Multiclass Classification

Alan Ritter

(many slides from Greg Durrett and Vivek Srikumar)

Administrivia

- Homework 1 due on Wednesday
- Prof. Wei Xu will present lectures next week

Recall: Binary Classification

- Logistic regression: $P(y=1|x) = \dfrac{\exp\left(\sum_{i=1}^{n} w_i x_i\right)}{\left(1+\exp\left(\sum_{i=1}^{n} w_i x_i\right)\right)}$

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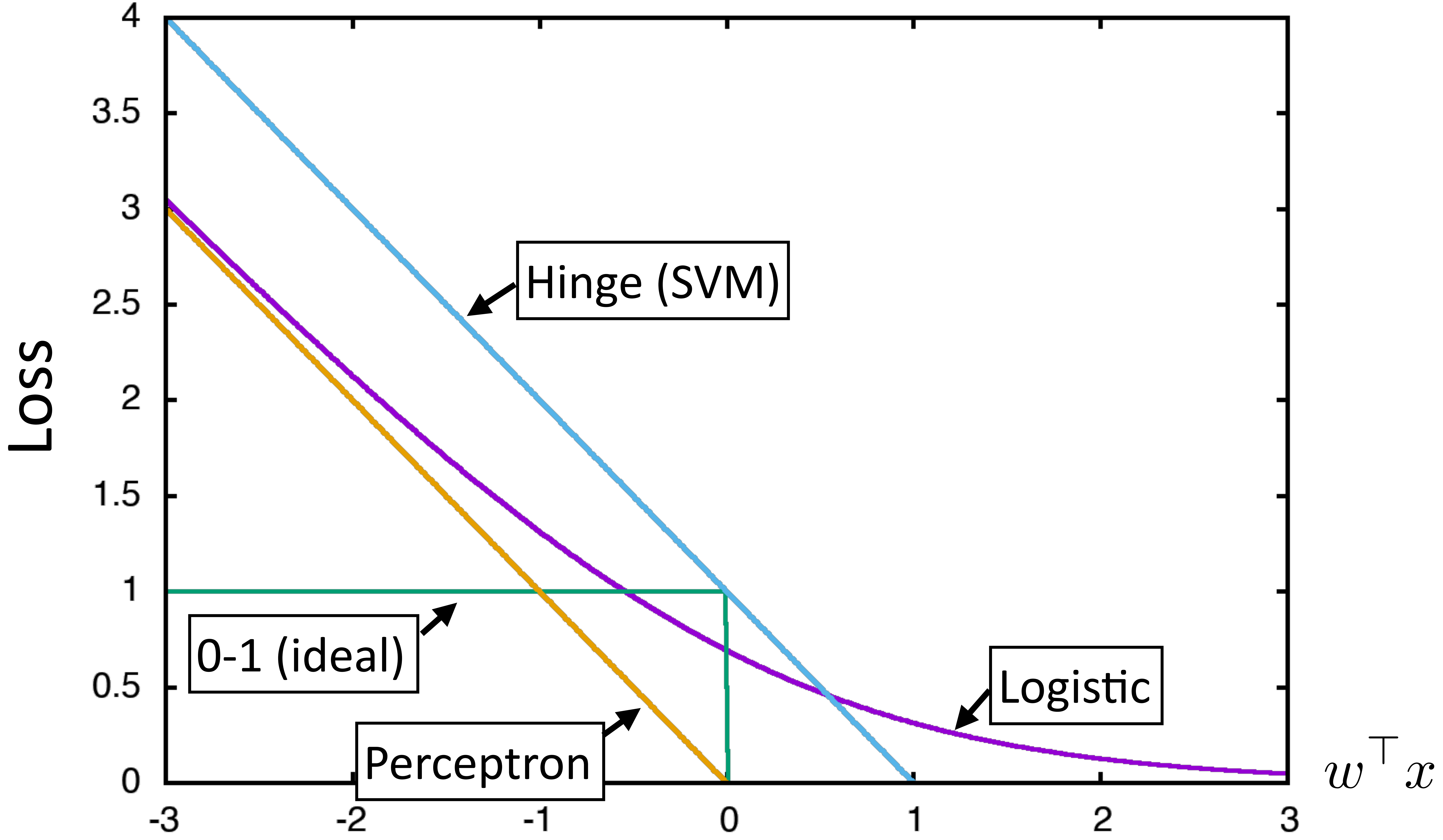
  (Sub)gradient (unregularized): 0 if correct with margin of 1, else $x(2y - 1)$

Loss Functions



This Lecture

- Multiclass fundamentals
- Feature extraction
- Multiclass logistic regression
- Multiclass SVM
- Optimization

Multiclass Fundamentals

Text Classification

A Cancer Conundrum: Too Many Drug Trials, Too Few Patients

Breakthroughs in immunotherapy and a rush to develop profitable new treatments have brought a crush of clinical trials scrambling for patients.

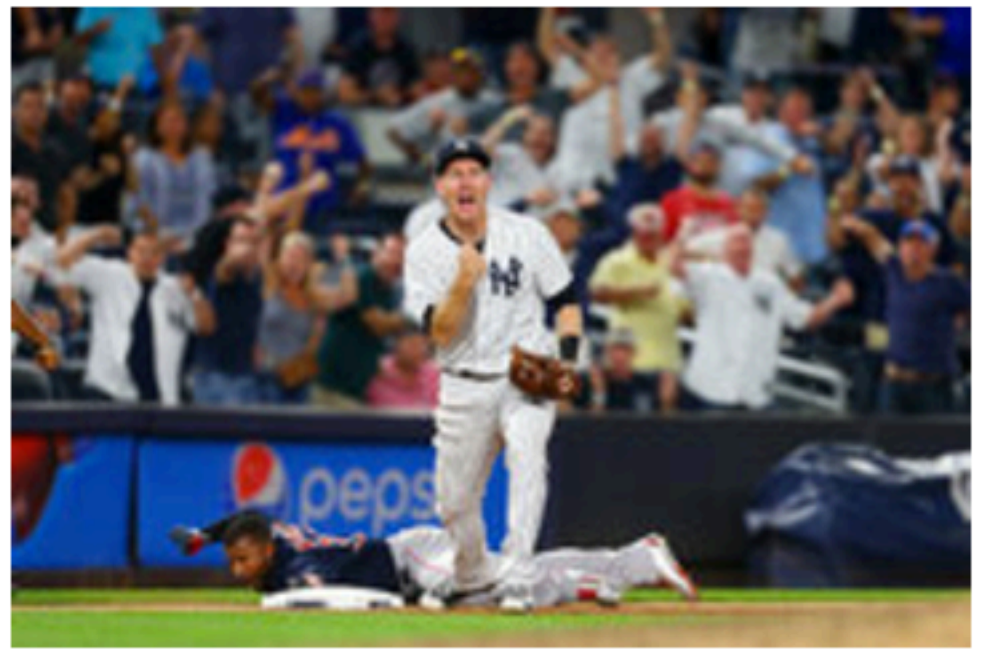
By GINA KOLATA

→ Health

Yankees and Mets Are on Opposite Tracks This Subway Series

As they meet for a four-game series, the Yankees are playing for a postseason spot, and the most the Mets can hope for is to play spoiler.

By FILIP BONDY



→ Sports

~20 classes

Image Classification

→ Dog

→ Car

- Thousands of classes (ImageNet)

Entity Linking

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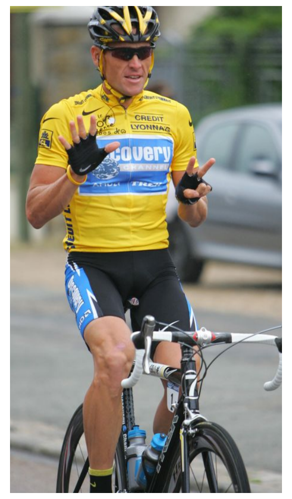

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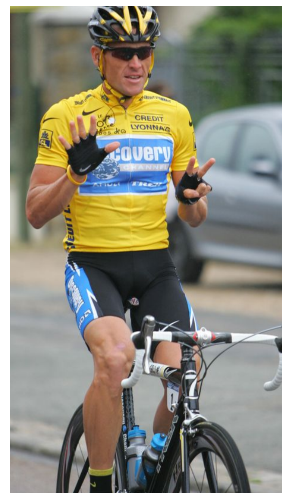

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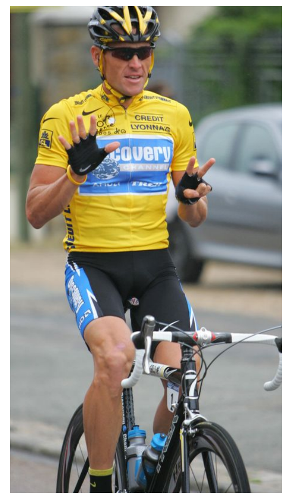

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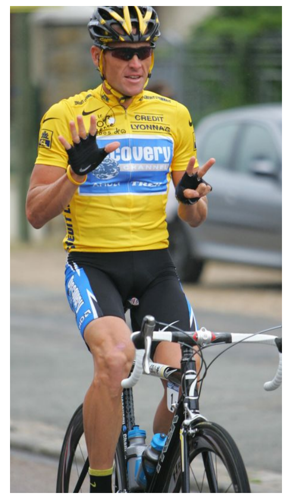

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- 4,500,000 classes (all articles in Wikipedia)

Reading Comprehension

One day, James thought he would go into town and see what kind of trouble he could get into. He went to the grocery store and pulled all the pudding off the shelves and ate two jars. Then he walked to the fast food restaurant and ordered 15 bags of fries. He didn't pay, and instead headed home.

3) Where did James go after he went to the grocery store?
A) his deck
B) his freezer
C) a fast food restaurant
D) his room

- Multiple choice questions, 4 classes (but classes change per example)

Richardson (2013)

