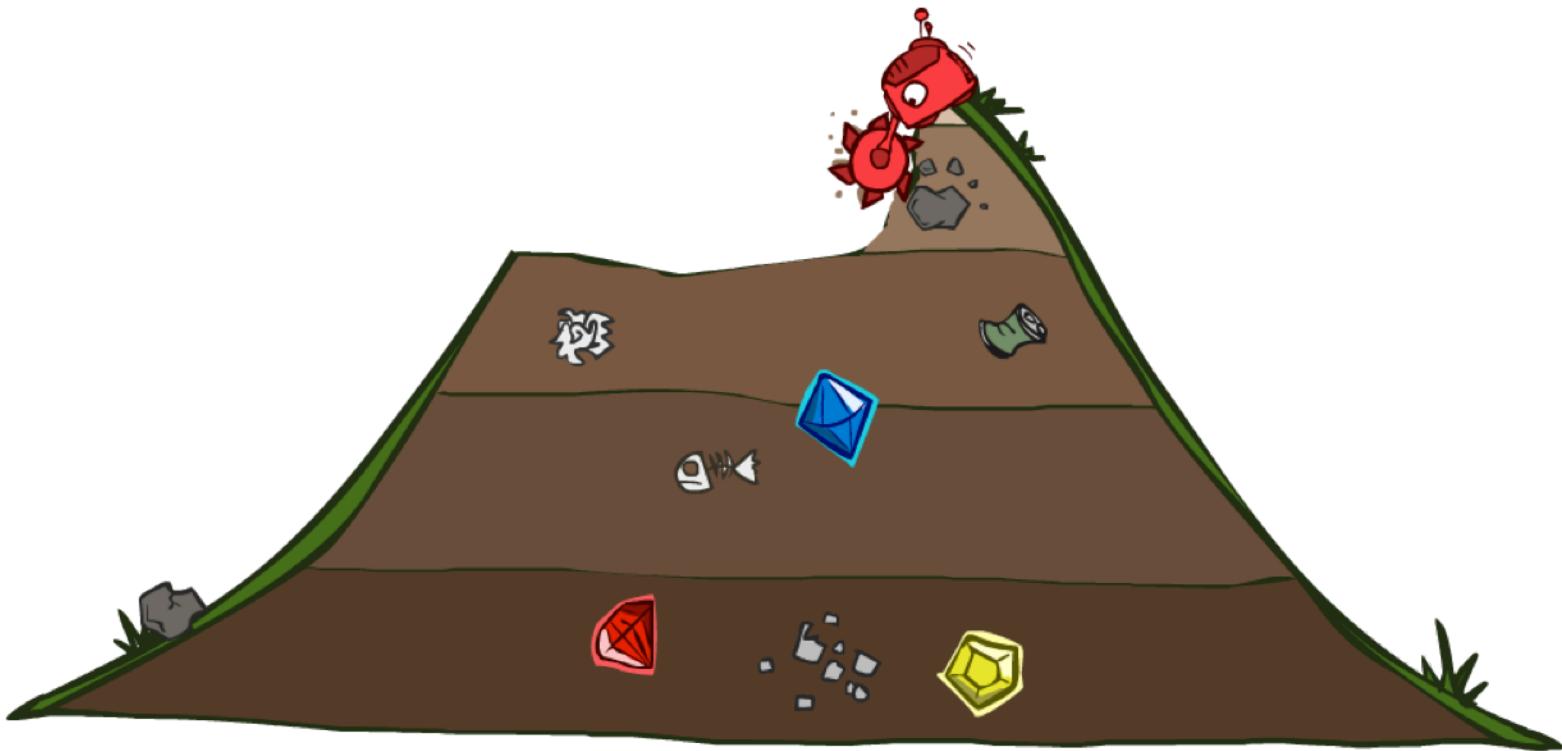
# Cost-Sensitive Search



BFS finds the shortest path in terms of number of actions.  
It does not find the least-cost path. We will now cover  
a similar algorithm which does find the least-cost path.

# Uniform Cost Search

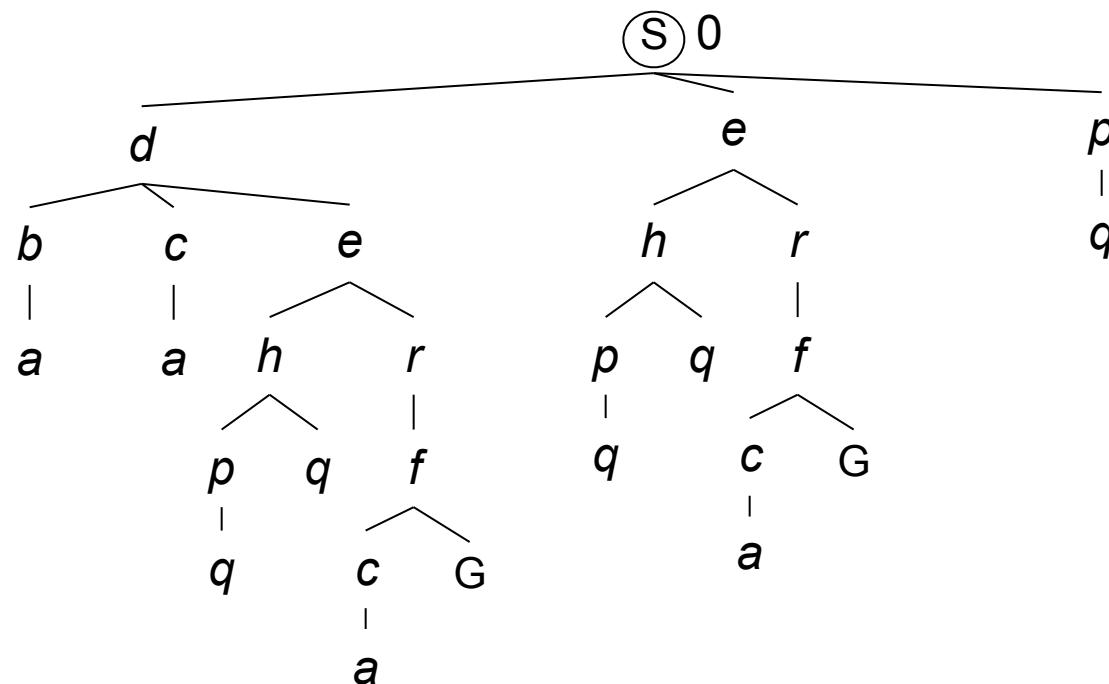
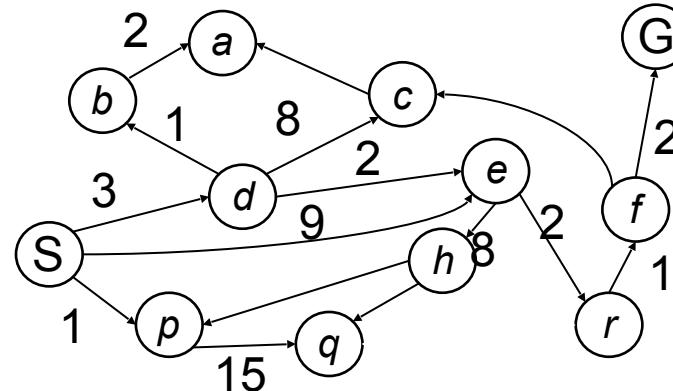
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# Uniform Cost Search

Strategy: expand a cheapest node first:

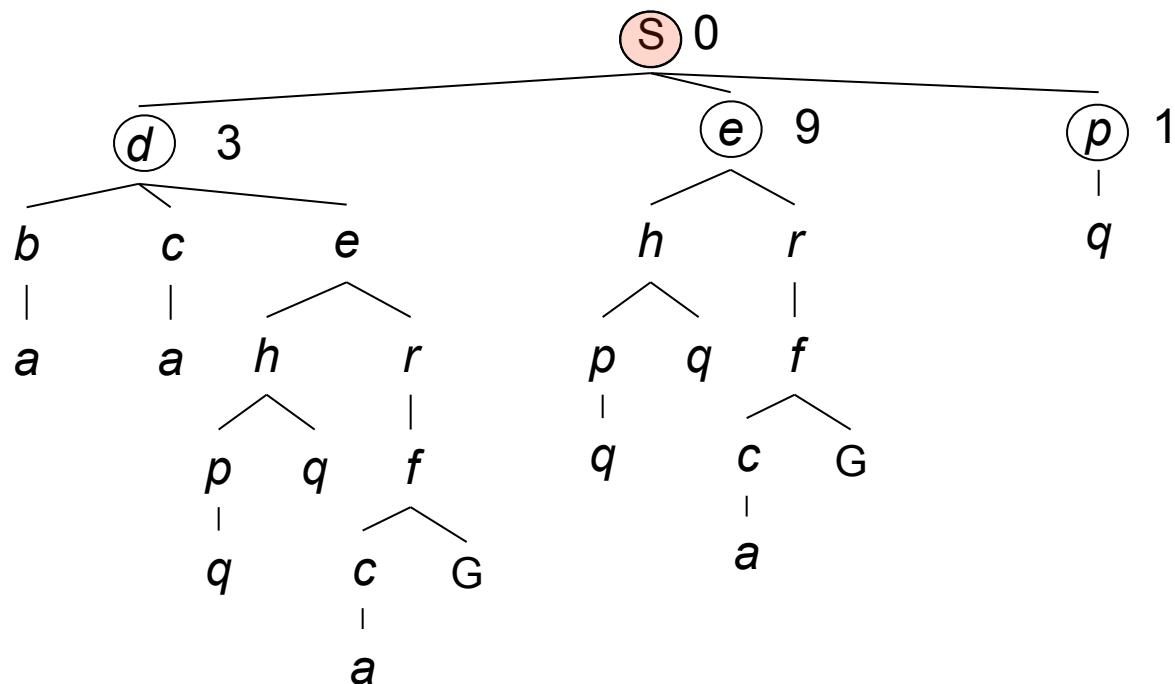
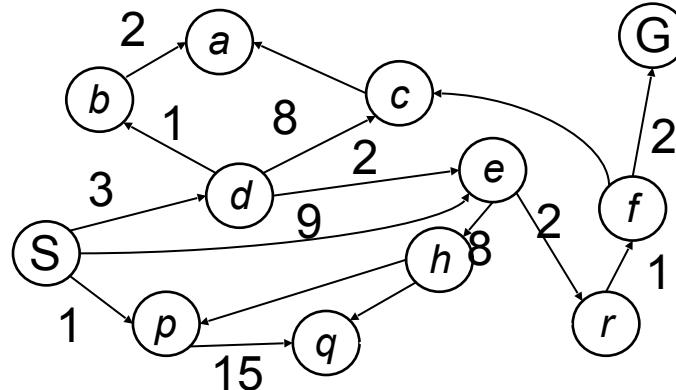
Fringe is a priority queue  
(priority: cumulative cost)



