

Lecture 17: Unsupervised Learning

Alan Ritter

(many slides from Greg Durrett)

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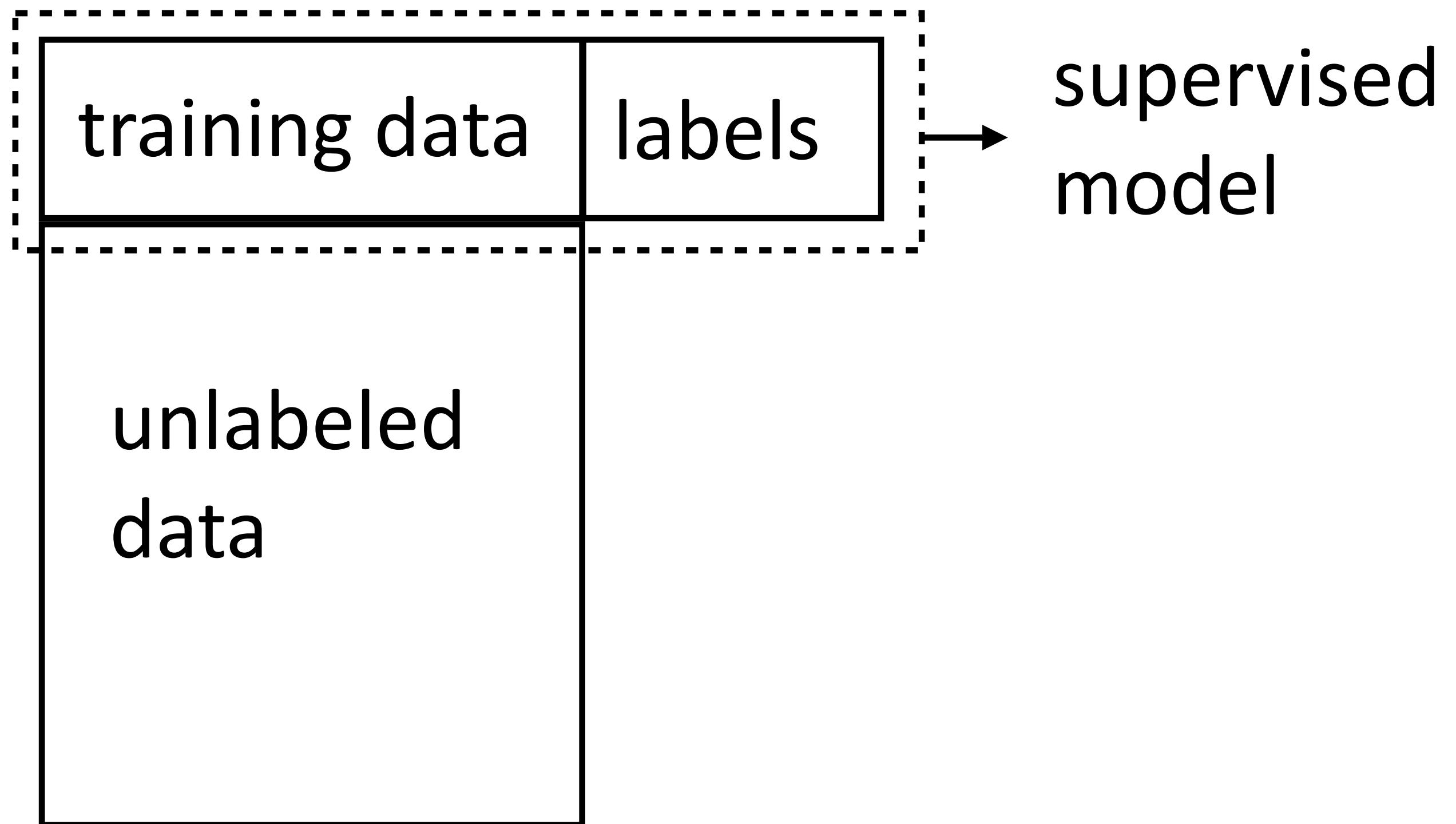
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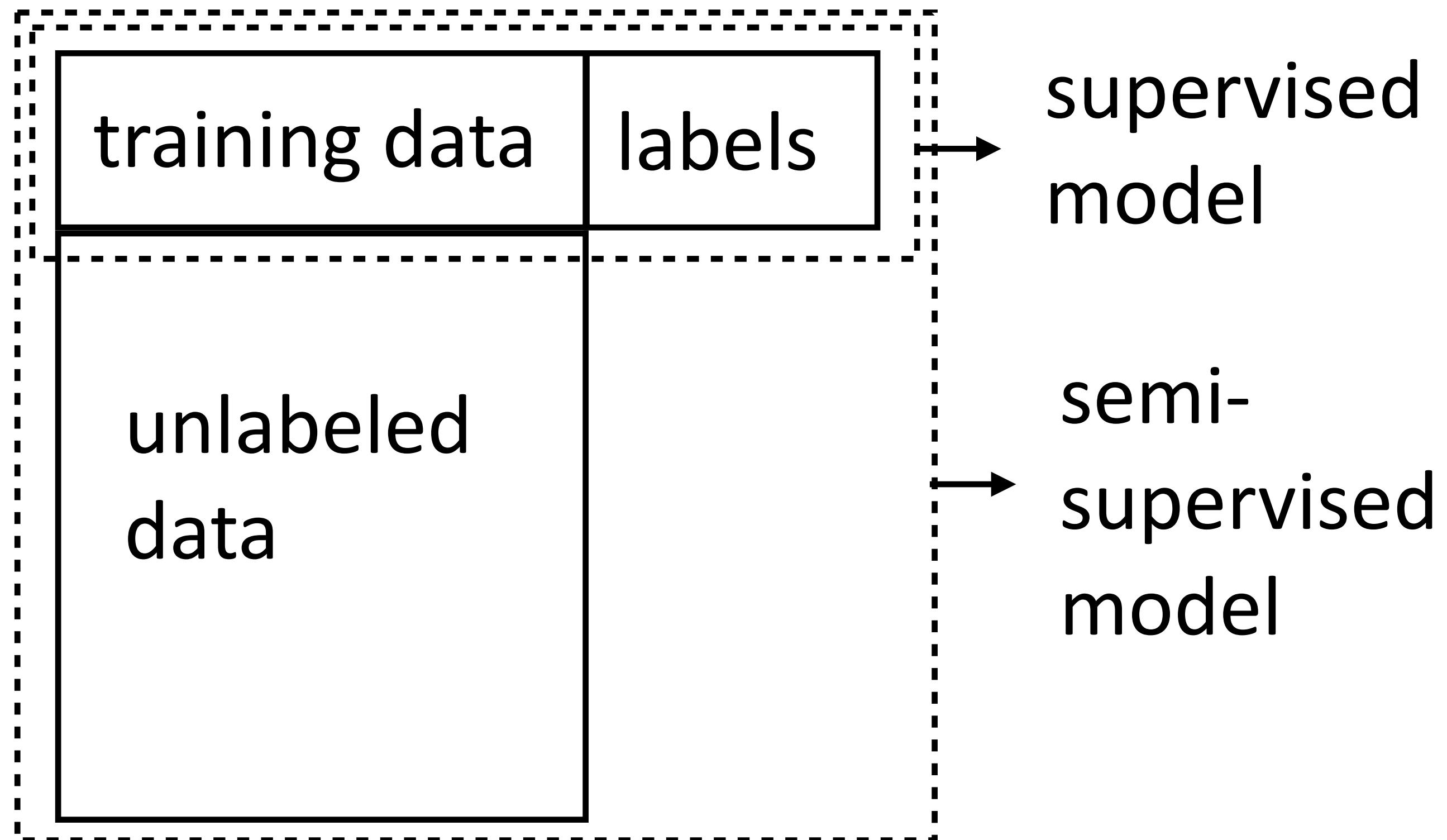
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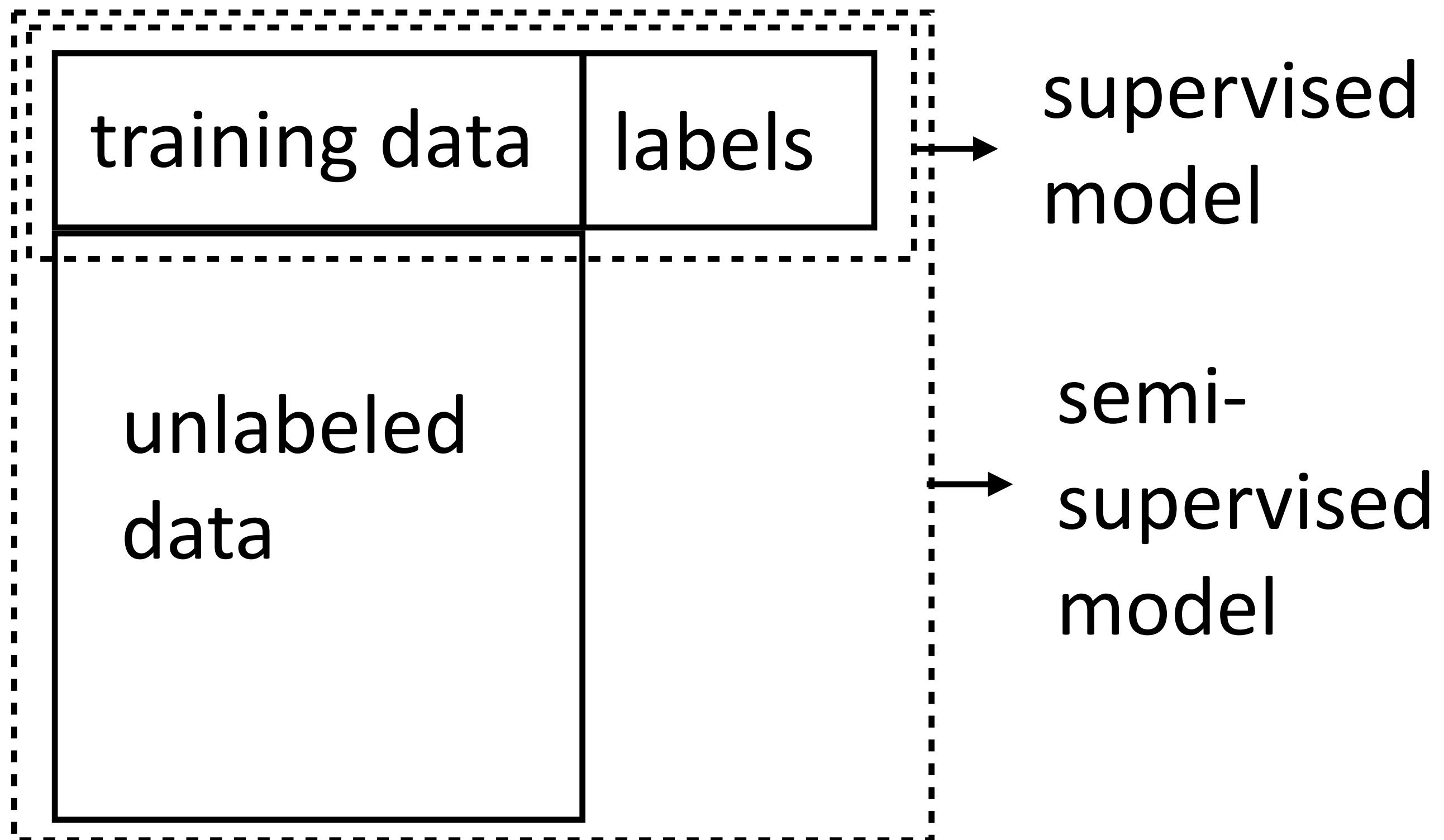
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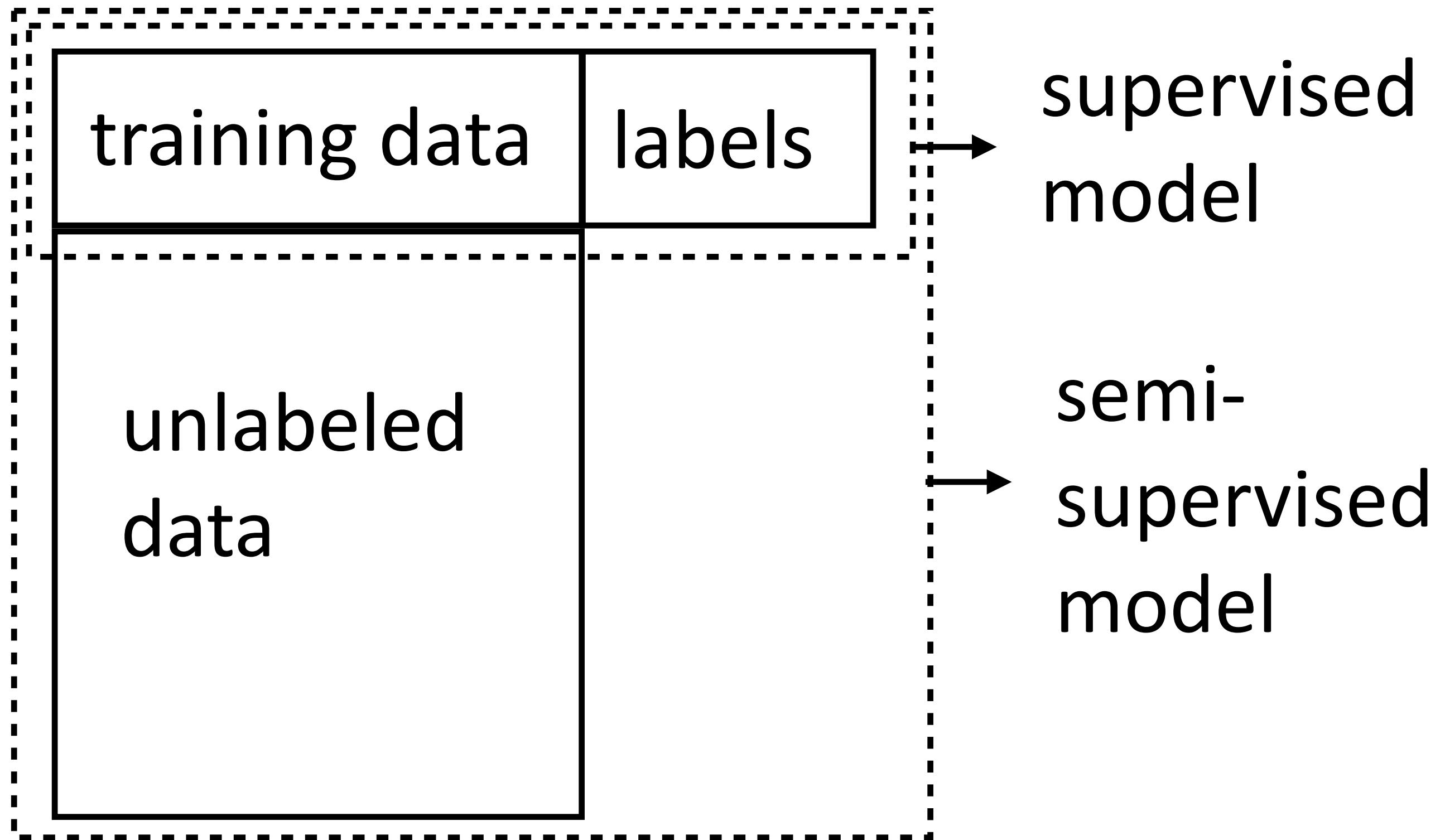
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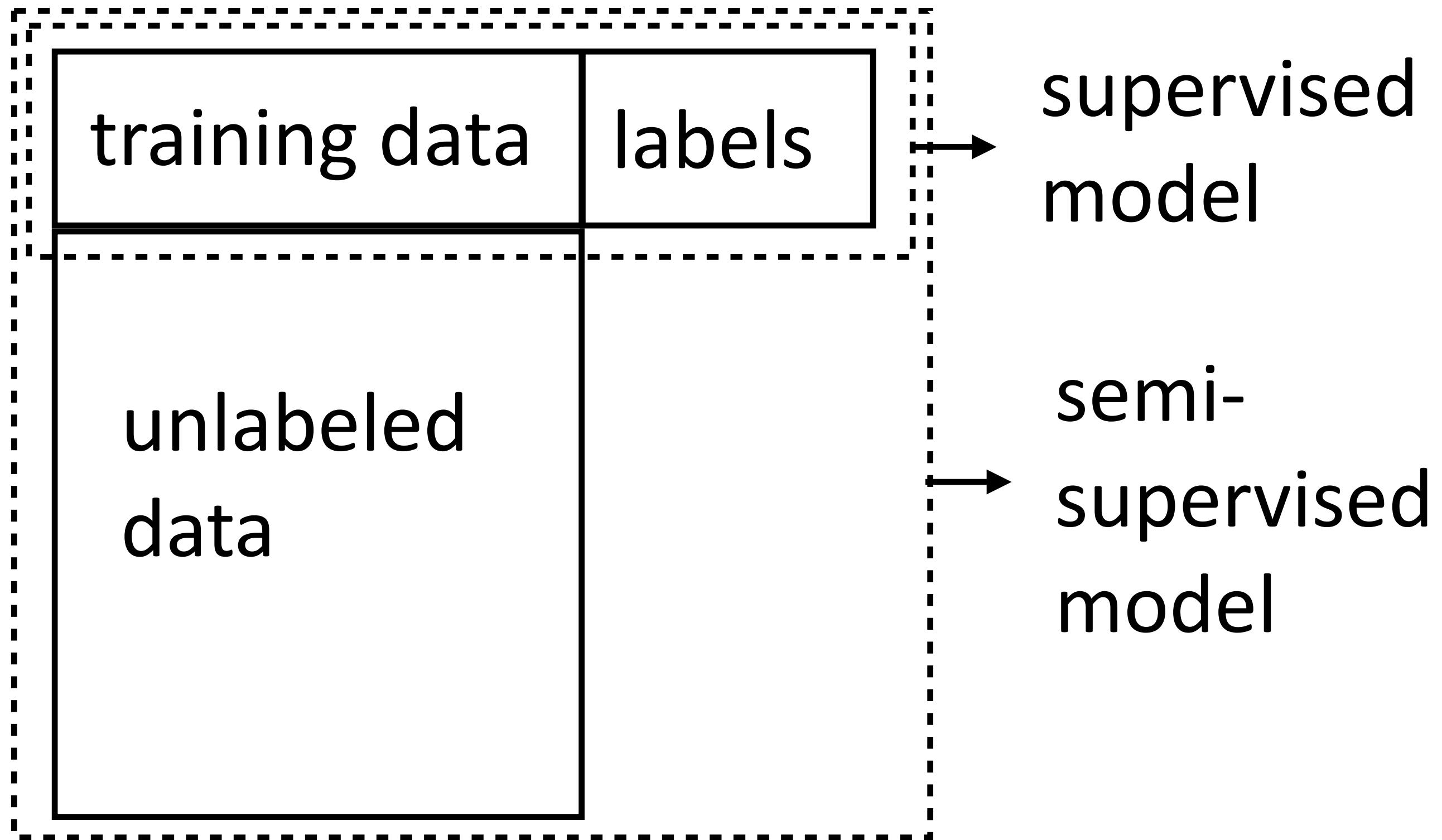
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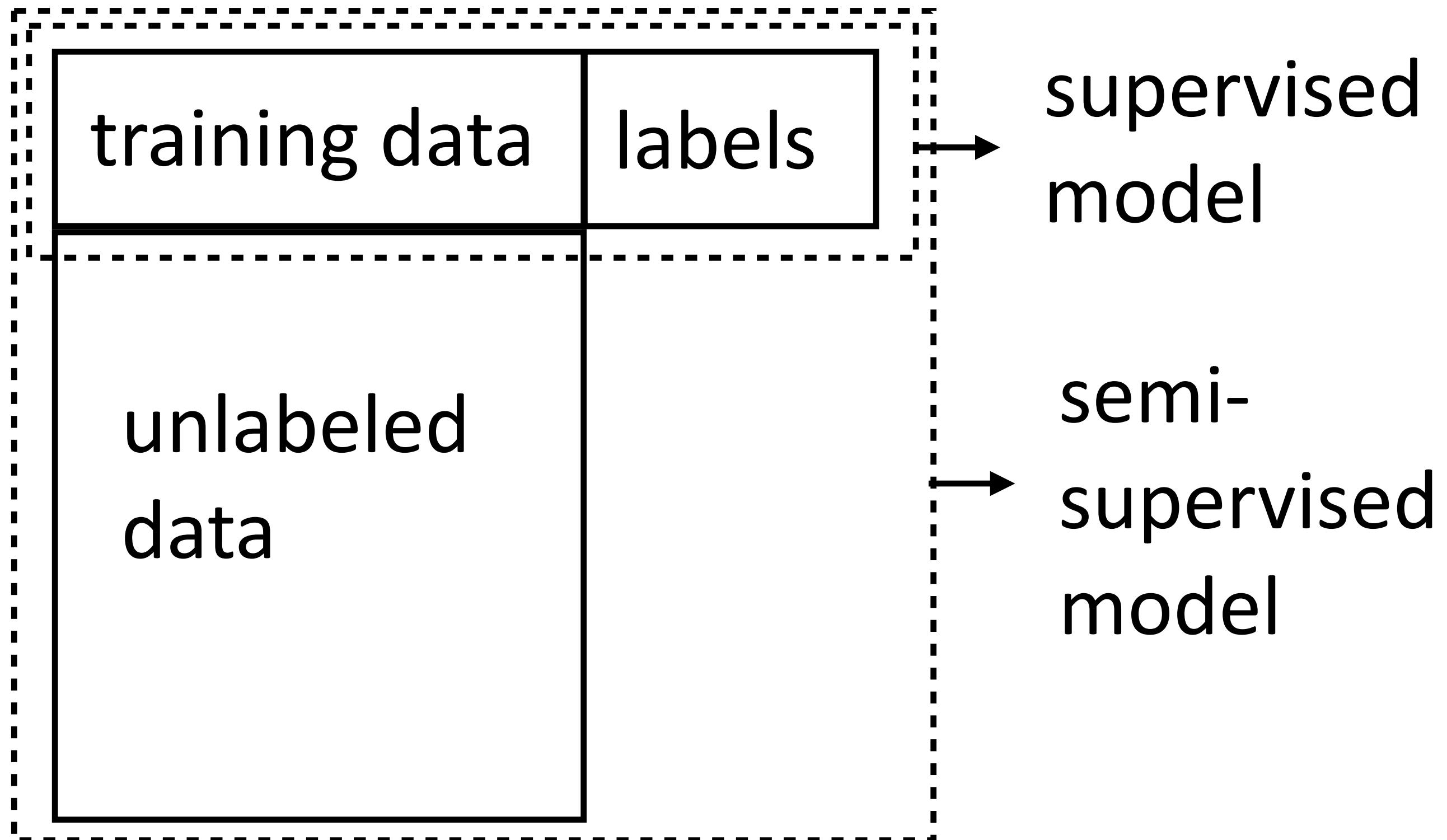
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 - ▶ Learn linguistic structure from unlabeled data and use it?