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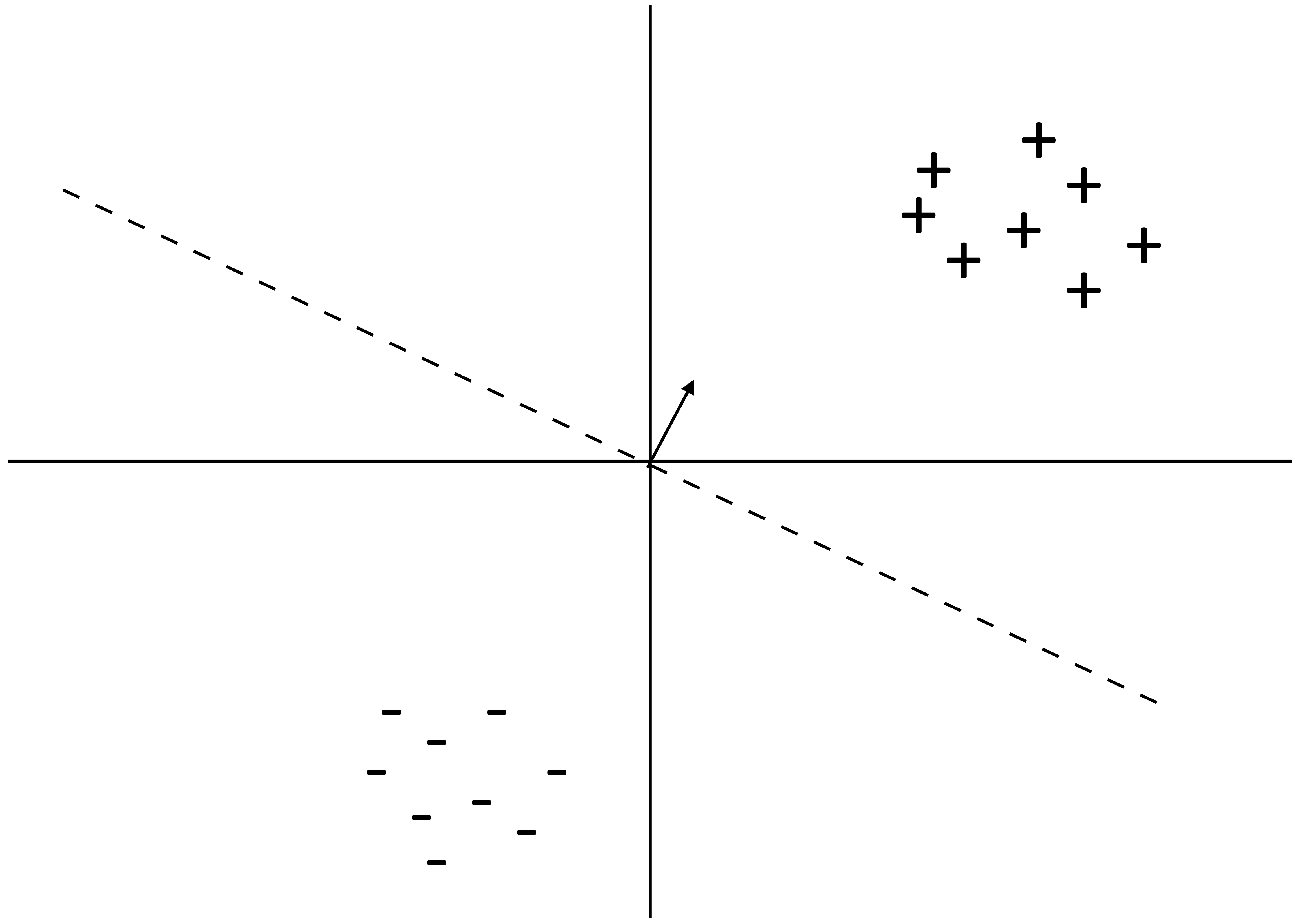
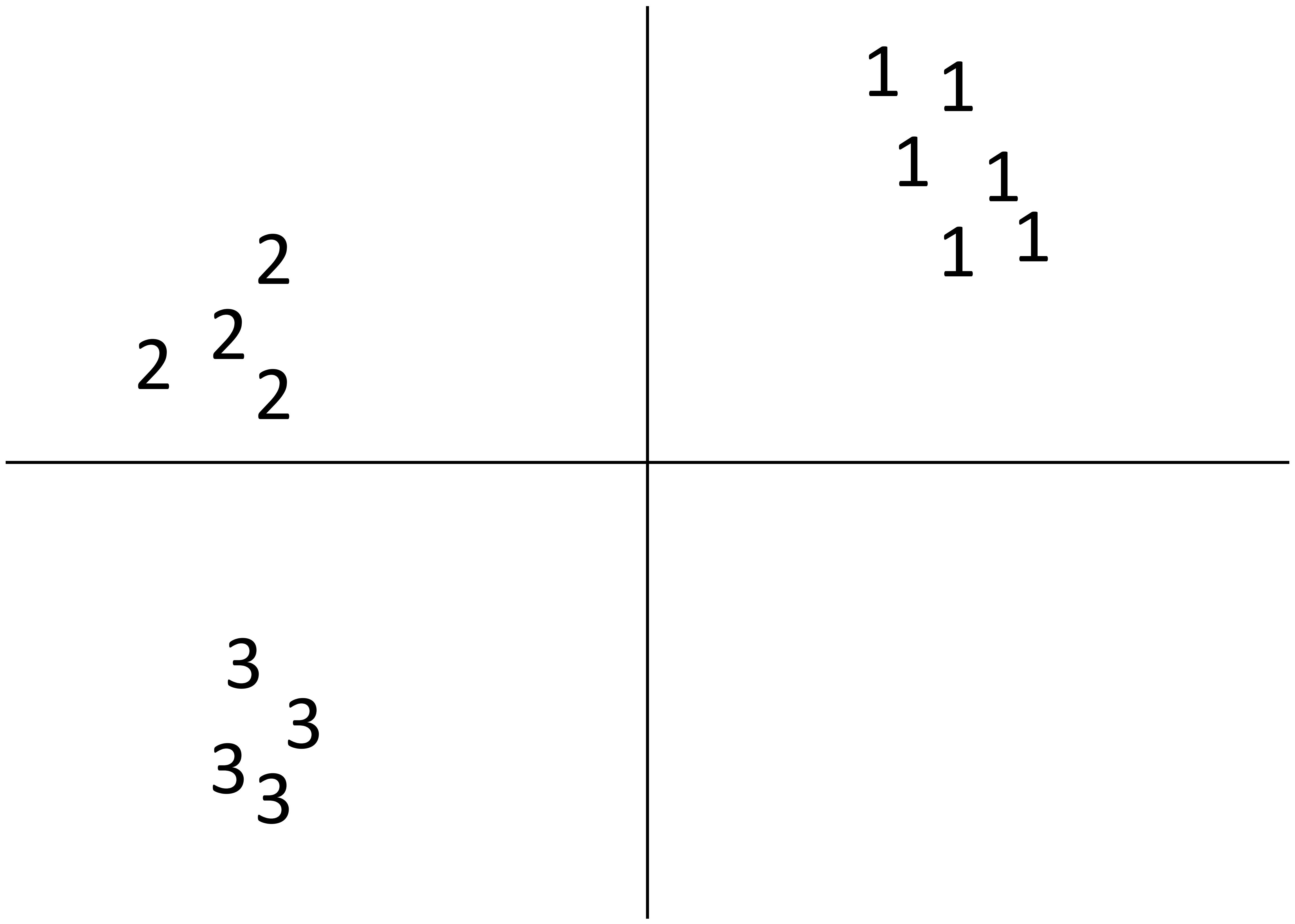
Binary Classification

- Binary classification: one weight vector defines positive and negative classes

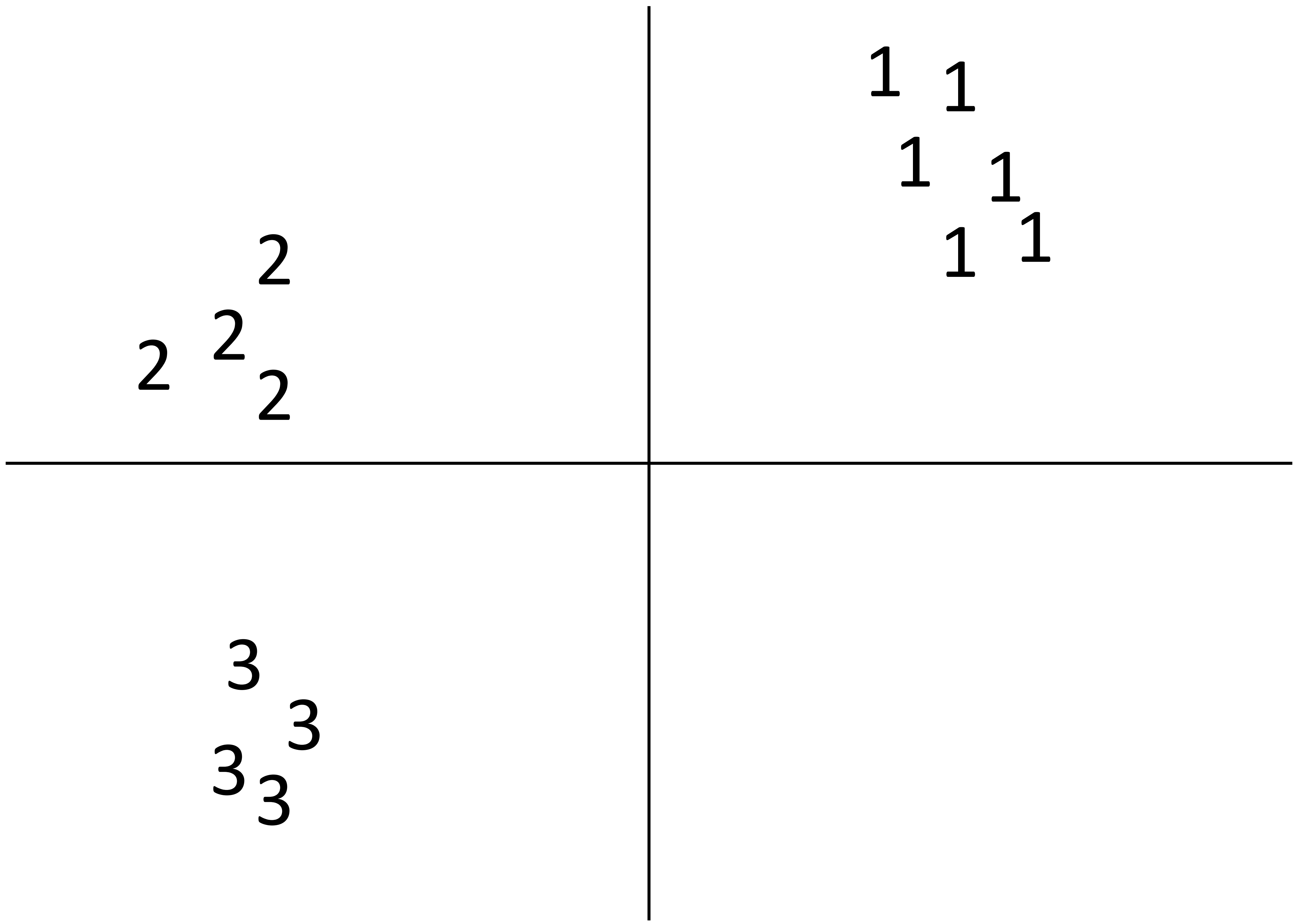


Multiclass Classification



Multiclass Classification

- Can we just use binary classifiers here?



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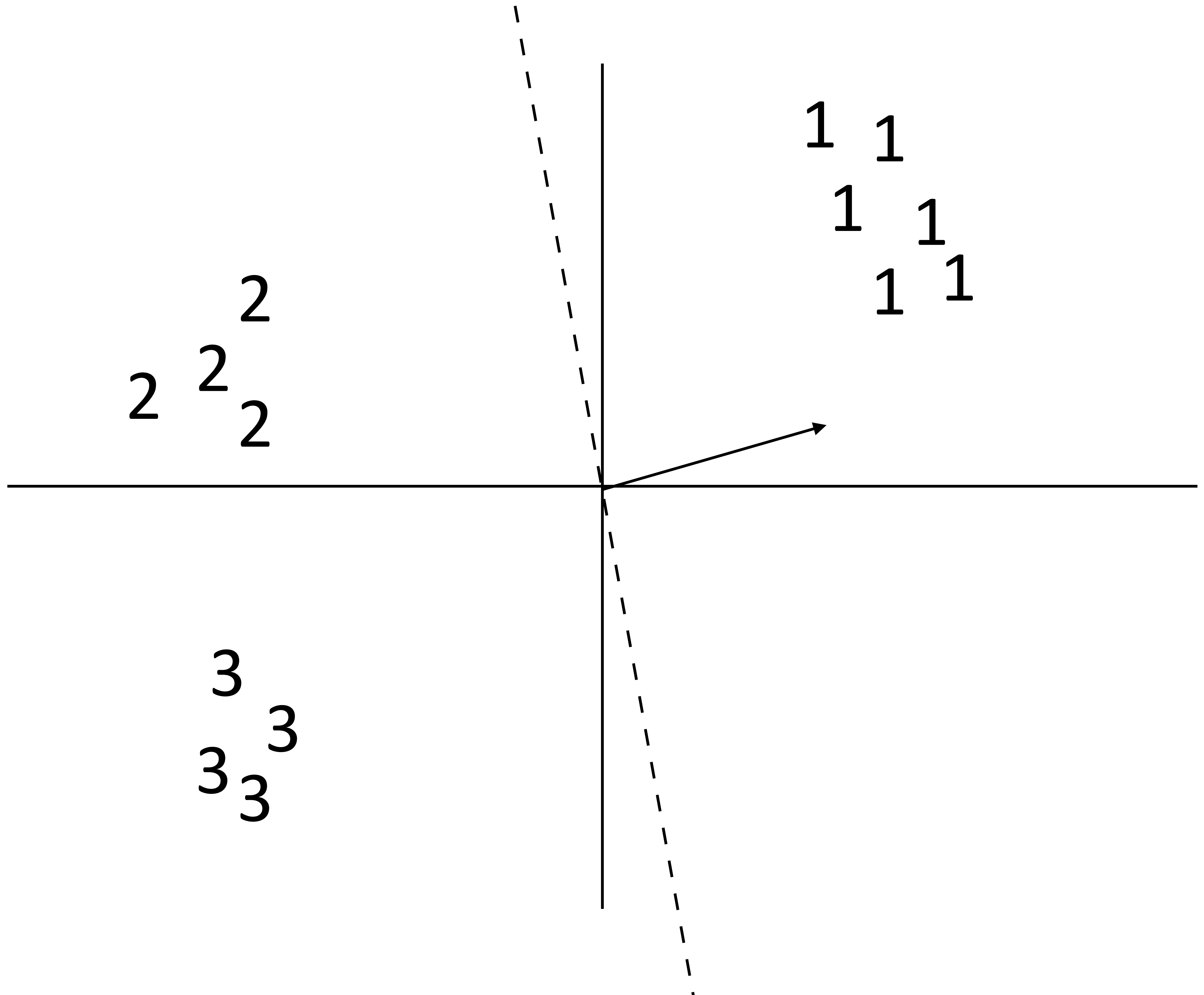