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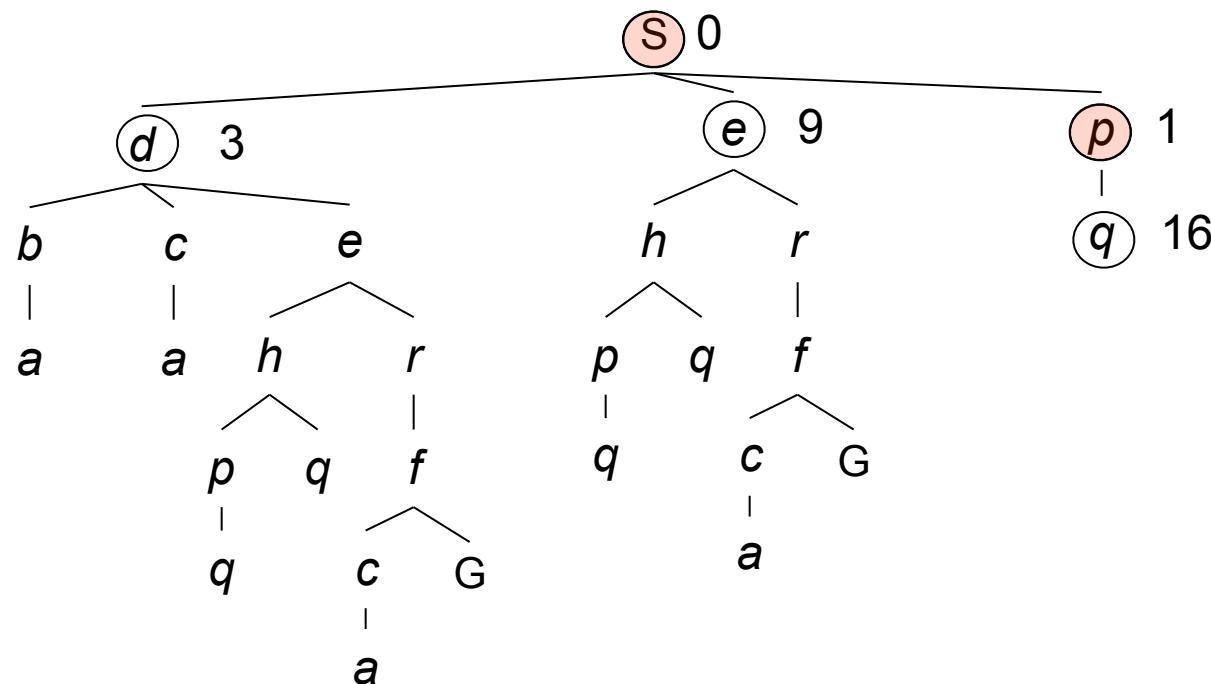
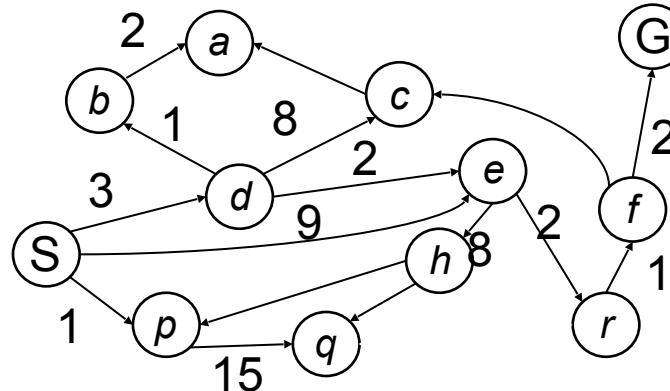
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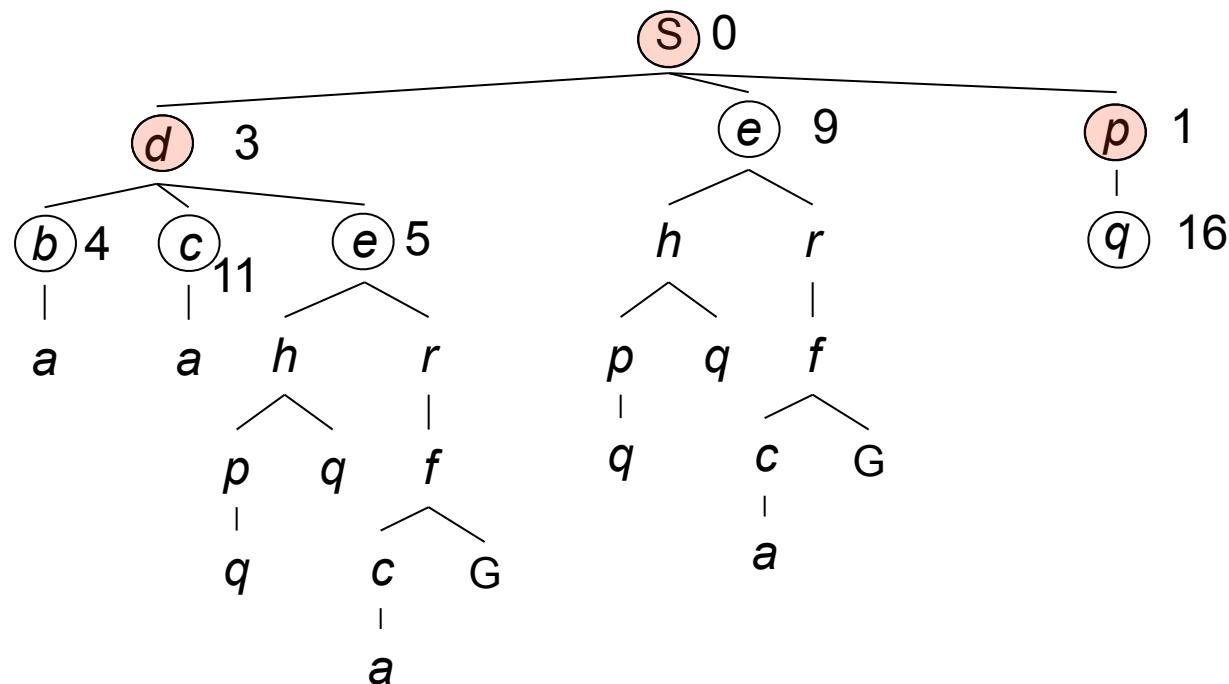
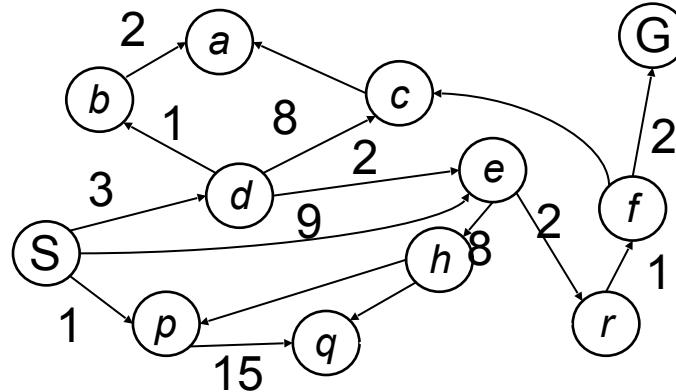
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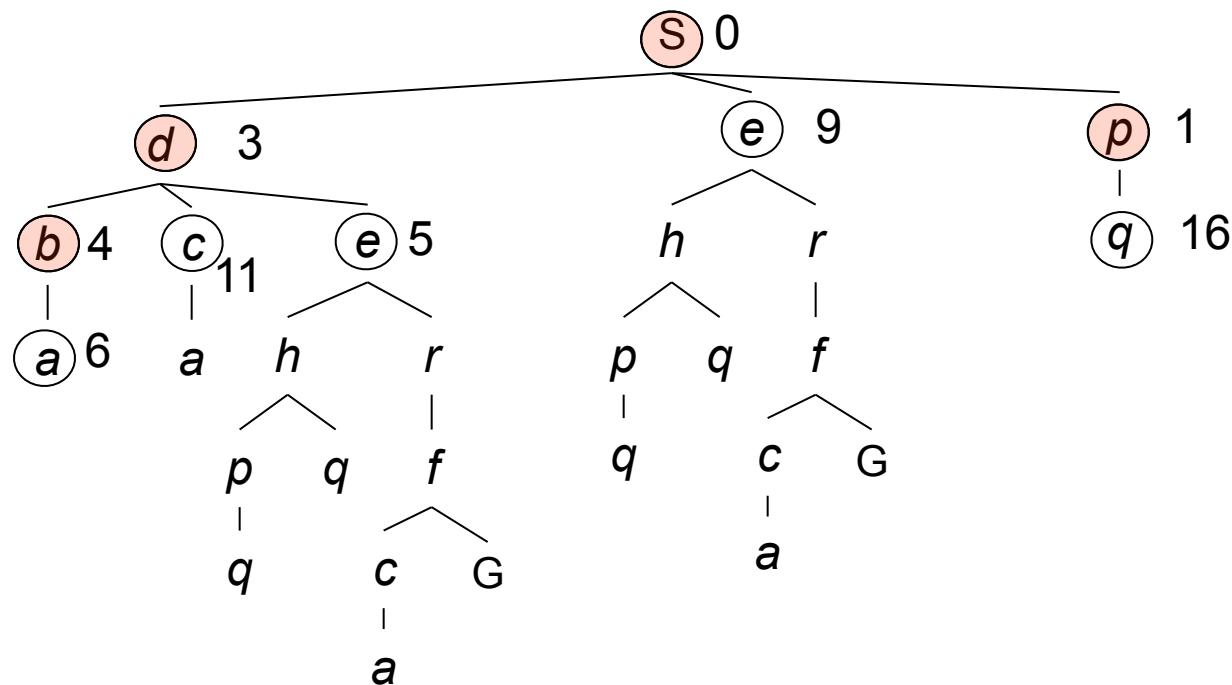
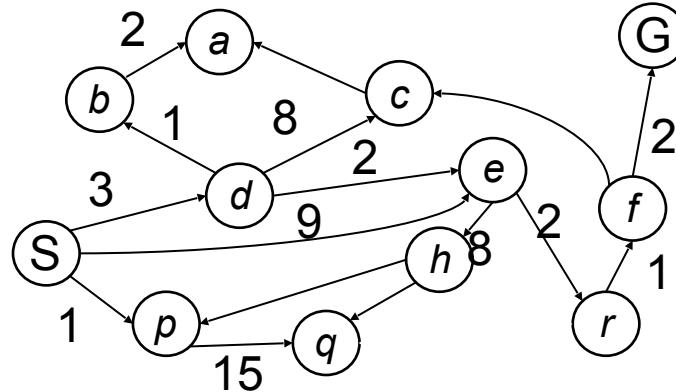
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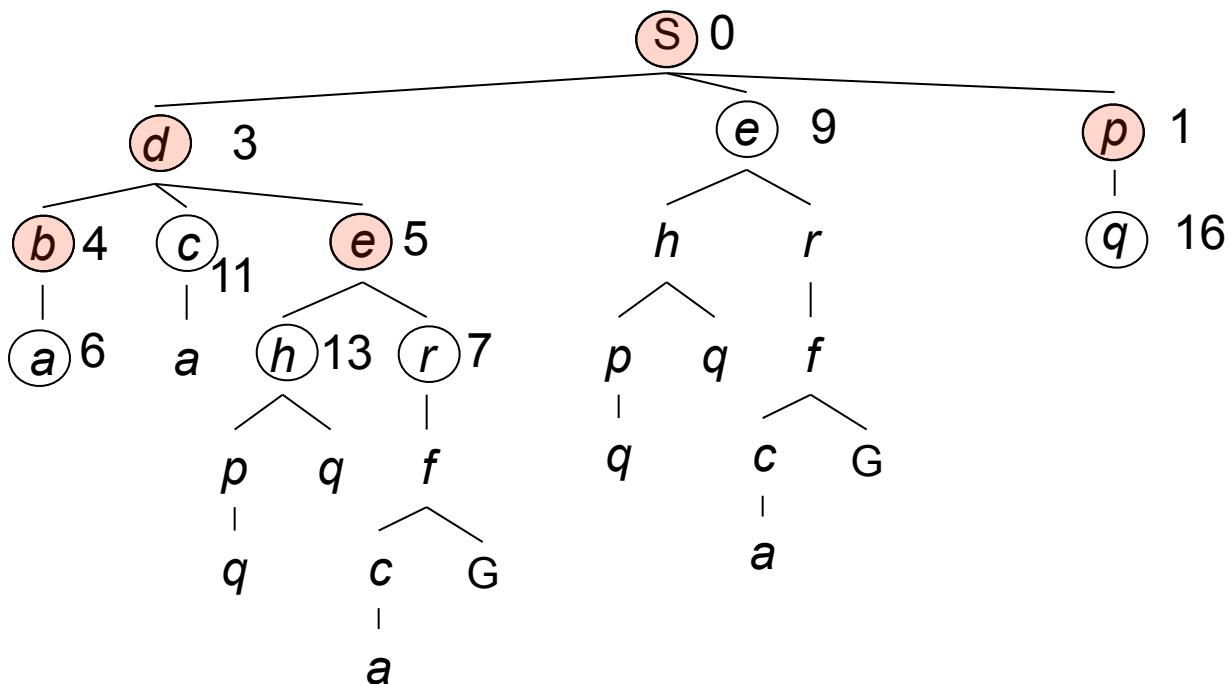
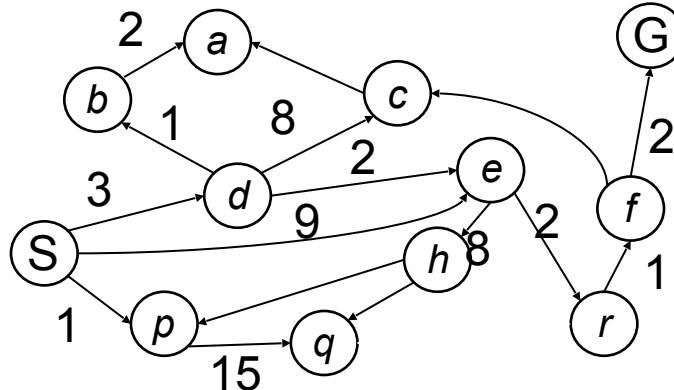
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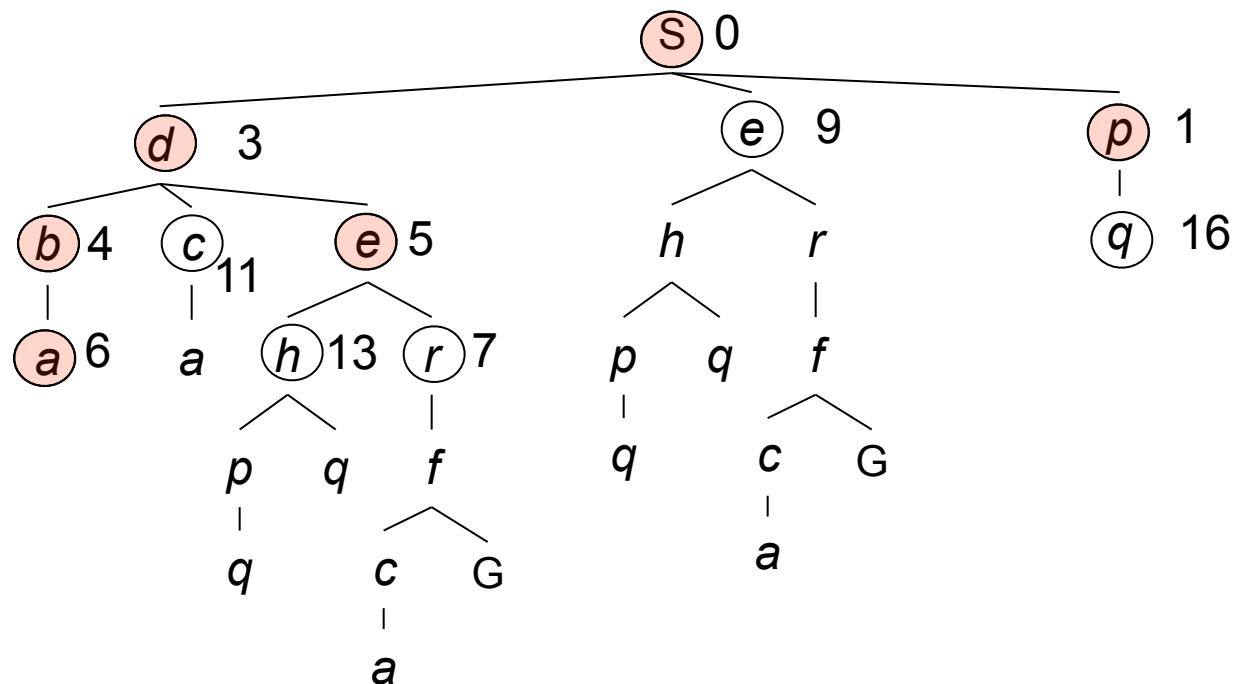
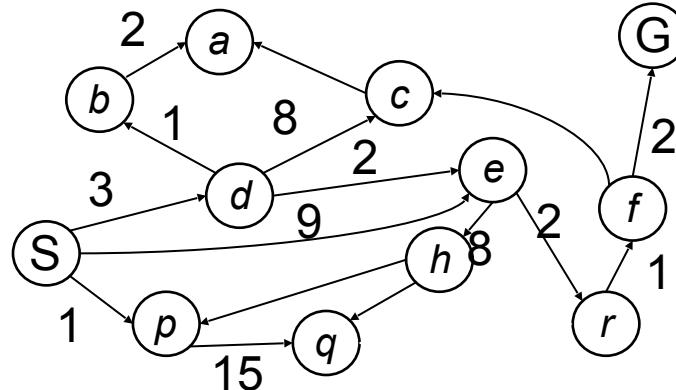
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# Uniform Cost Search

Strategy: expand a cheapest node first:

