

# CSE 5523 Homework 1: Math Review

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## Problem 1

Consider the following function:

$$f(x) = \frac{1}{1 + e^{-x}}$$

(a)

Draw a plot of  $f(x)$  - what are the minimum and maximum values of  $f$ . What values of  $x$  result in the largest or smallest values of  $f(x)$ ?

(b)

Show that the derivative of  $f(x)$  can be written simply in terms of the function's value like so:

$$\frac{df(x)}{dx} = f(x)(1 - f(x))$$

*Hint: start by computing the derivative of  $f(x)$  using the chain rule, then re-arrange terms to get the result into the form of the answer.*

## Problem 2

Assume the following joint distribution for  $P(A, B)$ :

$$P(A = 0, B = 0) = 0.2$$

$$P(A = 0, B = 1) = 0.2$$

$$P(A = 1, B = 0) = 0.6$$

$$P(A = 1, B = 1) = 0.0$$

- (a) What is the marginal probability of  $P(B = 0)$ ?
- (b) What is  $P(A = 1|B = 0)$ ?
- (c) What is  $P(A = B)$ ?

### Problem 3

Assume  $X$  is conditionally independent of  $Y$  given  $Z$ . Which of the following statements are always true?

- (a)  $P(X, Y) = P(X) + P(Y) - P(Z)$
- (b)  $P(X, Y, Z) = P(X) + P(Y) + P(Z)$
- (c)  $P(X, Y) = \sum_{c \in \mathcal{X}_Z} P(X, Y, Z = c)$
- (d)  $P(X, Y|Z) = P(X|Z)P(Y|Z)$
- (e)  $P(X, Y) = P(X)P(Y)$

### Problem 4

Derive Bayes Rule in the form:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

from the definition of conditional probability:

$$P(A, B) = P(A|B)P(B)$$

### Problem 5

Consider the following matrix,  $M$  and vector,  $v$ :

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 1 & 3 & 3 \end{bmatrix}, v = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

Compute the following matrix-vector and vector-vector products explaining how you arrived at each answer (show your work):

(a)

$$M \cdot v =$$

(b)

$$v^T \cdot M =$$

(c)

$$v^T \cdot v =$$

## Problem 6

Install Python (<https://www.python.org/>), Numpy (<http://www.numpy.org/>) and Scipy (<https://www.scipy.org/>) on your computer (assuming you have not already done so). Write a short program that defines  $M$  and  $v$  and computes the answers to Problem 5 above using Numpy. Turn in a print out of your code, shell commands to run it and output.

## Survey

- (a) What is your favorite programming language?
- (b) What programming languages do you feel comfortable using?
- (c) Are there any specific topics you would like to see covered in the class?