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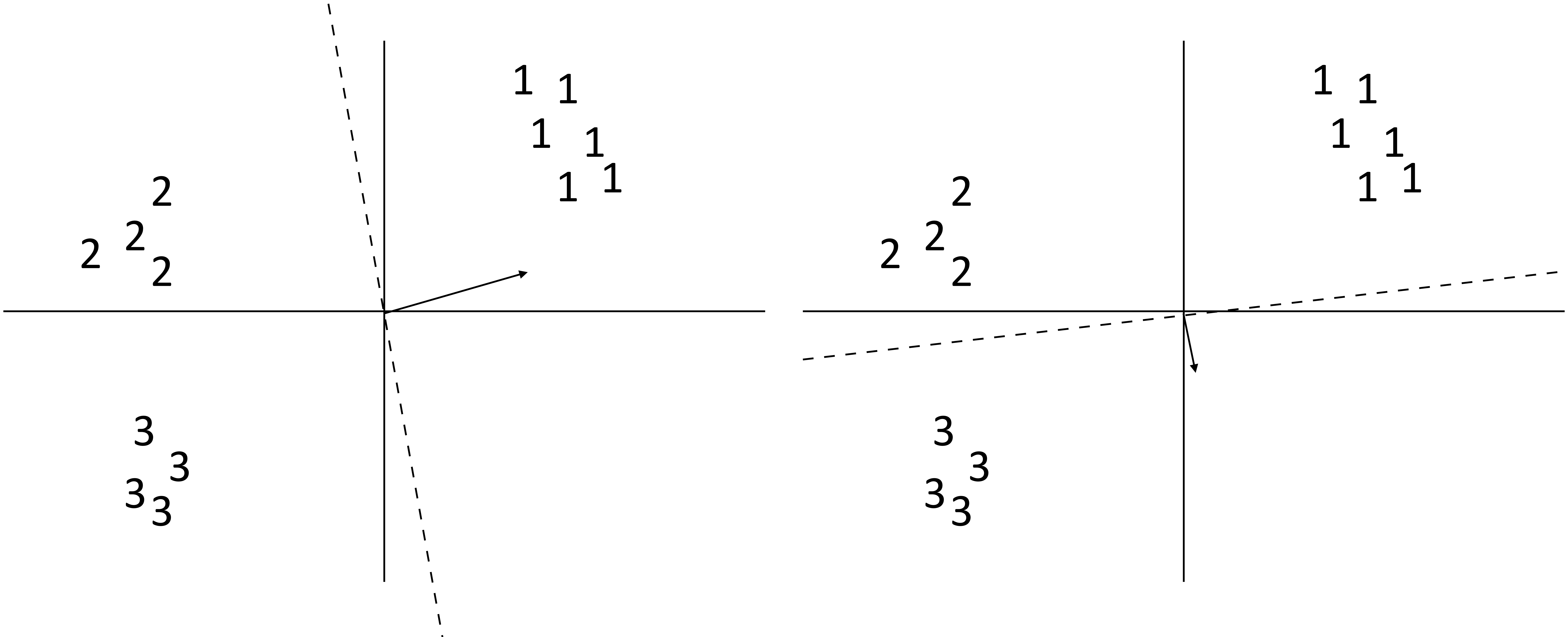
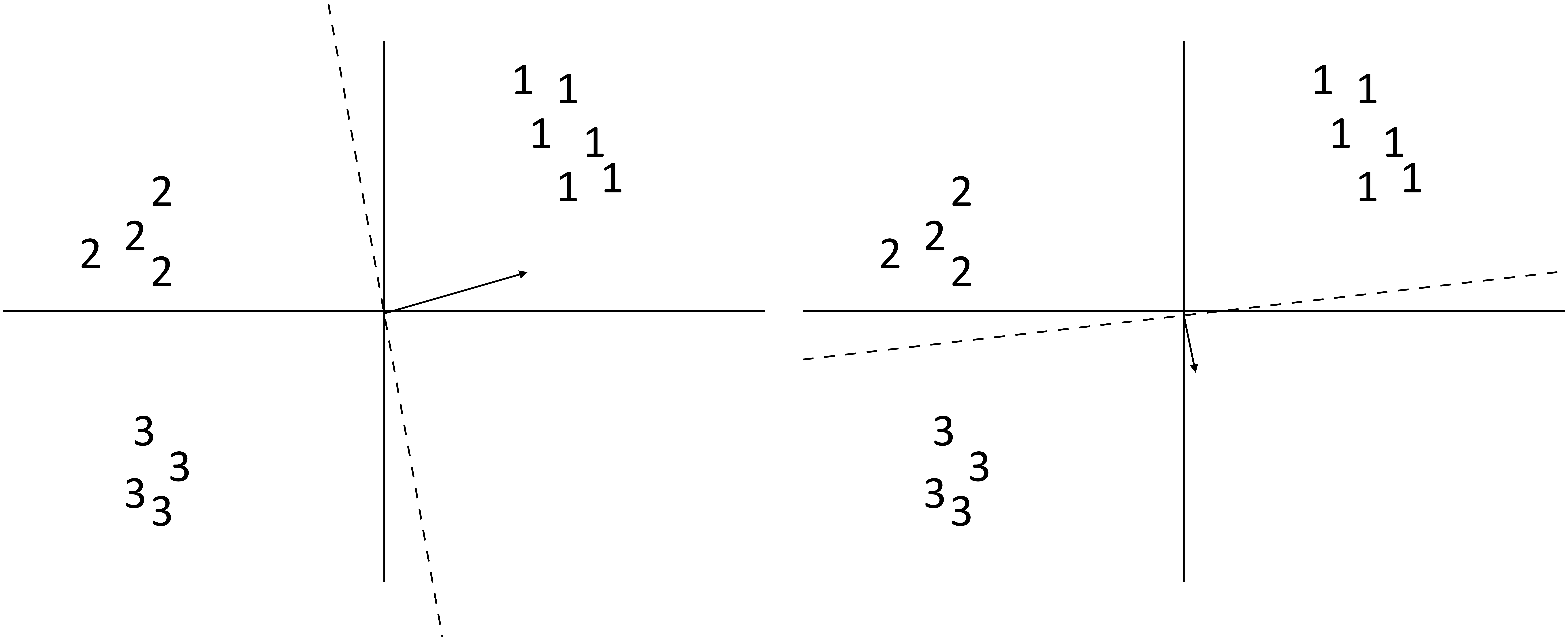


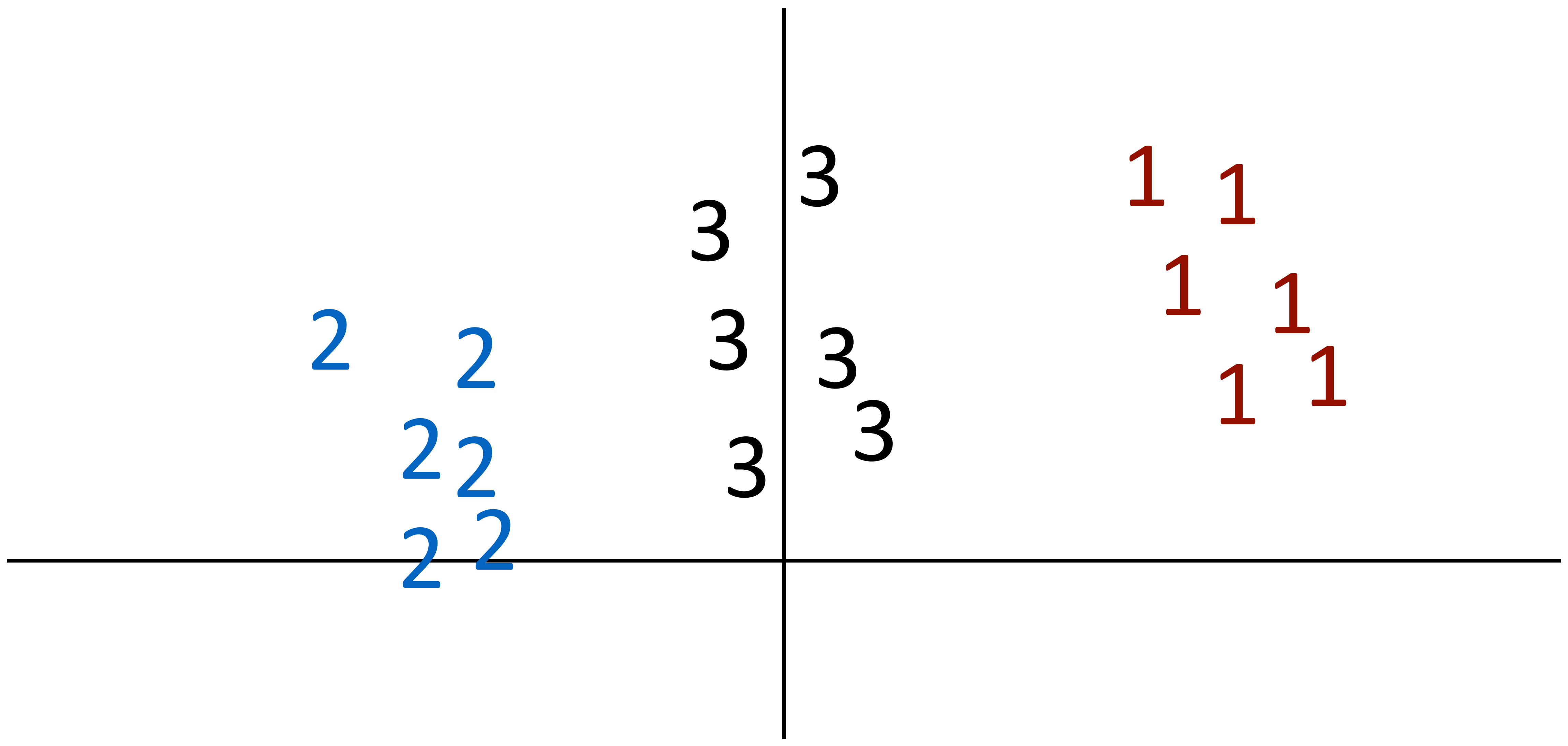
Multiclass Classification

- One-vs-all: train *k* classifiers, one to distinguish each class from all the rest
- How do we reconcile multiple positive predictions? Highest score?