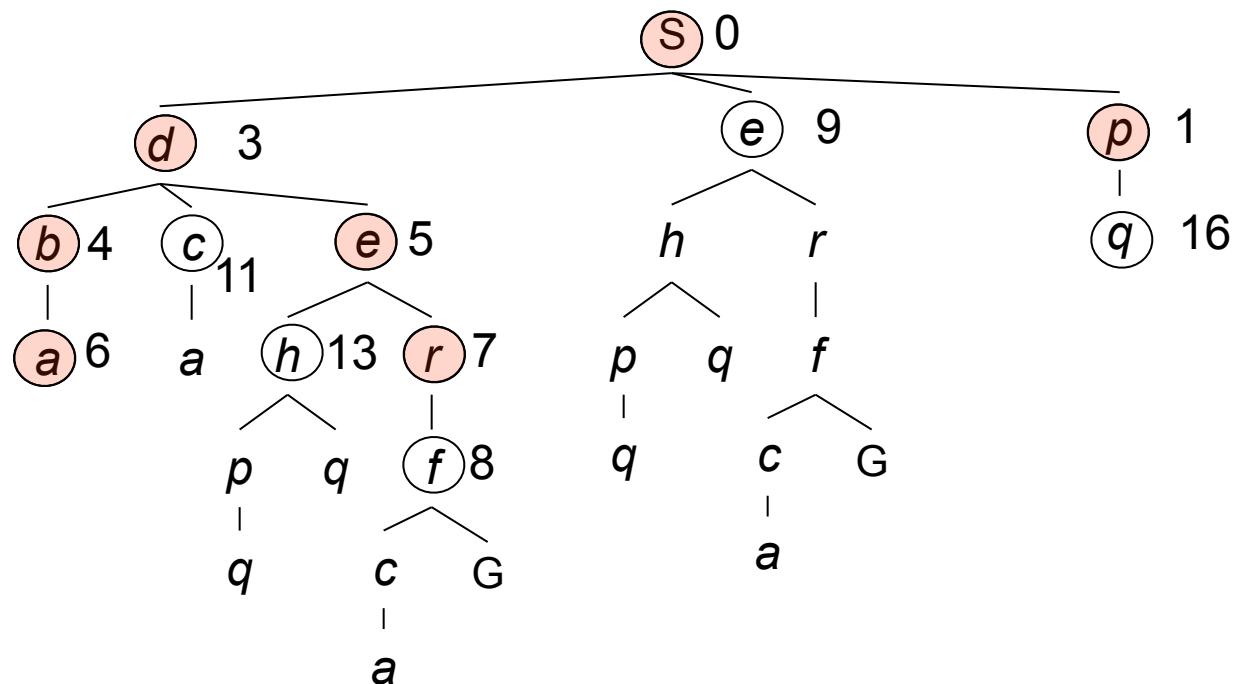
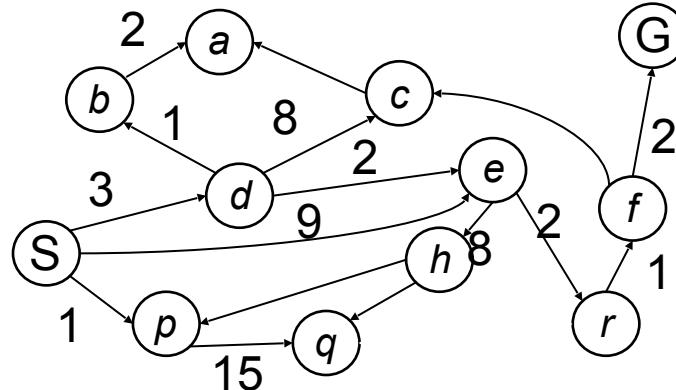
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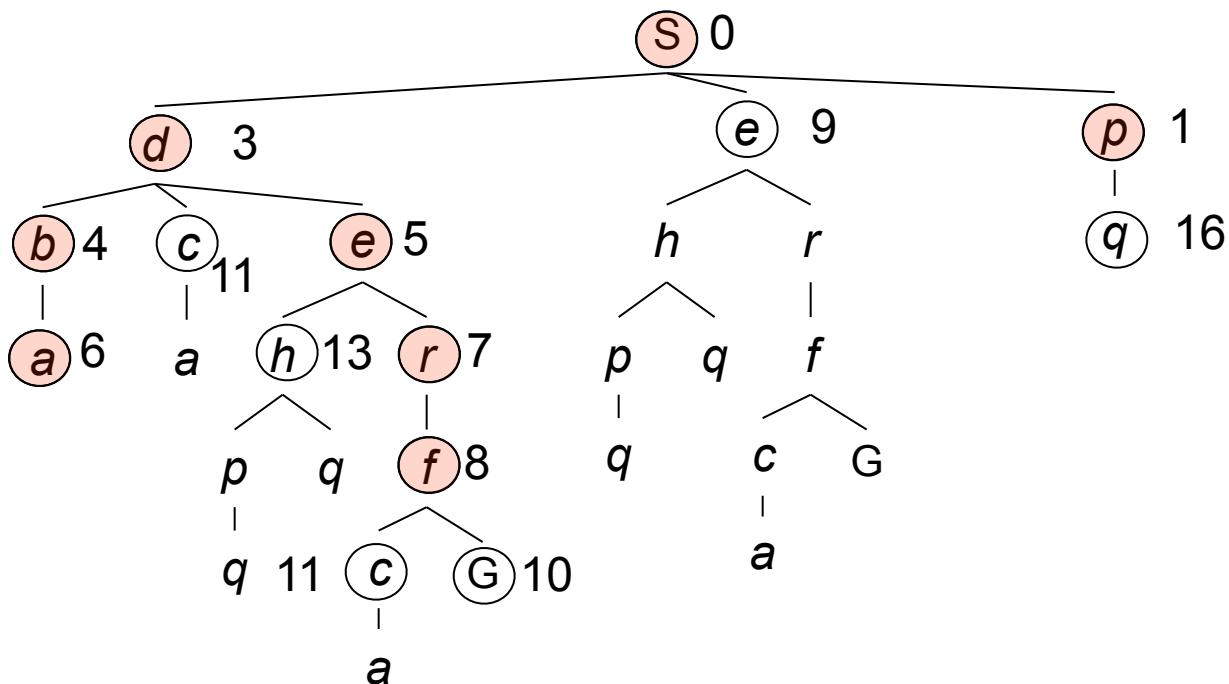
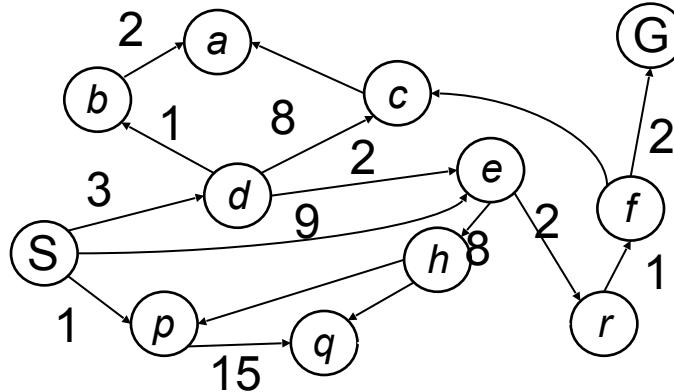
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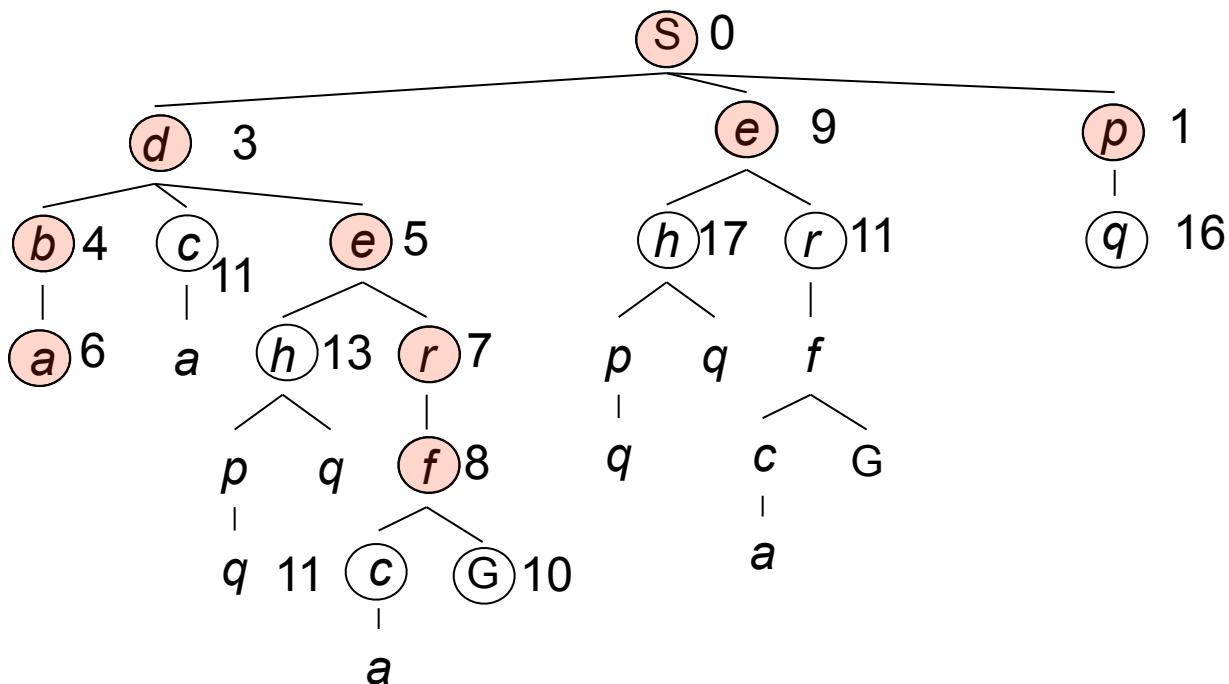
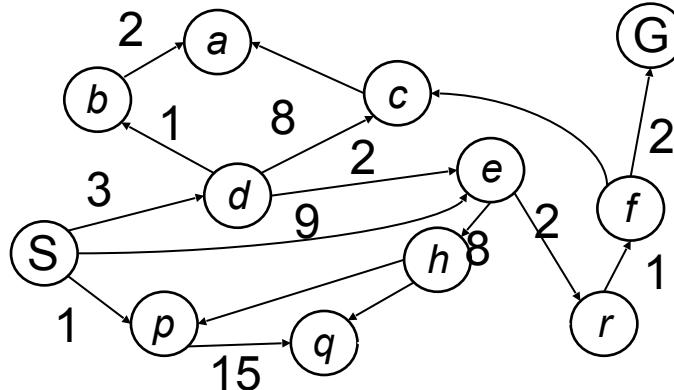
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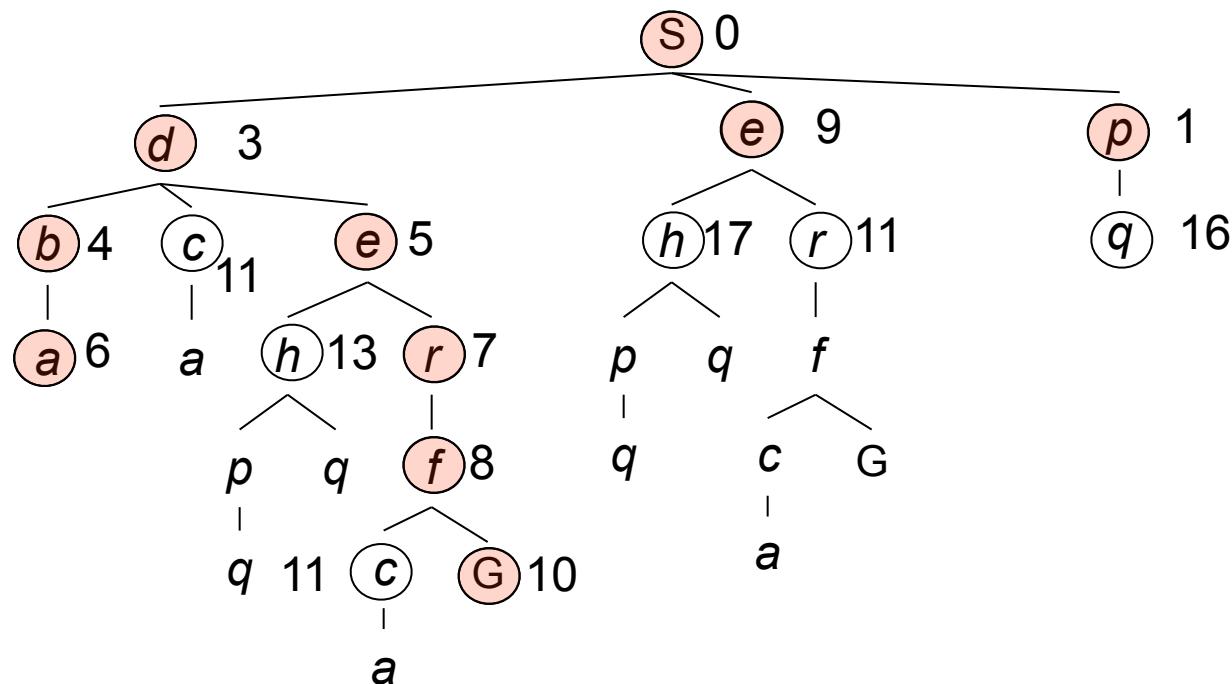
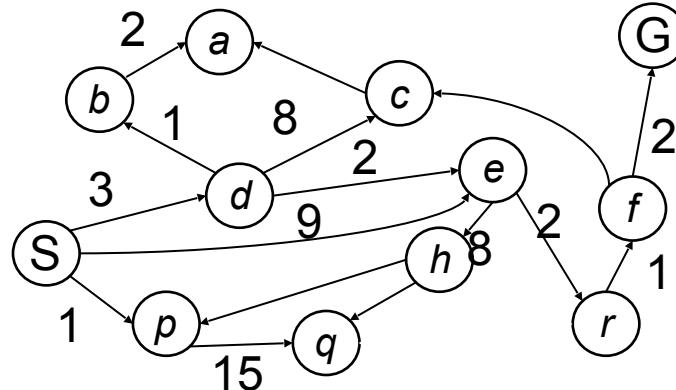
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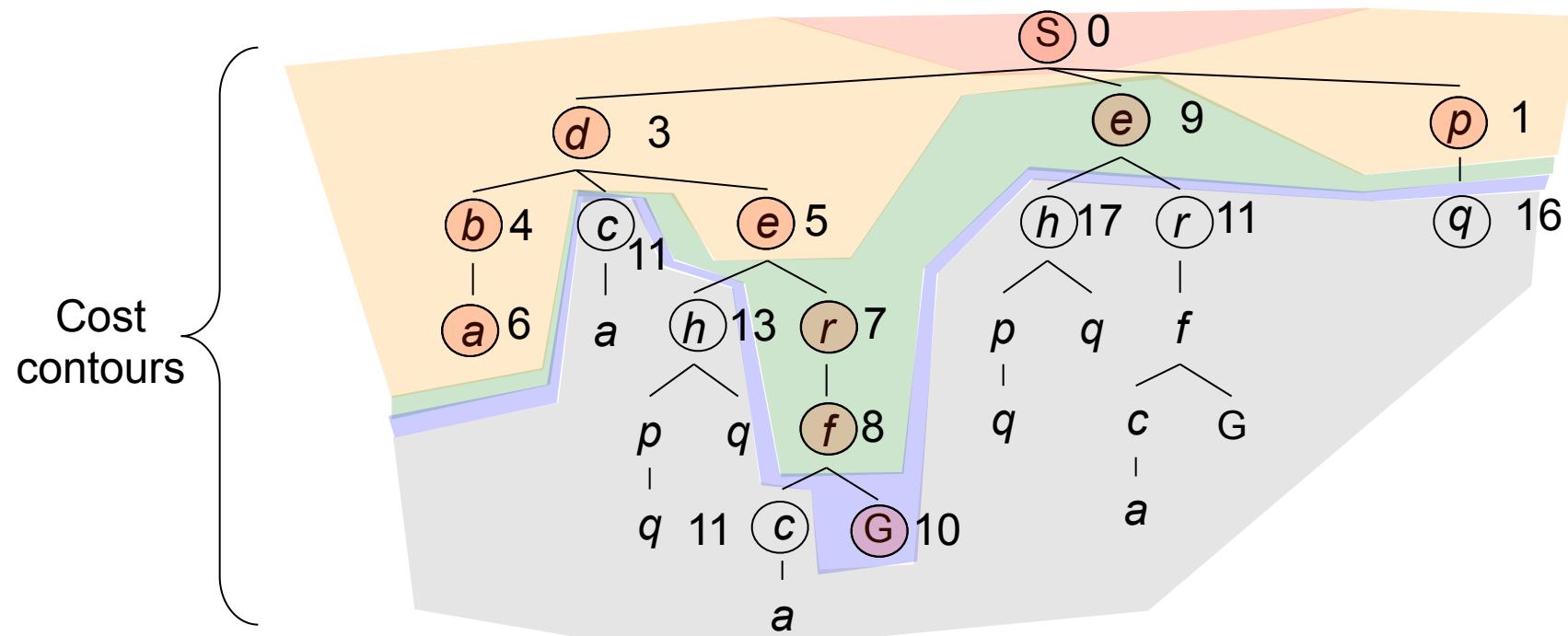
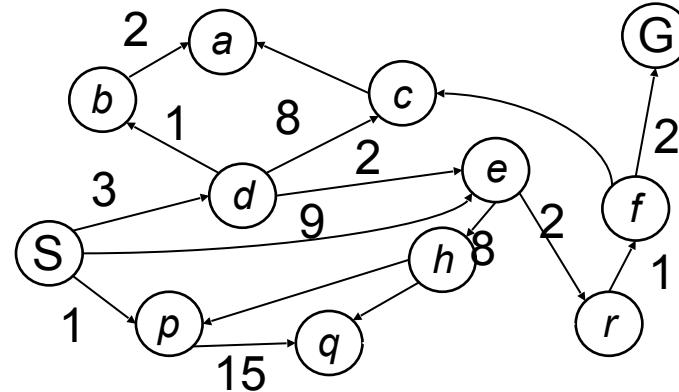
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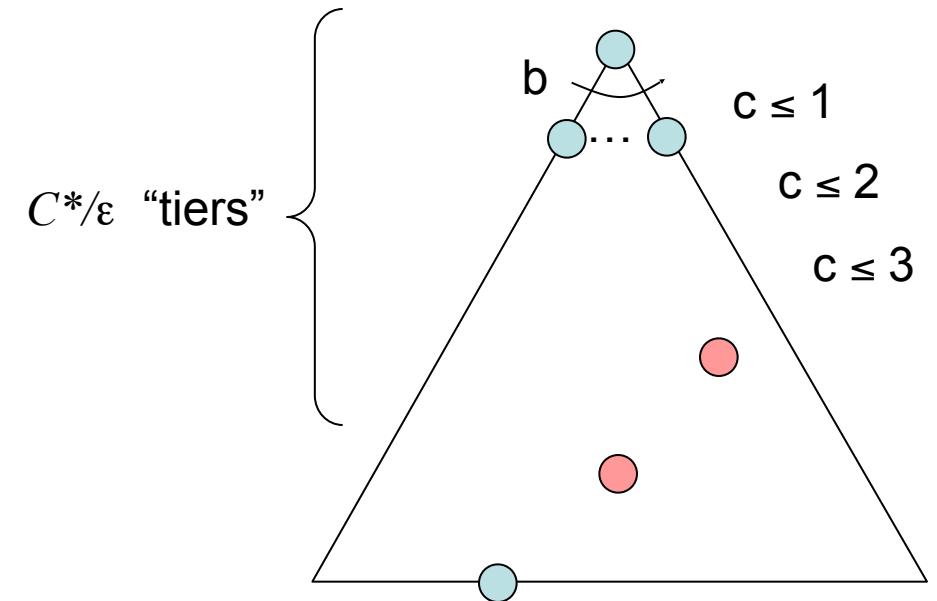
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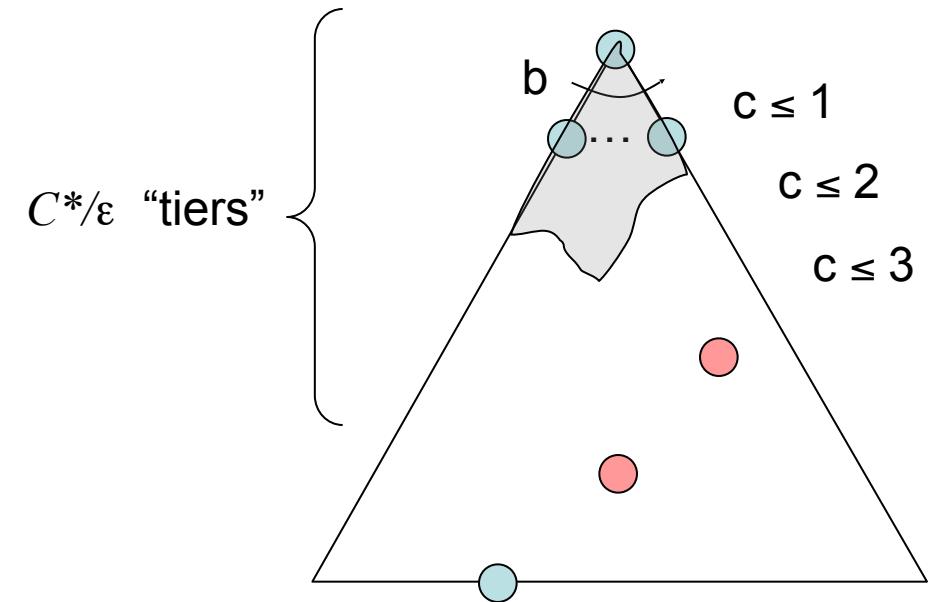
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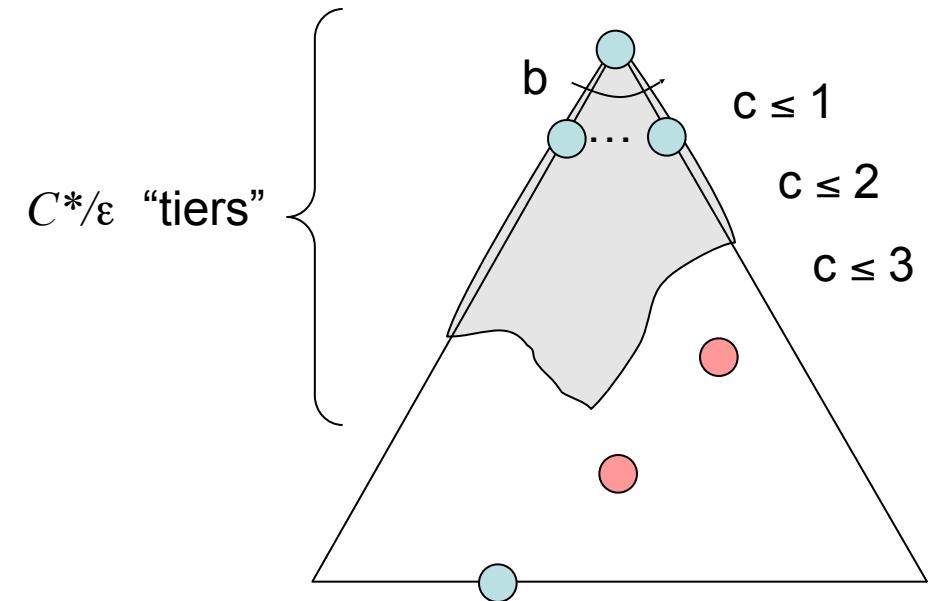
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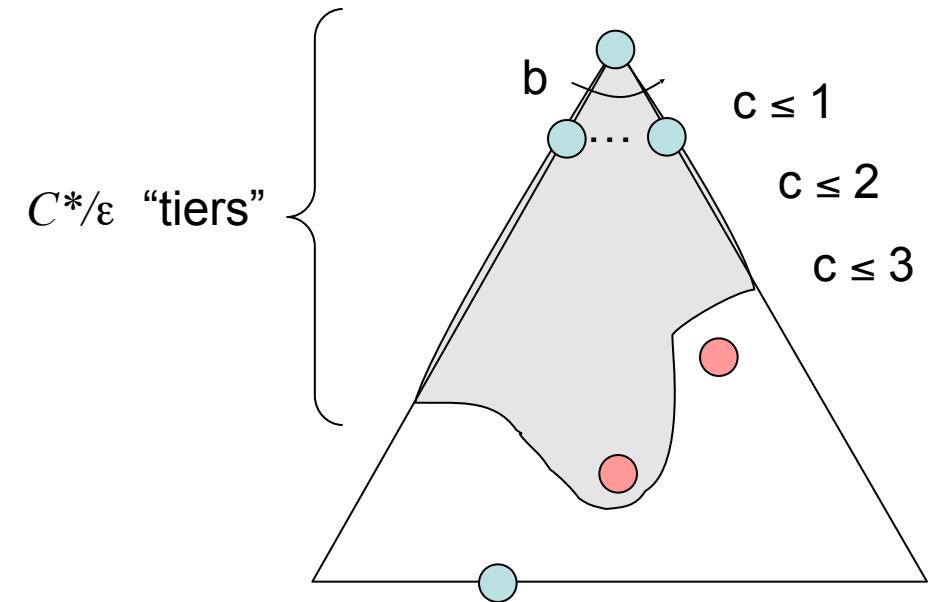
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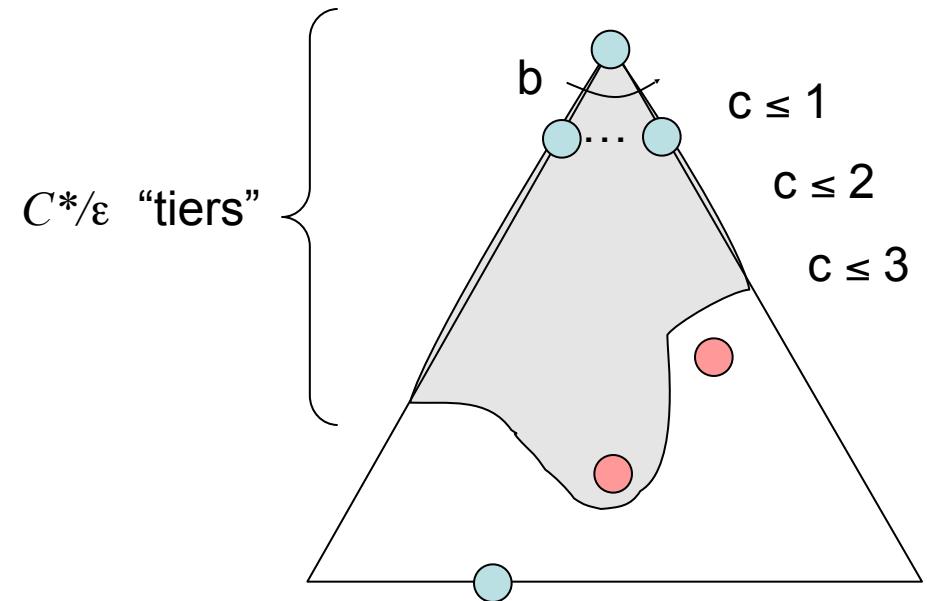
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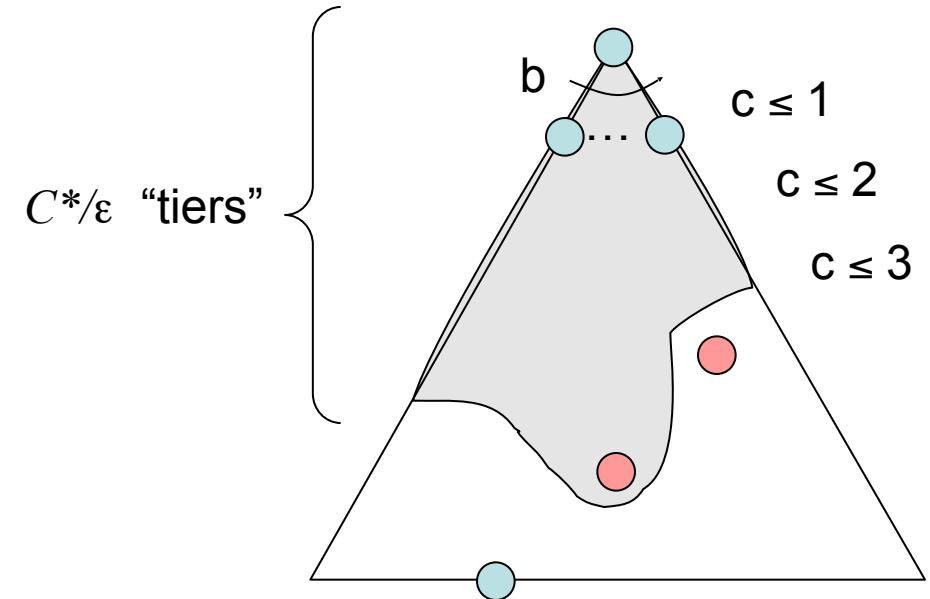
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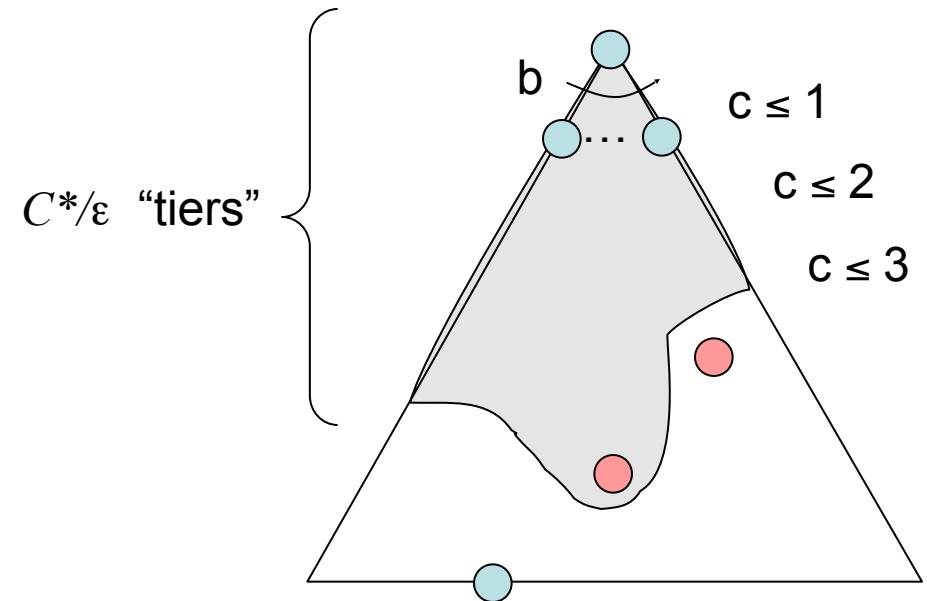
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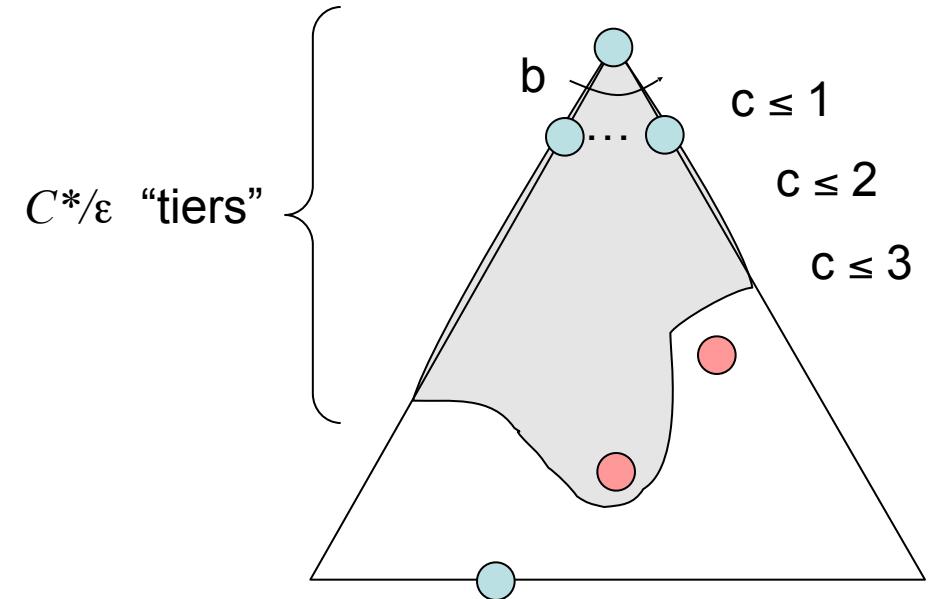


