

CSE 5523 Homework 1: Math Review

Alan Ritter

Problem 1

Consider the following function:

$$f(x) = \frac{1}{1 + e^{-x}}$$

(a)

Draw a plot of $f(x)$ - what are the minimum and maximum values of f . What values of x result in the largest or smallest values of $f(x)$?

(b)

Show that the derivative of $f(x)$ can be written simply in terms of the function's value like so:

$$\frac{df(x)}{dx} = f(x)(1 - f(x))$$

Hint: start by computing the derivative of $f(x)$ using the chain rule, then re-arrange terms to get the result into the form of the answer.

Problem 2

Assume the following joint distribution for $P(A, B)$:

$$\begin{aligned} P(A = 0, B = 0) &= 0.2 \\ P(A = 0, B = 1) &= 0.2 \\ P(A = 1, B = 0) &= 0.6 \\ P(A = 1, B = 1) &= 0.0 \end{aligned}$$

- (a) What is the marginal probability of $P(B = 0)$?
- (b) What is $P(A = 1|B = 0)$?
- (c) What is $P(A = B)$?

Problem 3

Assume X is conditionally independent of Y given Z . Which of the following statements are always true?

- (a) $P(X, Y) = P(X) + P(Y) - P(Z)$
- (b) $P(X, Y, Z) = P(X) + P(Y) + P(Z)$
- (c) $P(X, Y) = \sum_{c \in \mathcal{X}_Z} P(X, Y, Z = c)$
- (d) $P(X, Y|Z) = P(X|Z)P(Y|Z)$
- (e) $P(X, Y) = P(X)P(Y)$

Problem 4

Derive Bayes Rule in the form:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

from the definition of conditional probability:

$$P(A, B) = P(A|B)P(B)$$

Problem 5

Consider the following matrix, M and vector, v :

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 1 & 3 & 3 \end{bmatrix}, v = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

Compute the following matrix-vector and vector-vector products explaining how you arrived at each answer (show your work):

(a)

$$M \cdot v =$$

(b)

$$v^T \cdot M =$$

(c)

$$v^T \cdot v =$$

Problem 6

Install Python (<https://www.python.org/>), Numpy (<http://www.numpy.org/>) and Scipy (<https://www.scipy.org/>) on your computer (assuming you have not already done so). If you strongly prefer not to use Python for some reason you can use another programming language for example Matlab/Ocatave. Python is strongly recommended for the class, however - starter code will be provided for the homework in Python.

Write a short program that computes the answers to Problem 5 above. Turn in your code (can be hand-written).

Survey

- (a) What is your favorite programming language?
- (b) What programming languages do you feel comfortable using?
- (c) Are there any specific topics you would like to be covered in the class?