Multiclass Classification

- Not all classes may even be separable using this approach



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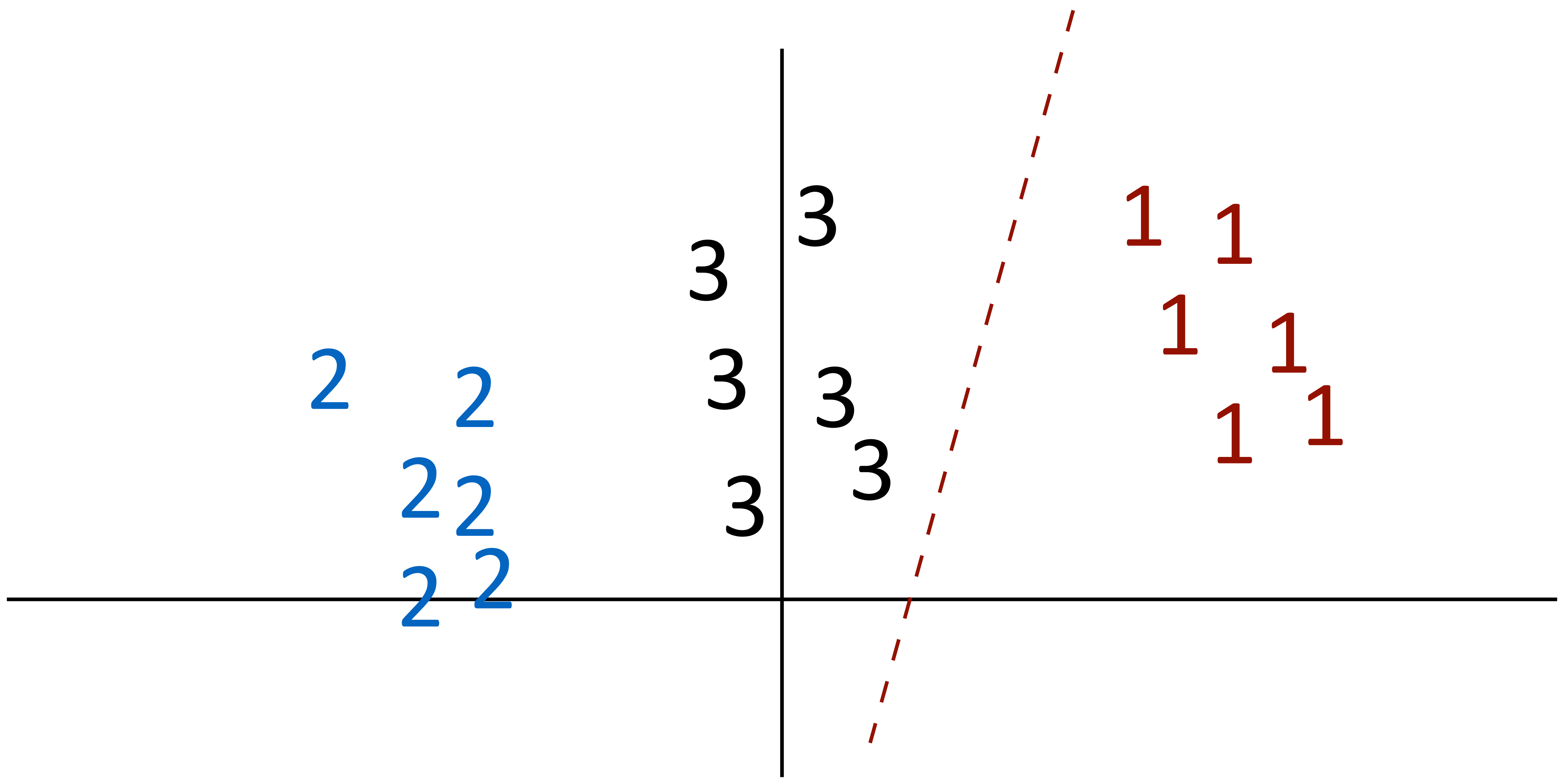

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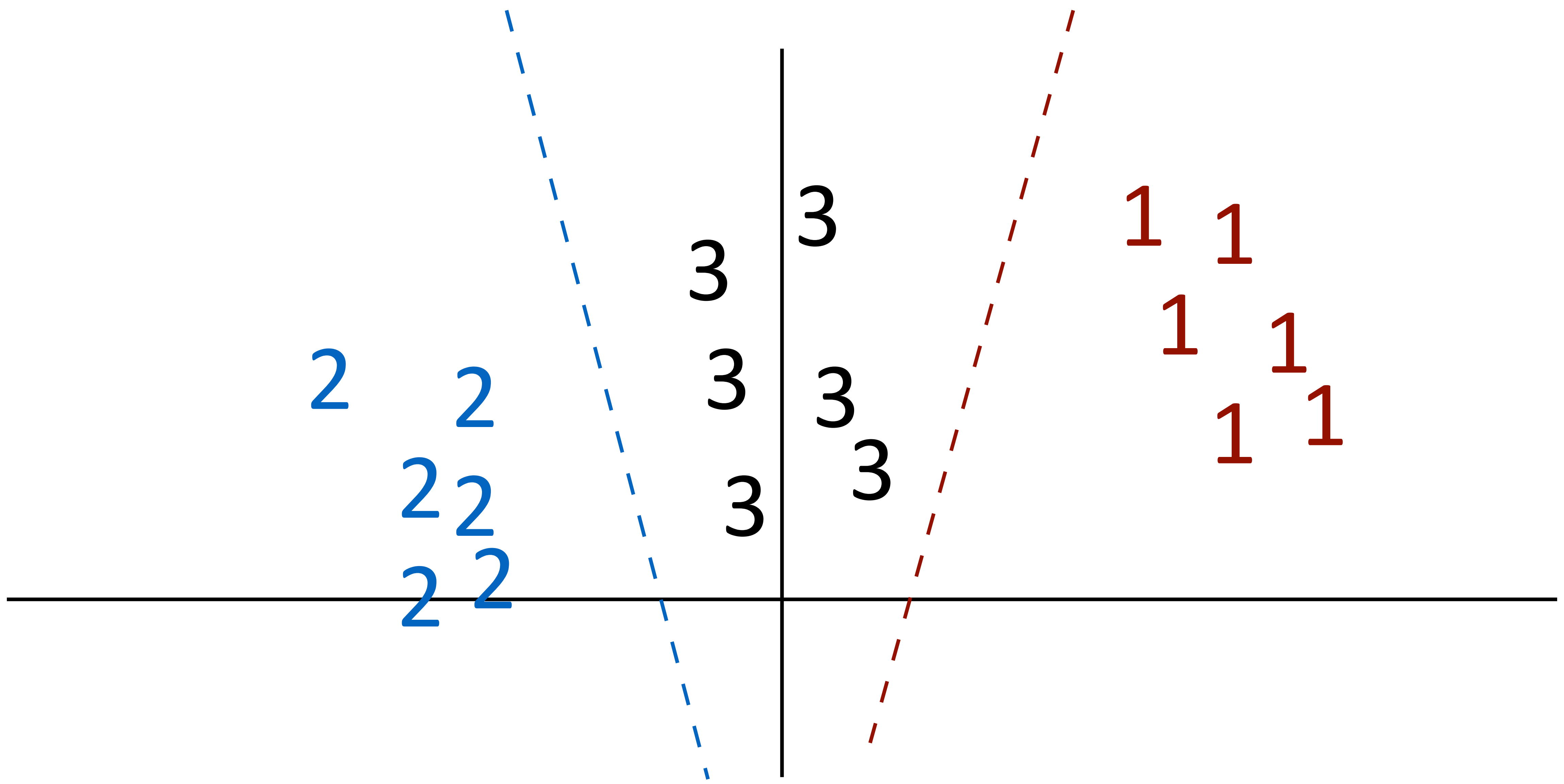

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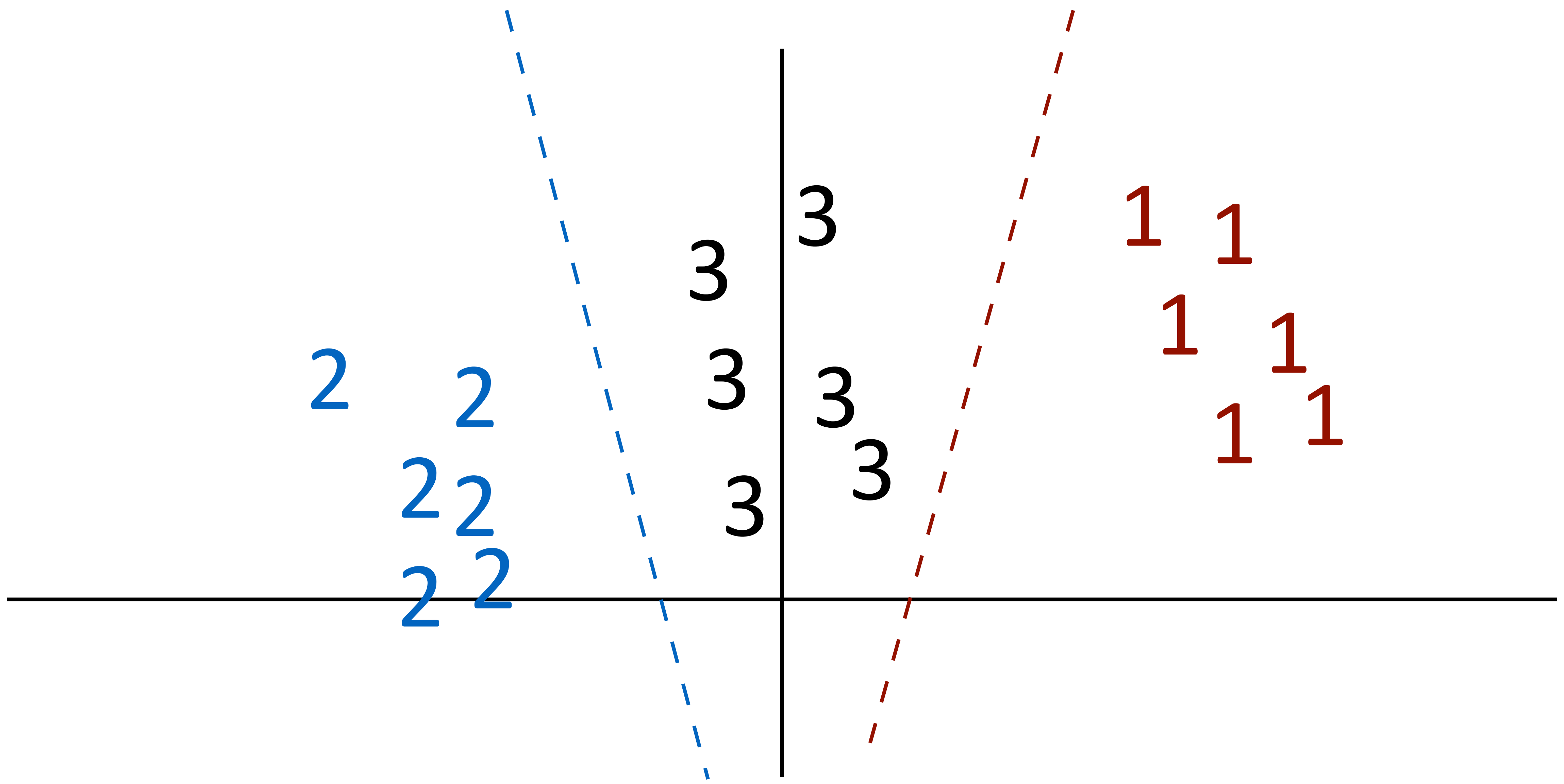

- Can separate 1 from 2+3 and 2 from 1+3 but not 3 from the others (with these features)