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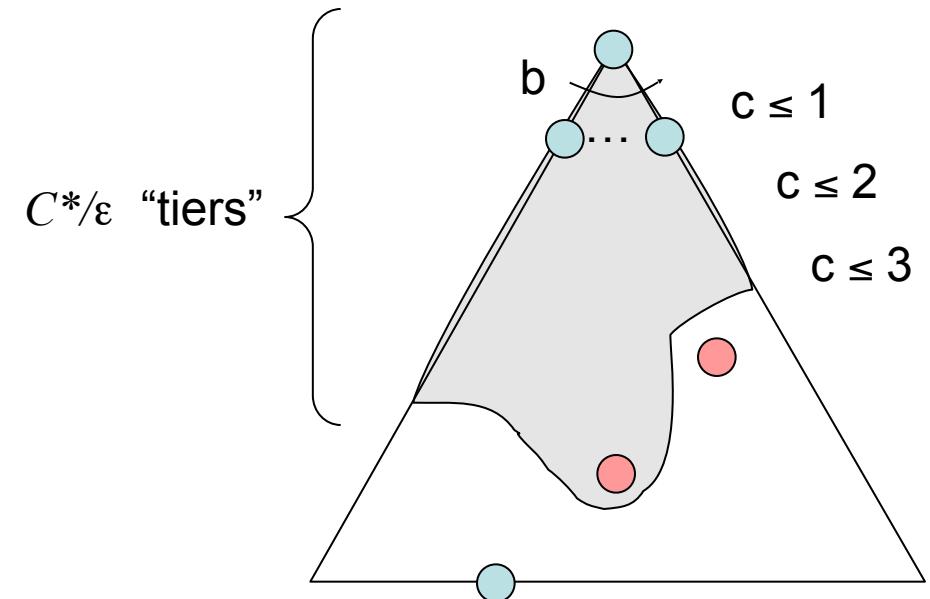
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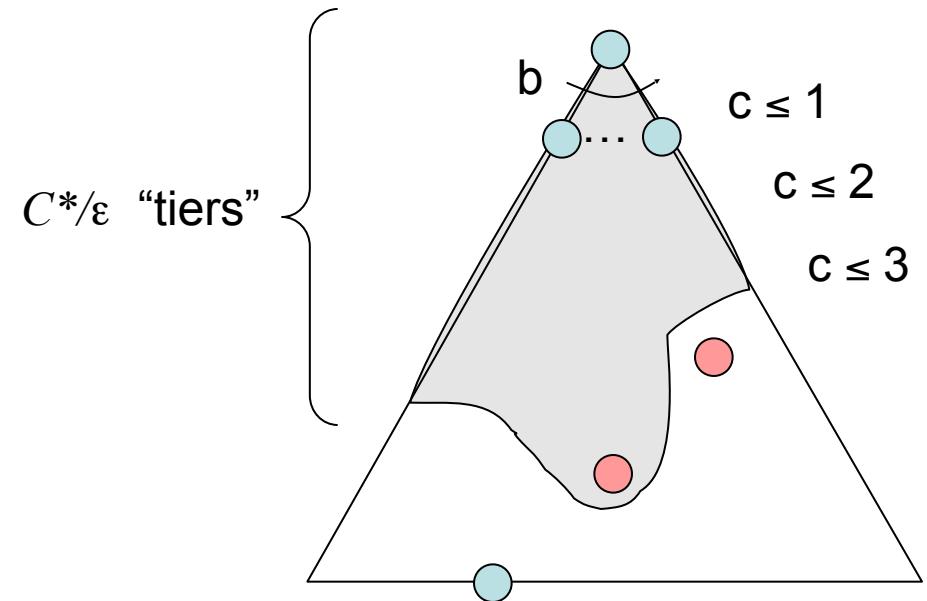
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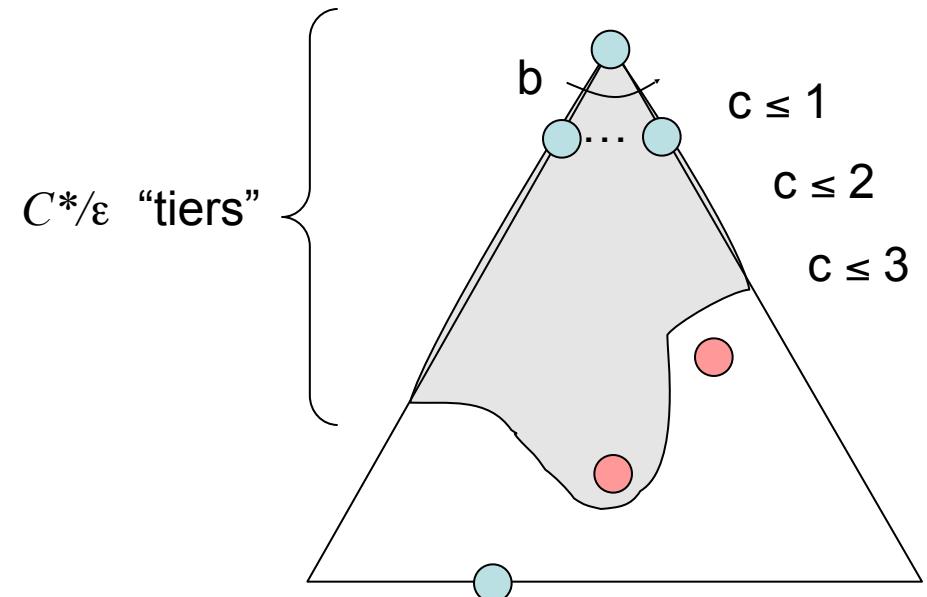
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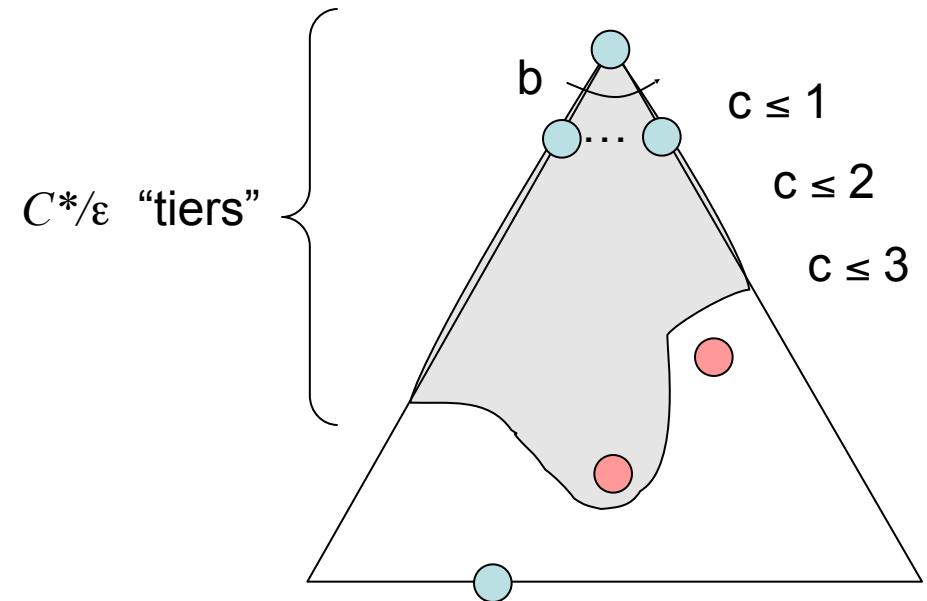
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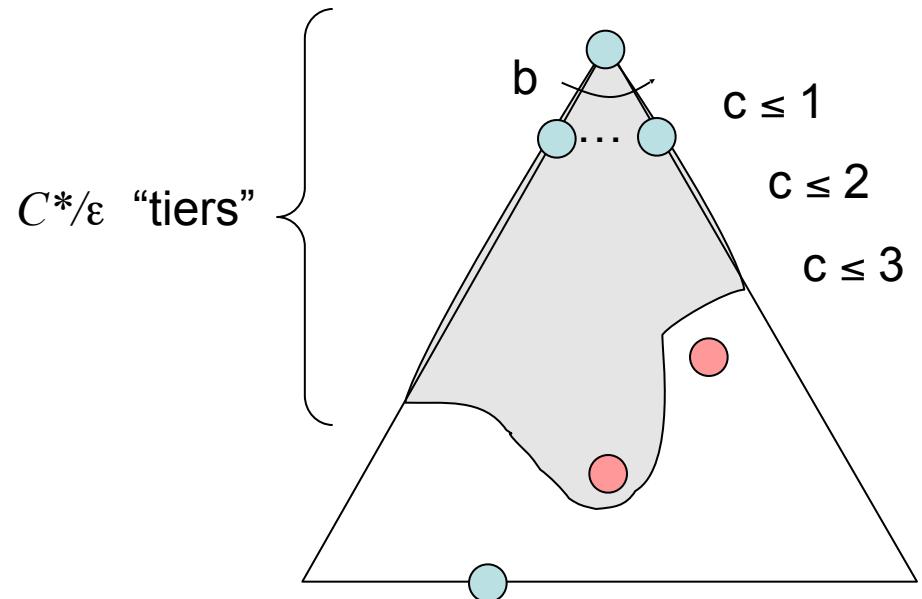
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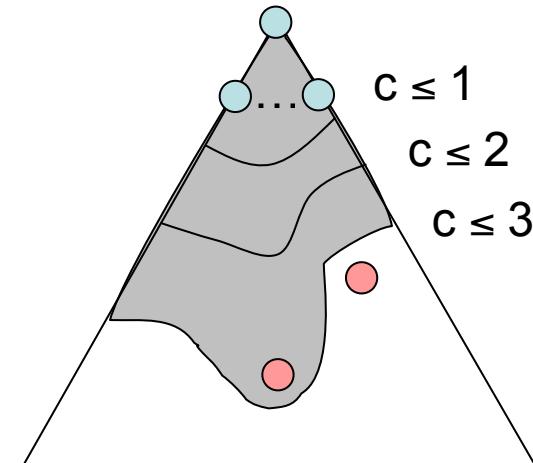
- Is it optimal?