This Lecture

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- ▶ Discrete linguistic structure from generative models: unsupervised POS induction

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 - ▶ Expectation maximization for learning HMMs
- ▶ Continuous structure with generative models: variational autoencoders
- ▶ Continuous structure with “discriminative” models: transfer learning

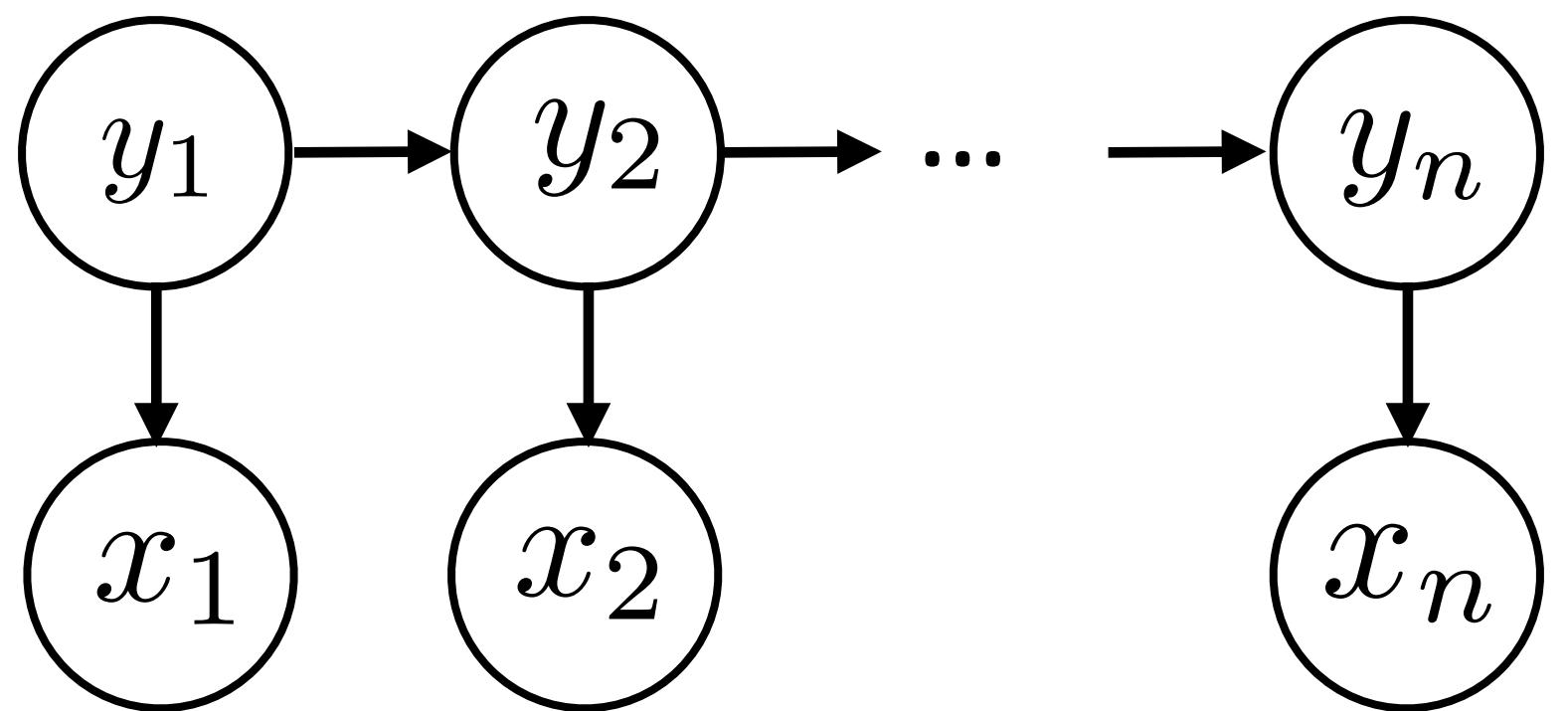
EM for HMMs

Recall: Hidden Markov Models

- ▶ Input $\mathbf{x} = (x_1, \dots, x_n)$ Output $\mathbf{y} = (y_1, \dots, y_n)$

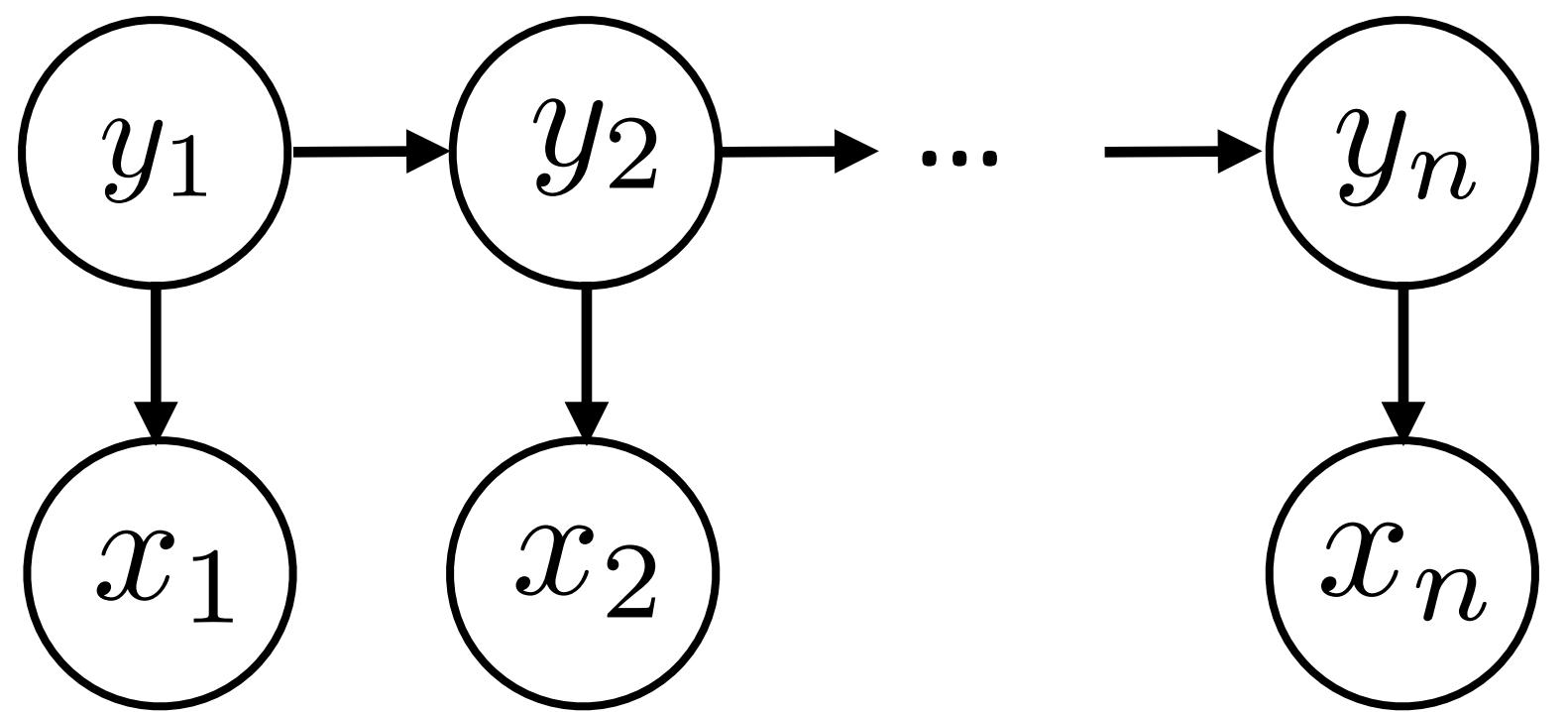
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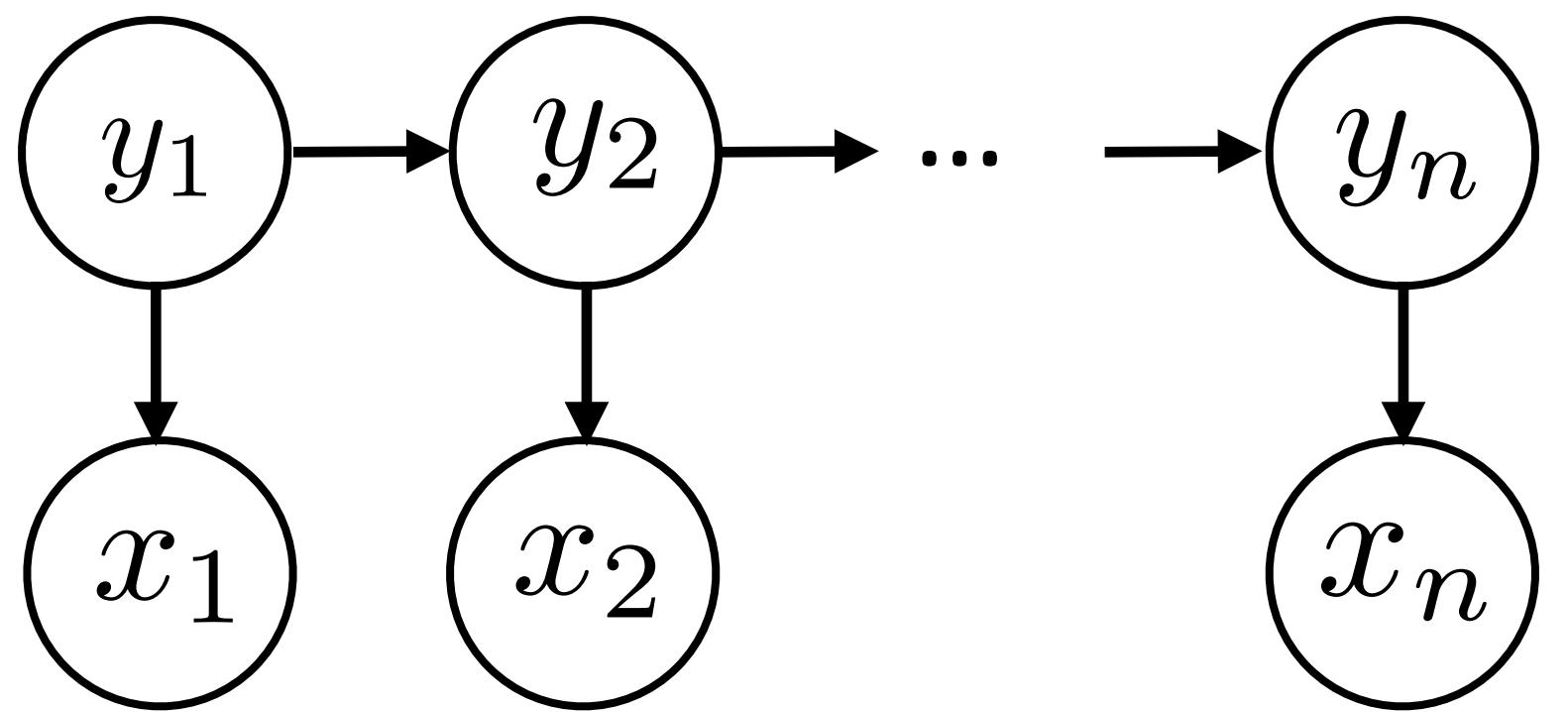
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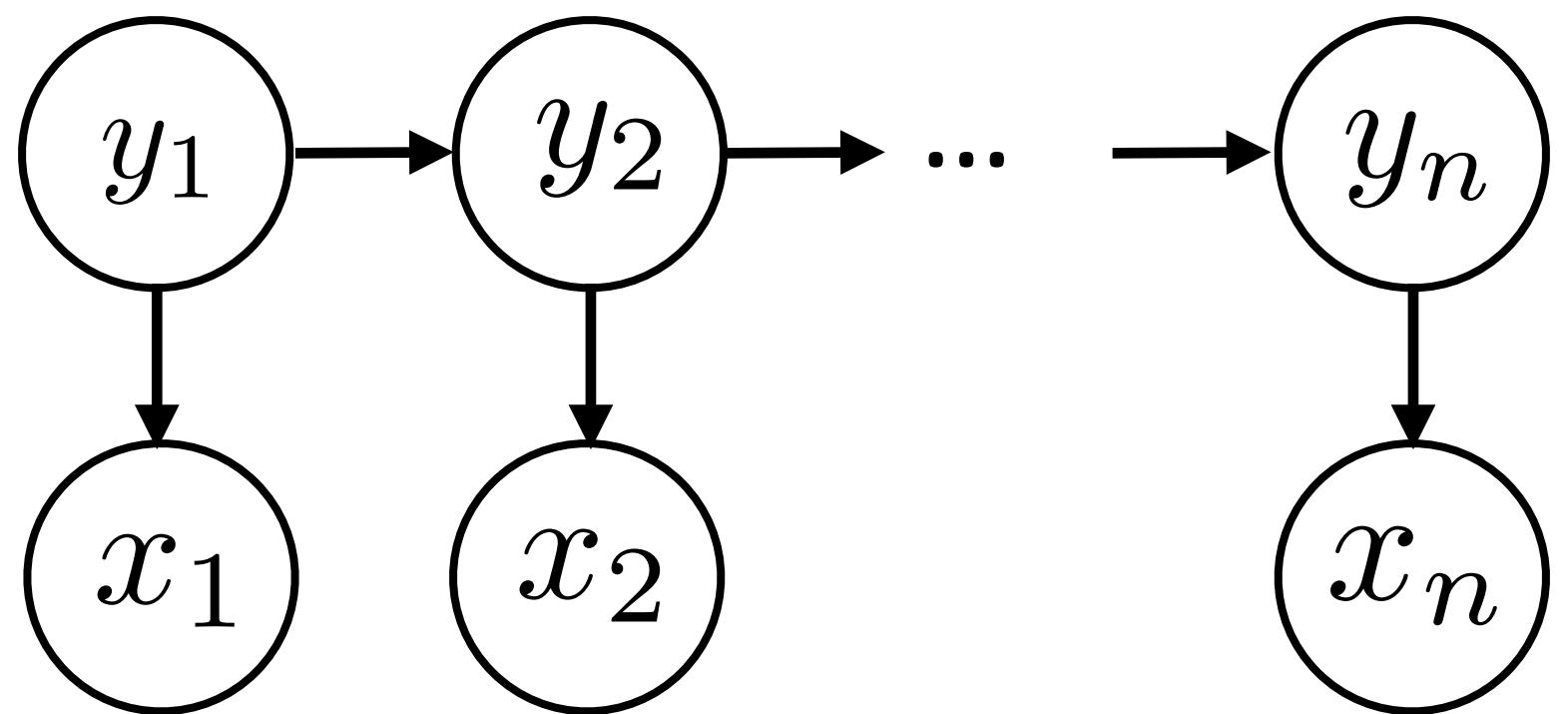


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Initial
distribution

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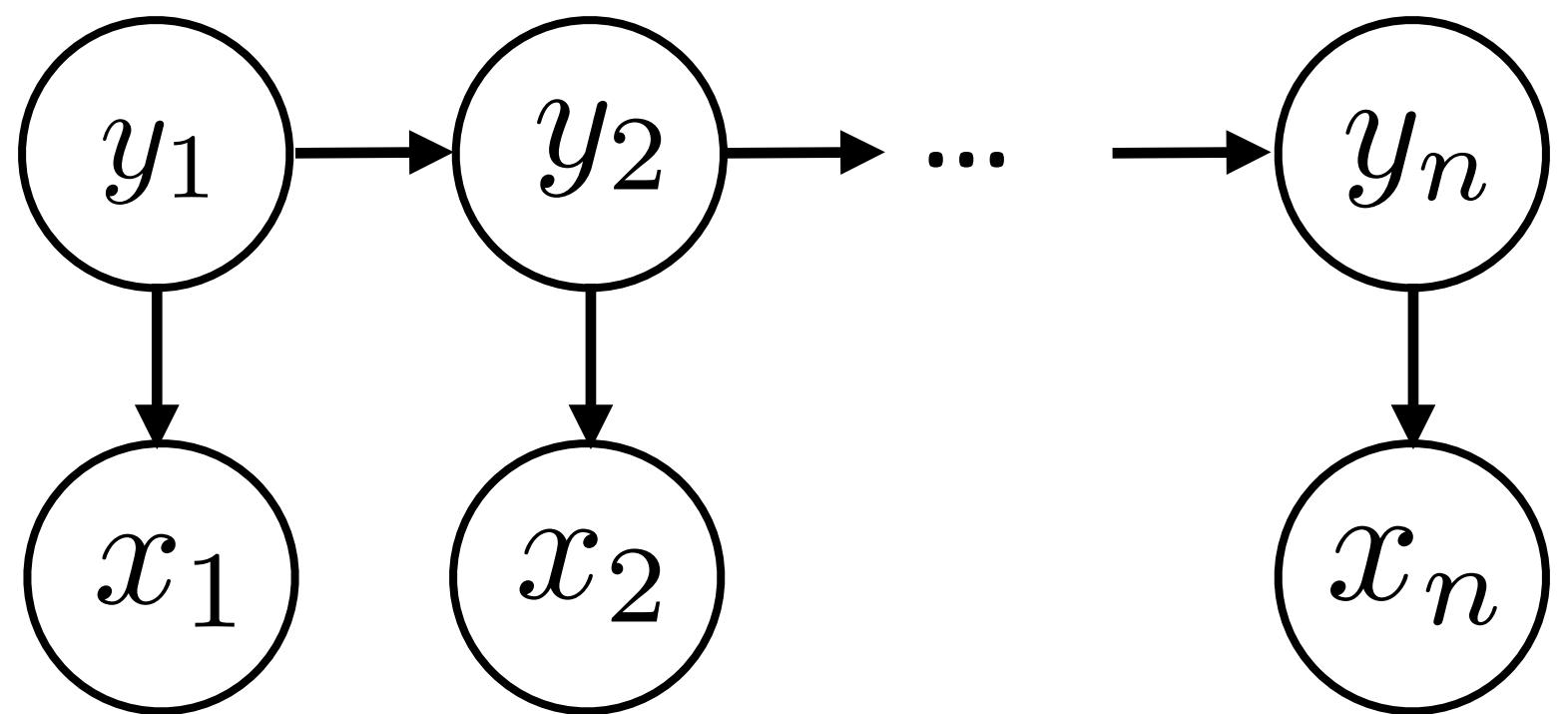
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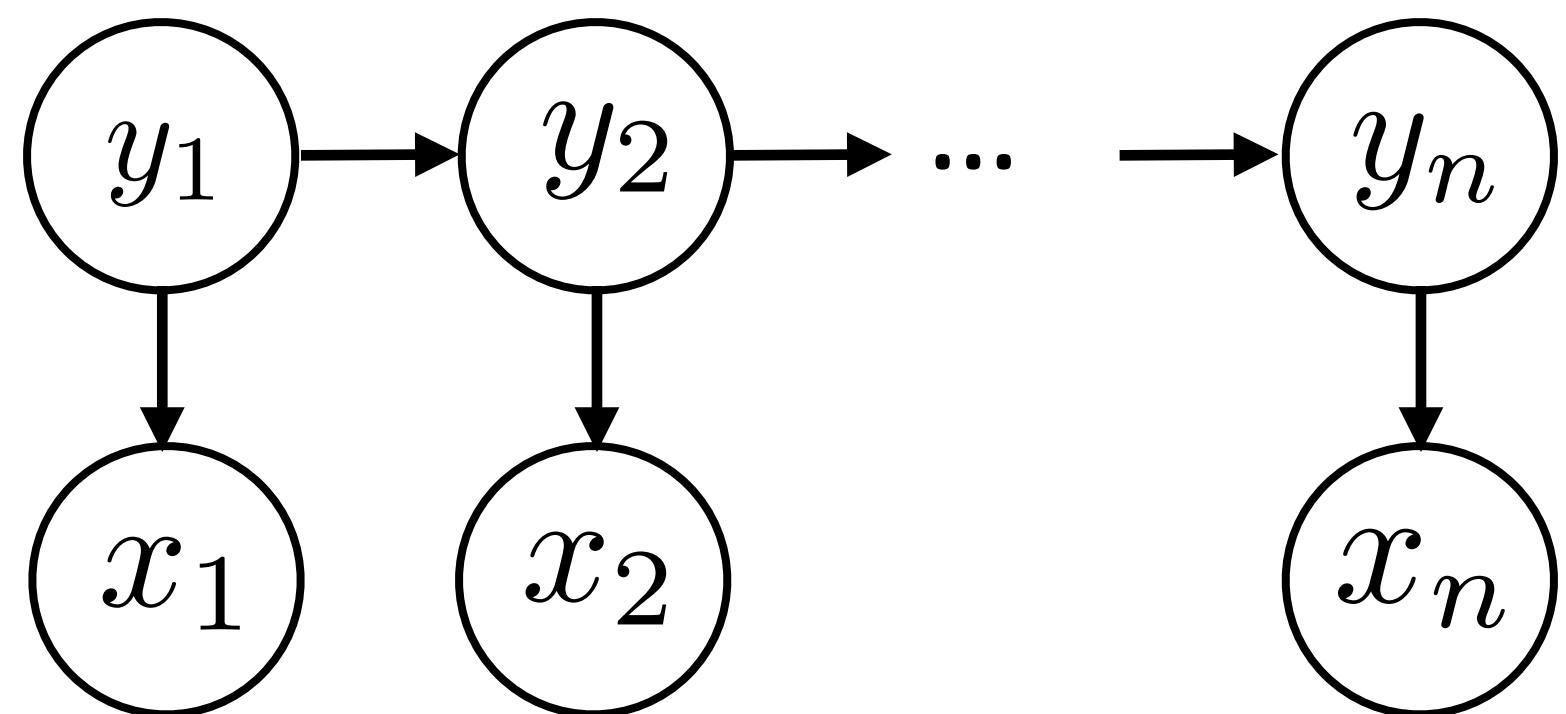
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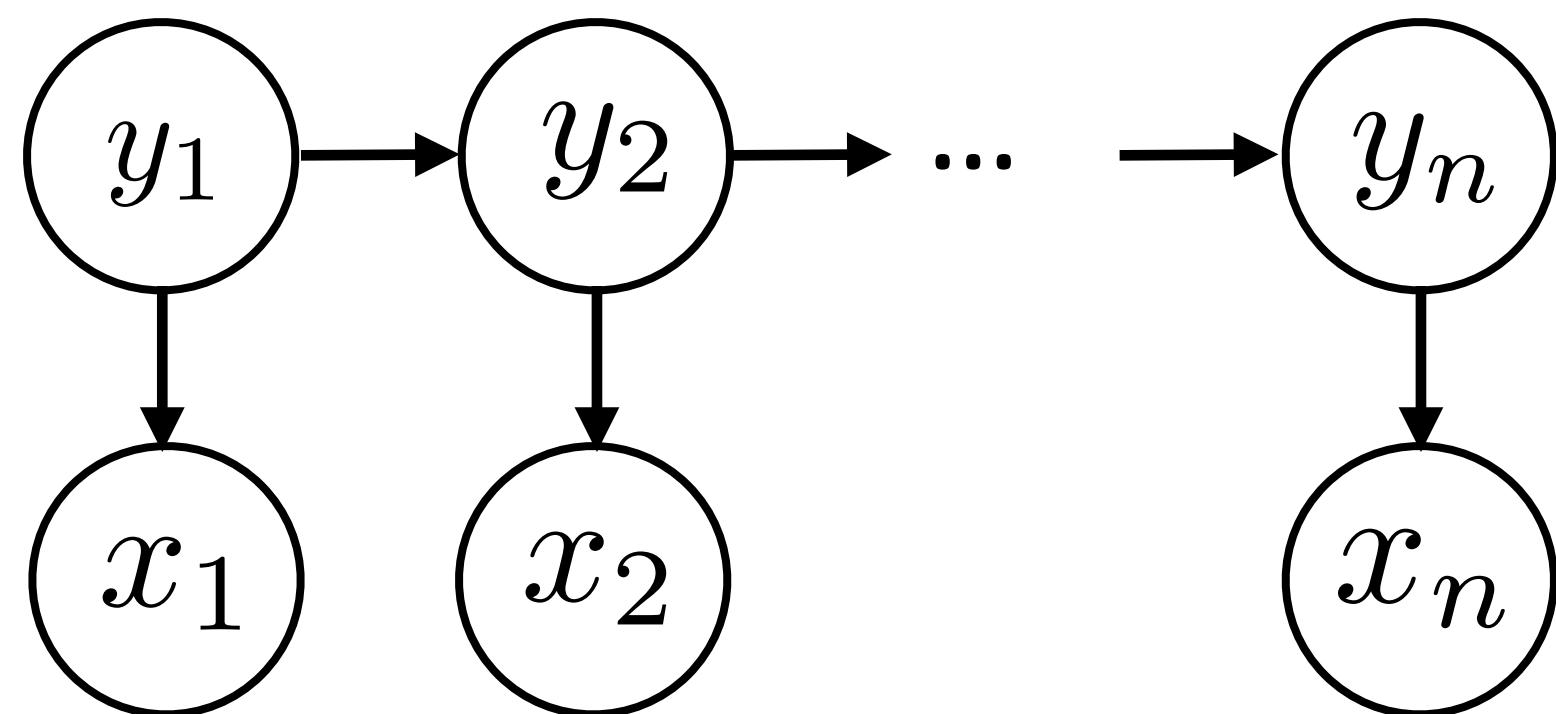


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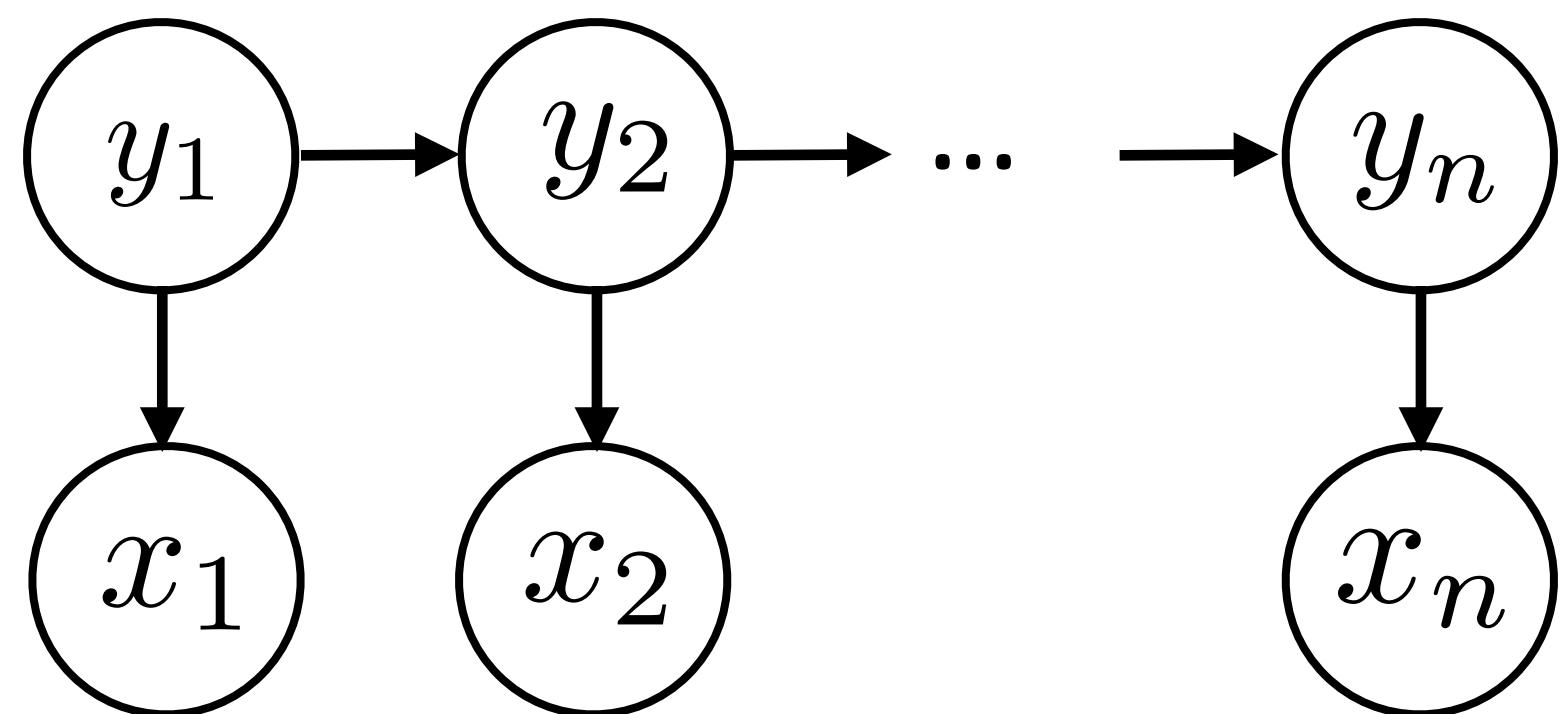


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- ▶ Observation (x) depends only on current state (y)
- ▶ Multinomials: tag x tag transitions, tag x word emissions
- ▶ $P(x|y)$ is a distribution over all words in the vocabulary
 - not a distribution over features (but could be!)