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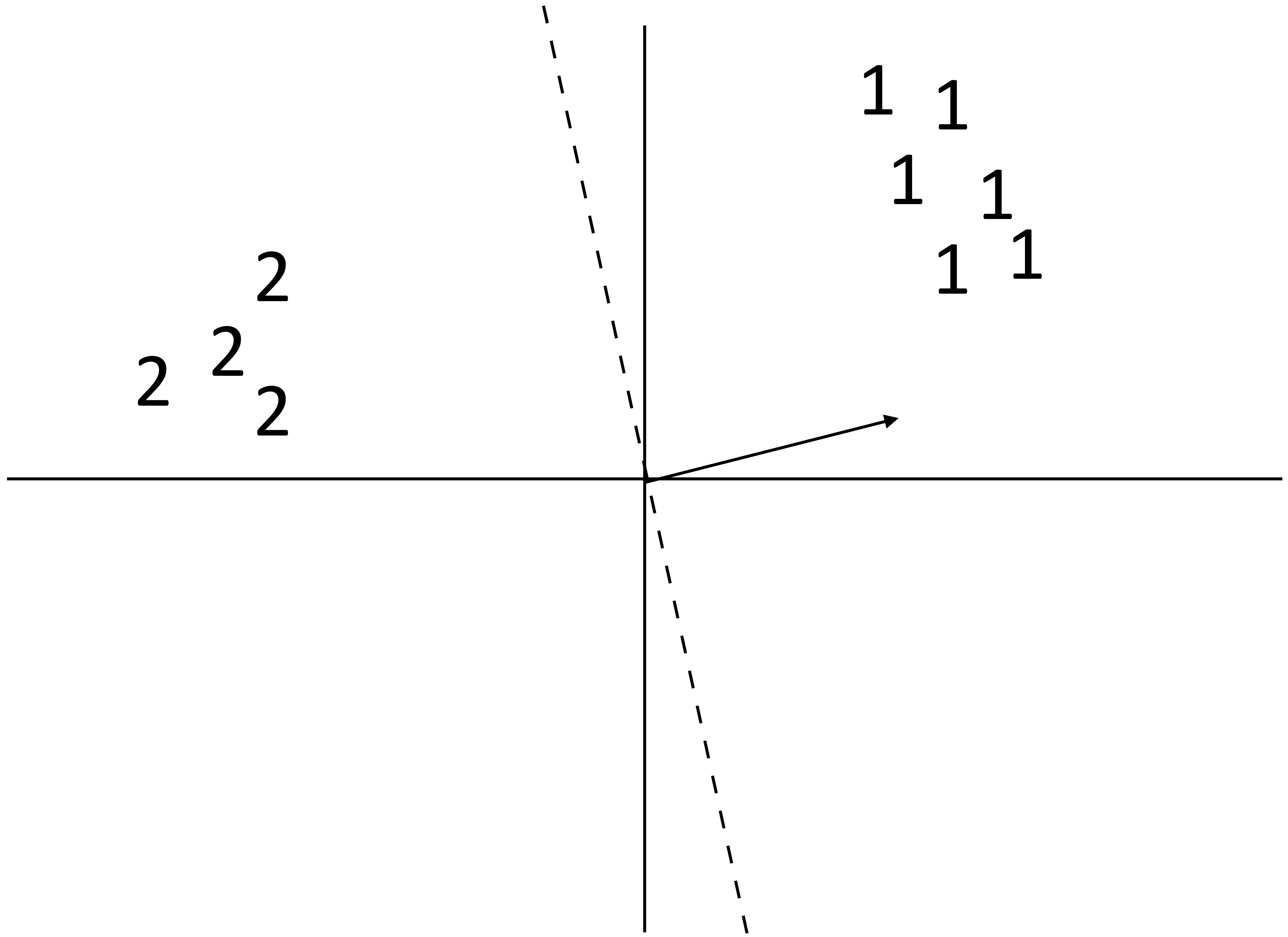

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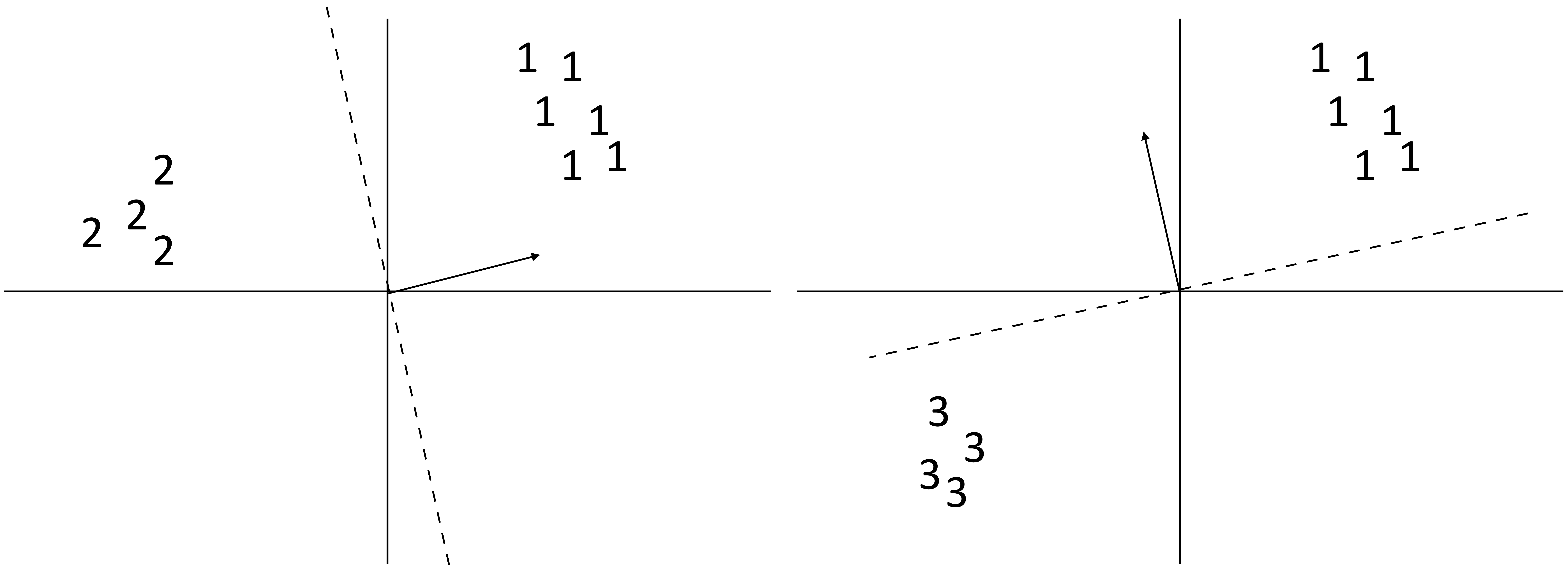
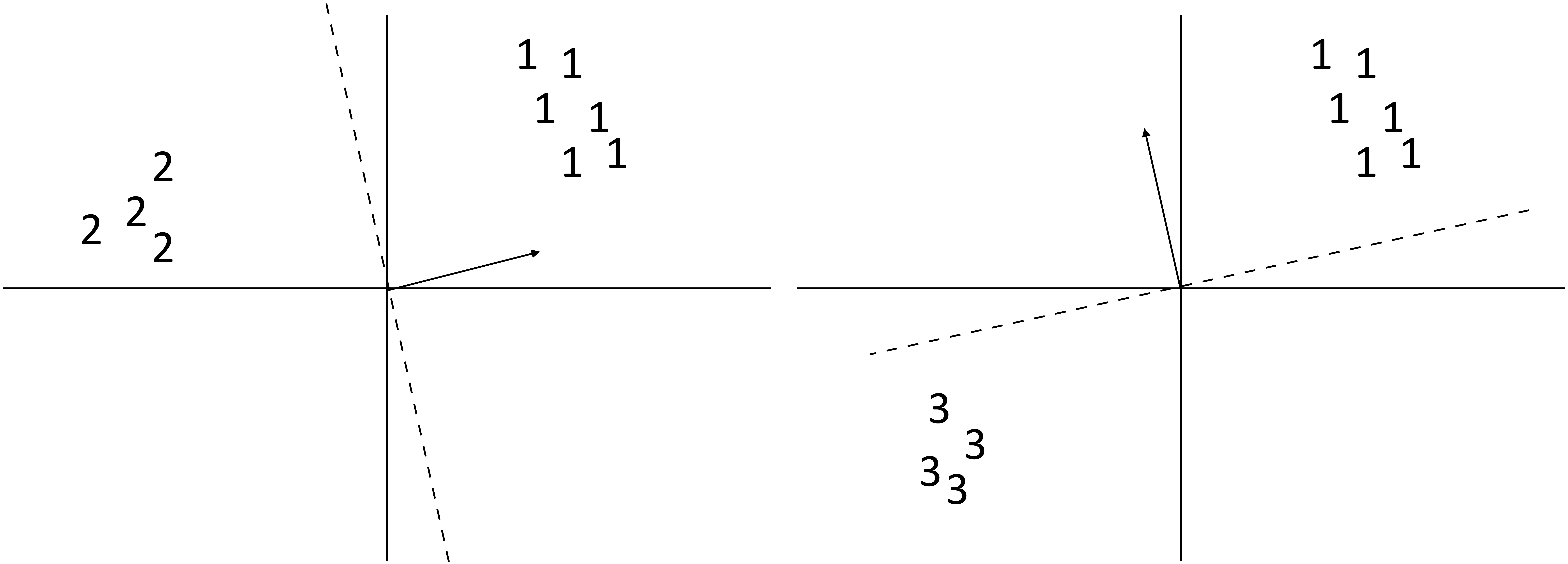


Multiclass Classification

- All-vs-all: train n(n-1)/2 classifiers to differentiate each pair of classes
- Again, how to reconcile?



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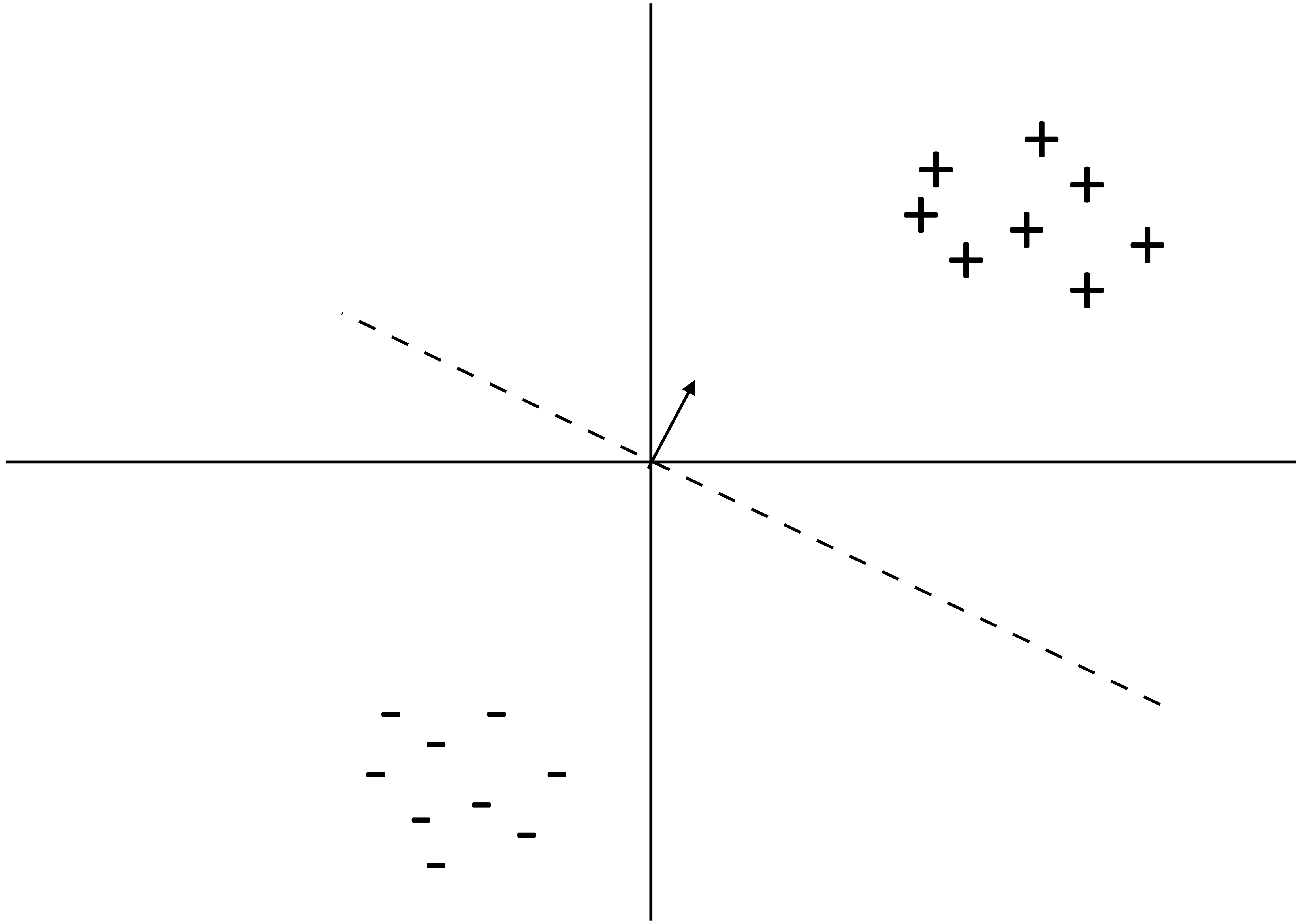