- Yes! (Proof next lecture via A\*)



# Uniform Cost Issues

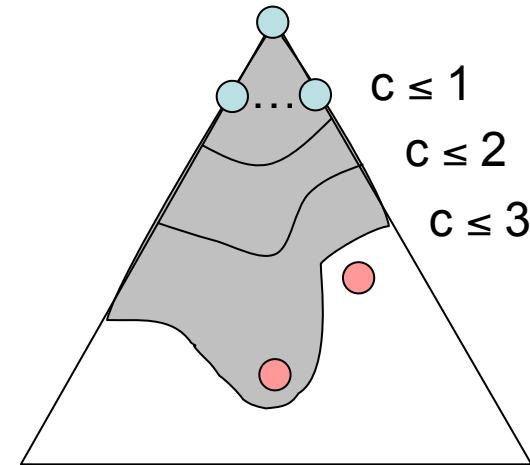
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[Demo: empty grid UCS (L2D5)]  
[Demo: maze with deep/shallow water DFS/BFS/UCS (L2D7)]

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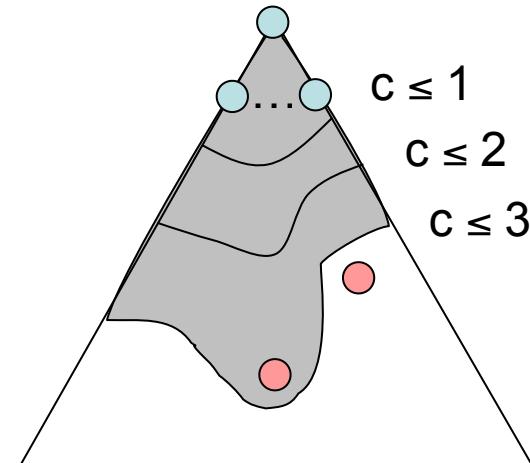
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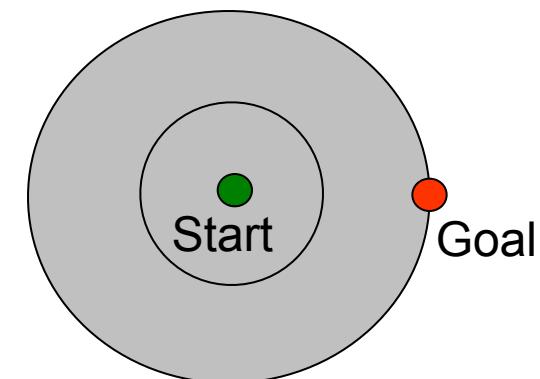
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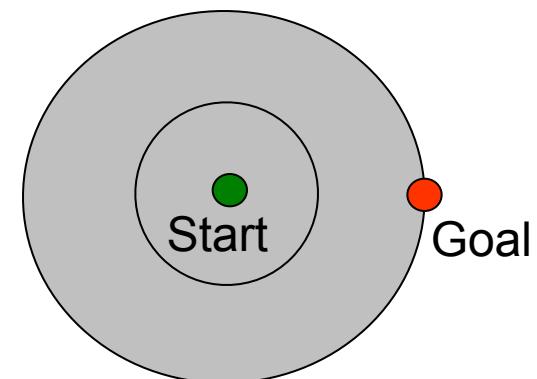
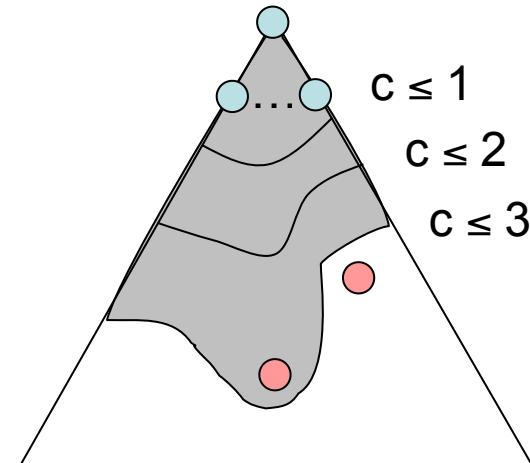
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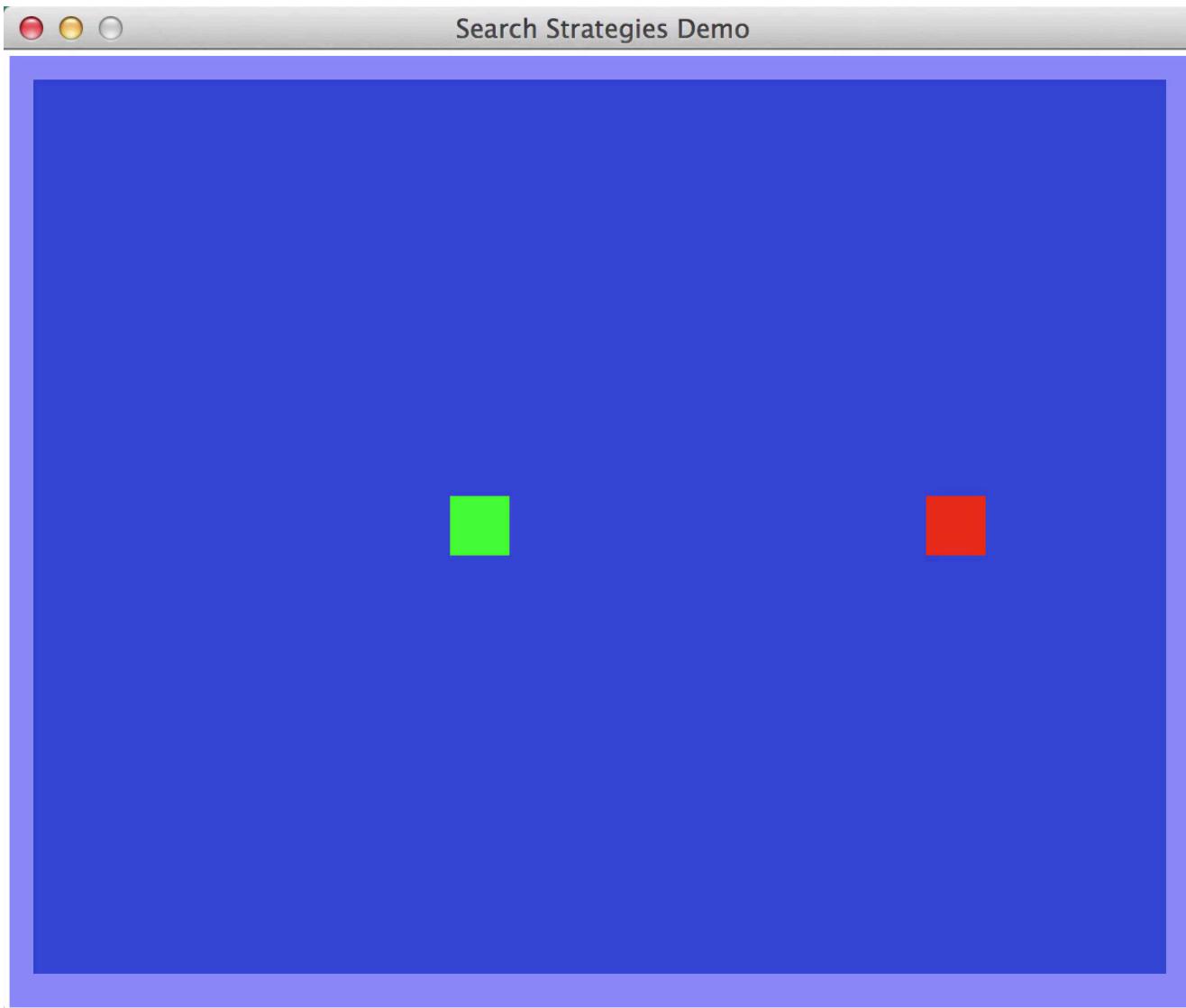
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- We’ll fix that soon!