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- ▶ What did you do? Use current model parameters + data to refine your model. This is what EM will do

Part-of-Speech Induction

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▶ non-convex optimization problem

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- ▶ EM is just one procedure for optimizing this kind of objective

Expectation Maximization

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$$\log \sum_{\mathbf{y}} P(\mathbf{x}, \mathbf{y} | \theta)$$

► Condition on parameters θ

Expectation Maximization

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- ▶ Can optimize this lower-bound on log likelihood instead of log-likelihood

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- ▶ E-step: for an HMM: run forward-backward with the given parameters
- ▶ Compute $P(y_i = s | \mathbf{x}, \theta^{t-1})$, $P(y_i = s_1, y_{i+1} = s_2 | \mathbf{x}, \theta^{t-1})$

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tag marginals at each position	tag pair marginals at each position
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 - tag marginals at each position
 - tag pair marginals at each position
- ▶ M-step: set parameters to optimize the crazy argmax term

M-Step

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- ▶ Recall how we maximized $\log P(x,y)$: read counts off data

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DT NN

the dog

M-Step

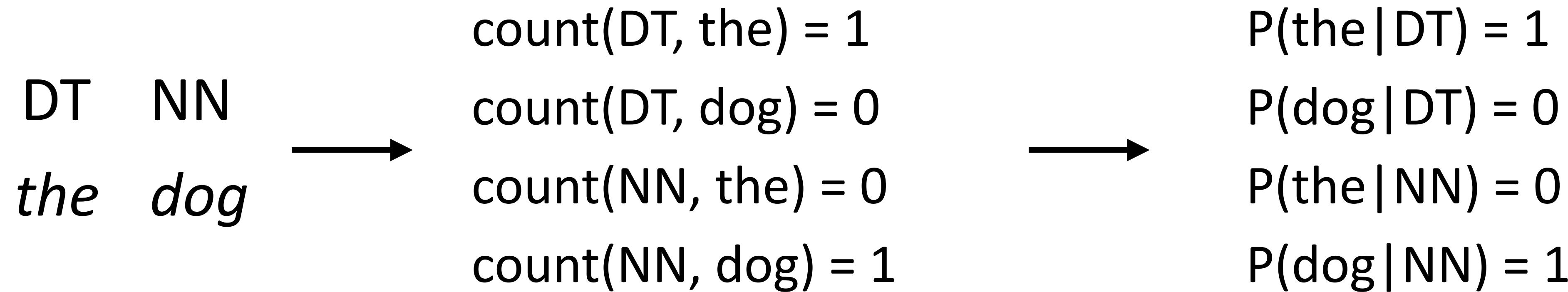
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DT NN
the *dog* →

count(DT, the) = 1
count(DT, dog) = 0
count(NN, the) = 0
count(NN, dog) = 1

M-Step

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M-Step

- ▶ Recall how we maximized $\log P(x,y)$: read counts off data

DT NN	\rightarrow	count(DT, the) = 1	$P(\text{the} \text{DT}) = 1$
<i>the dog</i>		count(DT, dog) = 0	$P(\text{dog} \text{DT}) = 0$
		count(NN, the) = 0	$P(\text{the} \text{NN}) = 0$
		count(NN, dog) = 1	$P(\text{dog} \text{NN}) = 1$

- ▶ Same procedure, but maximizing $P(x,y)$ in expectation under q means that q specifies *fractional counts*

M-Step

- Recall how we maximized $\log P(x,y)$: read counts off data

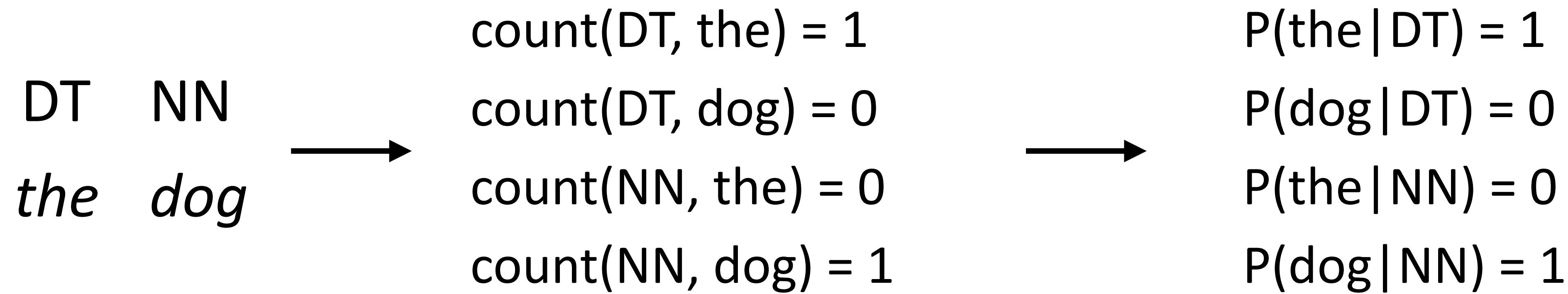
DT NN	\rightarrow	count(DT, the) = 1	$P(\text{the} \text{DT}) = 1$
<i>the dog</i>		count(DT, dog) = 0	$P(\text{dog} \text{DT}) = 0$
		count(NN, the) = 0	$P(\text{the} \text{NN}) = 0$
		count(NN, dog) = 1	$P(\text{dog} \text{NN}) = 1$

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the dog

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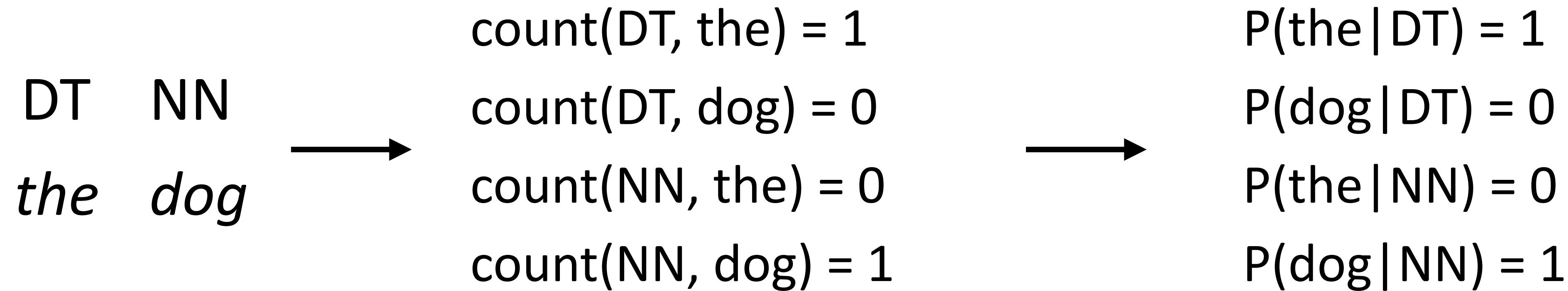


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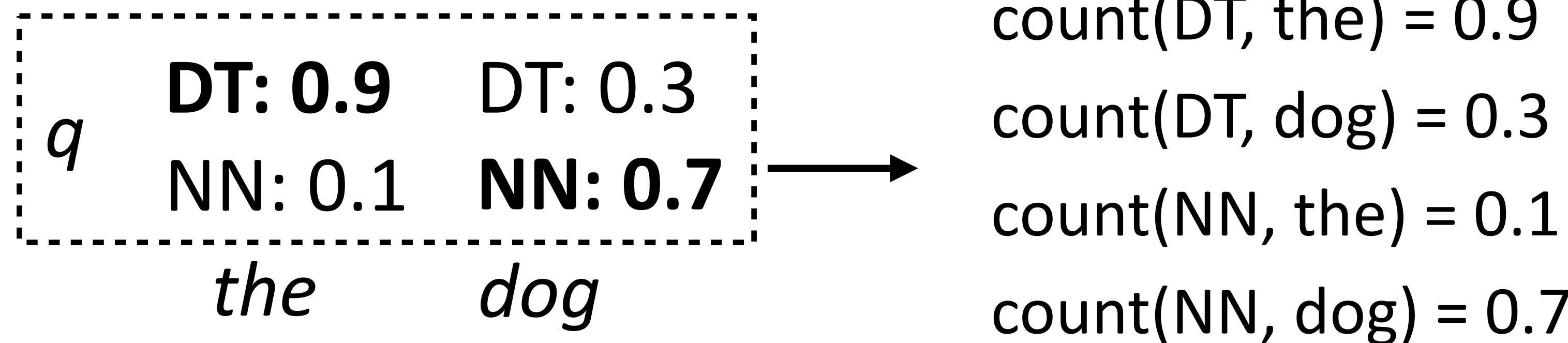
q	DT: 0.9	DT: 0.3
	NN: 0.1	NN: 0.7
	the	dog

M-Step

- Recall how we maximized $\log P(x,y)$: read counts off data

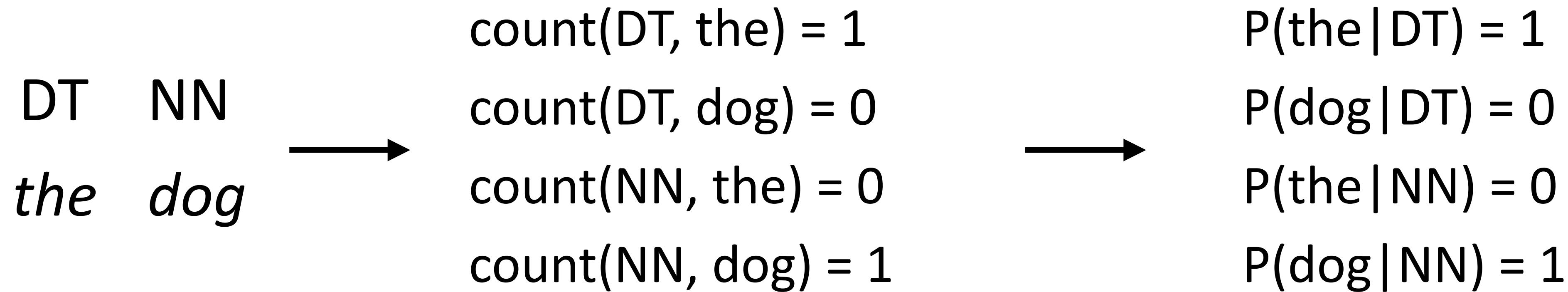


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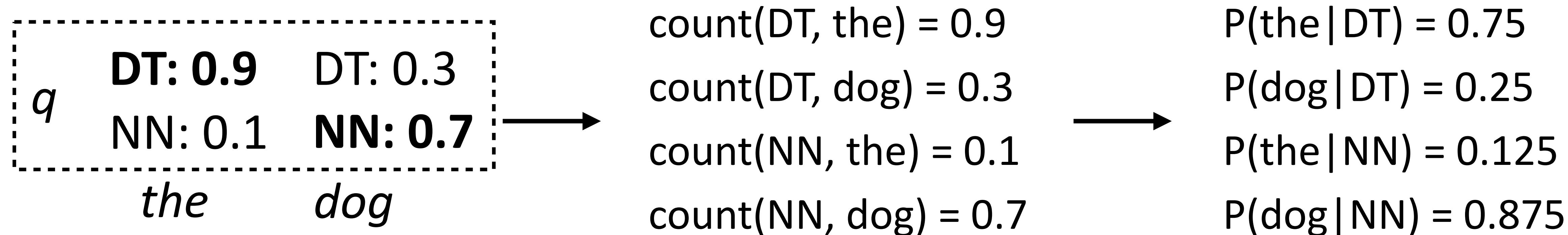


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M-Step

M-Step

- ▶ Same for transition probabilities

M-Step

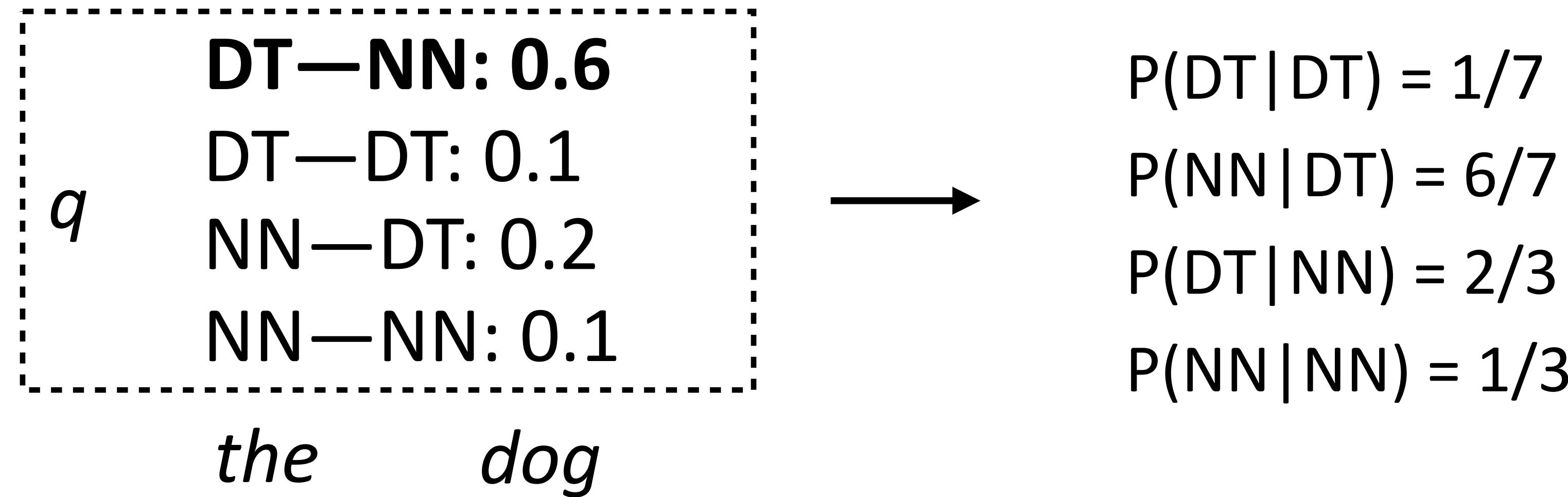
- ▶ Same for transition probabilities

q	DT—NN: 0.6
	DT—DT: 0.1
	NN—DT: 0.2
	NN—NN: 0.1

the dog

M-Step

- ▶ Same for transition probabilities



How does EM learn things?

How does EM learn things?

the dog

the marsupial

How does EM learn things?

- ▶ Initialize (M-step 0):
 - ▶ Emissions

the dog

the marsupial

How does EM learn things?

- ▶ Initialize (M-step 0):

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$$P(\text{the} \mid \text{DT}) = 0.9$$

$$P(\text{dog} \mid \text{DT}) = 0.05$$

$$P(\text{marsupial} \mid \text{DT}) = 0.05$$

the dog

the marsupial

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$$P(\text{the} \mid \text{NN}) = 0.05$$

$$P(\text{dog} \mid \text{NN}) = 0.9$$

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the dog

the marsupial

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$$P(\text{dog} \mid \text{NN}) = 0.9$$

$$P(\text{marsupial} \mid \text{NN}) = 0.05$$

- ▶ Transition probabilities: uniform

the

dog

the

marsupial

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- ▶ E-step 1: (all values are approximate)

the

dog

the

marsupial

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- ▶ Transition probabilities: uniform

- ▶ E-step 1: (all values are approximate)

DT: 0.95 DT: 0.05

NN: 0.05 NN: 0.95

the dog

the marsupial

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the *dog*

DT: 0.95 DT: 0.5

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the *marsupial*

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$$P(\text{the} \mid \text{NN}) = 0.05$$

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DT: 0.95 DT: 0.05

NN: 0.05 NN: 0.95

the

dog

DT: 0.95 DT: 0.5

NN: 0.05 NN: 0.5

the

marsupial

▶ uniform

How does EM learn things?

► E-step 1:

DT: 0.95 DT: 0.05

NN: 0.05 **NN: 0.95**

the *dog*

DT: 0.95 DT: 0.5

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the **marsupial**

How does EM learn things?

- ▶ E-step 1:

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the *dog*

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the **marsupial**

- ▶ M-step 1:

- ▶ Emissions aren't so different

How does EM learn things?

► E-step 1:

DT: **0.95** DT: 0.05

NN: 0.05 **NN: 0.95**

the *dog*

DT: **0.95** DT: 0.5

NN: 0.05 **NN: 0.5**

the **marsupial**

► M-step 1:

► Emissions aren't so different

► Transition probabilities (approx): $P(\text{NN} | \text{DT}) = 3/4$, $P(\text{DT} | \text{DT}) = 1/4$

How does EM learn things?

► E-step 2:

DT: 0.95 DT: 0.05

NN: 0.05 NN: 0.95

the *dog*

DT: 0.95 DT: 0.30

NN: 0.05 NN: 0.70

the *marsupial*

► M-step 1:

► Emissions aren't so different

► Transition probabilities (approx): $P(NN|DT) = 3/4$, $P(DT|DT) = 1/4$

How does EM learn things?

► E-step 2:

DT: 0.95 DT: 0.05

NN: 0.05 **NN: 0.95**

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the **marsupial**

► M-step 2:

How does EM learn things?

► E-step 2:

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the *marsupial*

► M-step 2:

► Emission $P(\text{marsupial} | \text{NN}) > P(\text{marsupial} | \text{DT})$

How does EM learn things?

► E-step 2:

DT: 0.95 DT: 0.05

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the *dog*

DT: 0.95 DT: 0.30

NN: 0.05 NN: 0.70

the *marsupial*

► M-step 2:

► Emission $P(\text{marsupial} | \text{NN}) > P(\text{marsupial} | \text{DT})$

► Remember to tag marsupial as NN in the future!

How does EM learn things?

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DT: 0.95 DT: 0.30

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► M-step 2:

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► Context constrained what we learned! That's how data helped us

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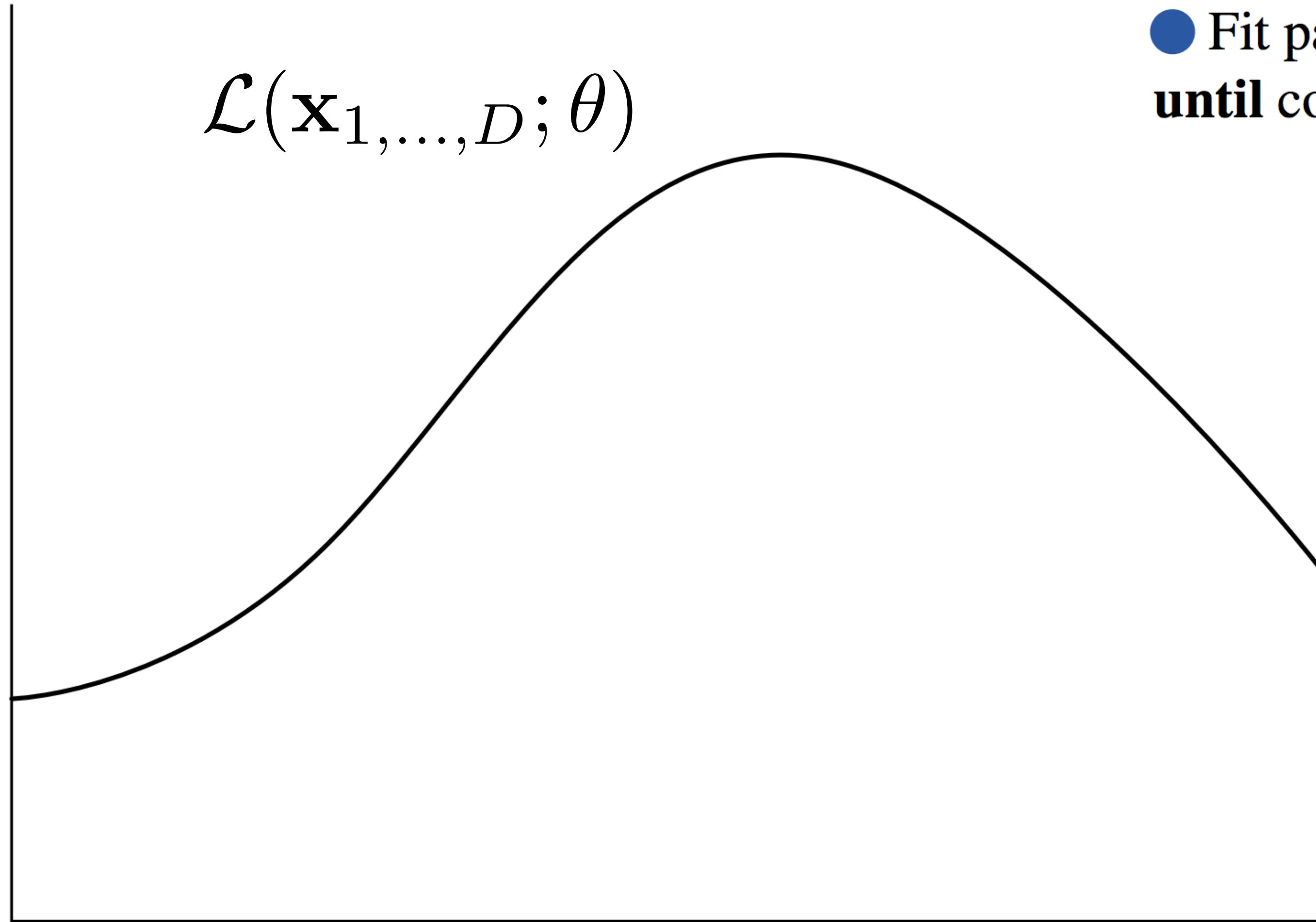
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How does EM learn things?

- ▶ Can think of q as a kind of “fractional annotation”
- ▶ E-step: compute annotations (posterior under current model)
- ▶ M-step: supervised learning with those fractional annotations
- ▶ Initialize with some reasonable weights, alternate E and M until convergence

EM's Lower Bound

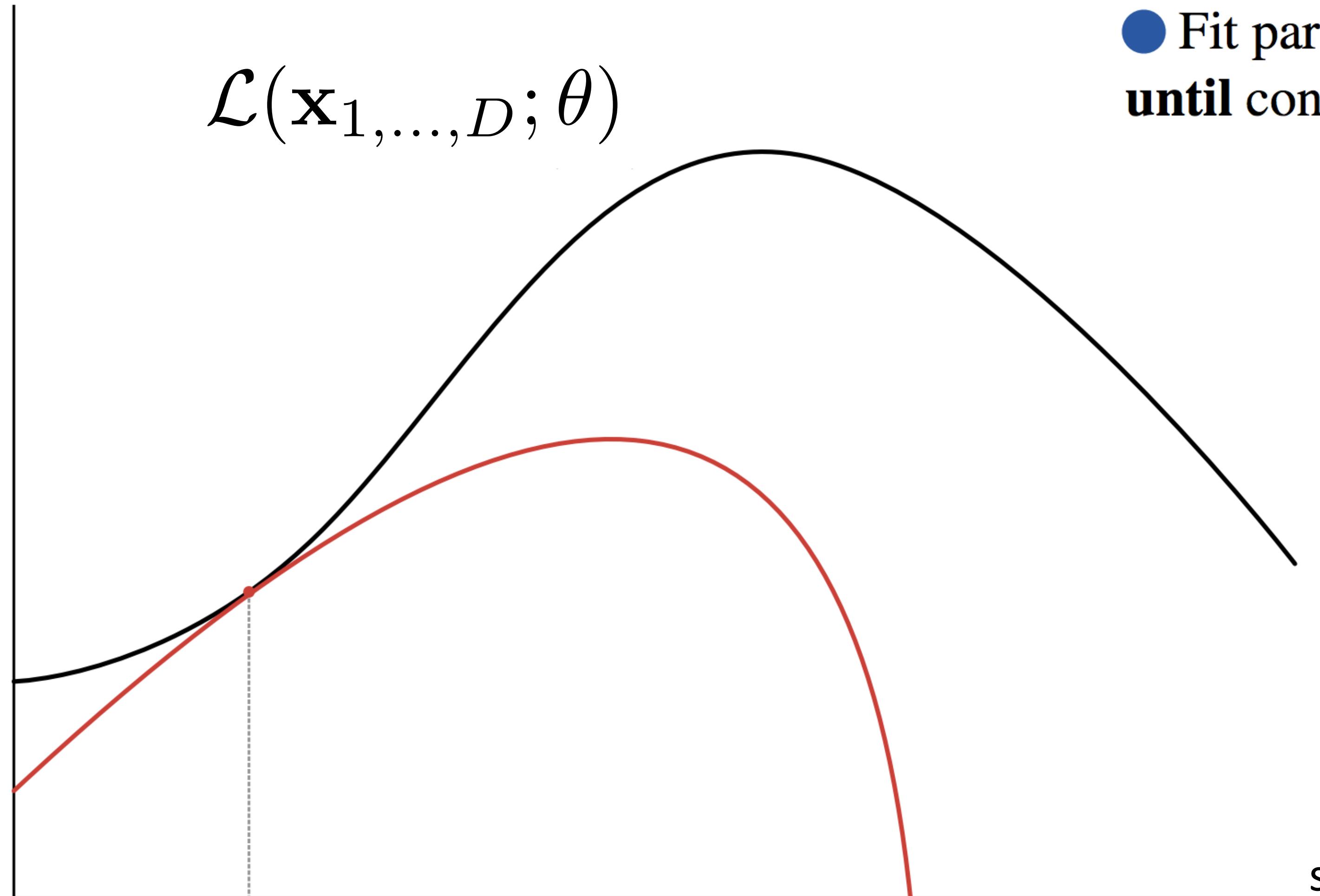
$$\mathcal{L}(\mathbf{x}_{1,\dots,D}) = \sum_{i=1}^D \log \sum_{\mathbf{y}} P(\mathbf{y}, \mathbf{x}_i)$$



Initialize probabilities θ
repeat
 ● Compute expected counts \mathbf{e}
 ● Fit parameters θ
until convergence