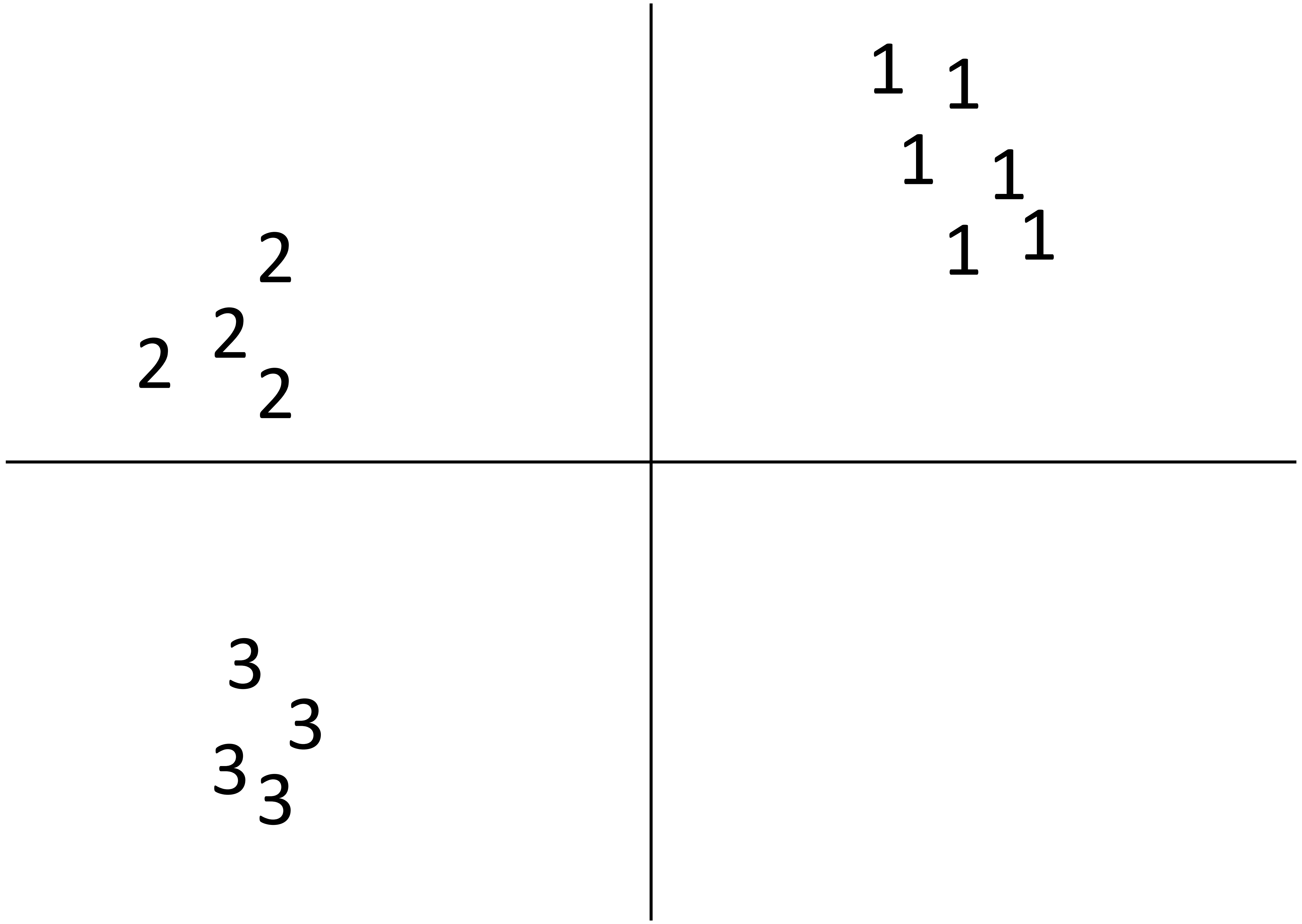

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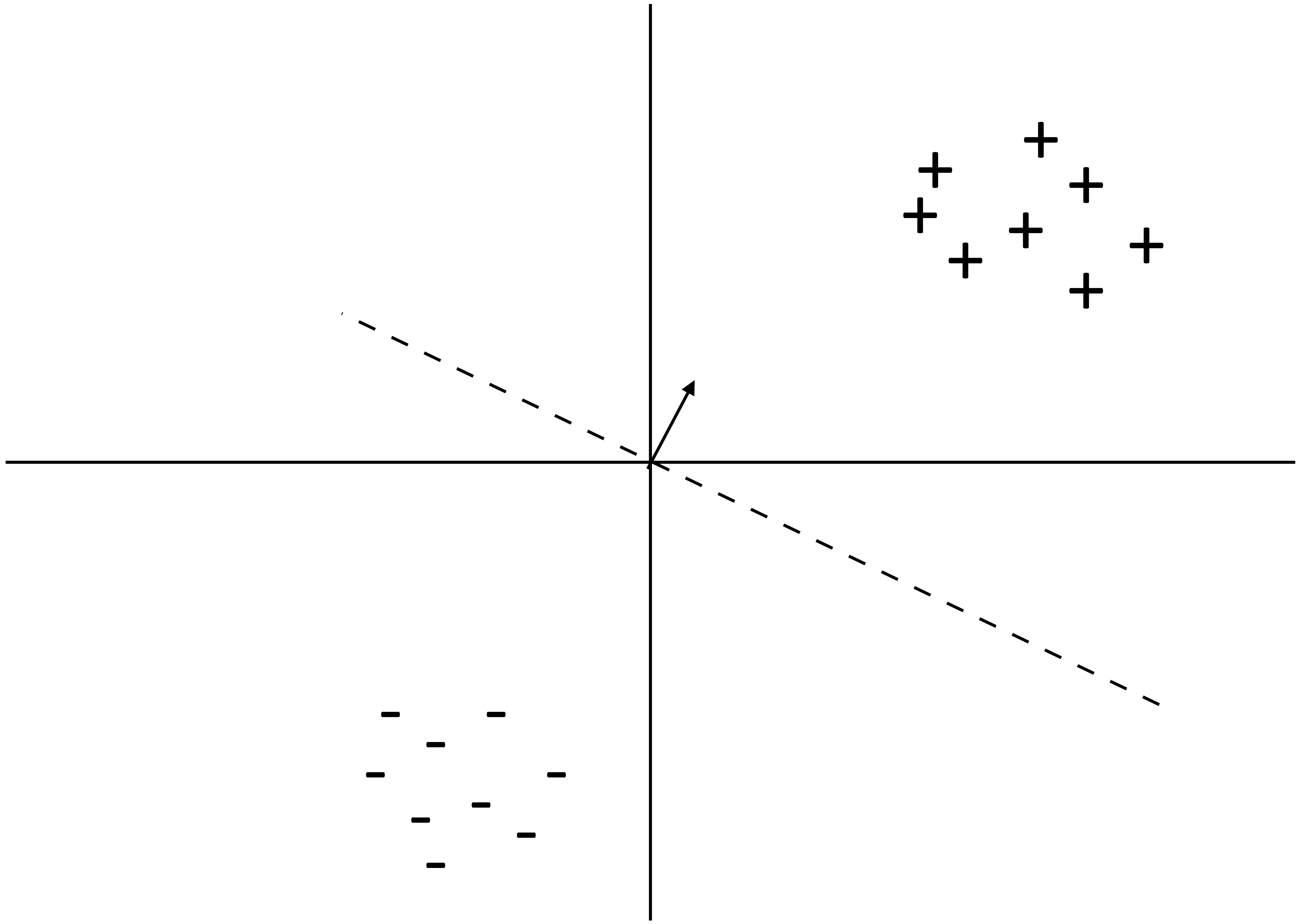
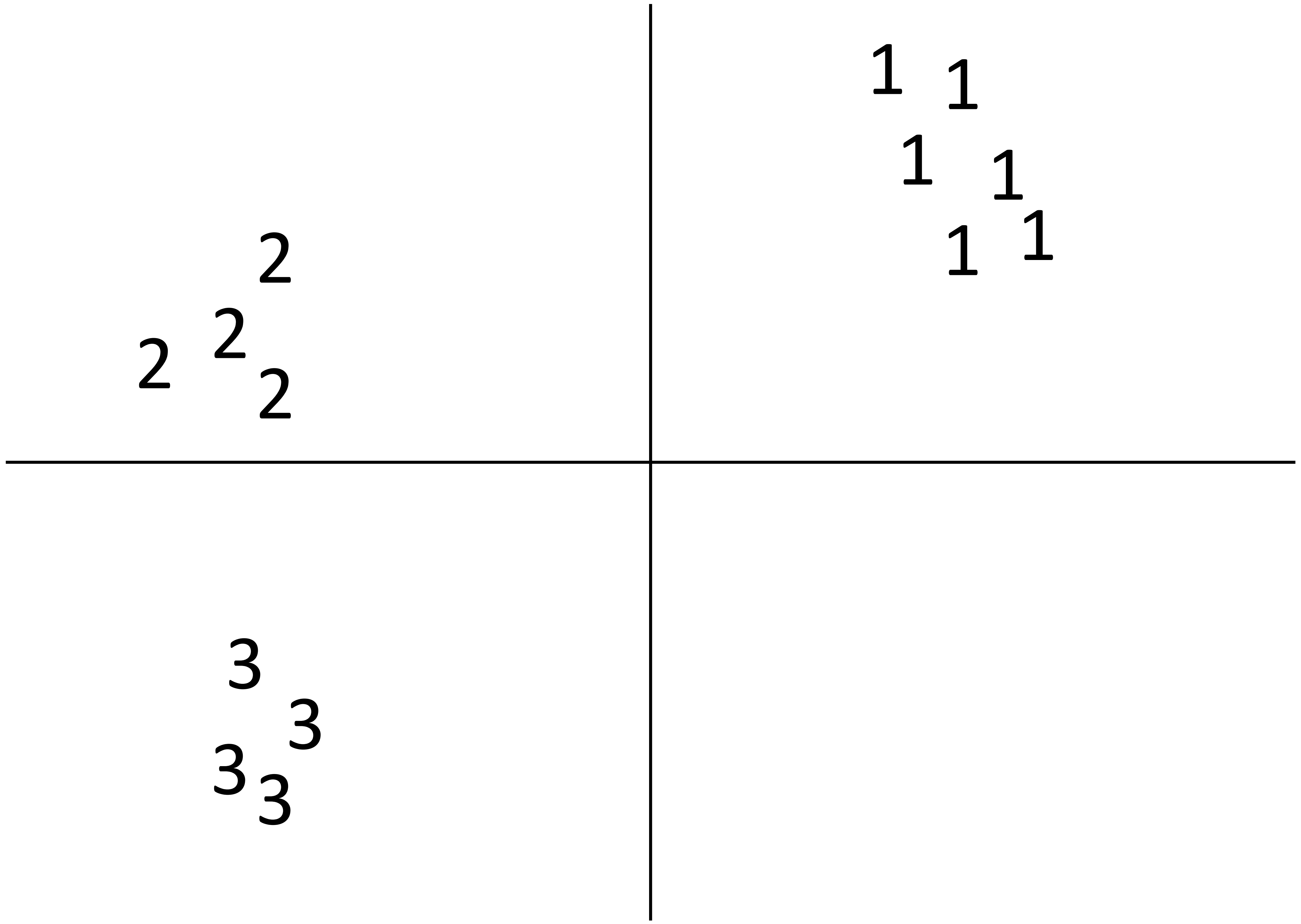