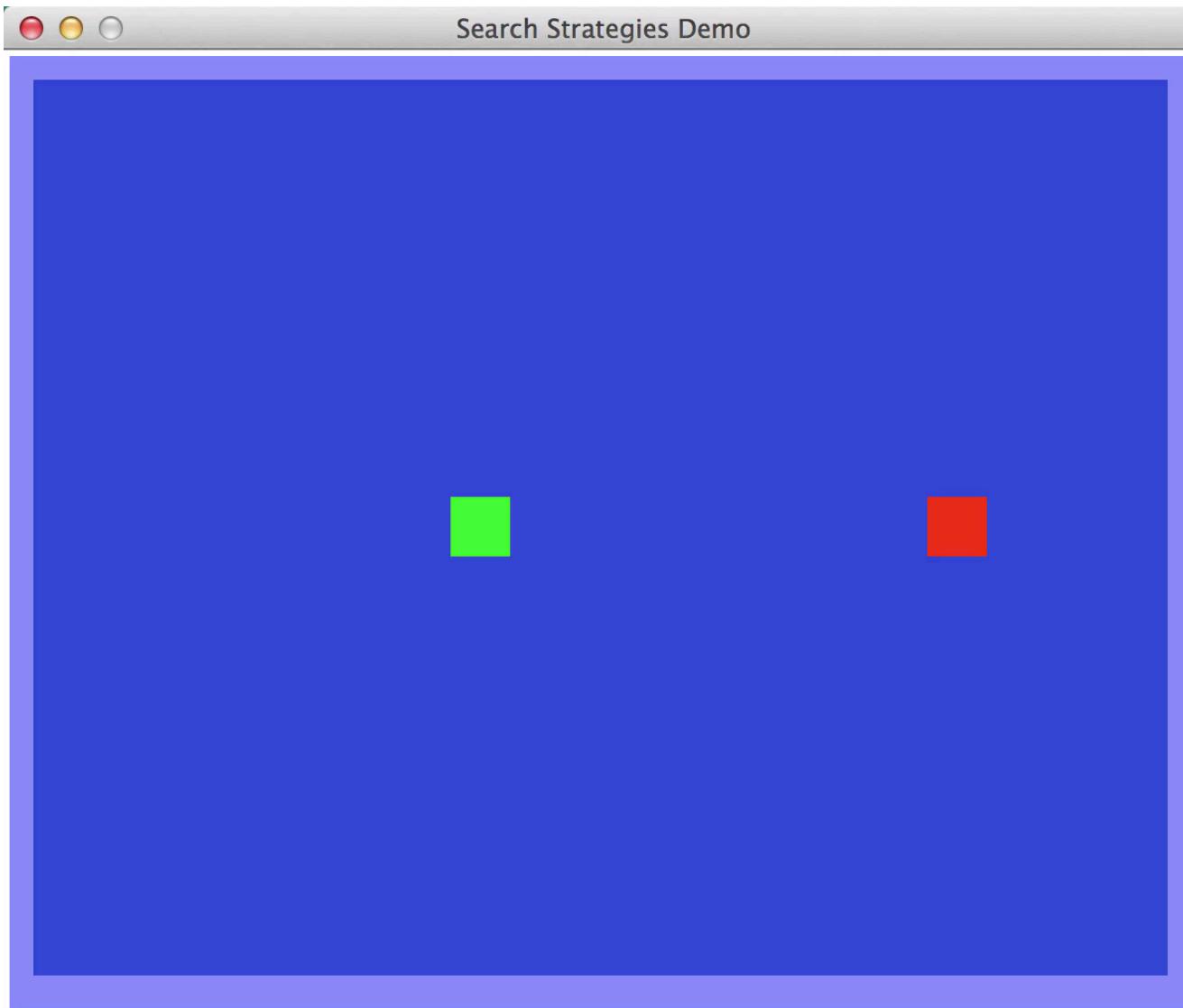


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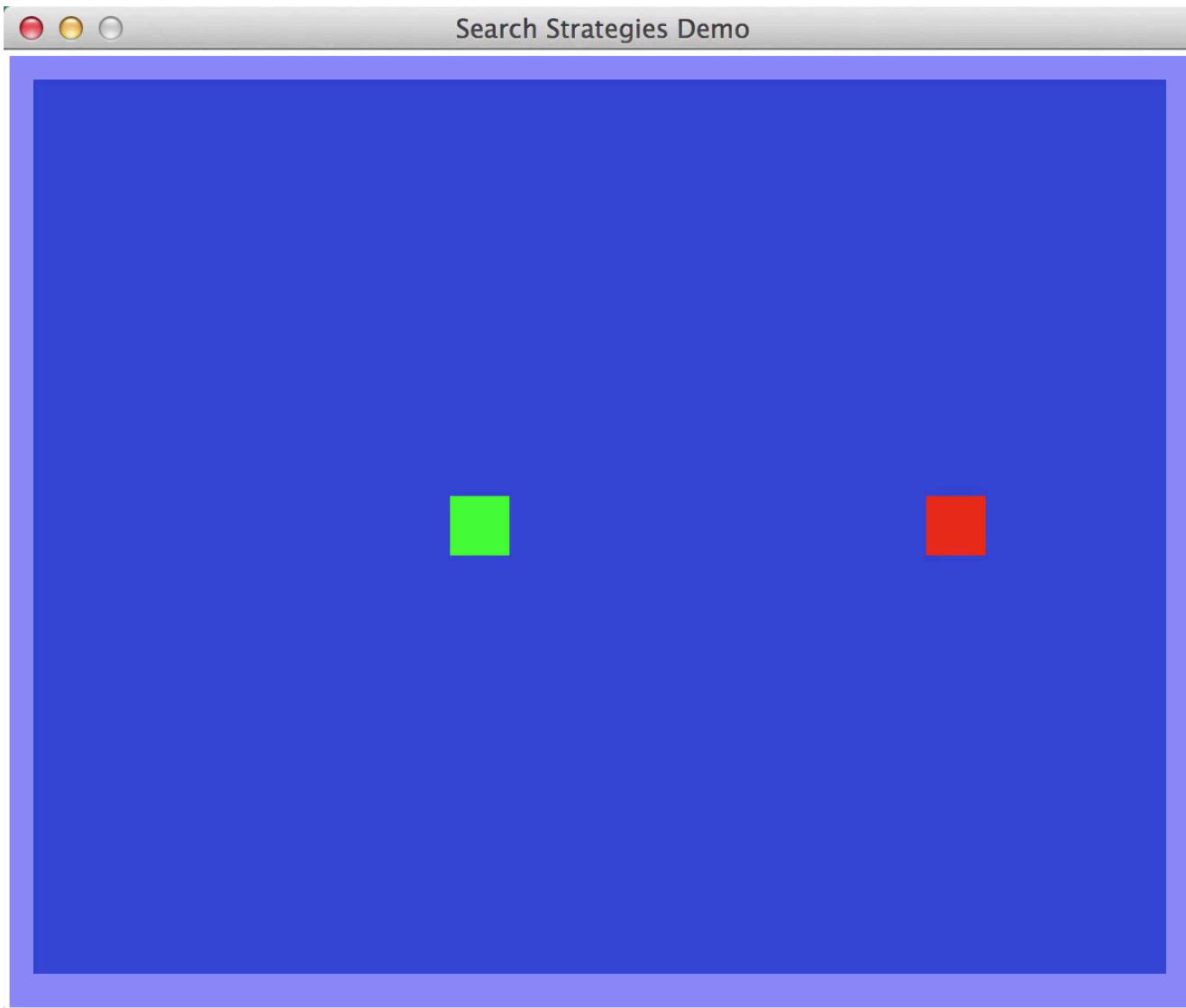
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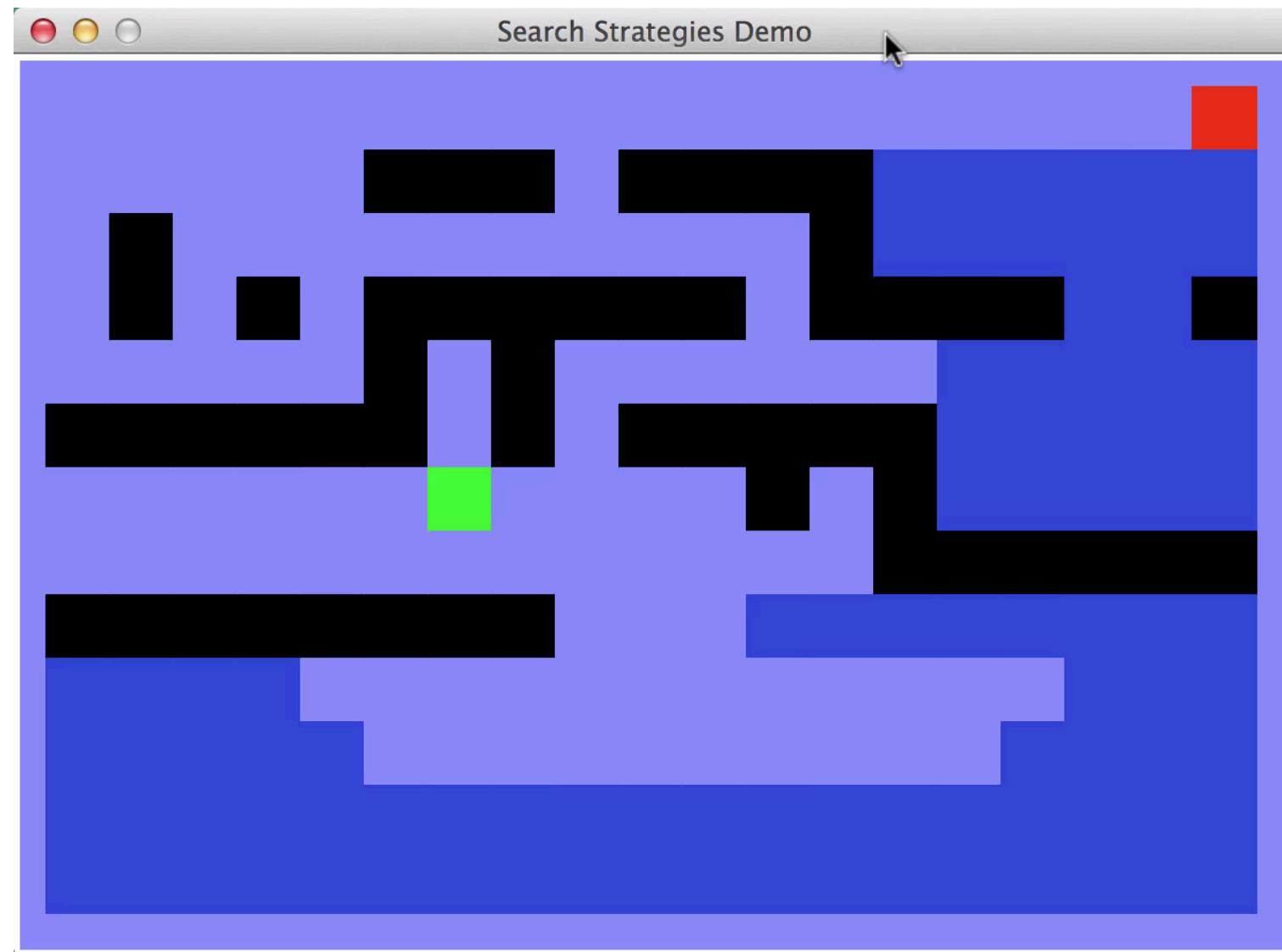
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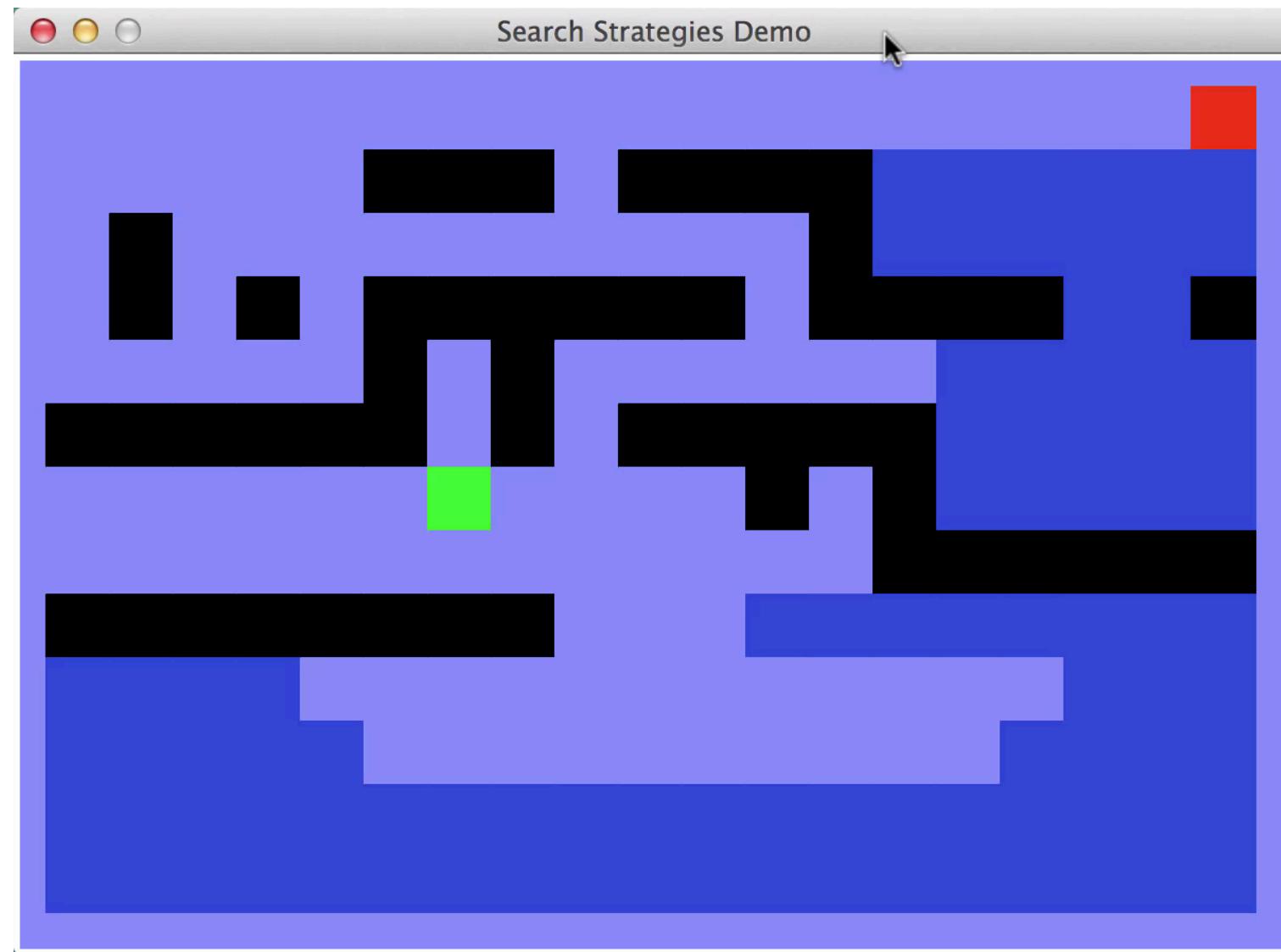
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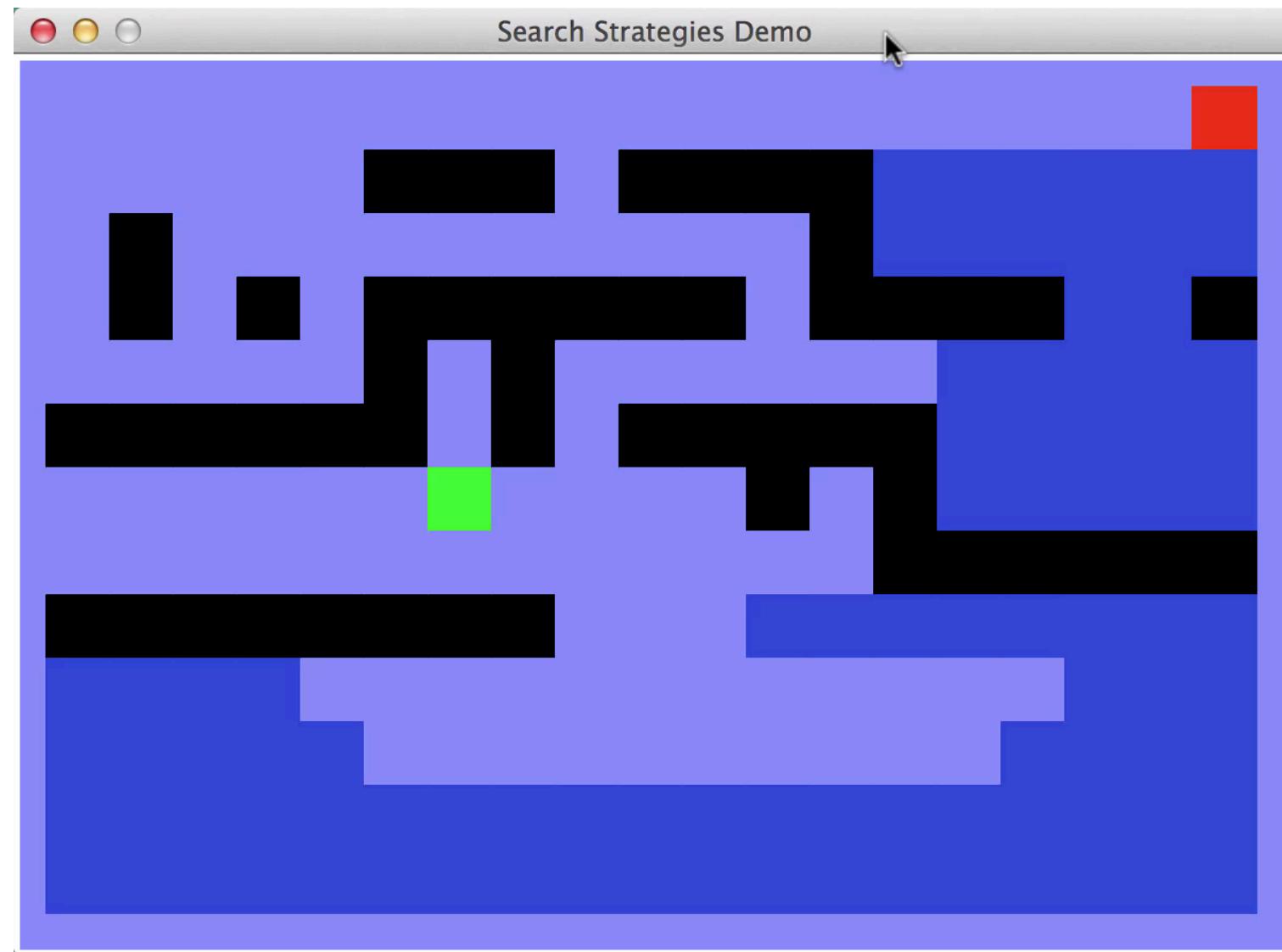
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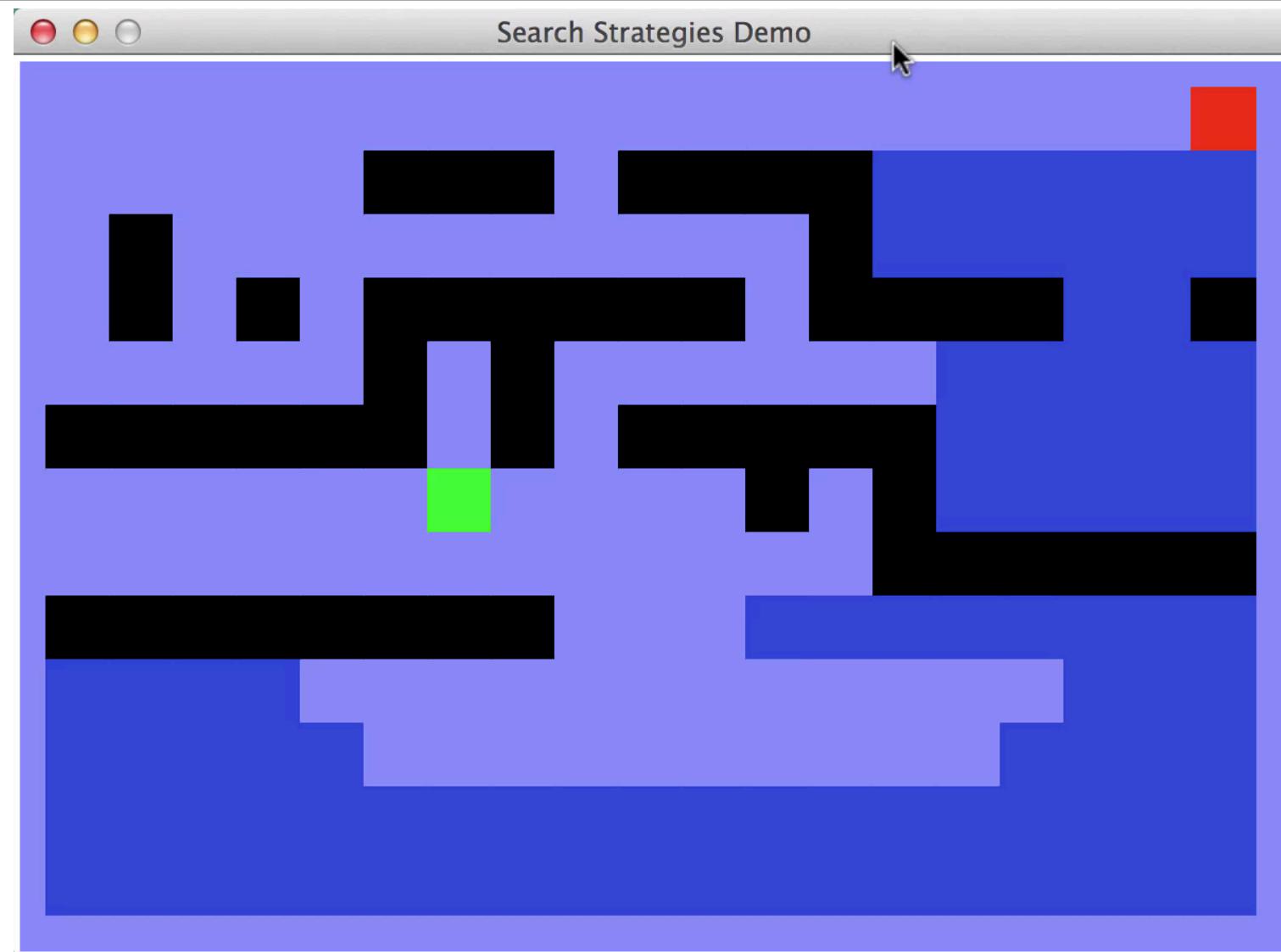