EM's Lower Bound

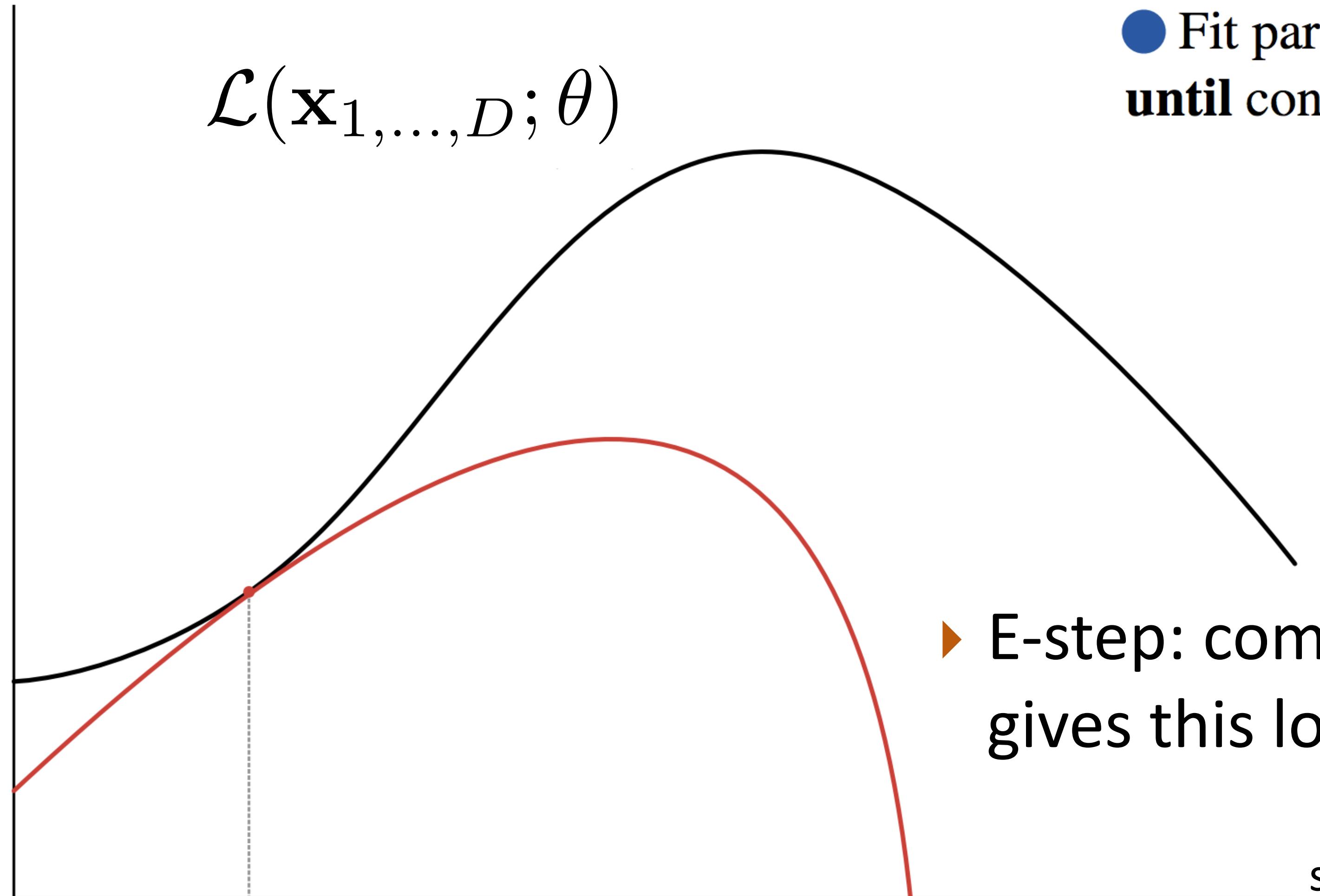
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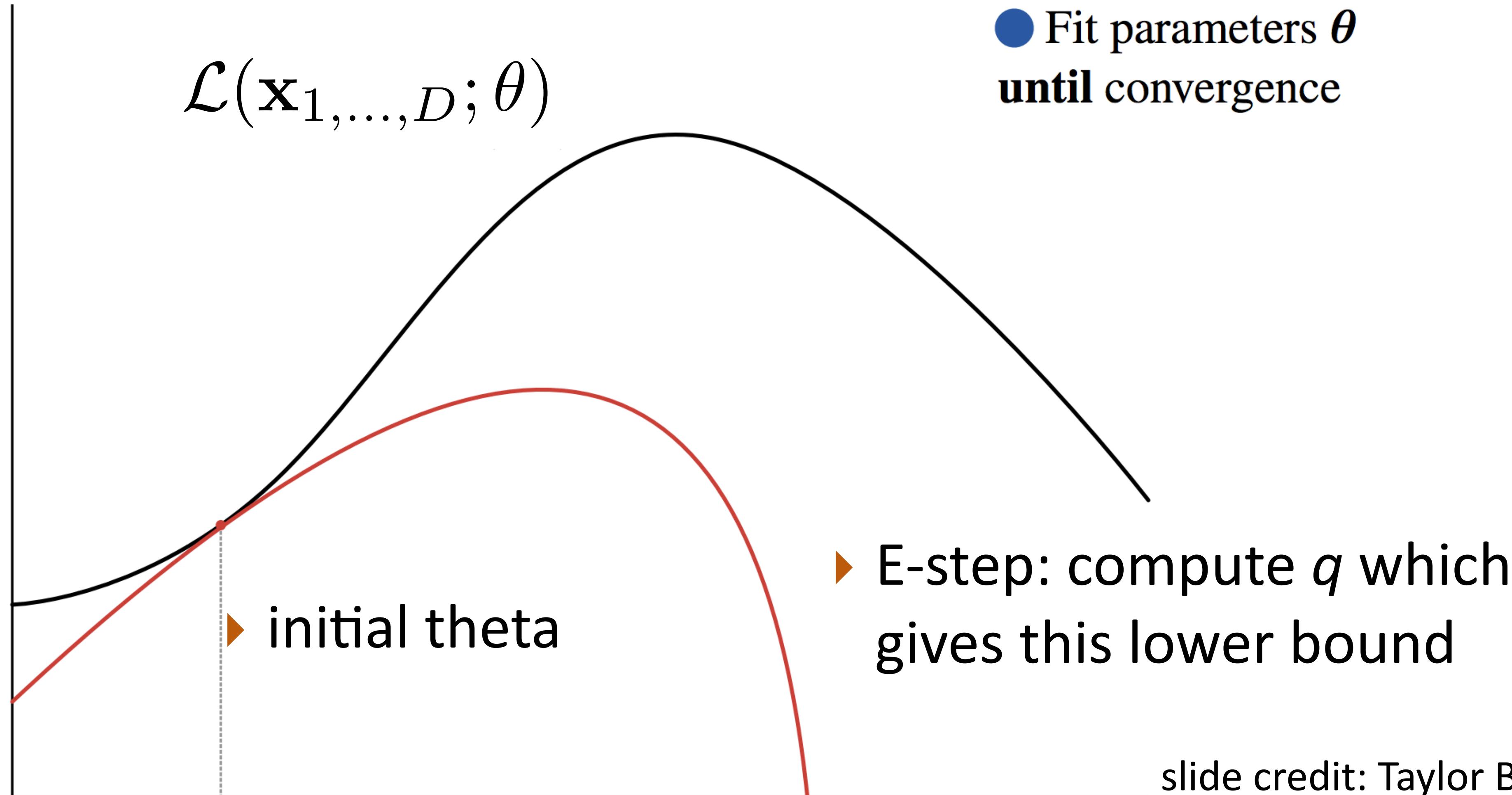


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► E-step: compute q which gives this lower bound

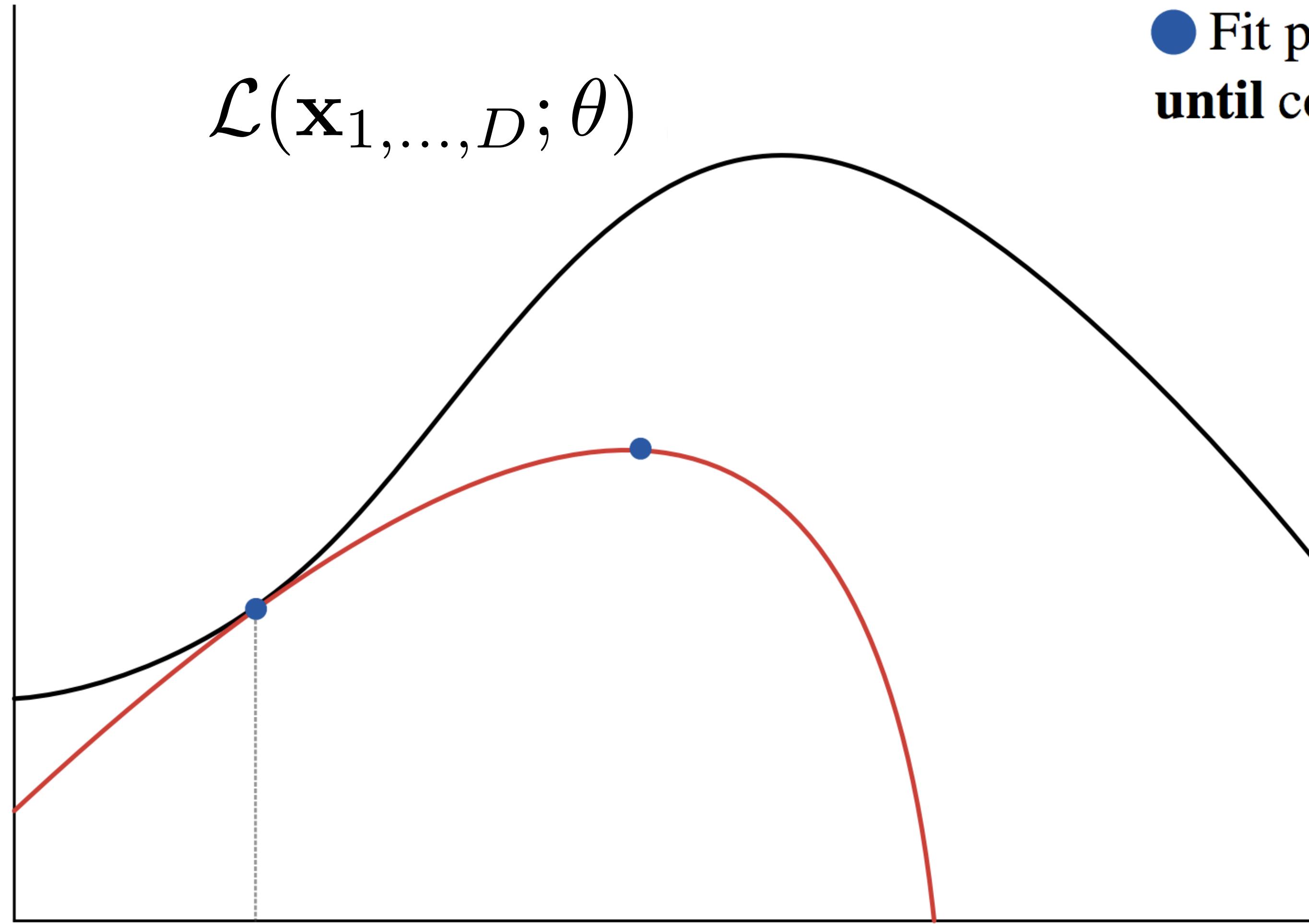
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EM's Lower Bound

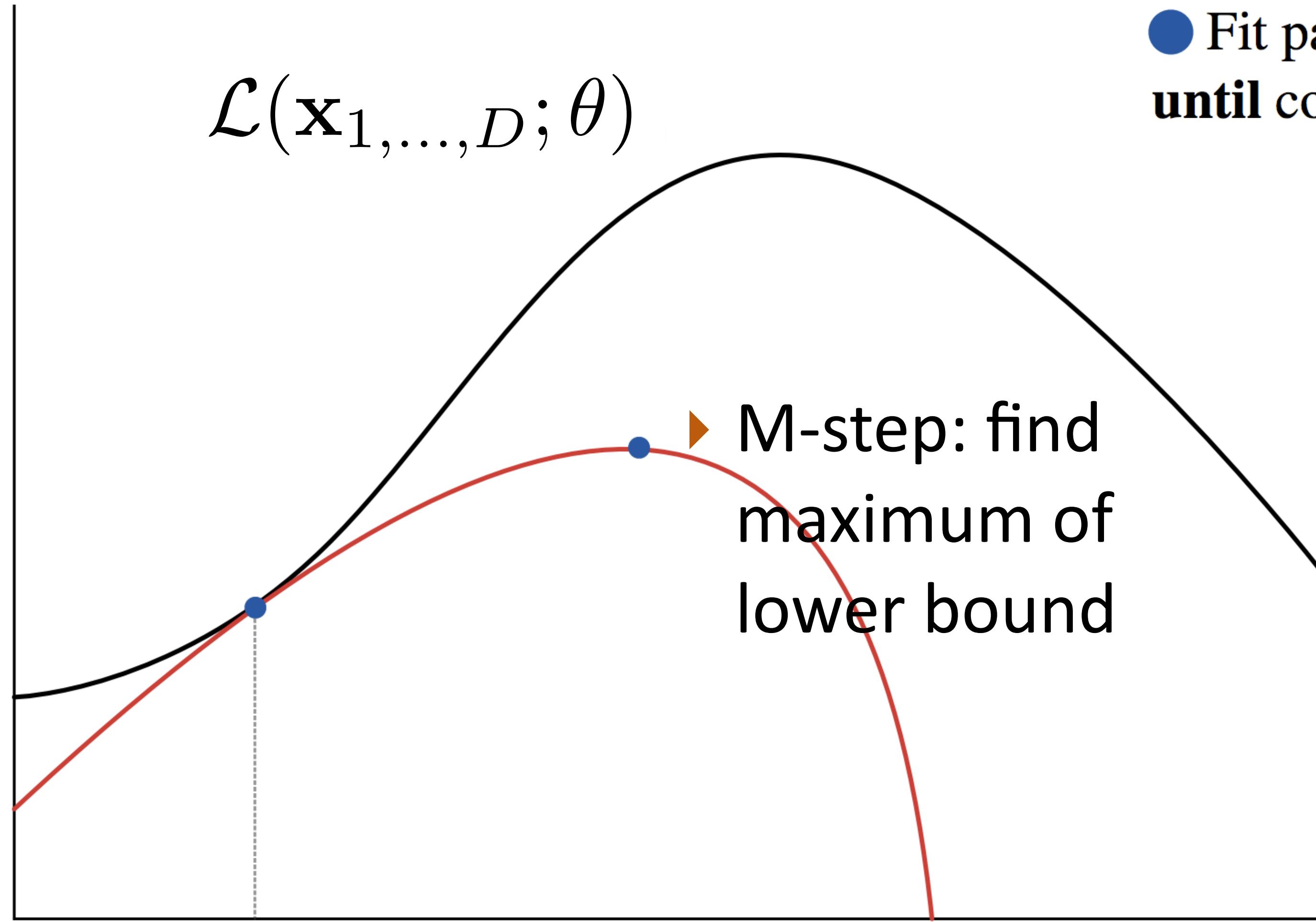
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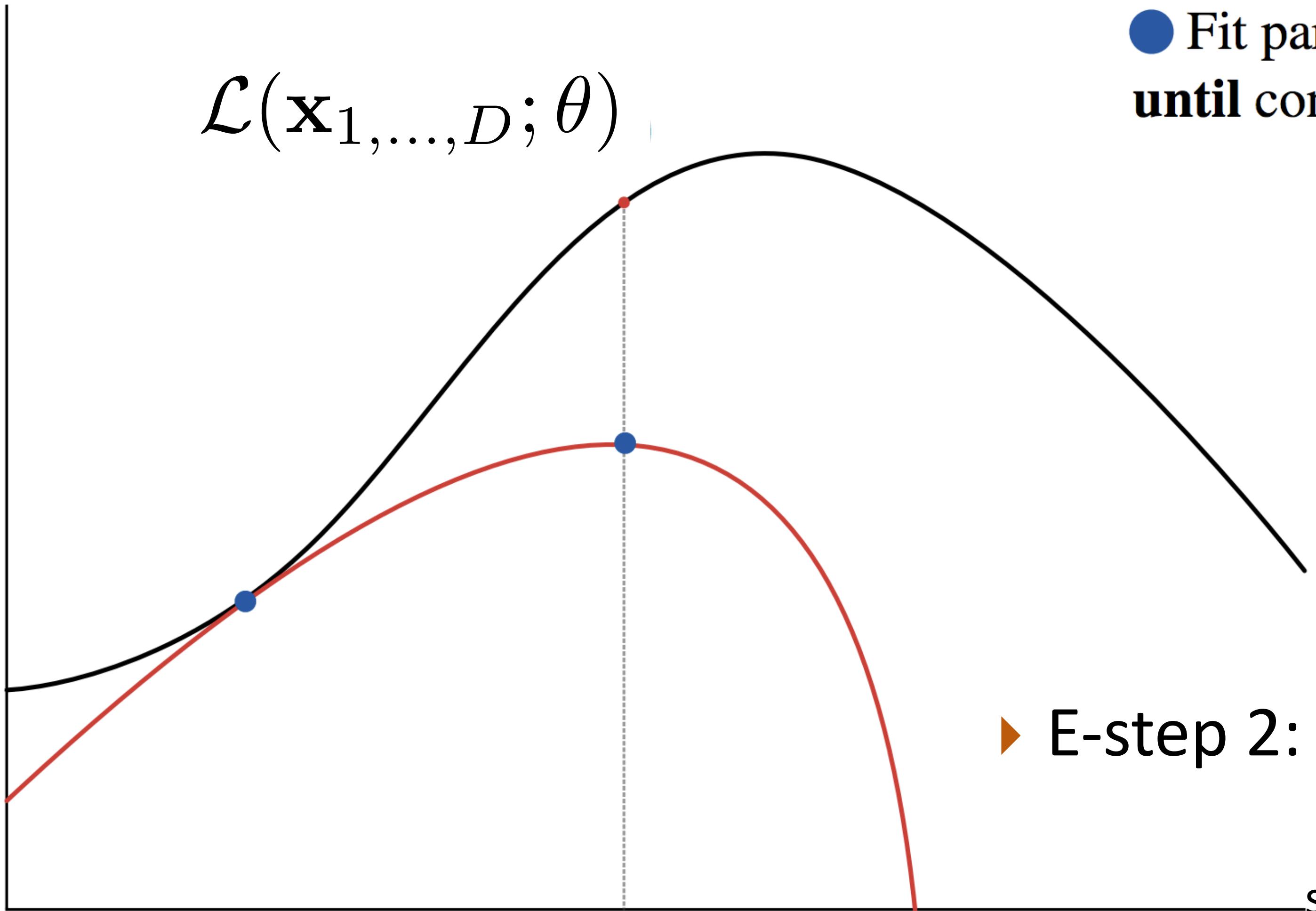
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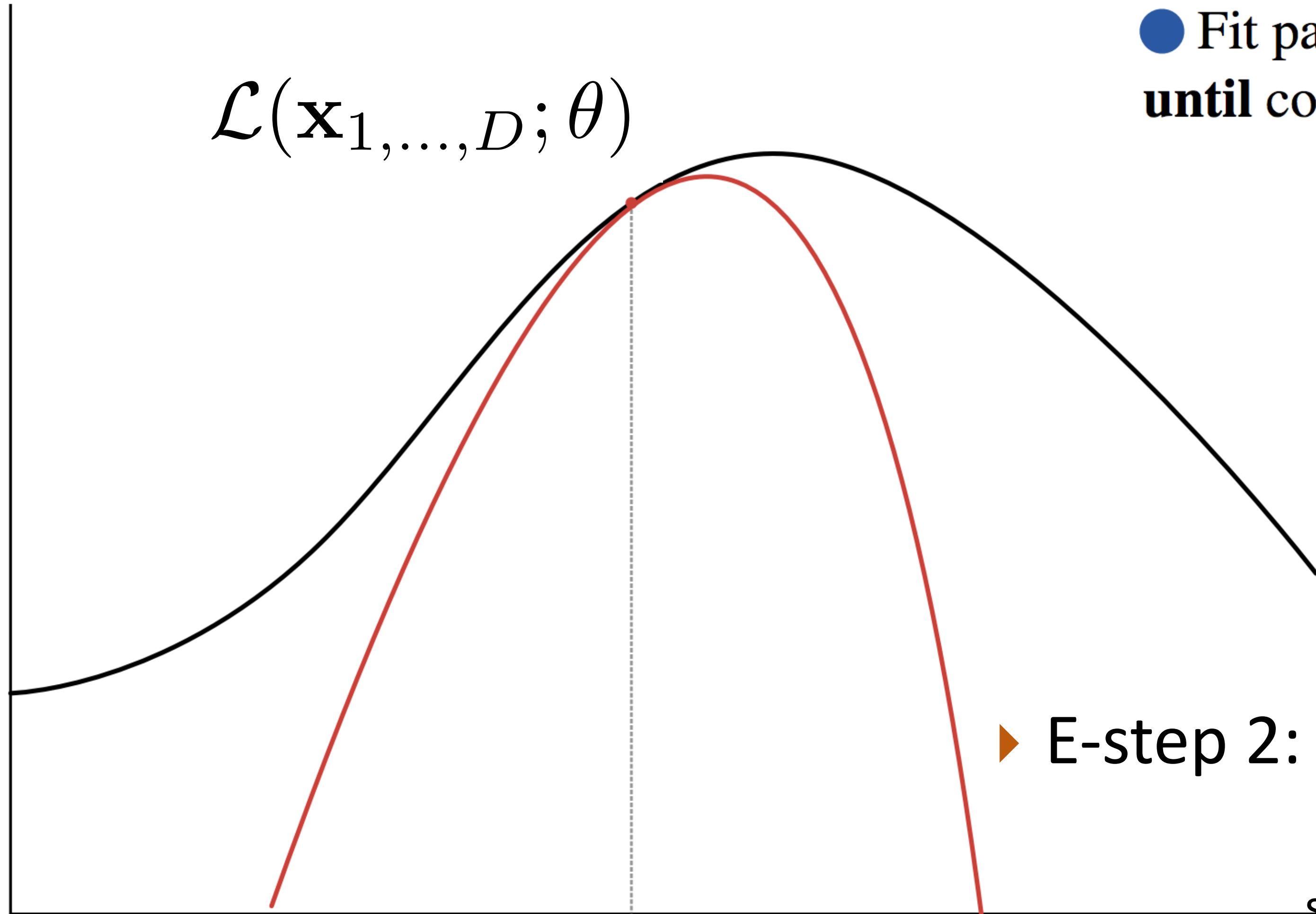


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► E-step 2: re-estimate q

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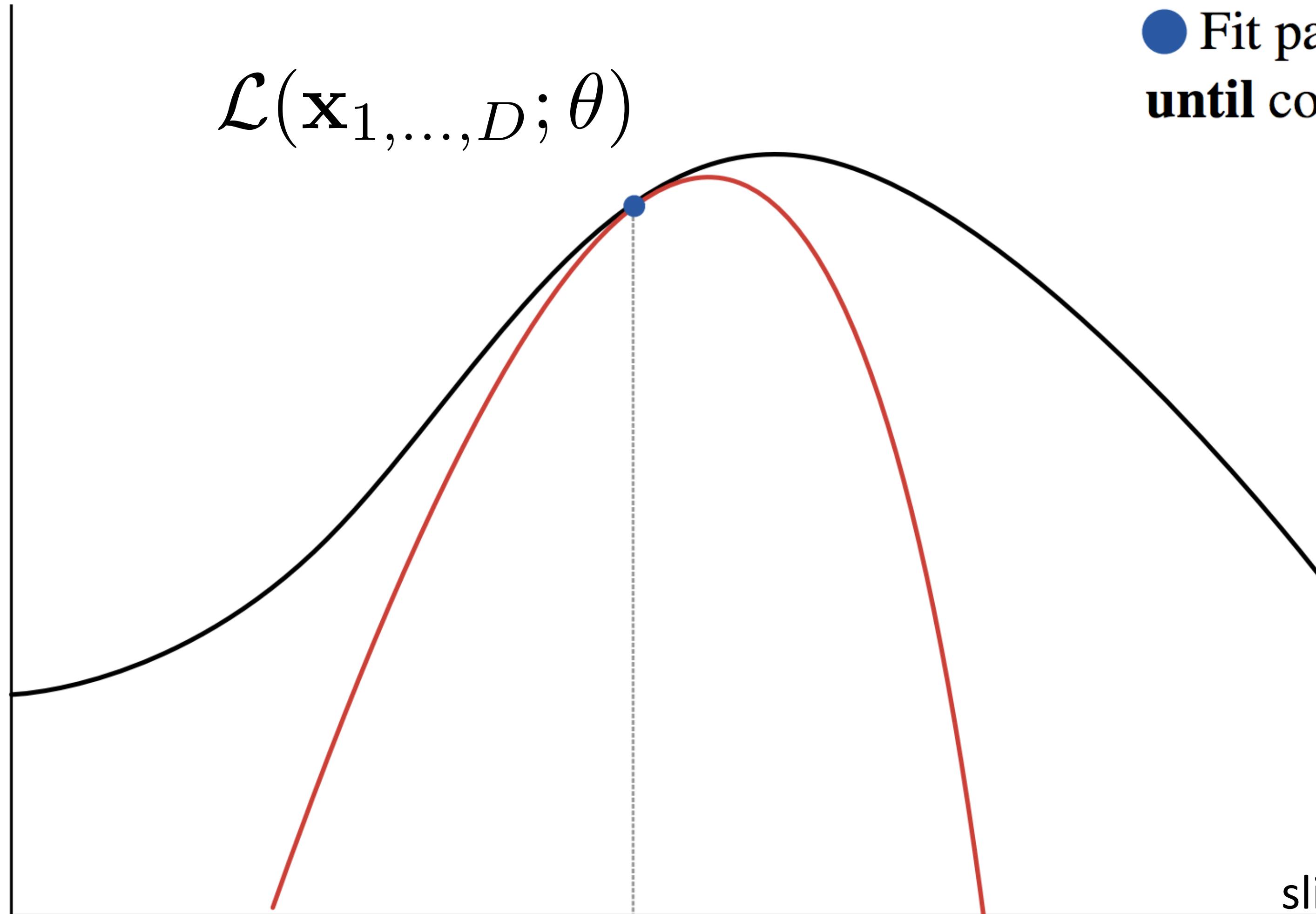