- Multiclass classification: different weights and/or features per class



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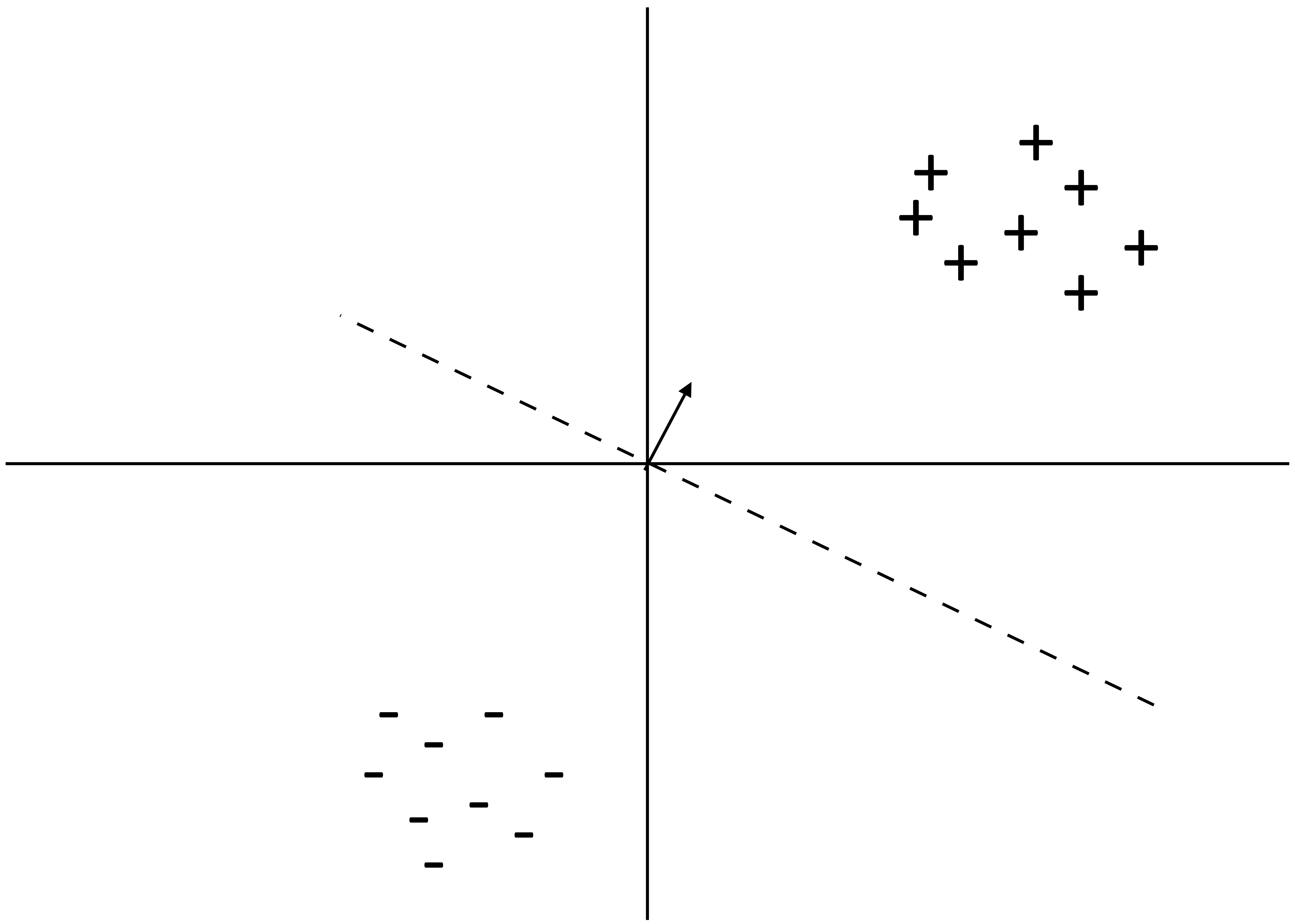

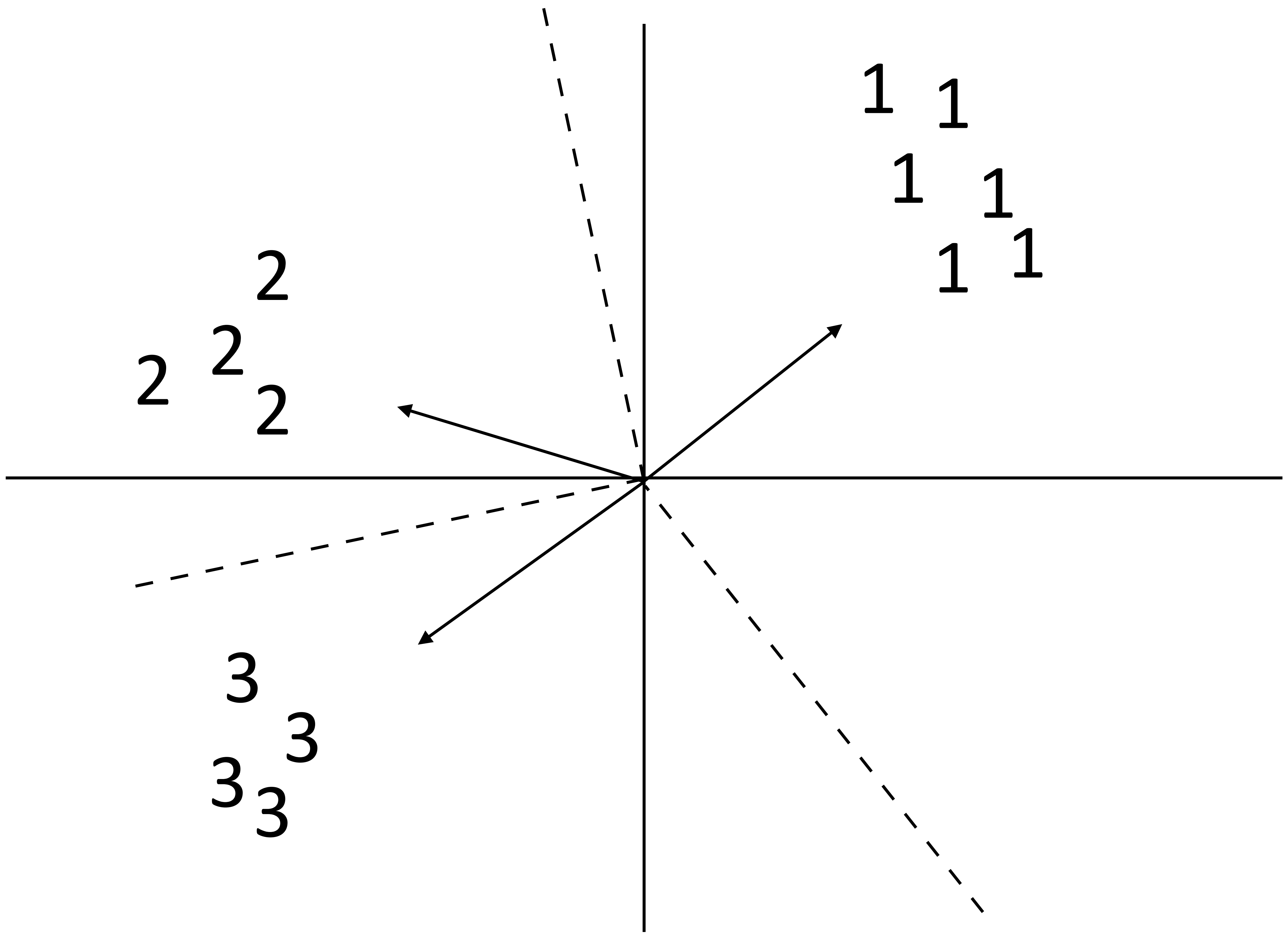

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  - Can also have one weight vector per class: $\text{argmax}_{y \in \mathcal{Y}} w_y^\top f(x)$
  - The single weight vector approach will generalize to structured output spaces, whereas per-class weight vectors won't

Feature Extraction

Block Feature Vectors

- Decision rule: $\mathrm{argmax}_{y \in \mathcal{Y}} w^\top f(x, y)$

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too many drug trials, too few patients

Health

Sports

Science

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feature vector blocks for each label

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I[contains *drug* & label = Health]

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- Equivalent to having three weight vectors in this case

Making Decisions

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$w =$ [+2.1, +2.3, -5, -2.1, -3.8, +5.2, +1.1, -1.7, -1.3]

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too many drug trials, too few patients $\rightarrow$ Health, Sports, Science

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"word drug in Science article" = +1.1

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argmax

Another example: POS tagging

blocks

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the router   blocks   the packets

Another example: POS tagging

the router *blocks* *the packets*

NNS
VBZ
NN
DT
...

Another example: POS tagging

- Classify *blocks* as one of 36 POS tags

the router *blocks* *the packets*

NNS
VBZ
NN
DT
…

Another example: POS tagging

- Classify *blocks* as one of 36 POS tags
- Example *x*: sentence with a word (in this case, *blocks*) highlighted

the router *blocks* *the packets*

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$f(x, y{=}\text{VBZ}) = $ I[curr_word=*blocks* & tag = VBZ],
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not saying that *the* is tagged as VBZ! saying that *the* follows the VBZ word

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- Example x: sentence with a word (in this case, *blocks*) highlighted

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$$
\begin{aligned}
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\end{aligned}
$$

not saying that *the* is tagged as VBZ! saying that *the* follows the VBZ word

- Next two lectures: sequence labeling!

Multiclass Logistic Regression

Multiclass Logistic Regression

$$P_w(y|x) = \frac{\exp\left(w^\top f(x,y)\right)}{\sum_{y' \in \mathcal{Y}} \exp\left(w^\top f(x,y')\right)}$$

Multiclass Logistic Regression

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sum over output
space to normalize

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- Compare to binary:

$$P(y=1|x) = \frac{\exp(w^\top f(x))}{1+\exp(w^\top f(x))}$$

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$$= \sum_{j=1}^{n} \left(w^\top f(x_j, y_j^*) - \log \sum_y \exp(w^\top f(x_j, y)) \right)$$

Training

- Multiclass logistic regression $P_w(y|x) = \frac{\exp\left(w^\top f(x,y)\right)}{\sum_{y' \in \mathcal{Y}} \exp\left(w^\top f(x,y')\right)}$

- Likelihood $\mathcal{L}(x_j, y_j^*) = w^\top f(x_j, y_j^*) - \log \sum_y \exp(w^\top f(x_j, y))$

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gold feature value

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model's expectation of feature value

gold feature value

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too many drug trials, too few patients

y* = Health

$f(x, y =$ Health $) =$ [1, 1, 0, 0, 0, 0, 0, 0, 0]

$f(x, y =$ Sports $) =$ [0, 0, 0, 1, 1, 0, 0, 0, 0]

Training

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[1, 1, 0, 0, 0, 0, 0, 0, 0] — 0.2 [1, 1, 0, 0, 0, 0, 0, 0, 0]

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— 0.3 [0, 0, 0, 0, 0, 0, 1, 1, 0]

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— 0.3 [0, 0, 0, 0, 0, 0, 1, 1, 0]

= [0.8, 0.8, 0, -0.5, -0.5, 0, -0.3, -0.3, 0]

Logistic Regression: Summary

- Model: $P_w(y|x) = \frac{\exp\left(w^\top f(x,y)\right)}{\sum_{y' \in \mathcal{Y}} \exp\left(w^\top f(x,y')\right)}$

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- Inference: $\text{argmax}_y P_w(y|x)$

- Learning: gradient ascent on the discriminative log-likelihood

$$f(x,y^*) - \mathbb{E}_y[f(x,y)] = f(x,y^*) - \sum_y [P_w(y|x) f(x,y)]$$

"towards gold feature value, away from expectation of feature value"

Training

Training

- Are all decisions equally costly?