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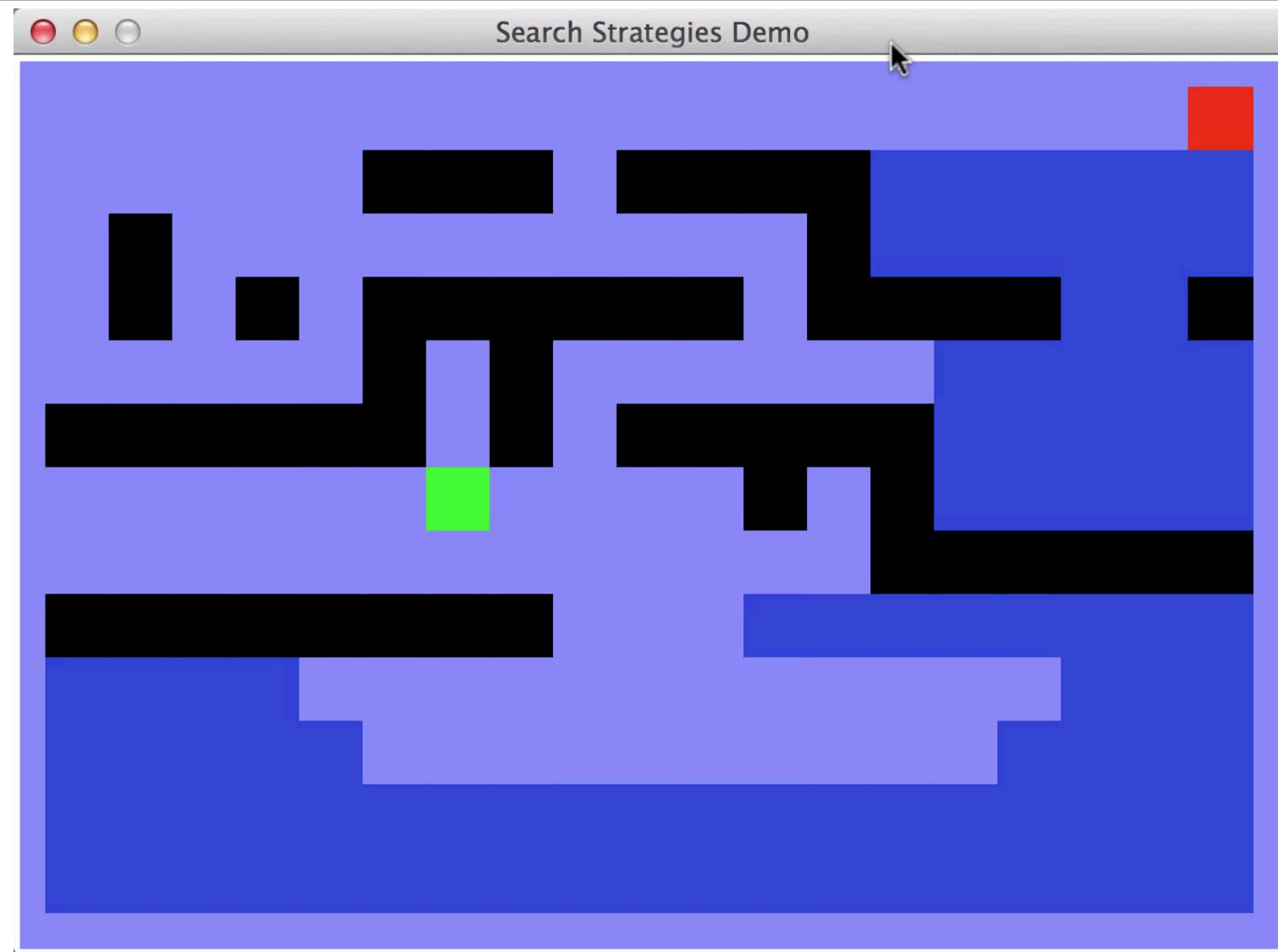
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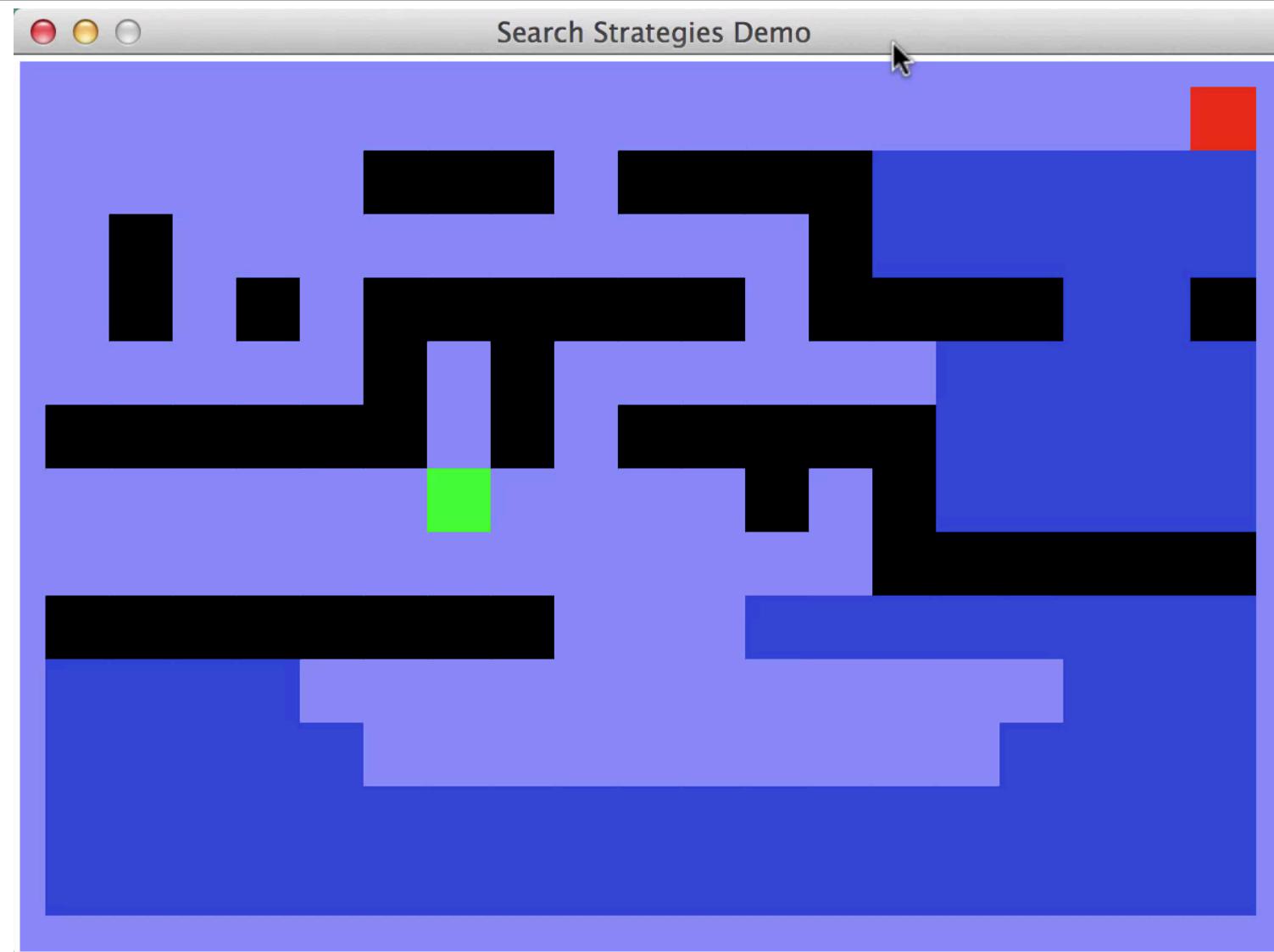
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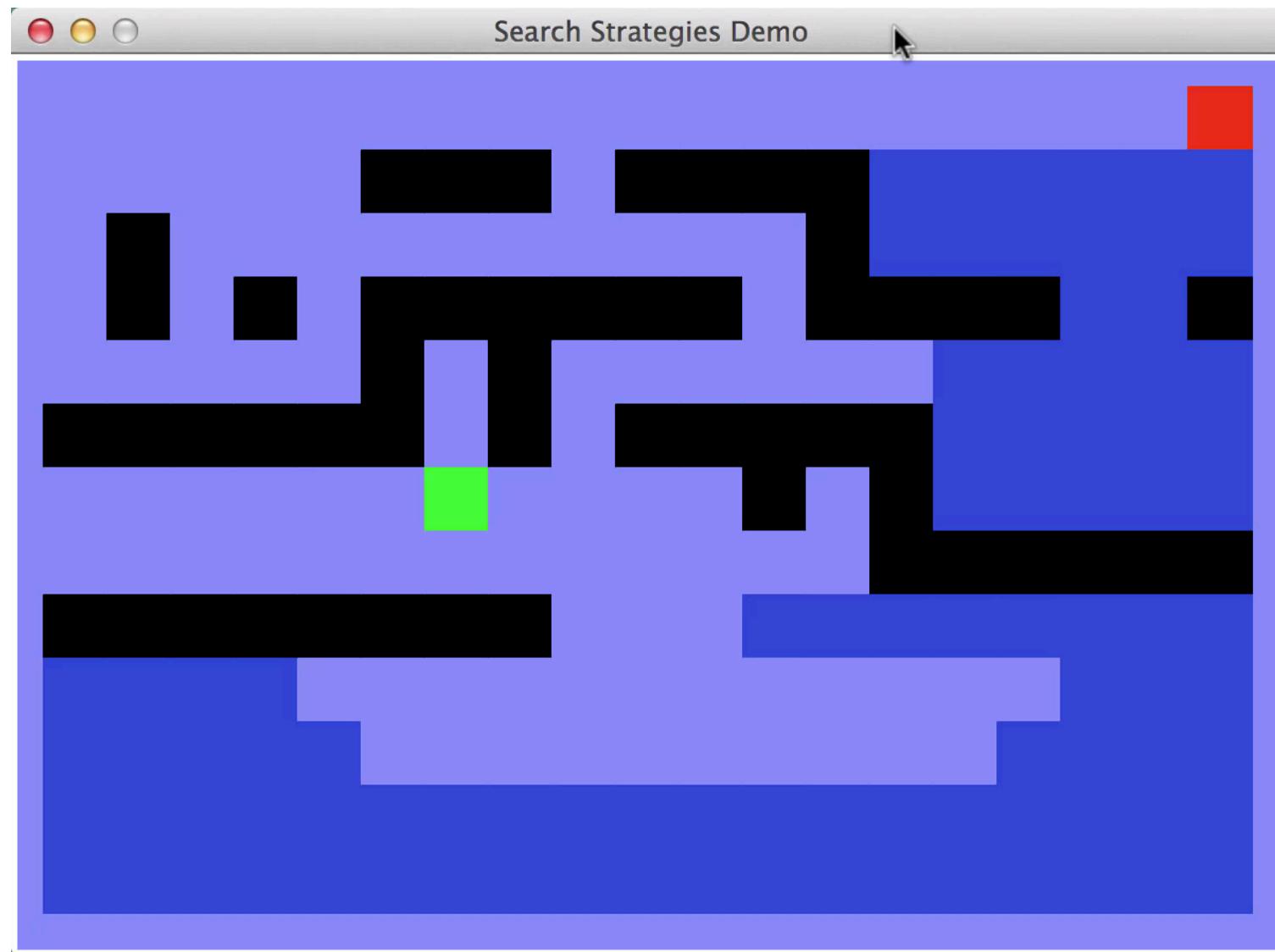
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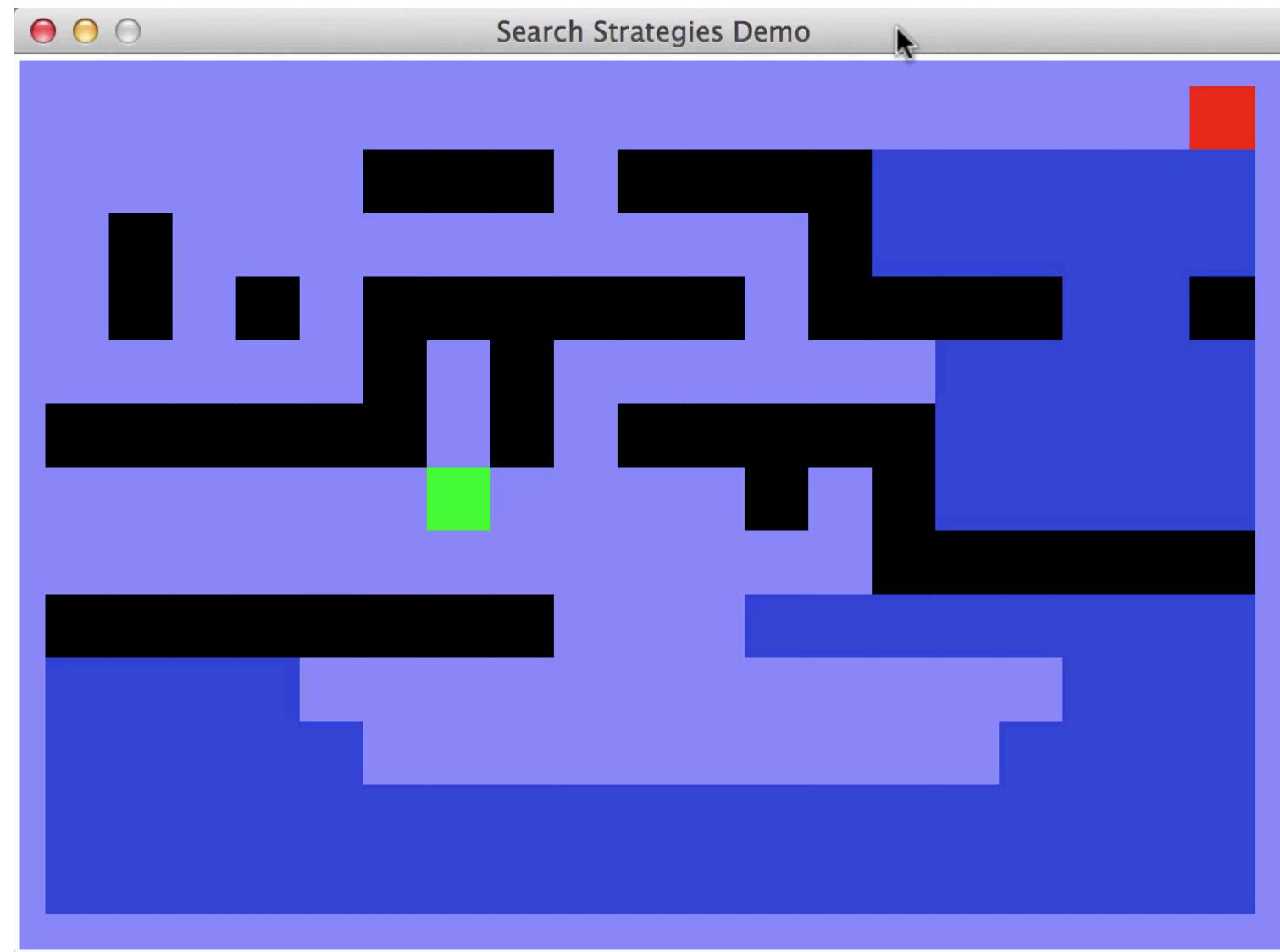
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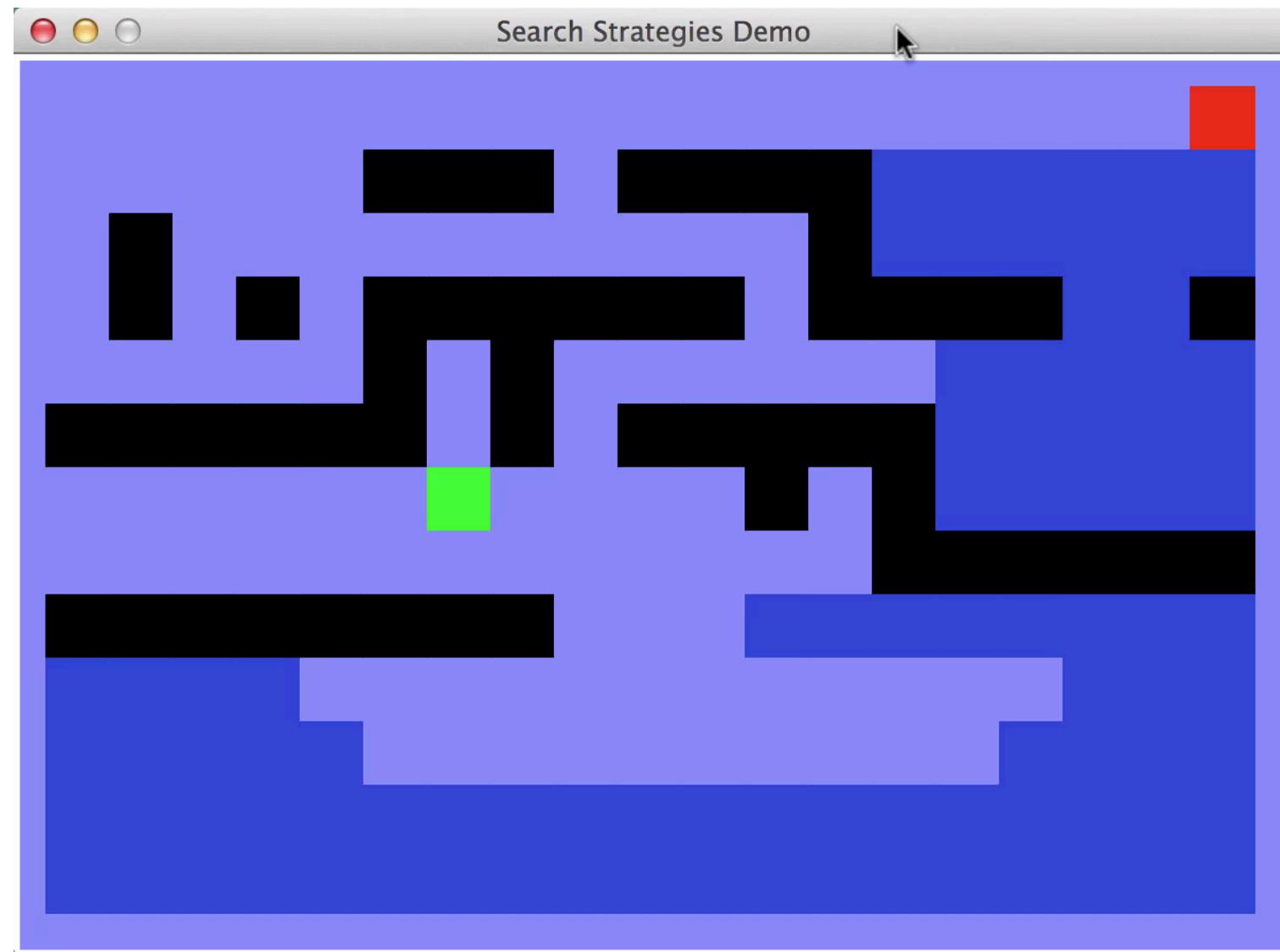
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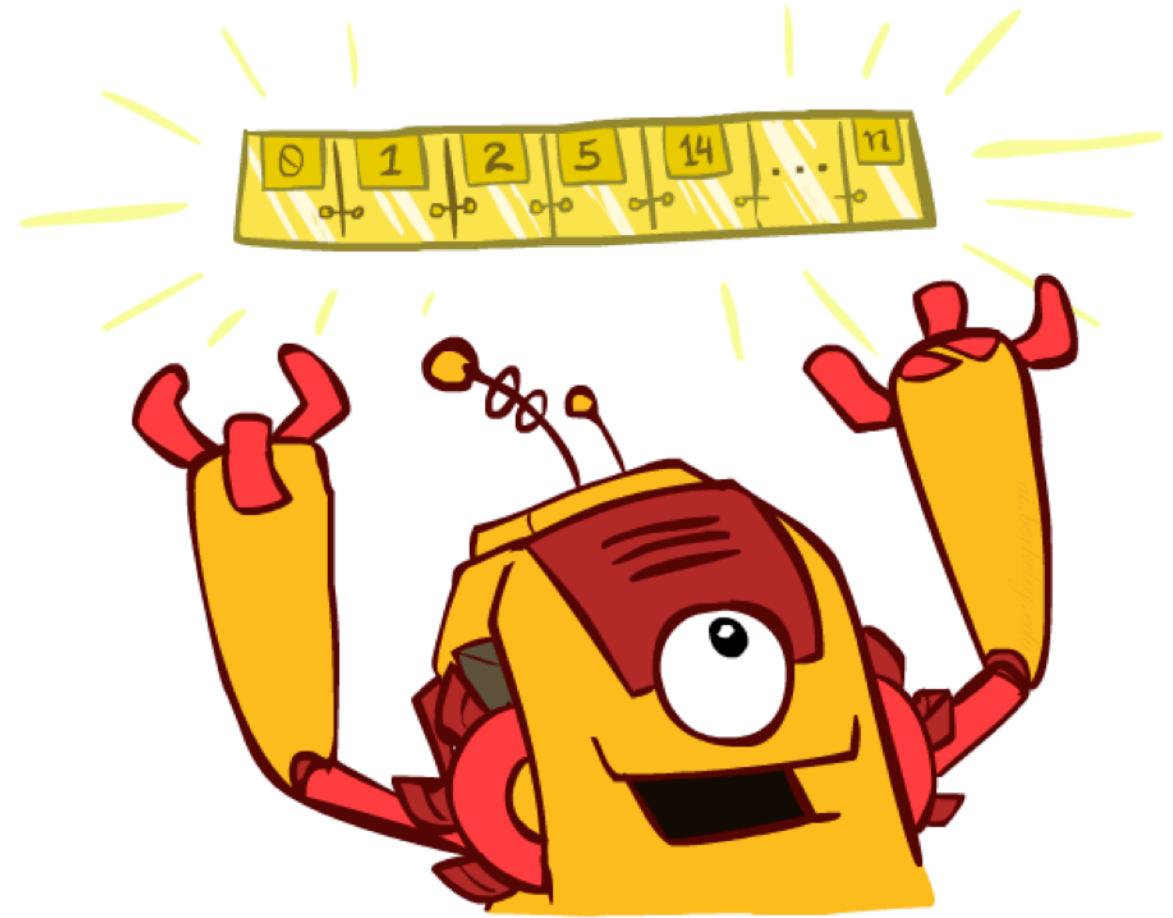