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EM's Lower Bound

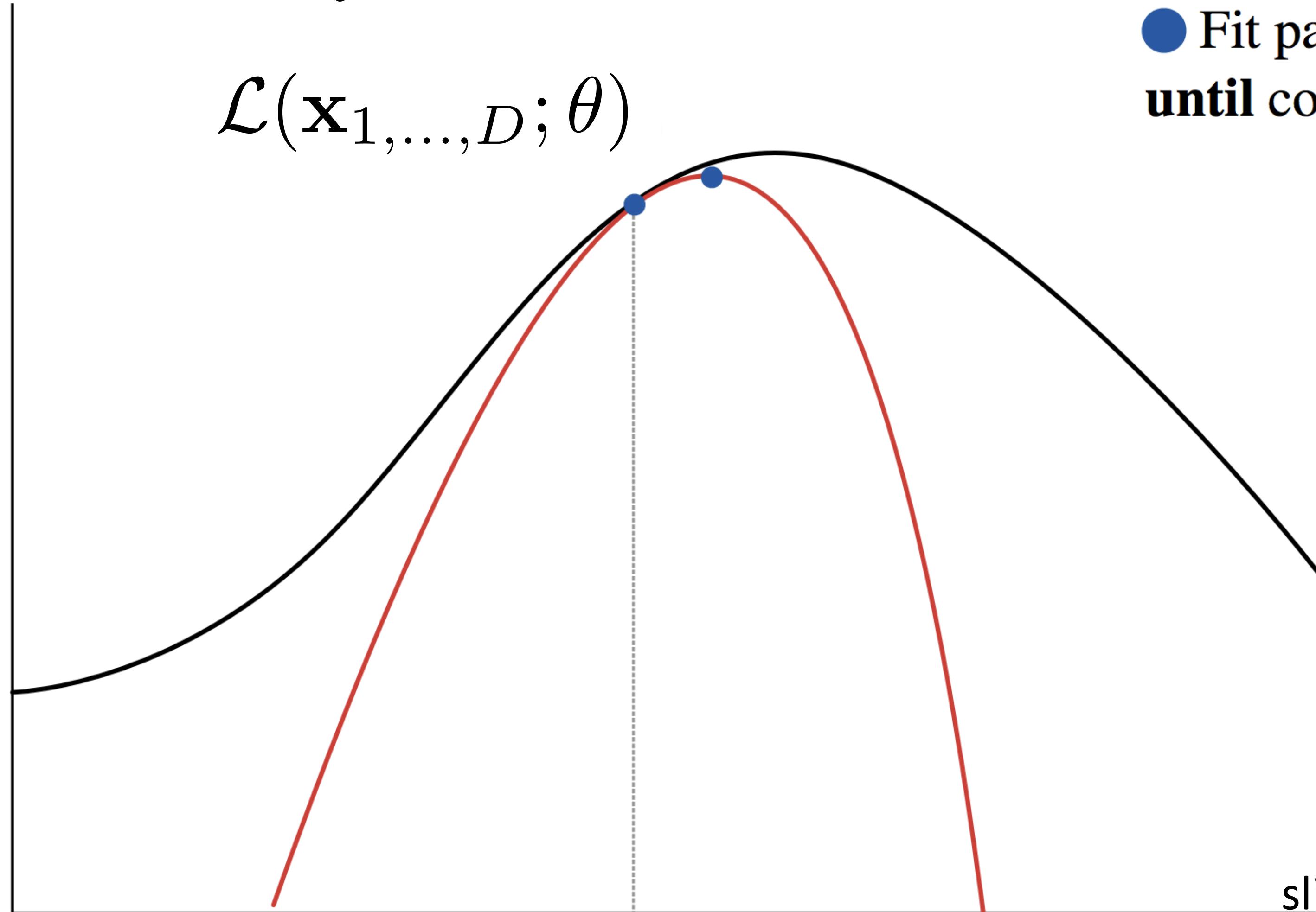
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Initialize probabilities θ

repeat

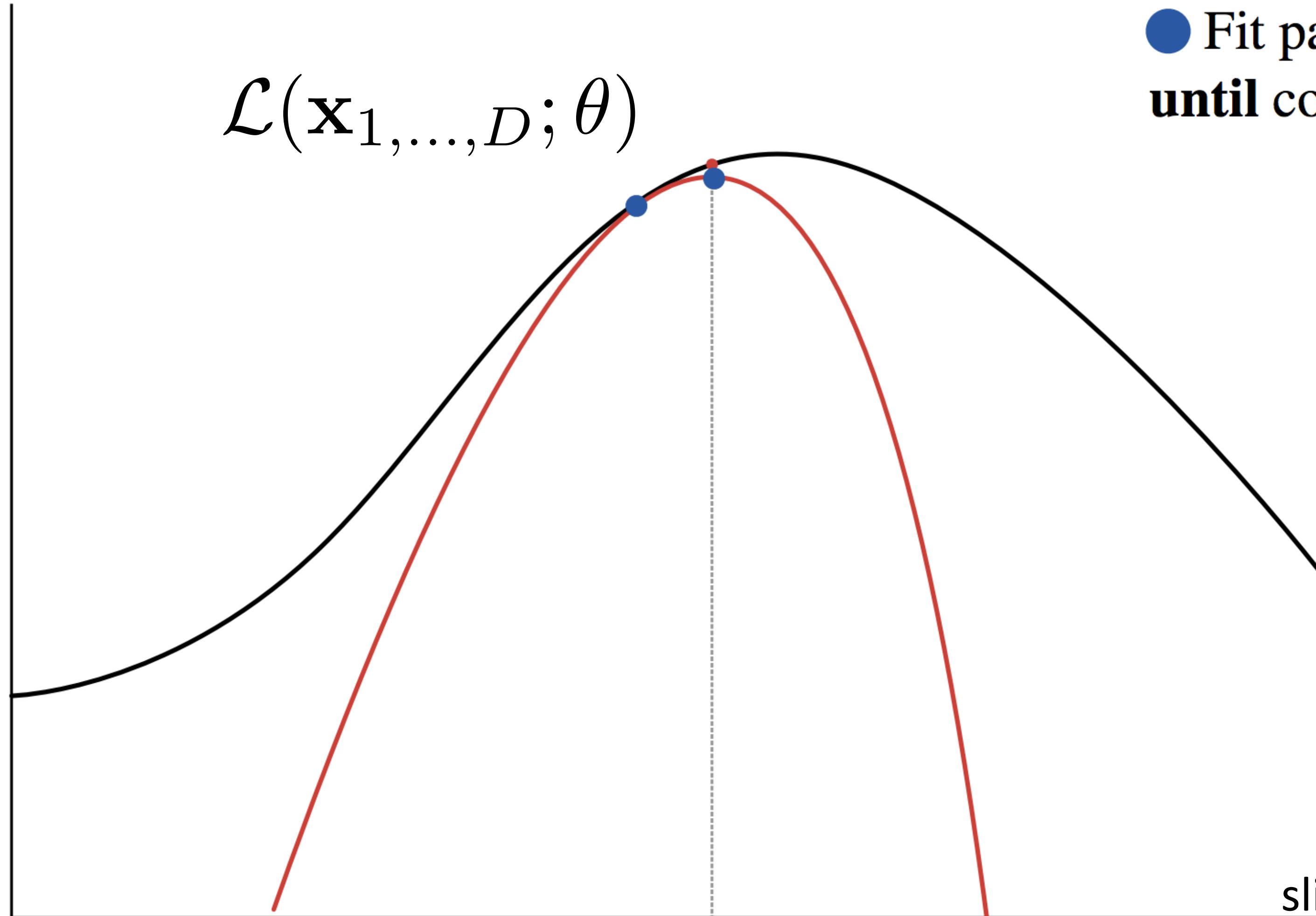
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Part-of-speech Induction

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- ▶ Learn parameters on k examples to start, use those to initialize EM, run on 1 million words of unlabeled data
- ▶ Tag dictionary + data should get us started in the right direction...

Part-of-speech Induction

Number of tagged sentences used for the initial model							
	0	100	2000	5000	10000	20000	all
Iter	Correct tags (% words) after ML on 1M words						
0	77.0	90.0	95.4	96.2	96.6	96.9	97.0
1	80.5	92.6	95.8	96.3	96.6	96.7	96.8
2	81.8	93.0	95.7	96.1	96.3	96.4	96.4
3	83.0	93.1	95.4	95.8	96.1	96.2	96.2
4	84.0	93.0	95.2	95.5	95.8	96.0	96.0
5	84.8	92.9	95.1	95.4	95.6	95.8	95.8
6	85.3	92.8	94.9	95.2	95.5	95.6	95.7
7	85.8	92.8	94.7	95.1	95.3	95.5	95.5
8	86.1	92.7	94.6	95.0	95.2	95.4	95.4
9	86.3	92.6	94.5	94.9	95.1	95.3	95.3
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- ▶ Small amounts of data > large amounts of unlabeled data
- ▶ Running EM *hurts* performance once you have labeled data

Two Hours of Annotation

Human Annotations	0. No EM			1. EM only			2. With LP		
Initial data	T	K	U	T	K	U	T	K	U
KIN tokens A	72	90	58	55	82	32	71	86	58
KIN types A				63	77	32	78	83	69
MLG tokens B	74	89	49	68	87	39	74	89	49
MLG types B				71	87	46	72	81	57
ENG tokens A	63	83	38	62	83	37	72	85	55
ENG types A				66	76	37	75	81	56
ENG tokens B	70	87	44	70	87	43	78	90	60
ENG types B				69	83	38	75	82	61

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ENG tokens A	63	83	38	62	83	37	72	85	55
ENG types A				66	76	37	75	81	56
ENG tokens B	70	87	44	70	87	43	78	90	60
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- ▶ Kinyarwanda and Malagasy (two actual low-resource languages)

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ENG types A				66	76	37	75	81	56
ENG tokens B	70	87	44	70	87	43	78	90	60
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- ▶ Kinyarwanda and Malagasy (two actual low-resource languages)
- ▶ Label propagation (technique for using dictionary labels) helps a lot, with data that was collected in two hours

Variational Autoencoders

Continuous Latent Variables

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- ▶ Can use EM here when $P(z)$ and $P(\mathbf{x}|z)$ are Gaussians

Continuous Latent Variables

- ▶ For discrete latent variables y , we optimized: $P(\mathbf{x}) = \sum_y P(y, \mathbf{x})$
- ▶ What if we want to use continuous latent variables?

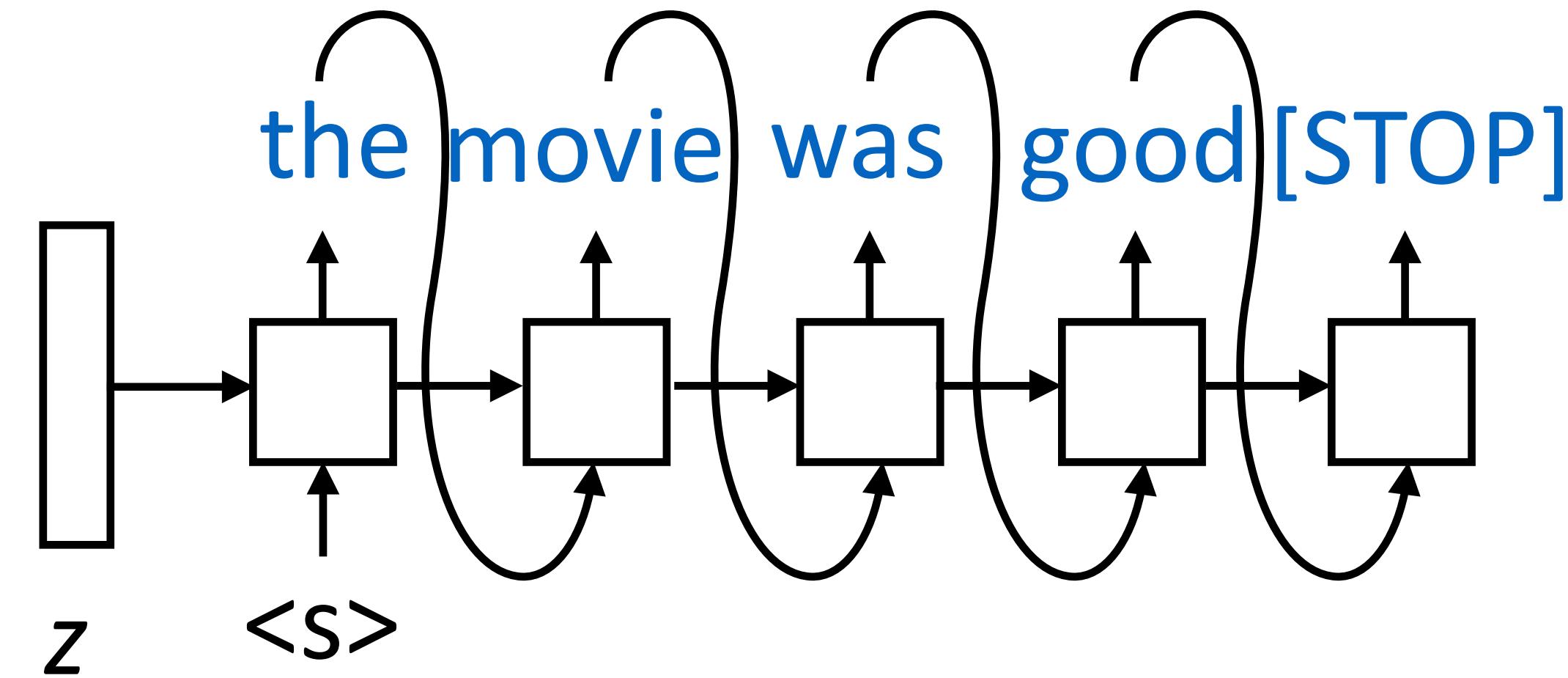
$$P(z, \mathbf{x}) = P(z)P(\mathbf{x}|z)$$

$$P(\mathbf{x}) = \int P(z)P(\mathbf{x}|z)\partial z$$

- ▶ Can use EM here when $P(z)$ and $P(\mathbf{x}|z)$ are Gaussians
- ▶ What if we want $P(\mathbf{x}|z)$ to be something more complicated, like an LSTM with z as the initial state?

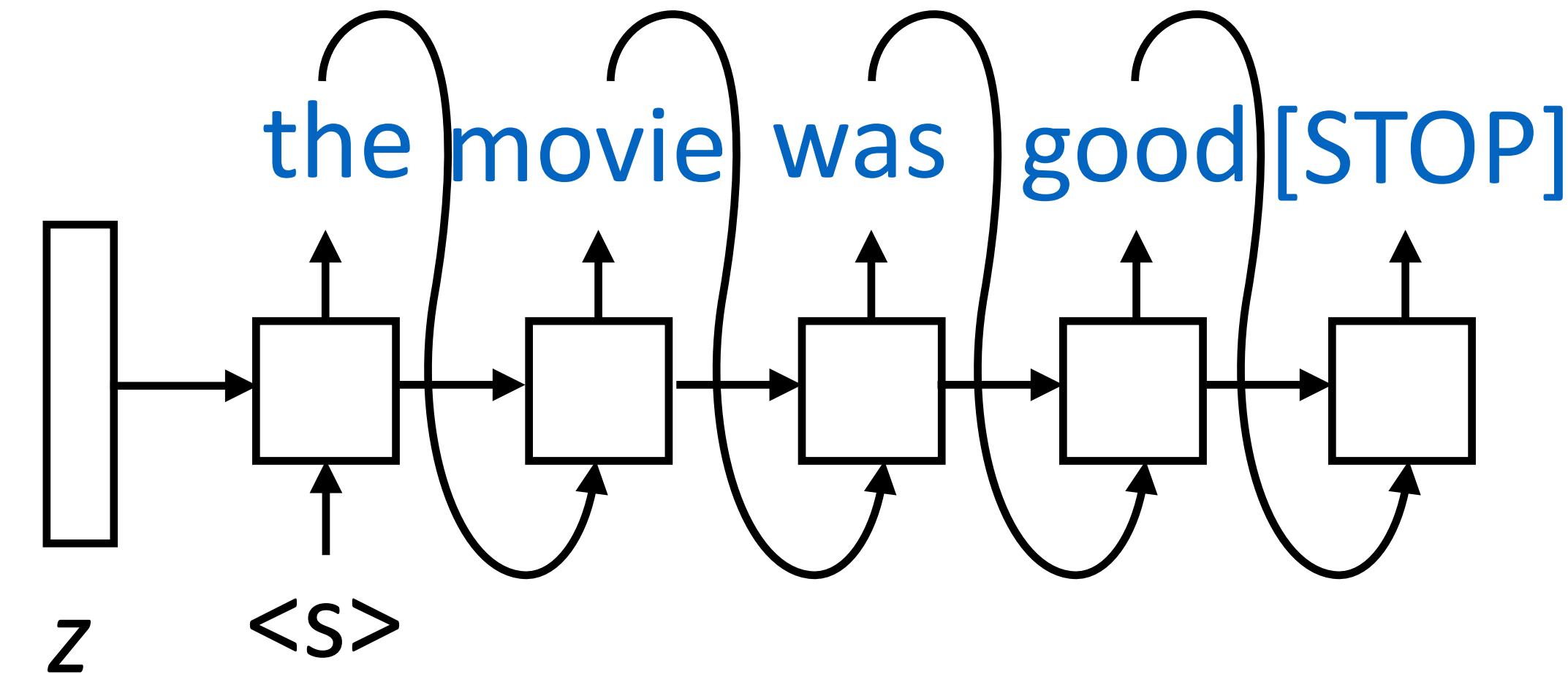
Deep Generative Models

$$P(z, \mathbf{x}) = P(z)P(\mathbf{x}|z)$$



Deep Generative Models

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- ▶ z is a latent variable which should control the generation of the sentence, maybe capture something about its topic

Deep Generative Models

$$\log \int_z P(\mathbf{x}, z|\theta)$$

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“make the data likely under q” “make q close to the prior”
(discriminative)

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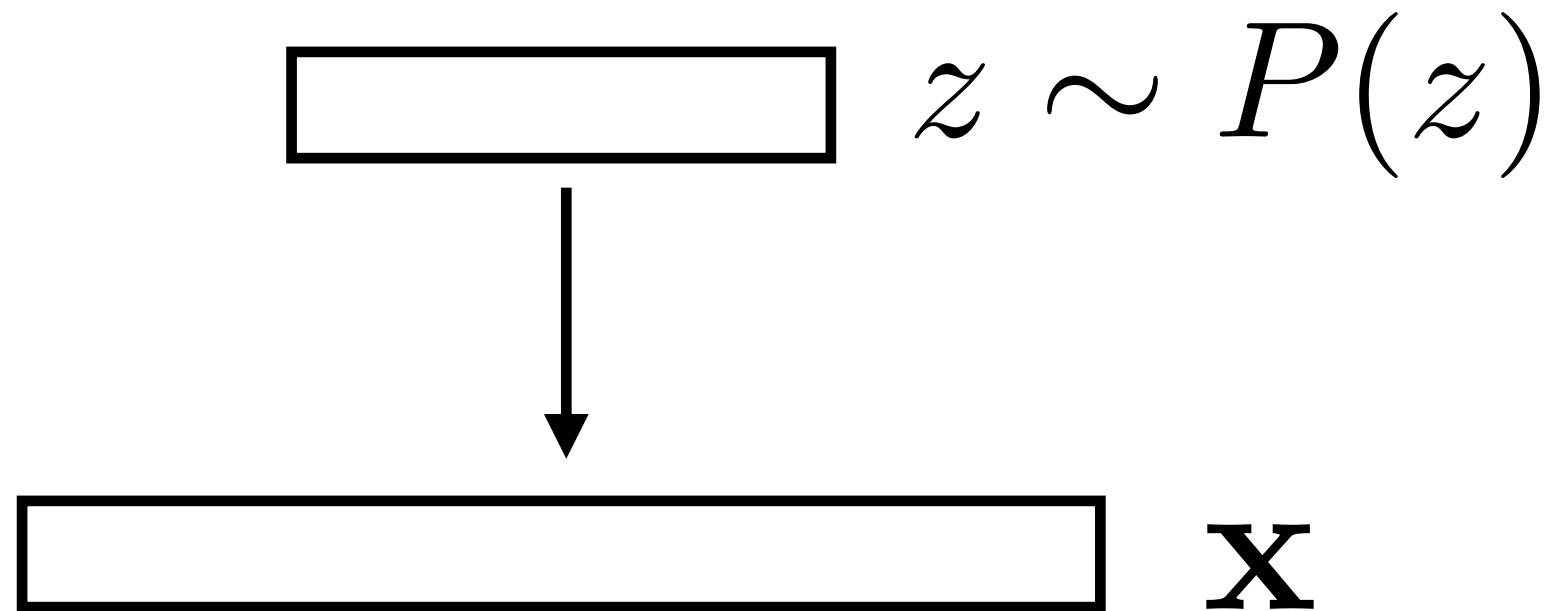
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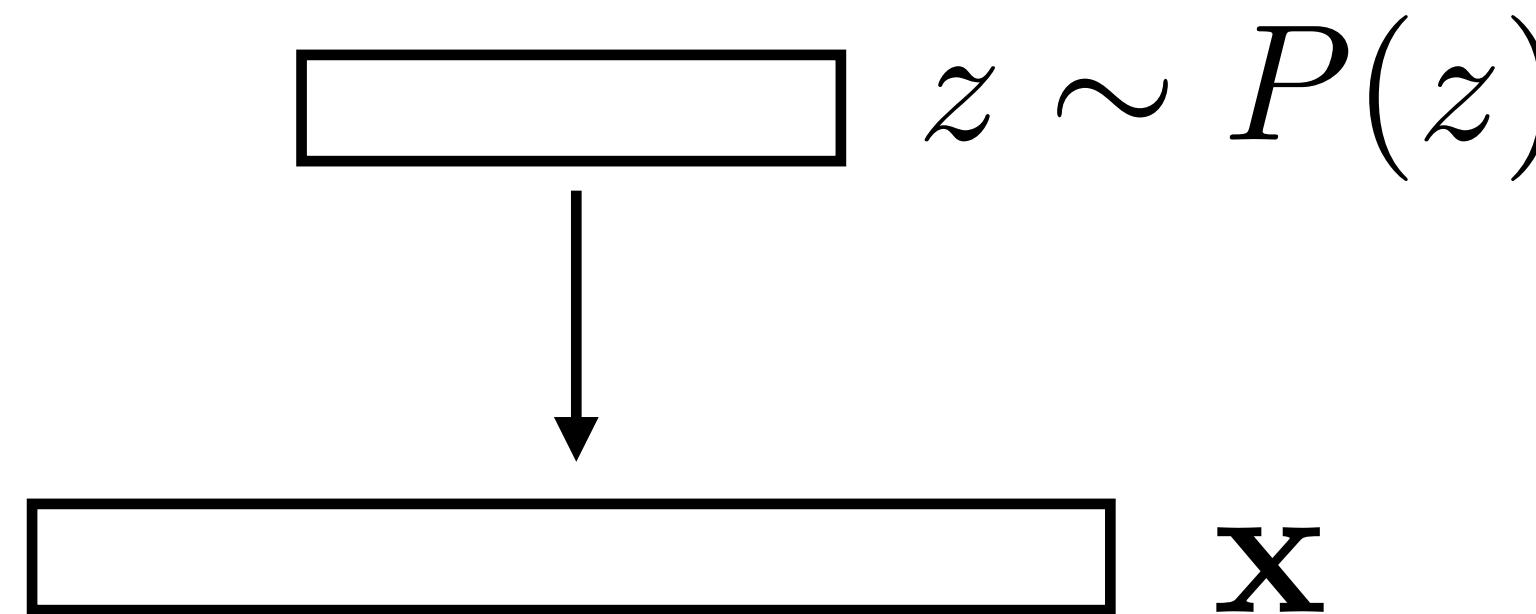
Generative model (test):



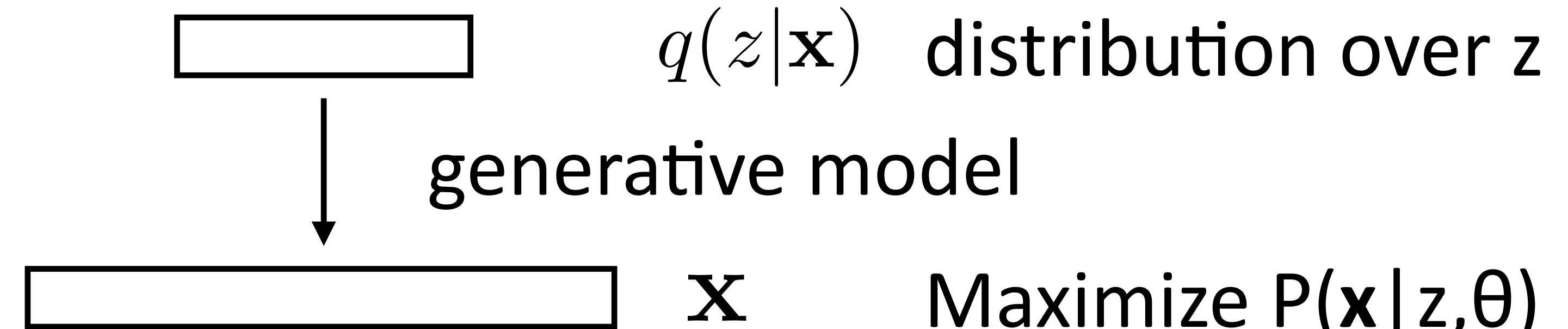
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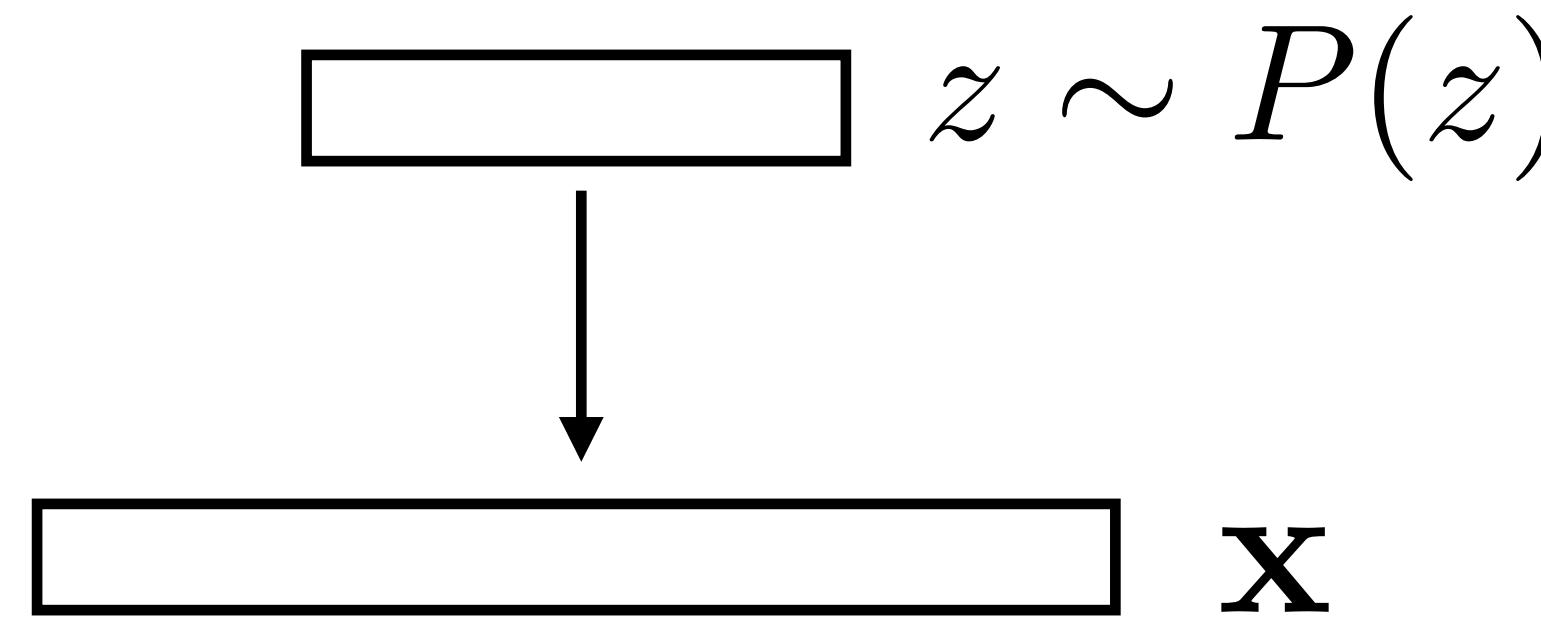
Autoencoder (training):