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# The One Queue

- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the  $\log(n)$  overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object



# Search and Models

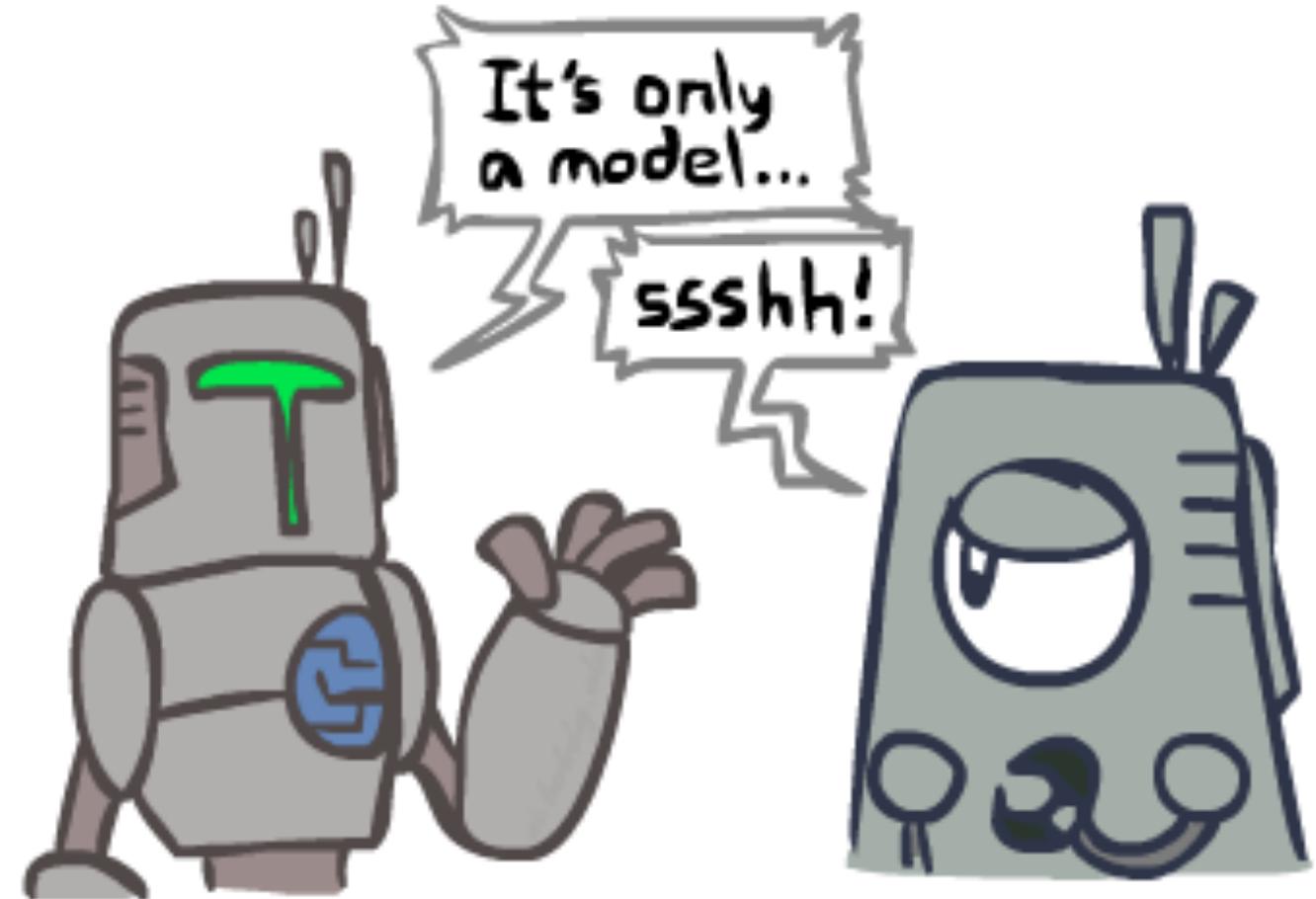
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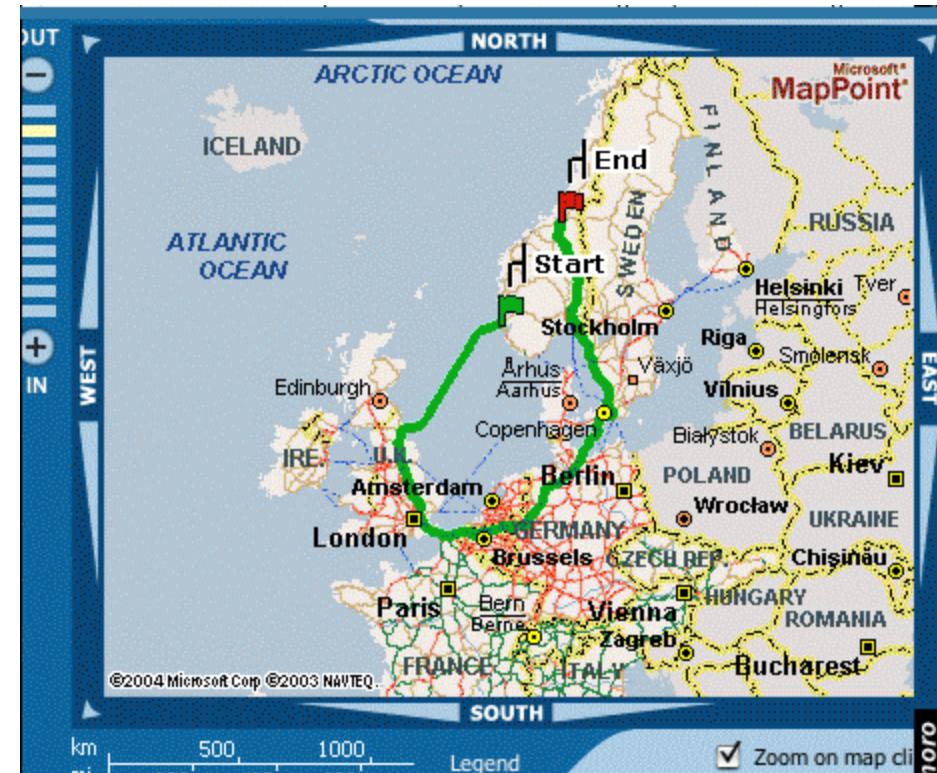
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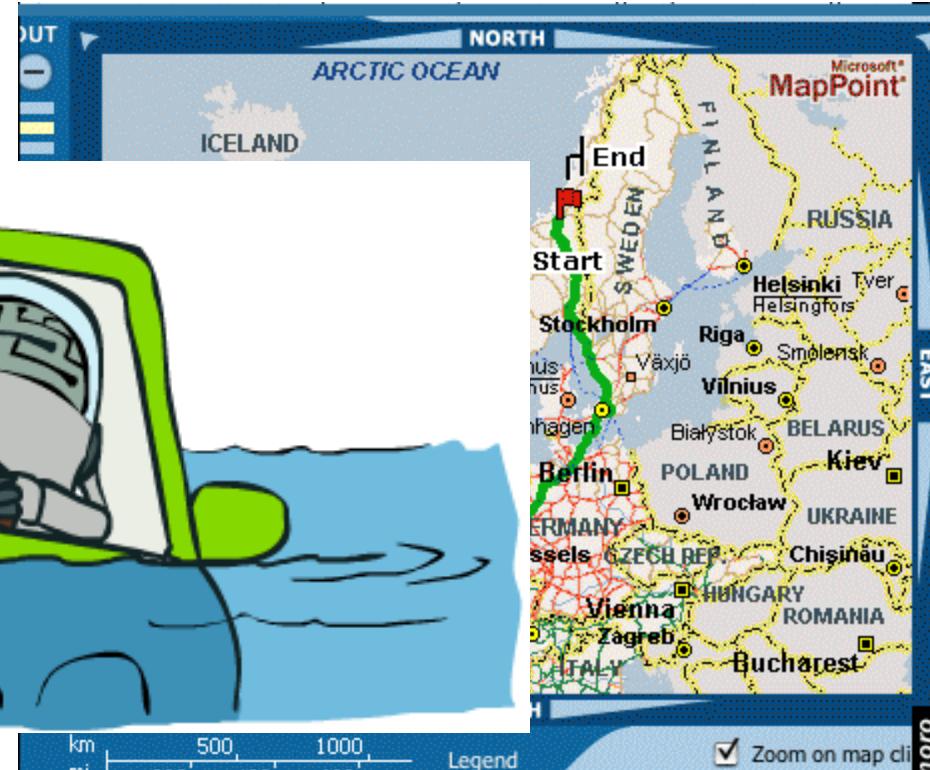
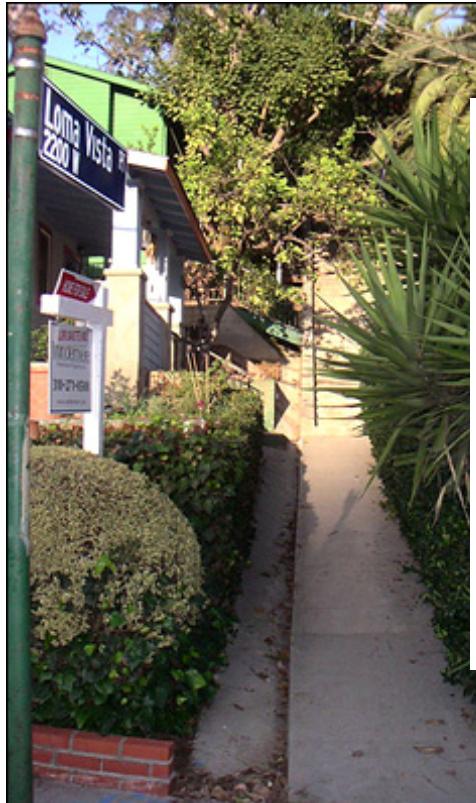
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