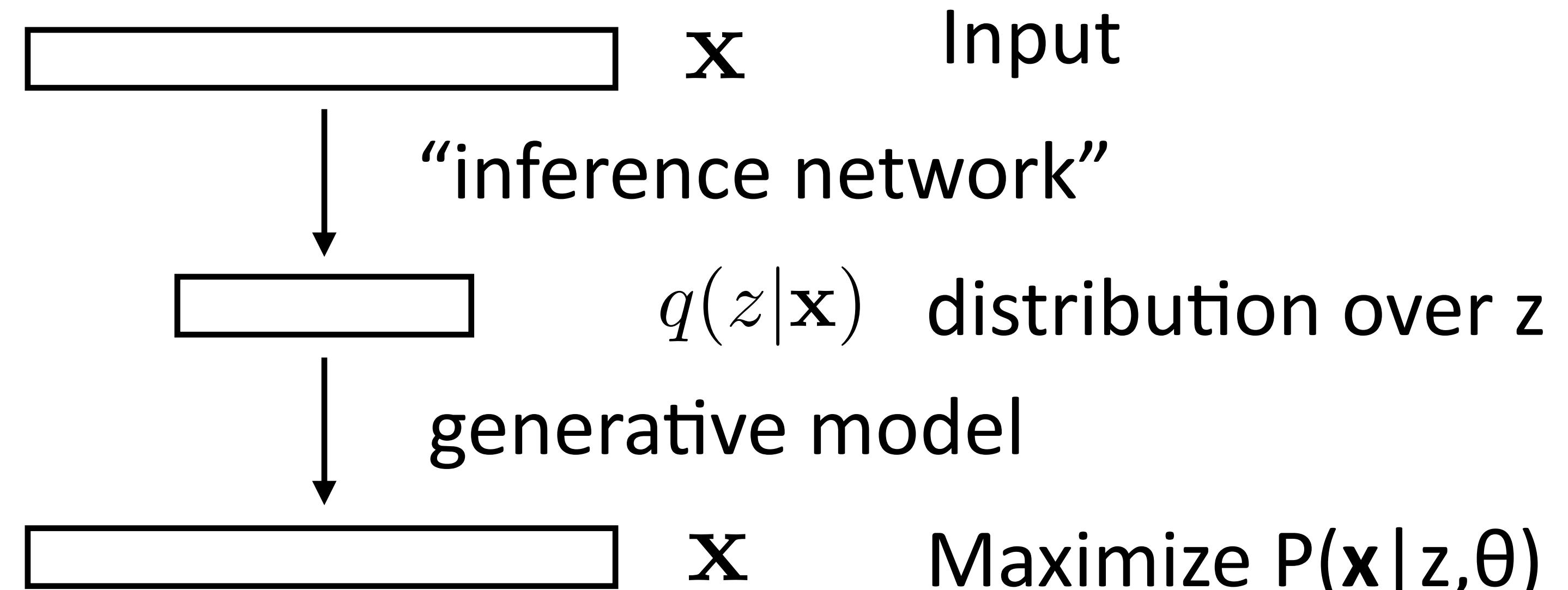
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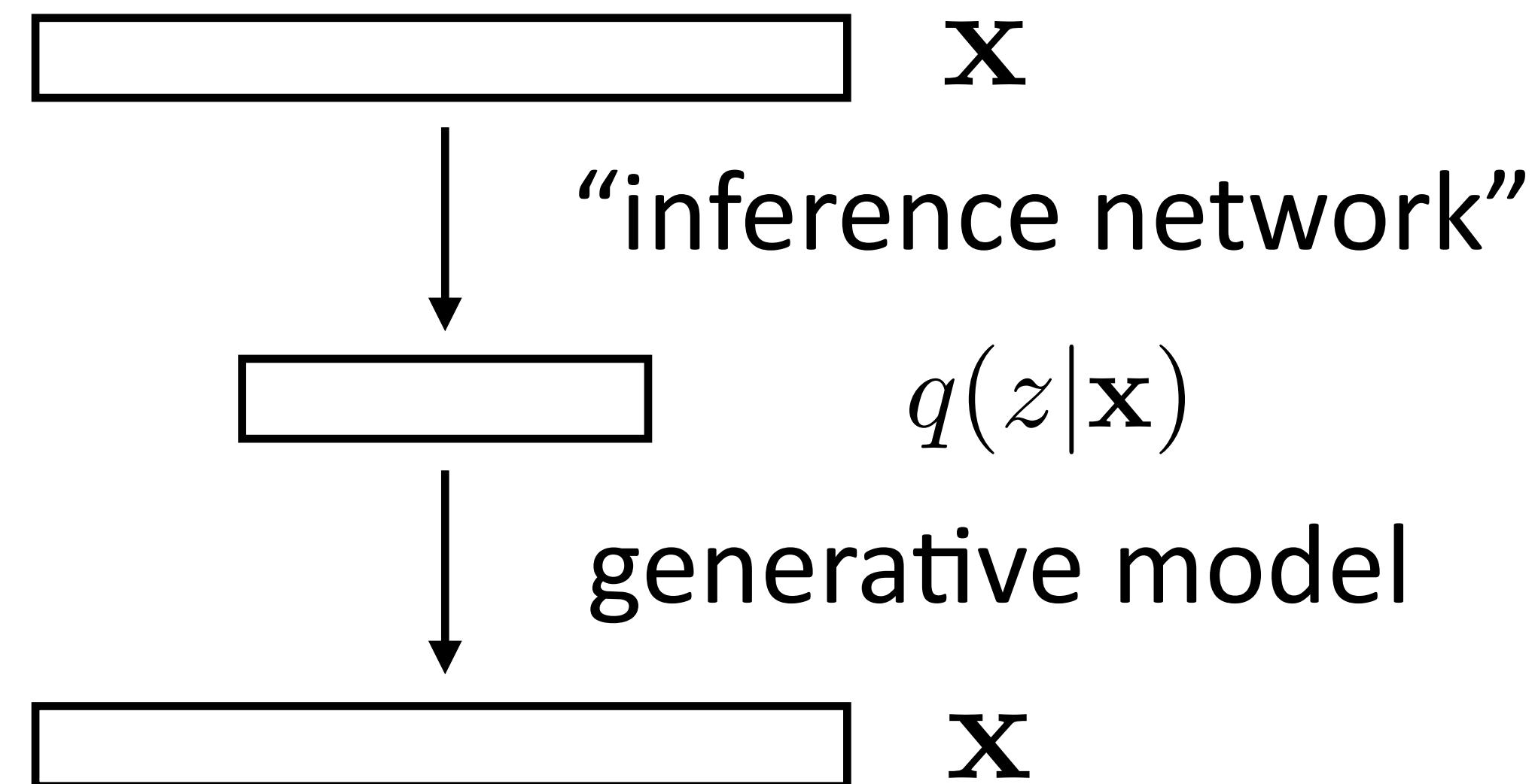


Training VAEs

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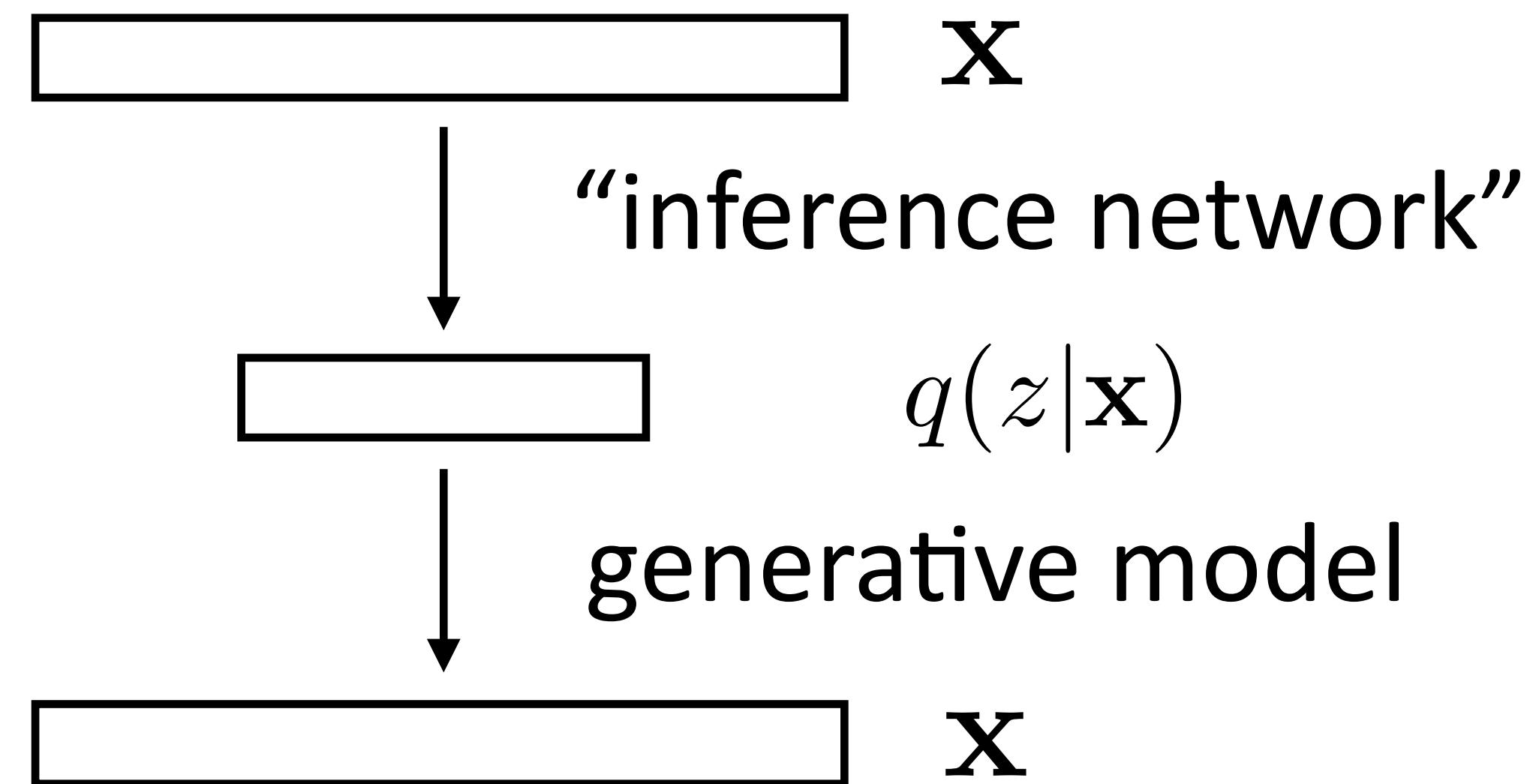
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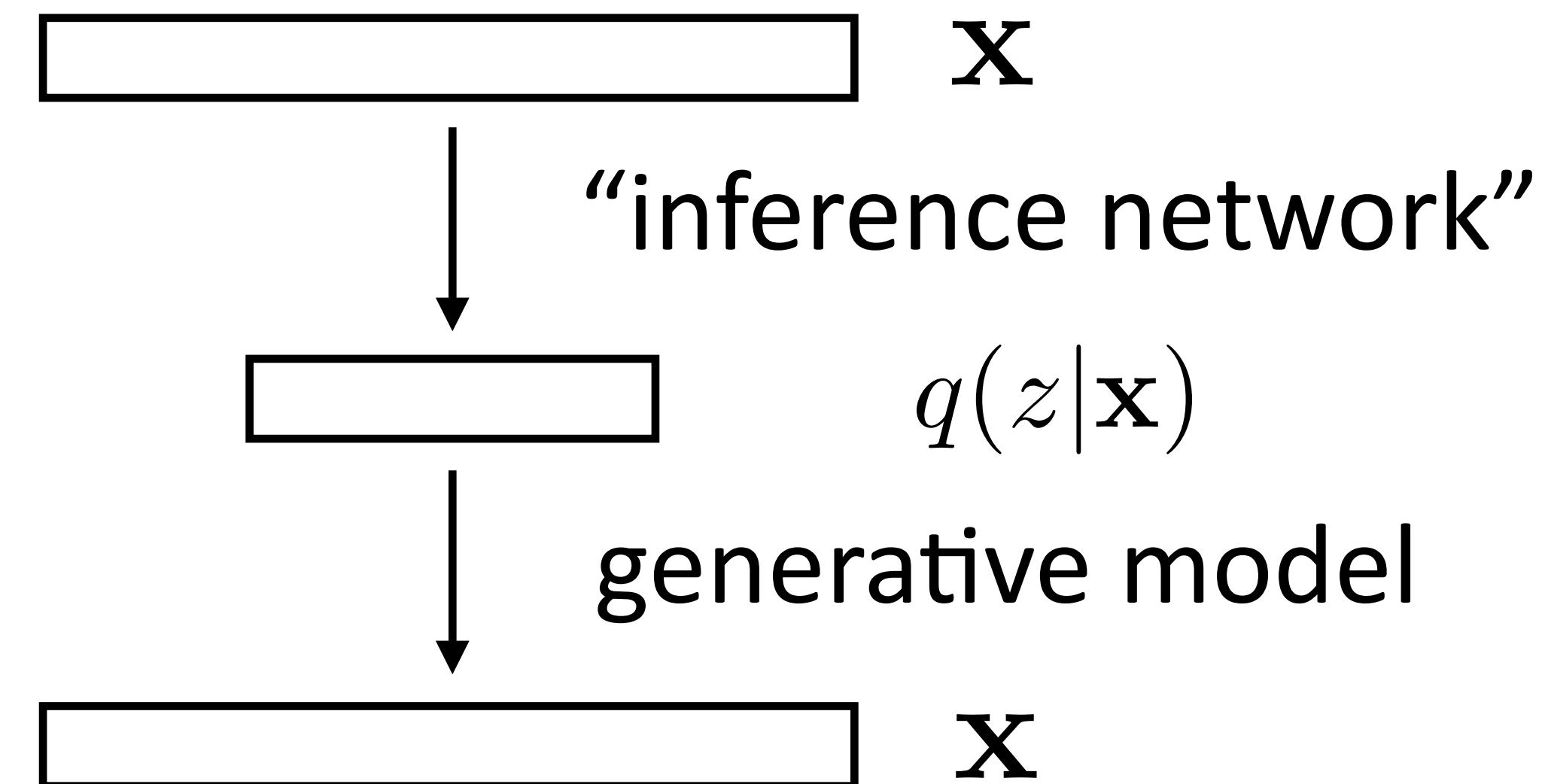
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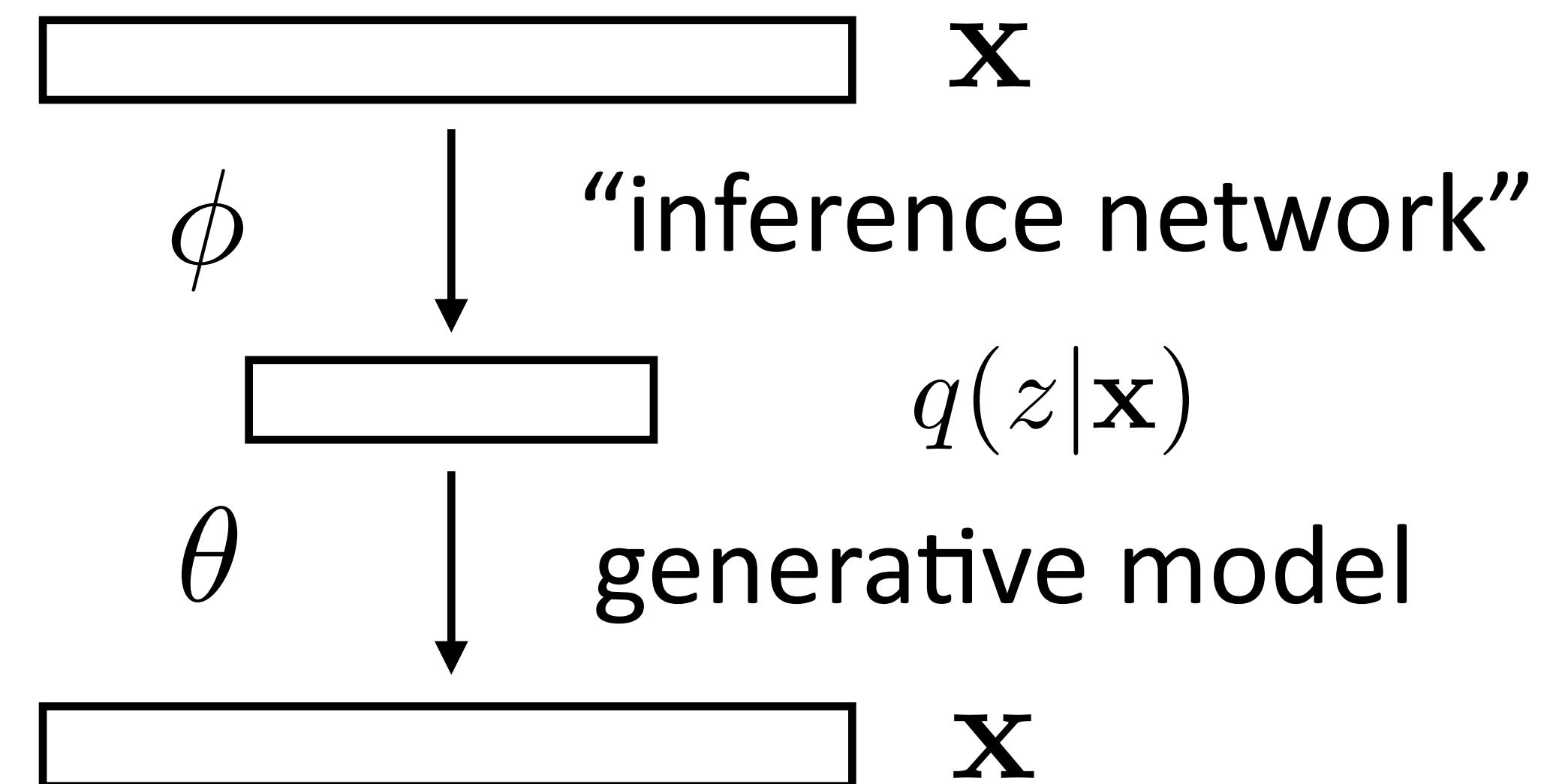
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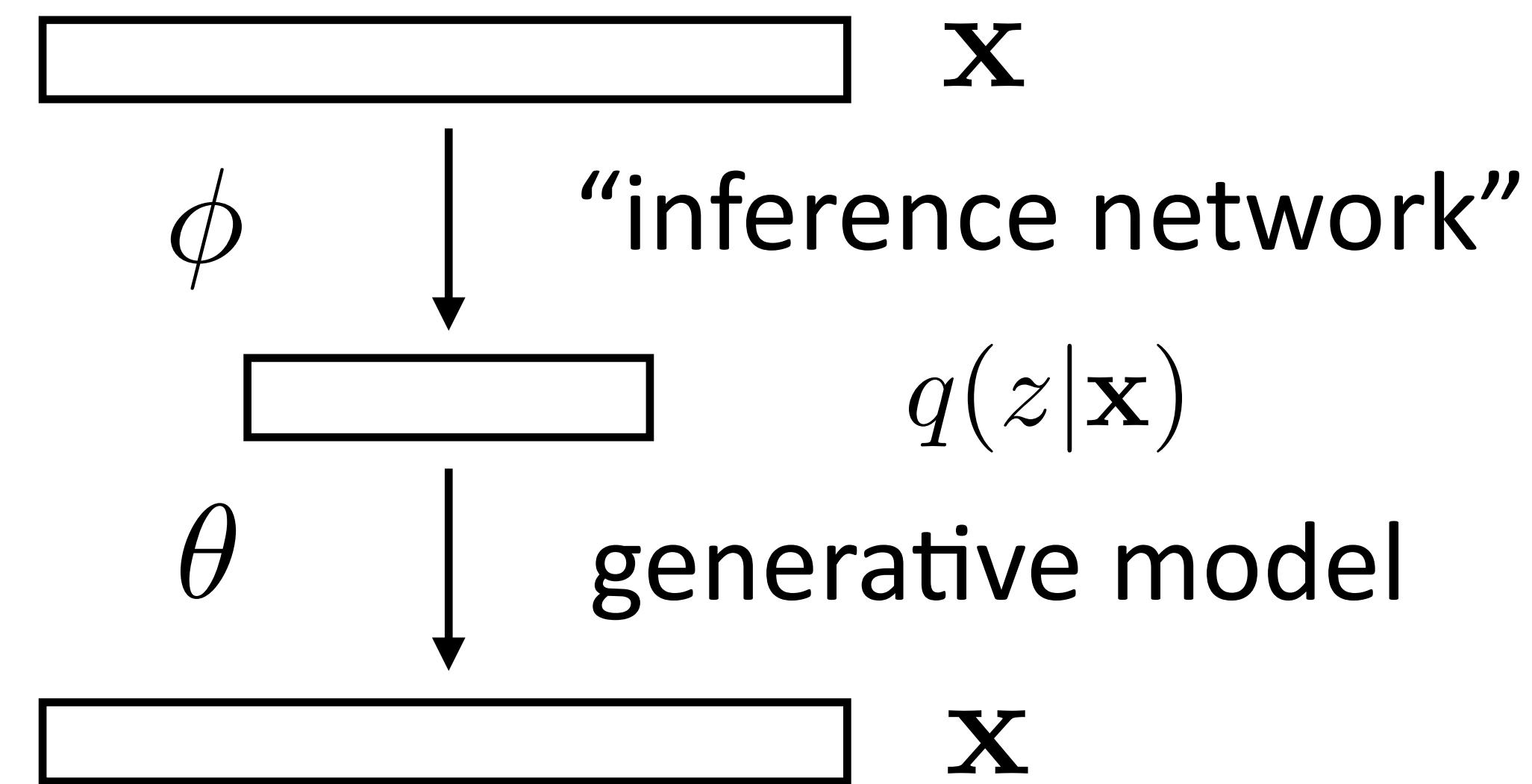
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- ▶ How to handle the expectation?
Sampling

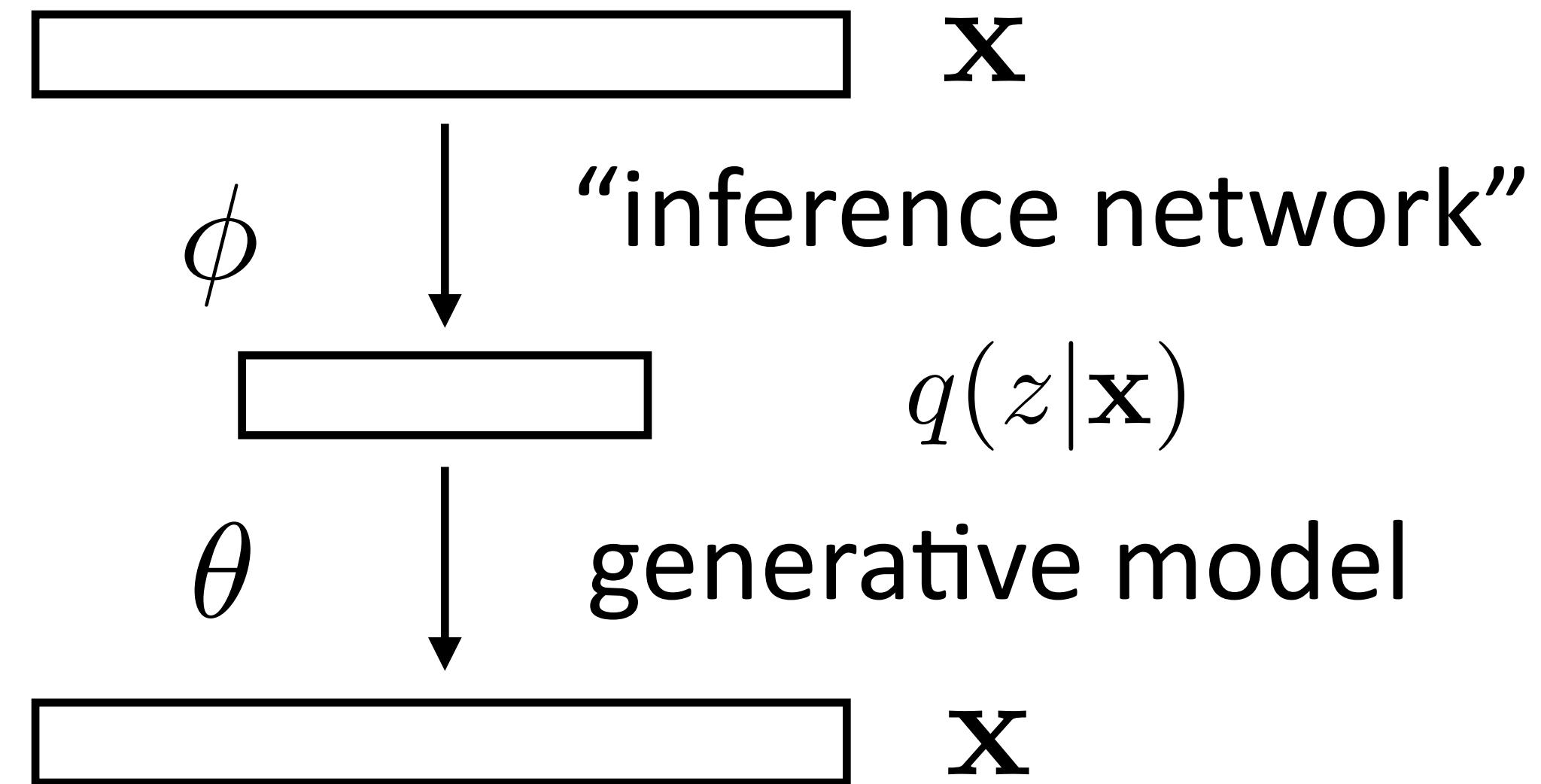
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Training VAEs

For each example \mathbf{x}

Autoencoder (training):

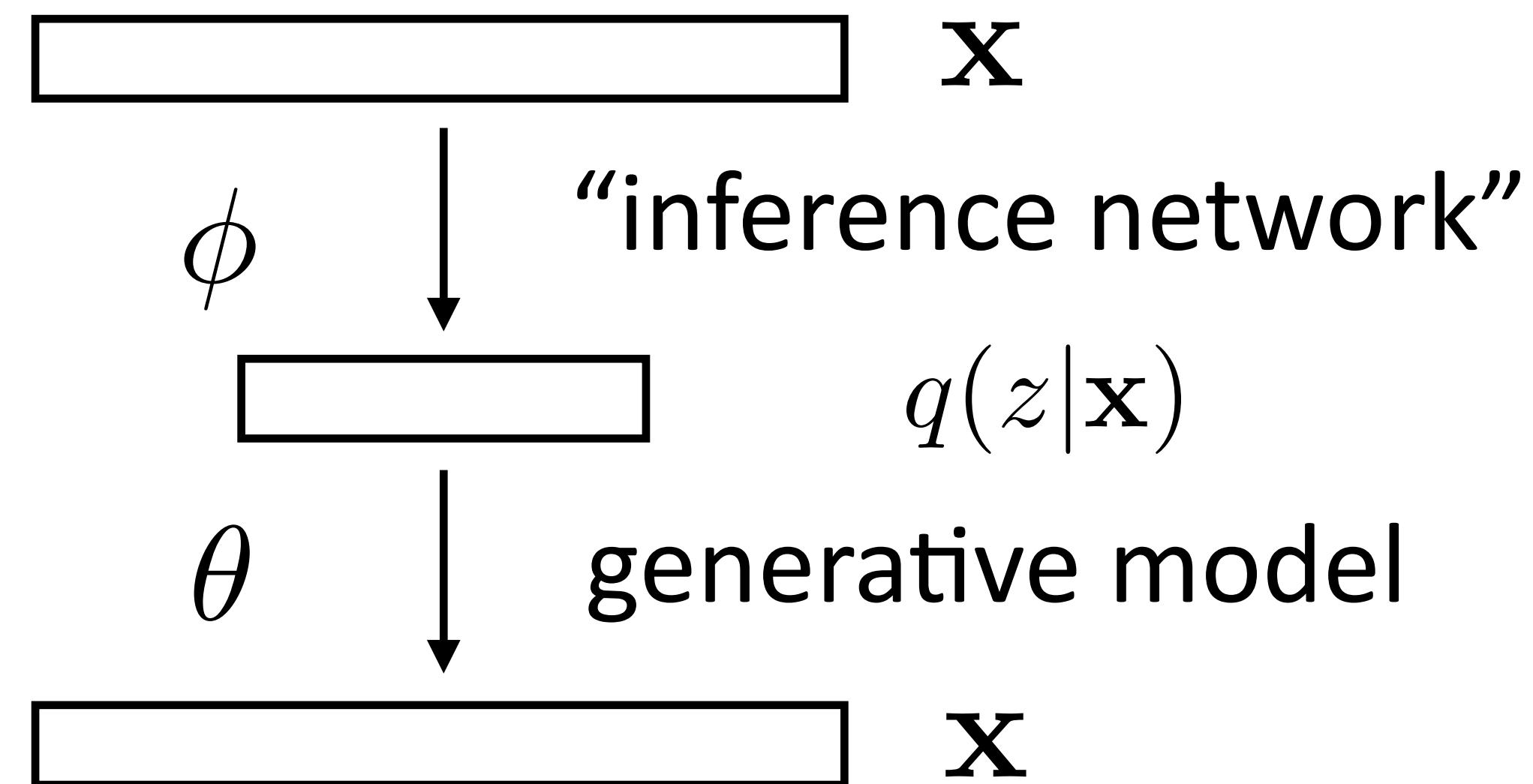


Training VAEs

For each example \mathbf{x}

Compute q (run forward pass to
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Autoencoder (training):



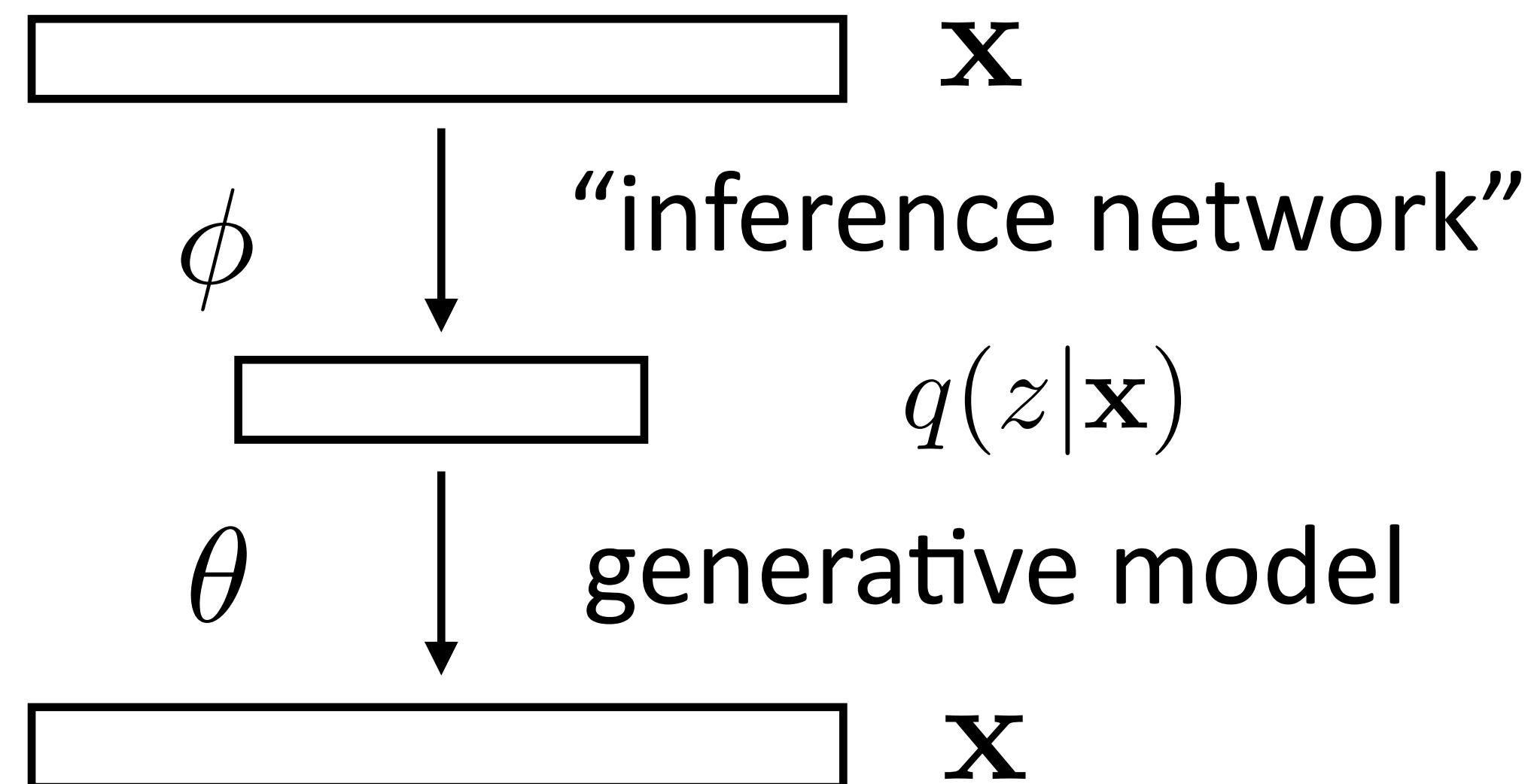
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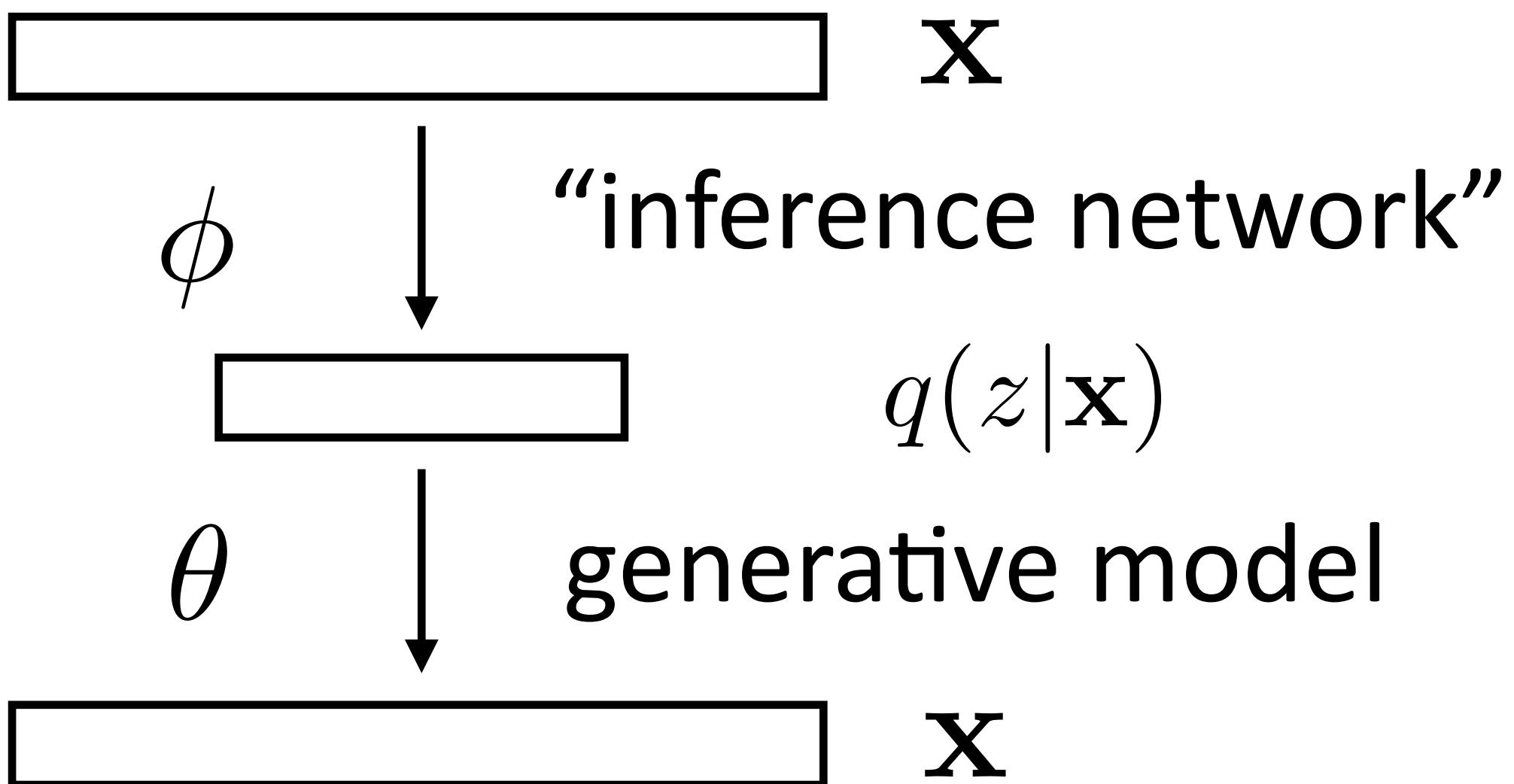
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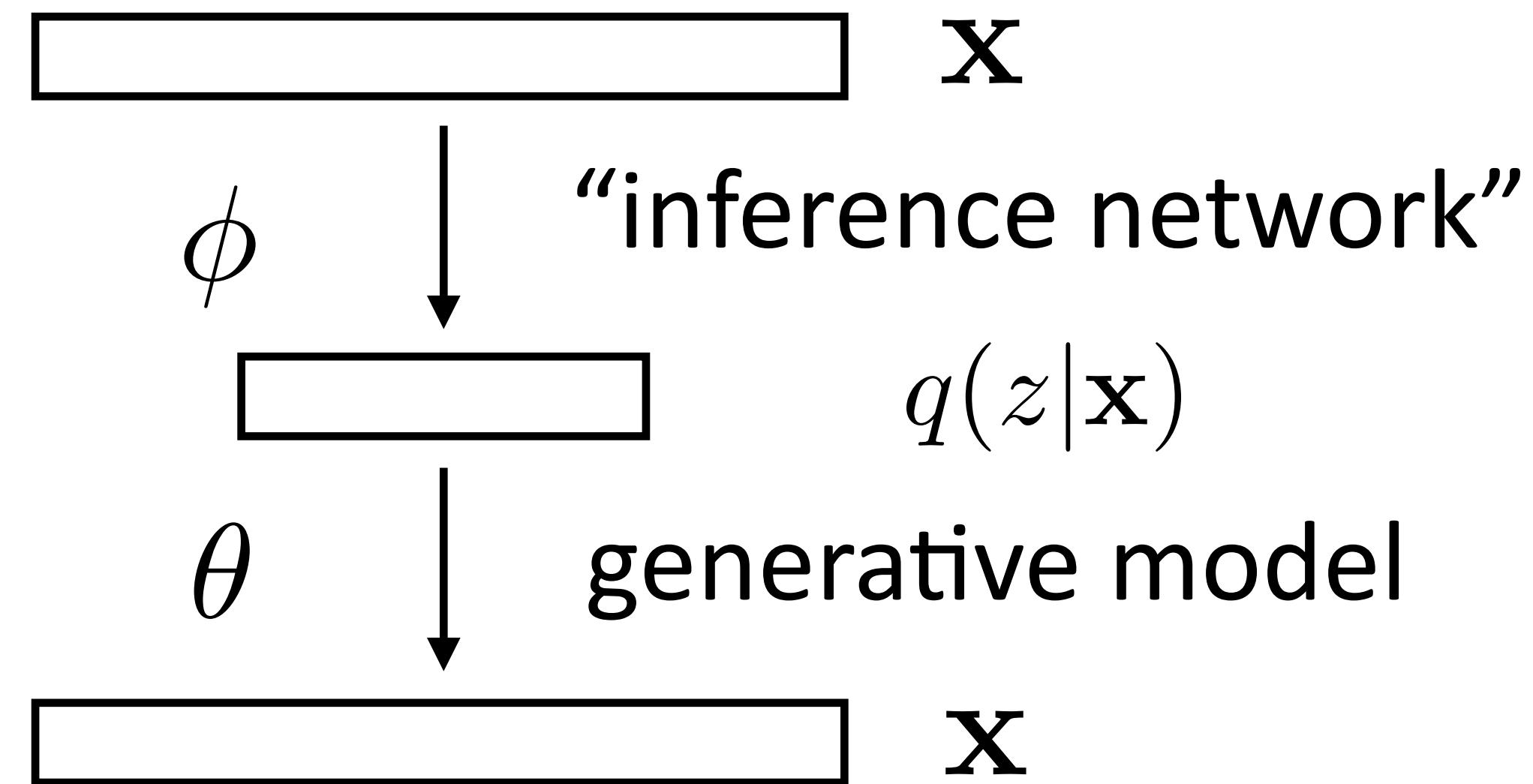
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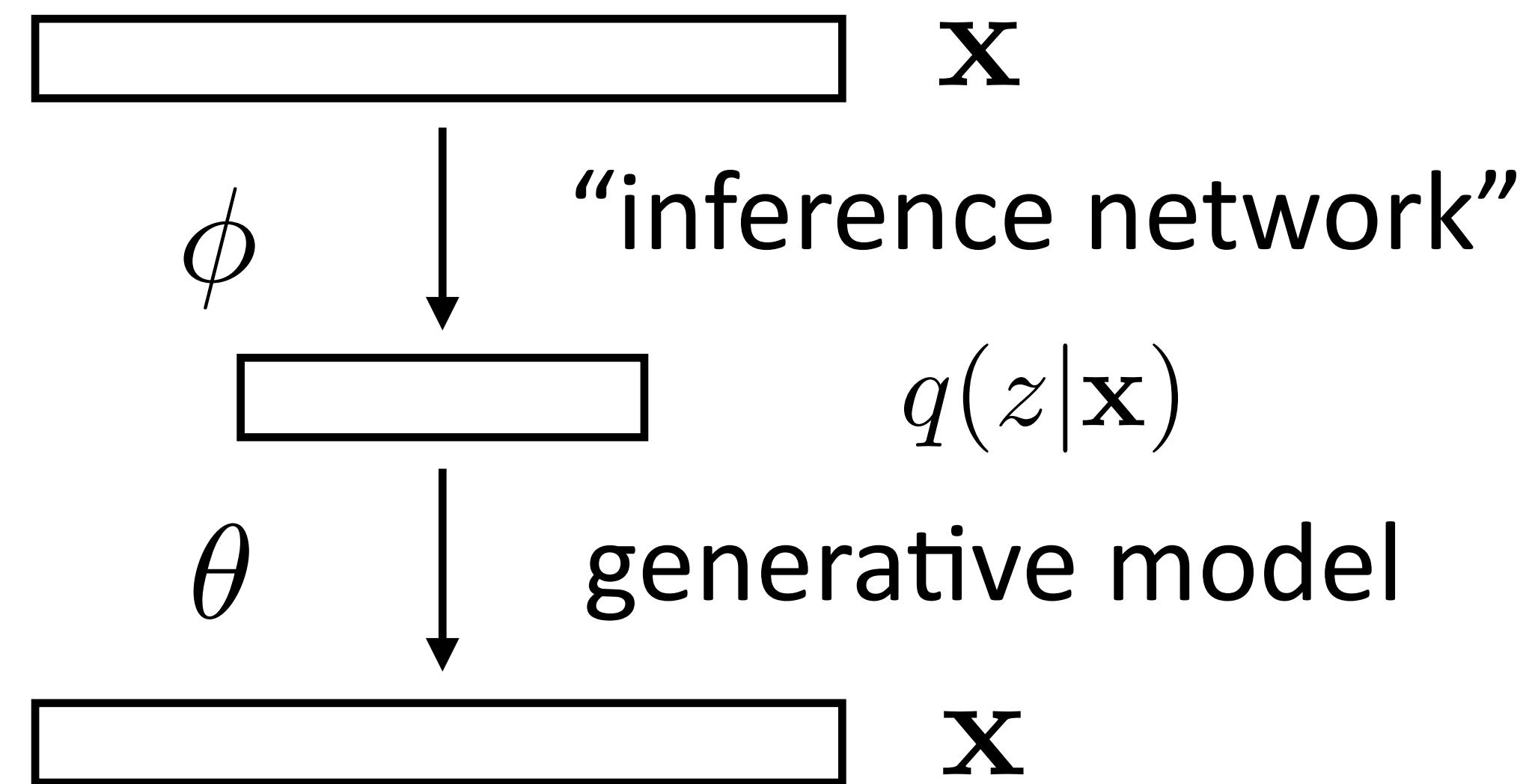
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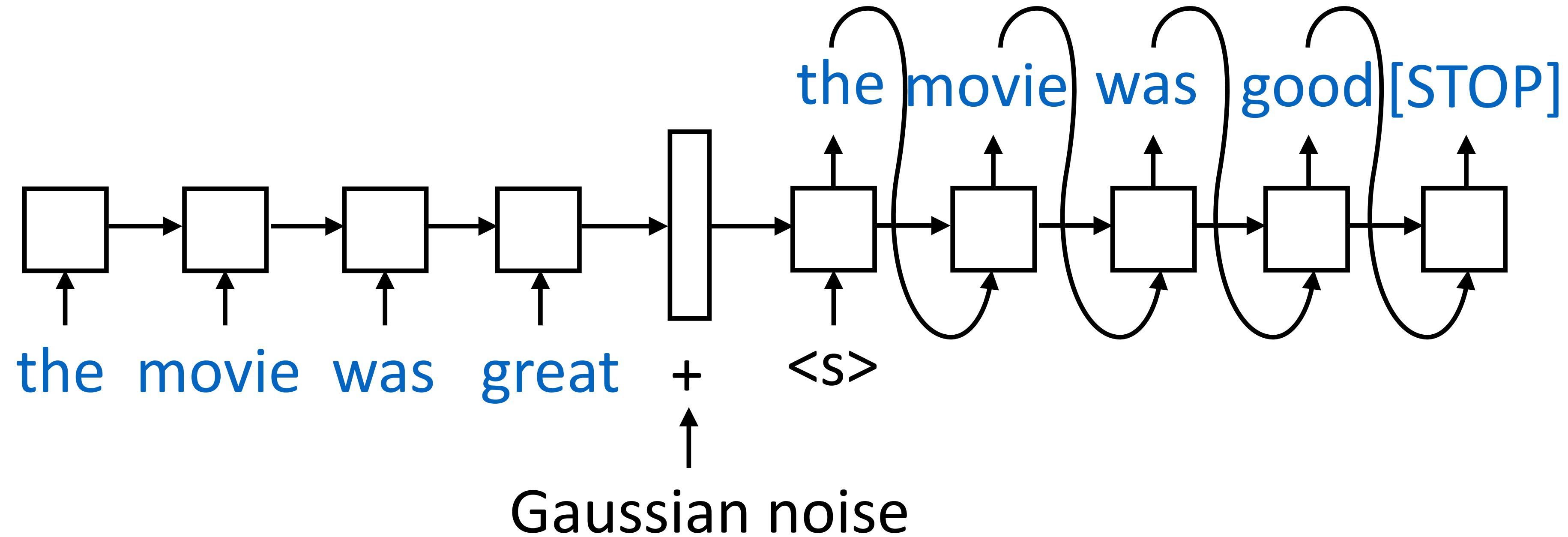
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Backpropagate to update phi, theta

Autoencoder (training):

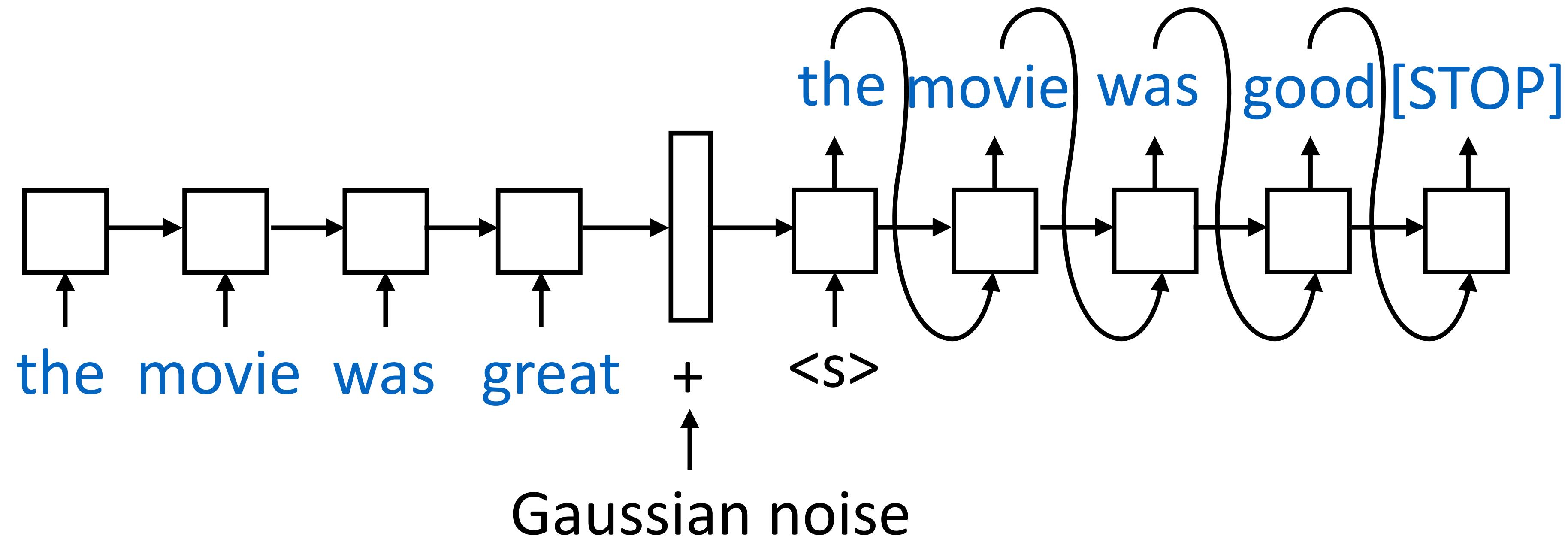


Autoencoders



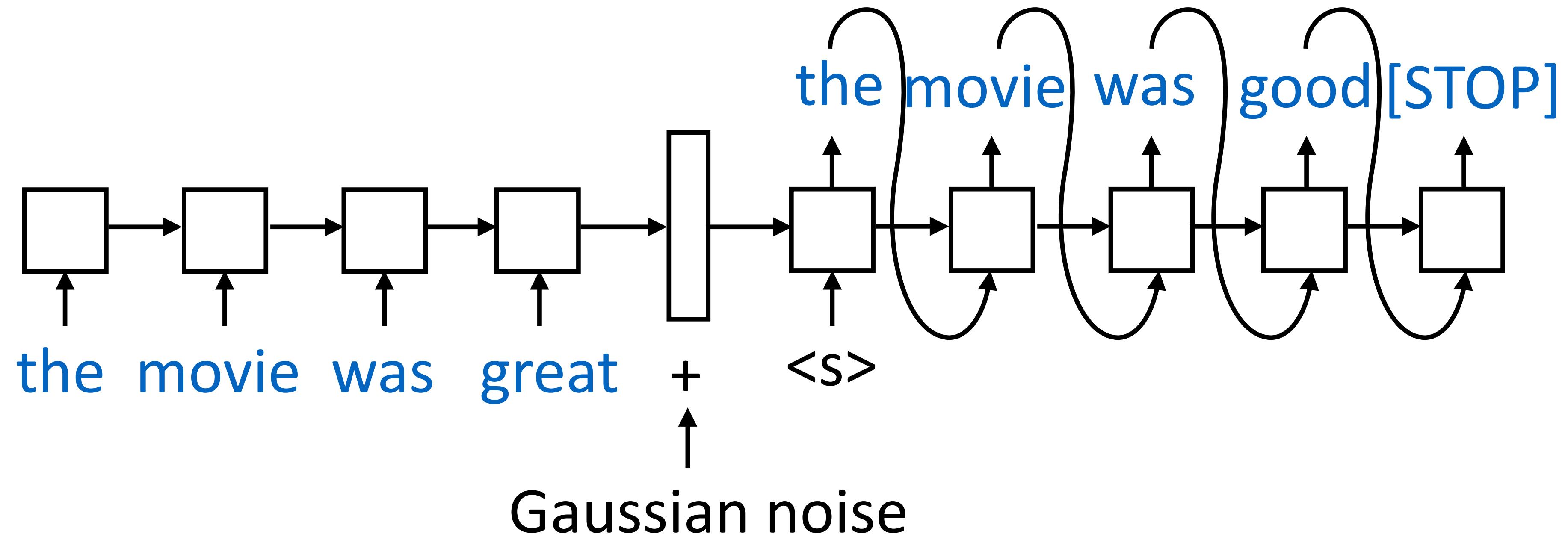
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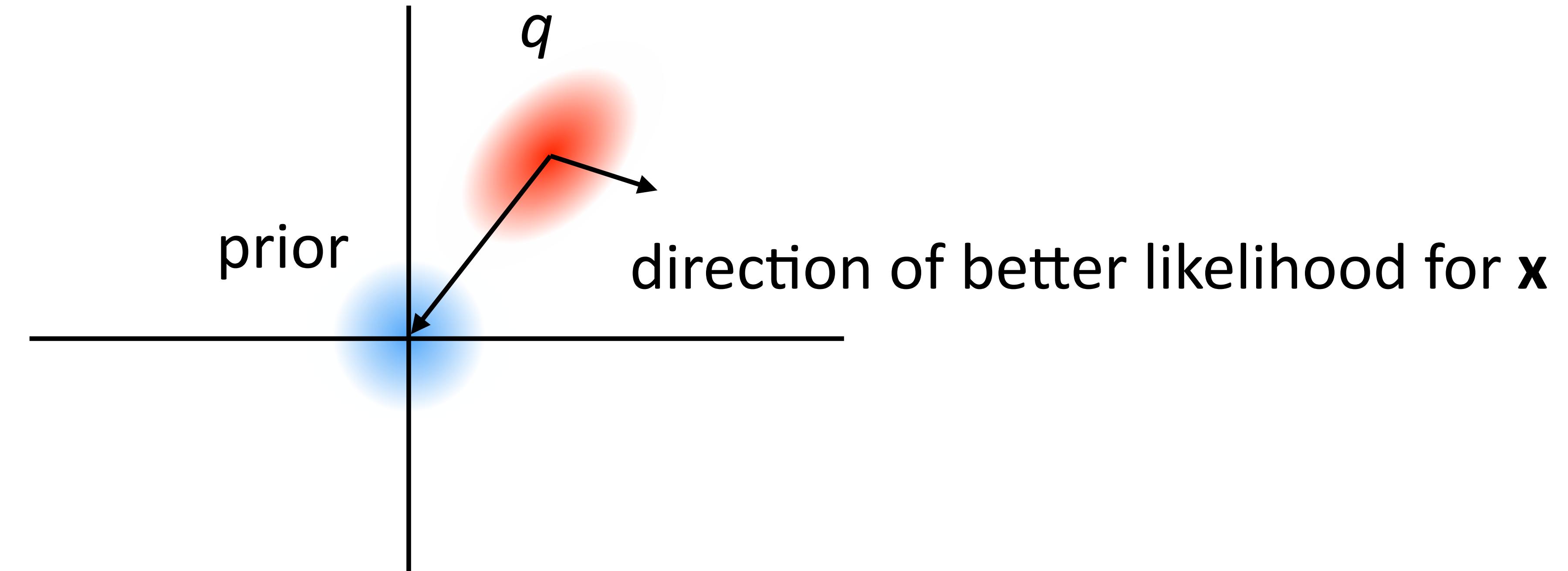


- ▶ Another interpretation: train an autoencoder and add Gaussian noise
- ▶ Same computation graph as VAE, add KL divergence term to make the objective the same
- ▶ Inference network (q) is the encoder and generator is the decoder

Visualization

$$\mathbb{E}_{q(z|\mathbf{x})}[\log P(\mathbf{x}|z, \theta)] + \text{KL}(q(z|\mathbf{x})||P(z))$$

- ▶ What does gradient encourage latent space to do?



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Positive ⇒ ARAE ⇒ Cross-AE	great indoor mall . no smoking mall . terrible outdoor urine .
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- ▶ Style transfer: also condition on sentiment, change sentiment
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⇒ ARAE
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- ▶ ...or use the latent representations for semi-supervised learning
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Self-Supervision / Transfer Learning

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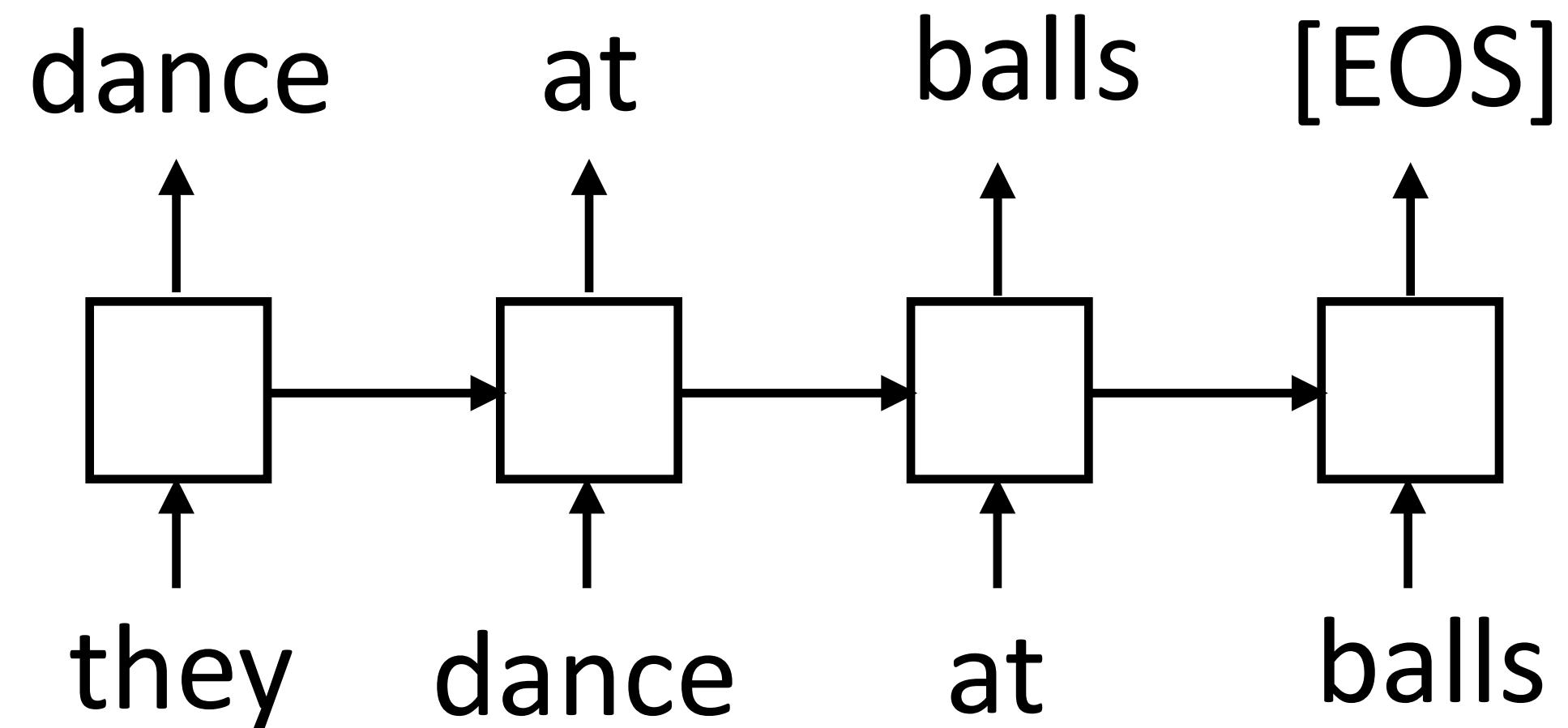
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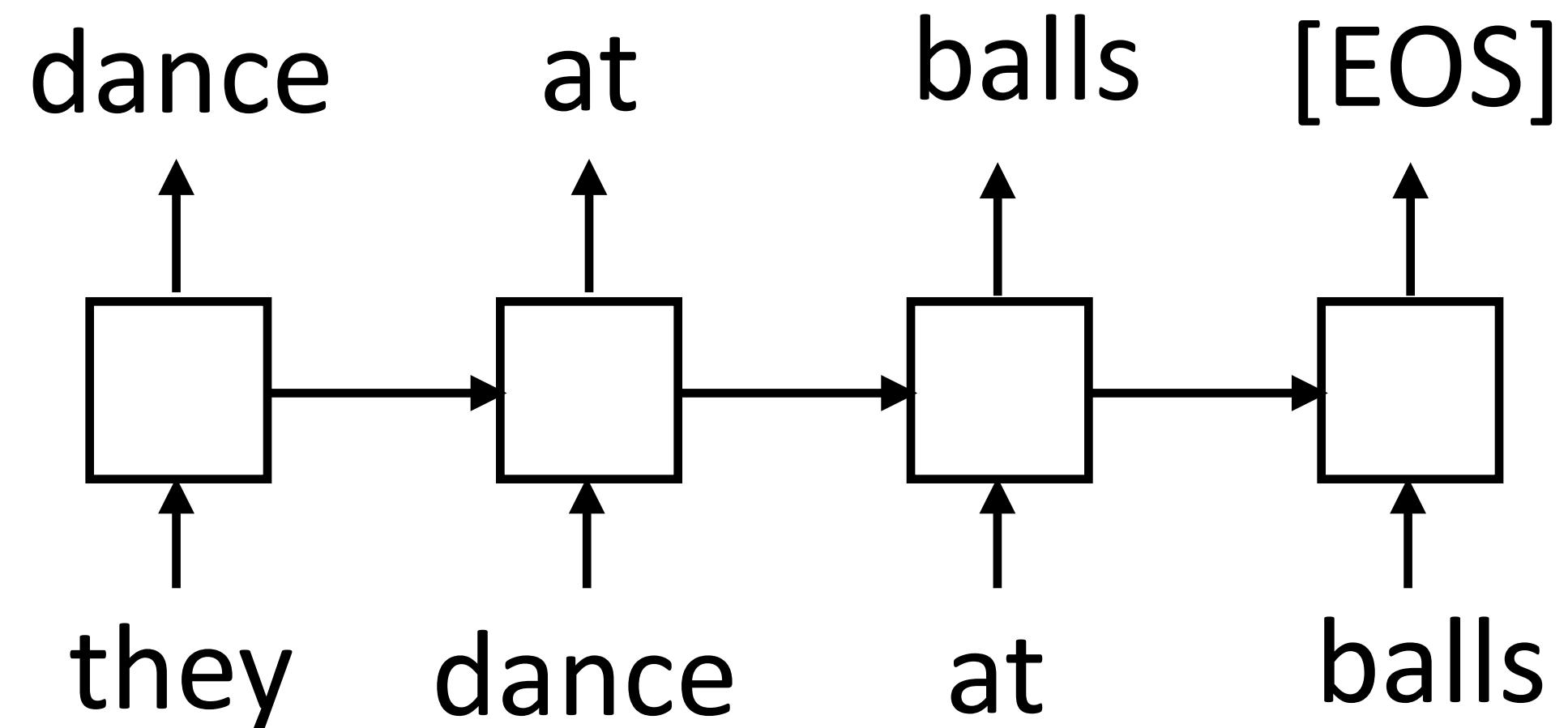
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- ▶ Language modeling is a “more contextualized” form of word2vec

ELMo

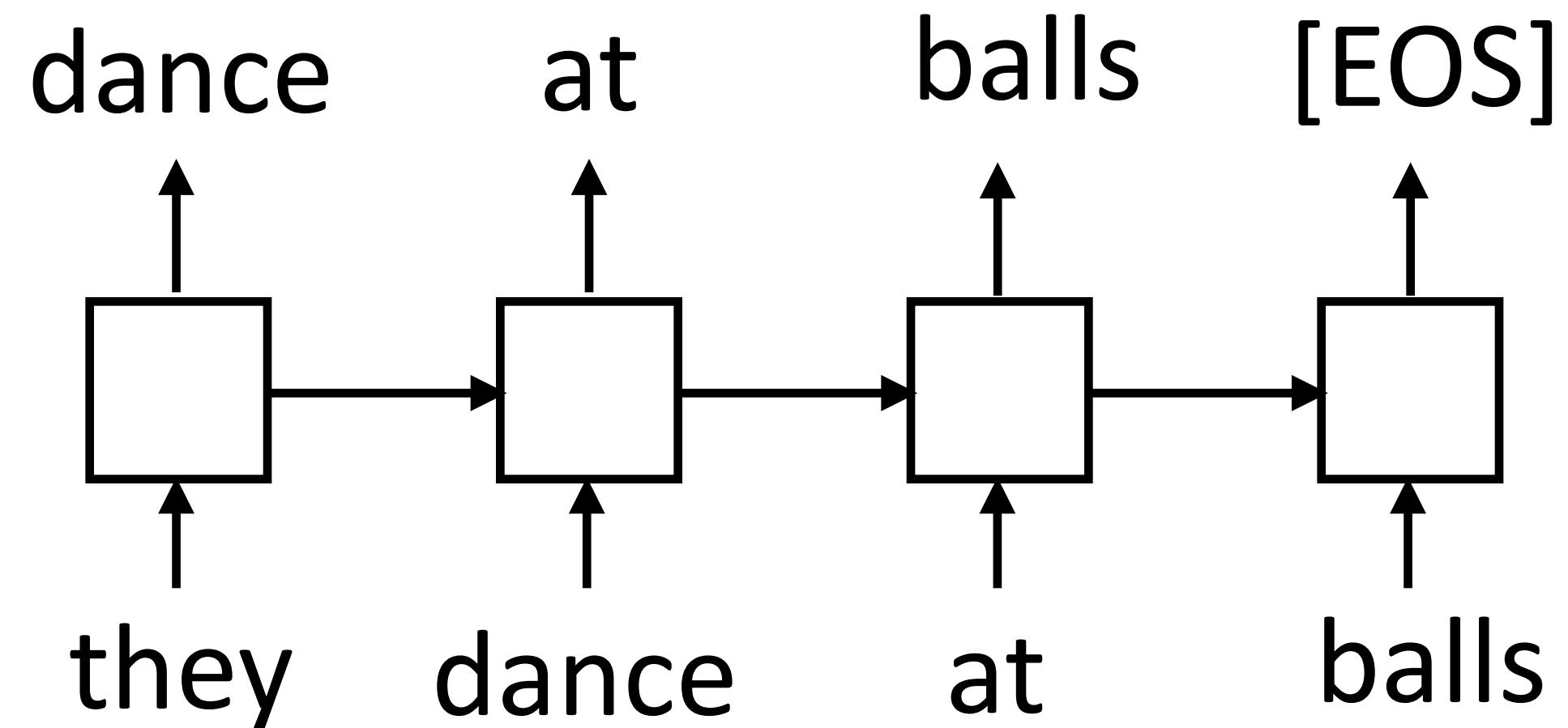


ELMo



$$P(x_i | x_1, \dots, x_{i-1}) = \text{LSTM}(x_1, \dots, x_{i-1})$$

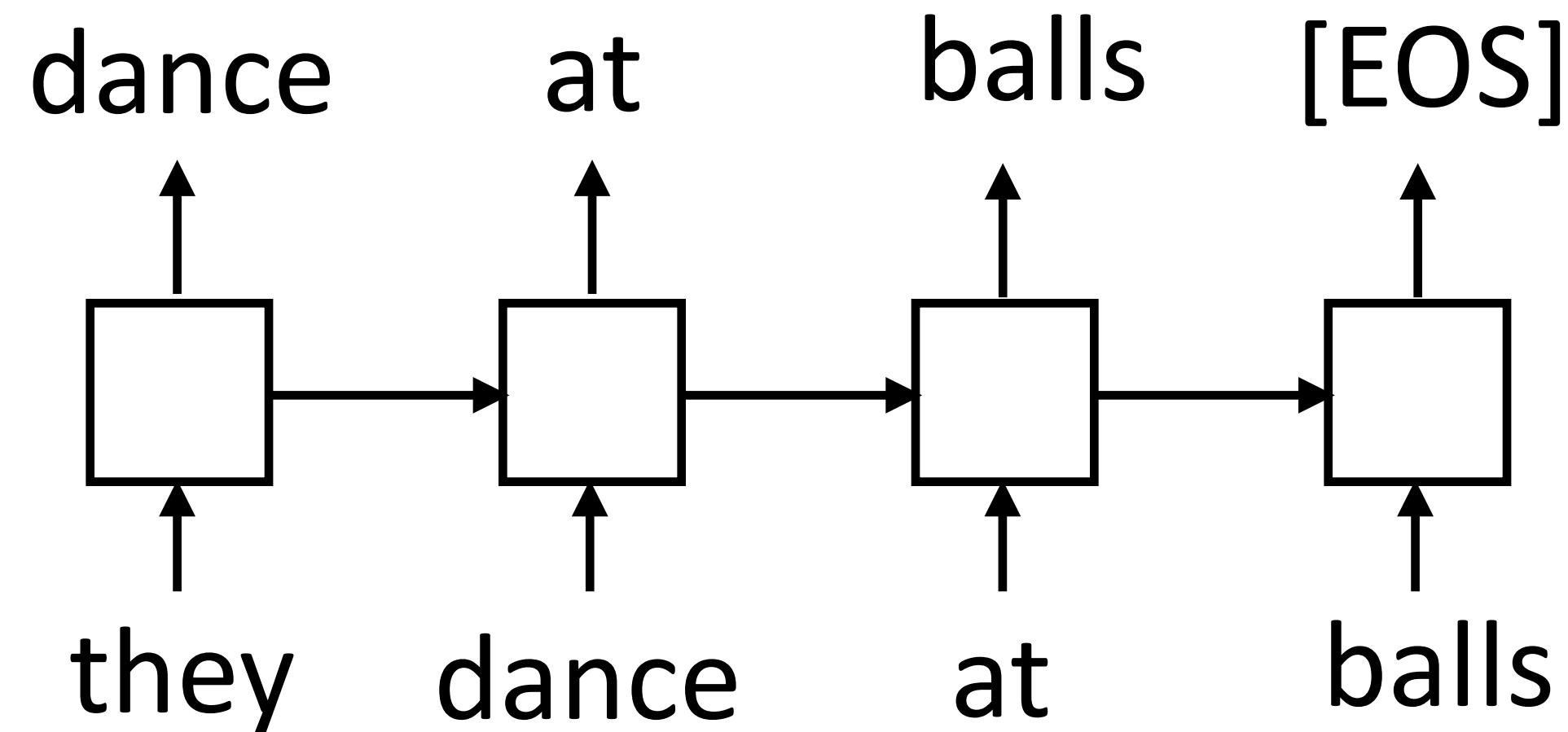
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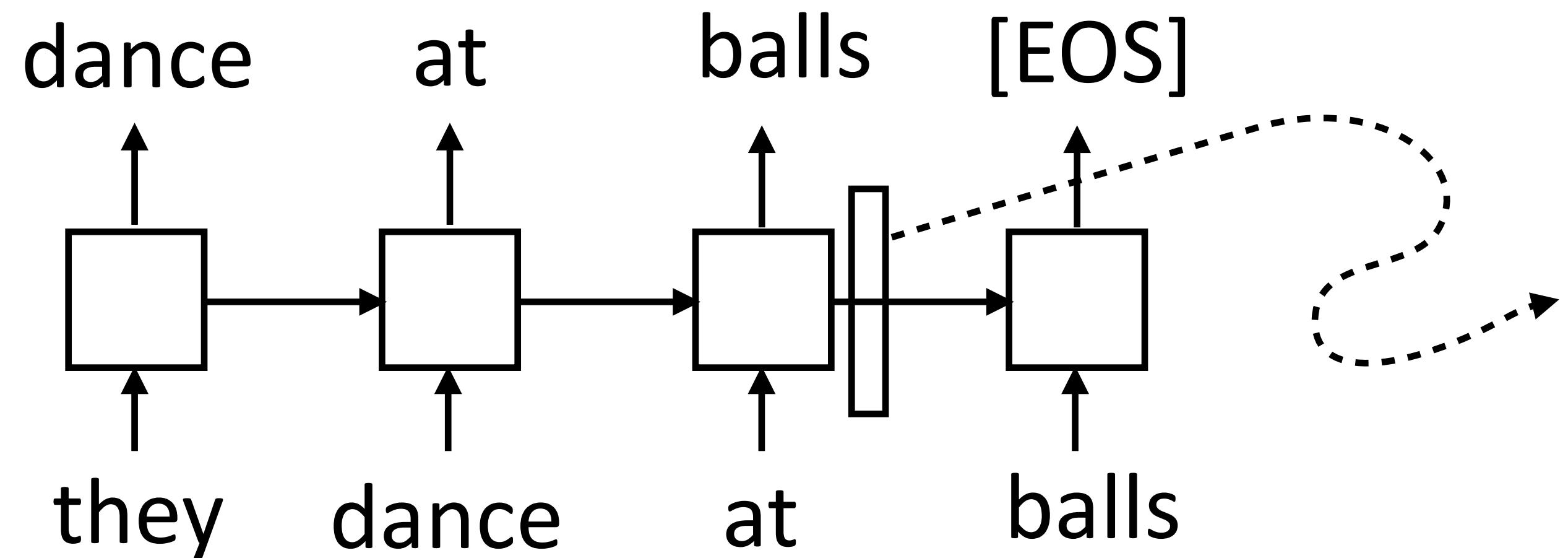
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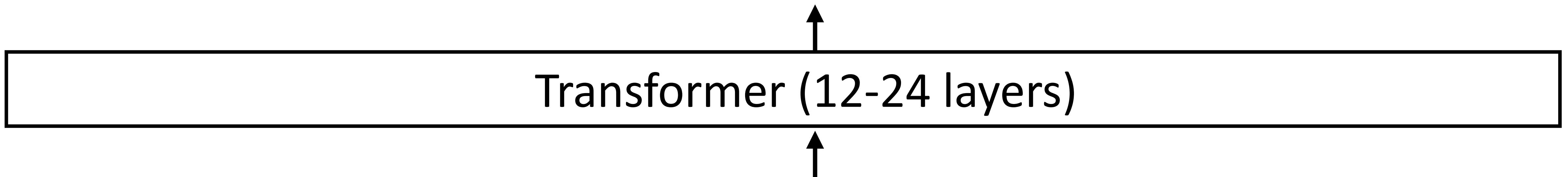
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Results

System	MNLI-(m/mm)	QQP	QNLI	SST-2	CoLA	STS-B	MRPC	RTE	Average
	392k	363k	108k	67k	8.5k	5.7k	3.5k	2.5k	-
Pre-OpenAI SOTA	80.6/80.1	66.1	82.3	93.2	35.0	81.0	86.0	61.7	74.0
BiLSTM+ELMo+Attn	76.4/76.1	64.8	79.9	90.4	36.0	73.3	84.9	56.8	71.0
OpenAI GPT	82.1/81.4	70.3	88.1	91.3	45.4	80.0	82.3	56.0	75.2
BERT _{BASE}	84.6/83.4	71.2	90.1	93.5	52.1	85.8	88.9	66.4	79.6
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- ▶ Dramatic gains on a range of sentence pair / single sentence tasks: paraphrase identification, entailment, sentiment, textual similarity, ...

Results

System	MNLI-(m/mm)	QQP	QNLI	SST-2	CoLA	STS-B	MRPC	RTE	Average
	392k	363k	108k	67k	8.5k	5.7k	3.5k	2.5k	-
Pre-OpenAI SOTA	80.6/80.1	66.1	82.3	93.2	35.0	81.0	86.0	61.7	74.0
BiLSTM+ELMo+Attn	76.4/76.1	64.8	79.9	90.4	36.0	73.3	84.9	56.8	71.0
OpenAI GPT	82.1/81.4	70.3	88.1	91.3	45.4	80.0	82.3	56.0	75.2
BERT _{BASE}	84.6/83.4	71.2	90.1	93.5	52.1	85.8	88.9	66.4	79.6
BERT _{LARGE}	86.7/85.9	72.1	91.1	94.9	60.5	86.5	89.3	70.1	81.9

- ▶ Dramatic gains on a range of sentence pair / single sentence tasks: paraphrase identification, entailment, sentiment, textual similarity, ...
- ▶ Not a generative model! But learns really effective representations...

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- ▶ Continuous structure with “discriminative” models
 - ▶ ELMo / BERT seem extremely useful

Takeaways

- ▶ EM sort of works for POS induction
- ▶ VAE can learn sentence representations
- ▶ Language modeling or text infilling as pretraining seems best — arguably not “unsupervised” but the annotation is free
- ▶ Using unlabeled data effectively seems like one of the most important directions in NLP right now
- ▶ Next time: Jessy Li guest lecture on discourse