Training

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too many drug trials, too few patients → Health

→ Sports

→ Science

Training

- Are all decisions equally costly?

too many drug trials, too few patients → Health / Sports / Science

Predicted Sports: bad error

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$$\ell(\text{Sports}, \text{Health}) = 3$$

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Predicted Sports: bad error

Predicted Science: not so bad

- We can define a loss function $\ell(y, y^*)$

$$\ell(\text{Sports}, \text{Health}) = 3$$

$$\ell(\text{Science}, \text{Health}) = 1$$

Multiclass SVM

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Minimize $\lambda \|w\|_2^2 + \sum_{j=1}^{m} \xi_j$

slack variables > 0
iff example is
support vector

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s.t. $\forall j \quad \xi_j \geq 0$

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Correct prediction now has to beat every other class

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The 1 that was here is replaced by a loss function

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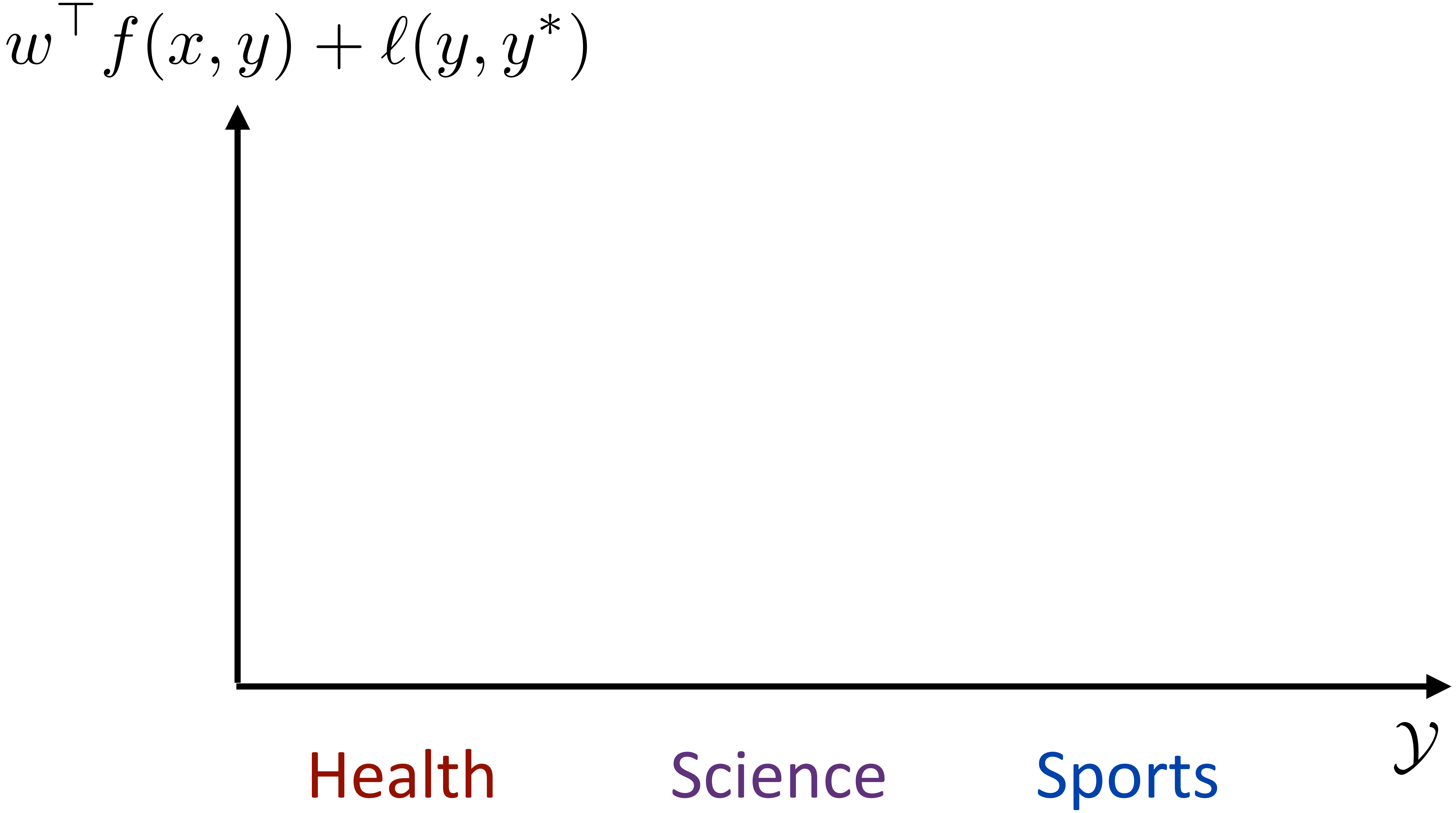

Multiclass SVM

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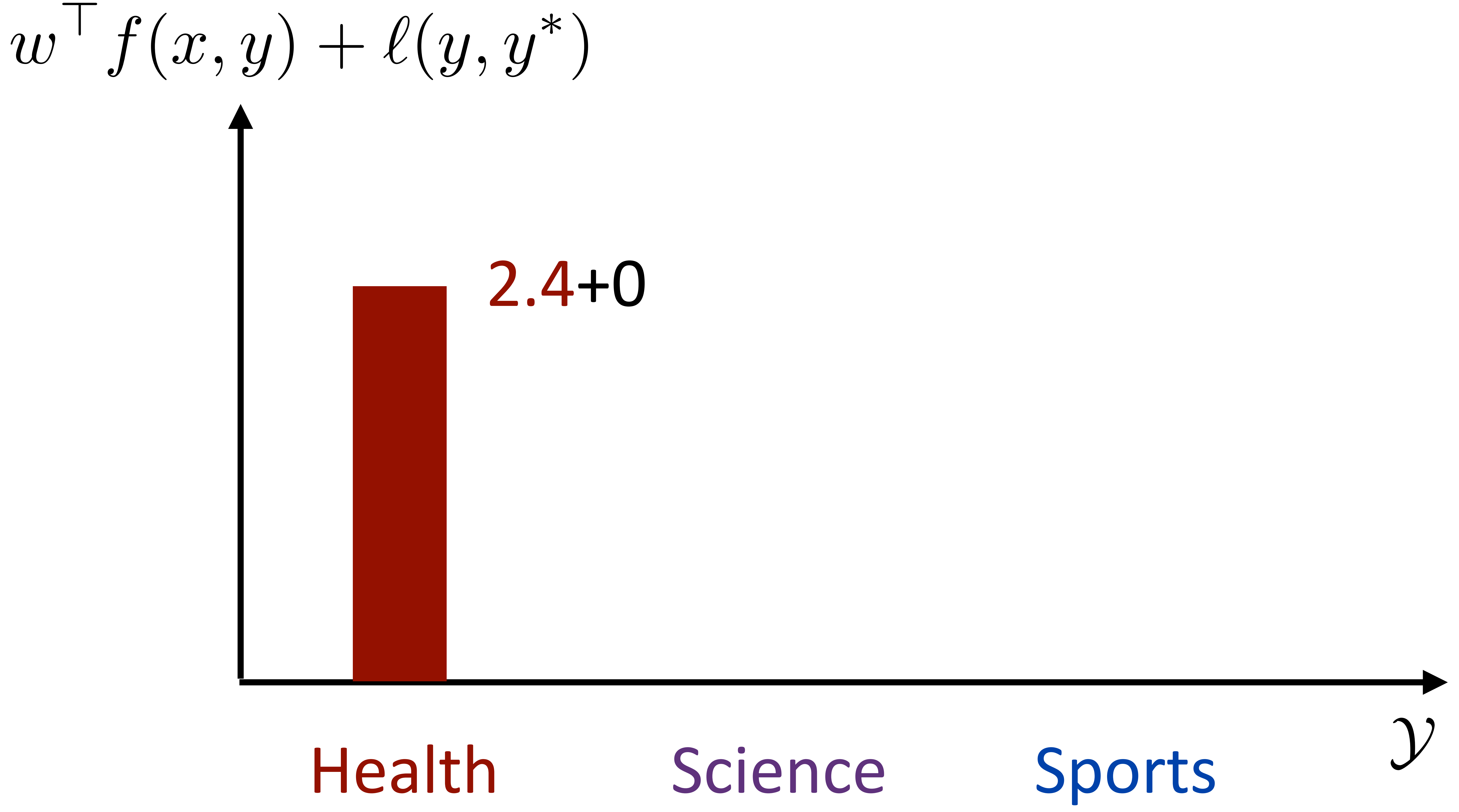

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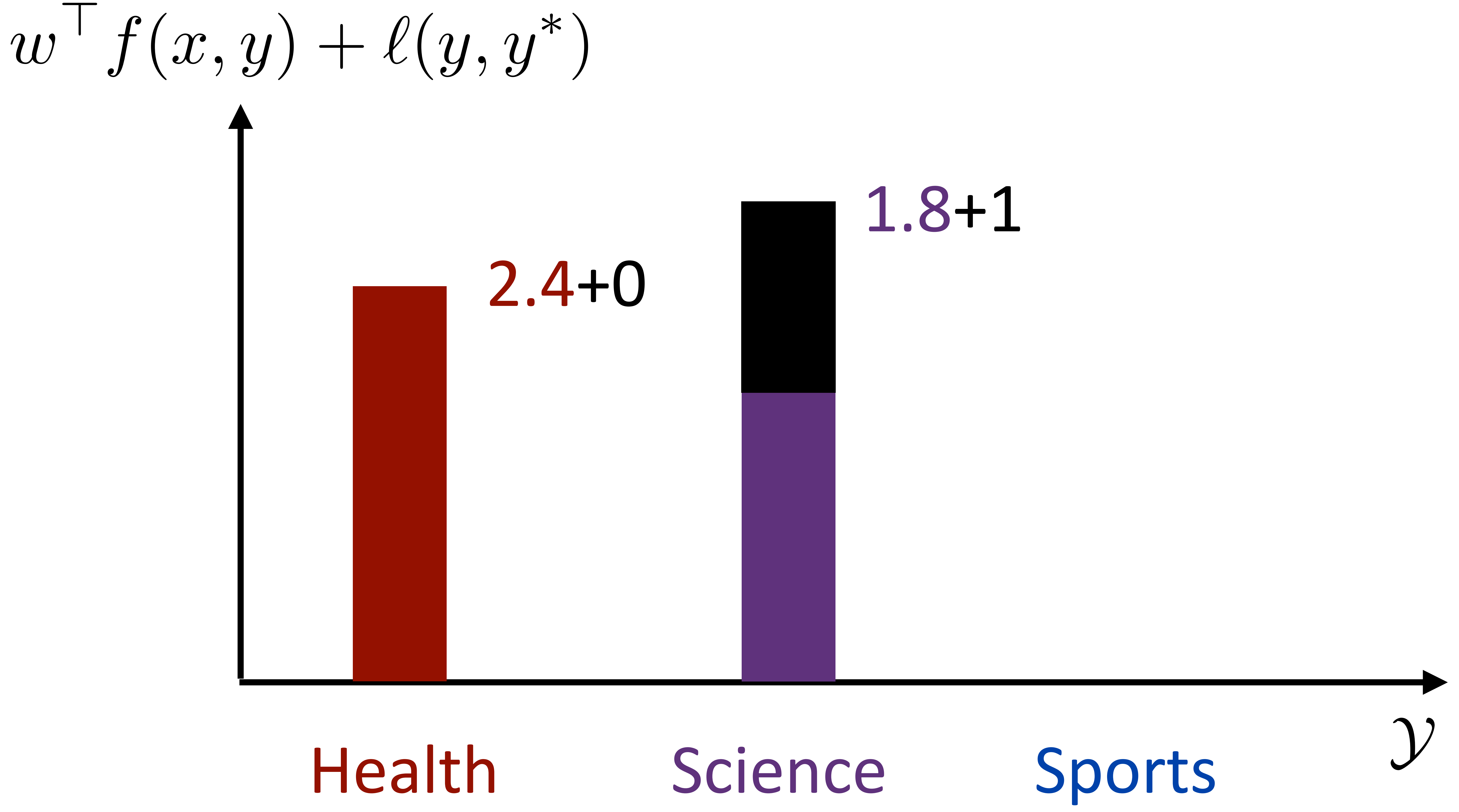

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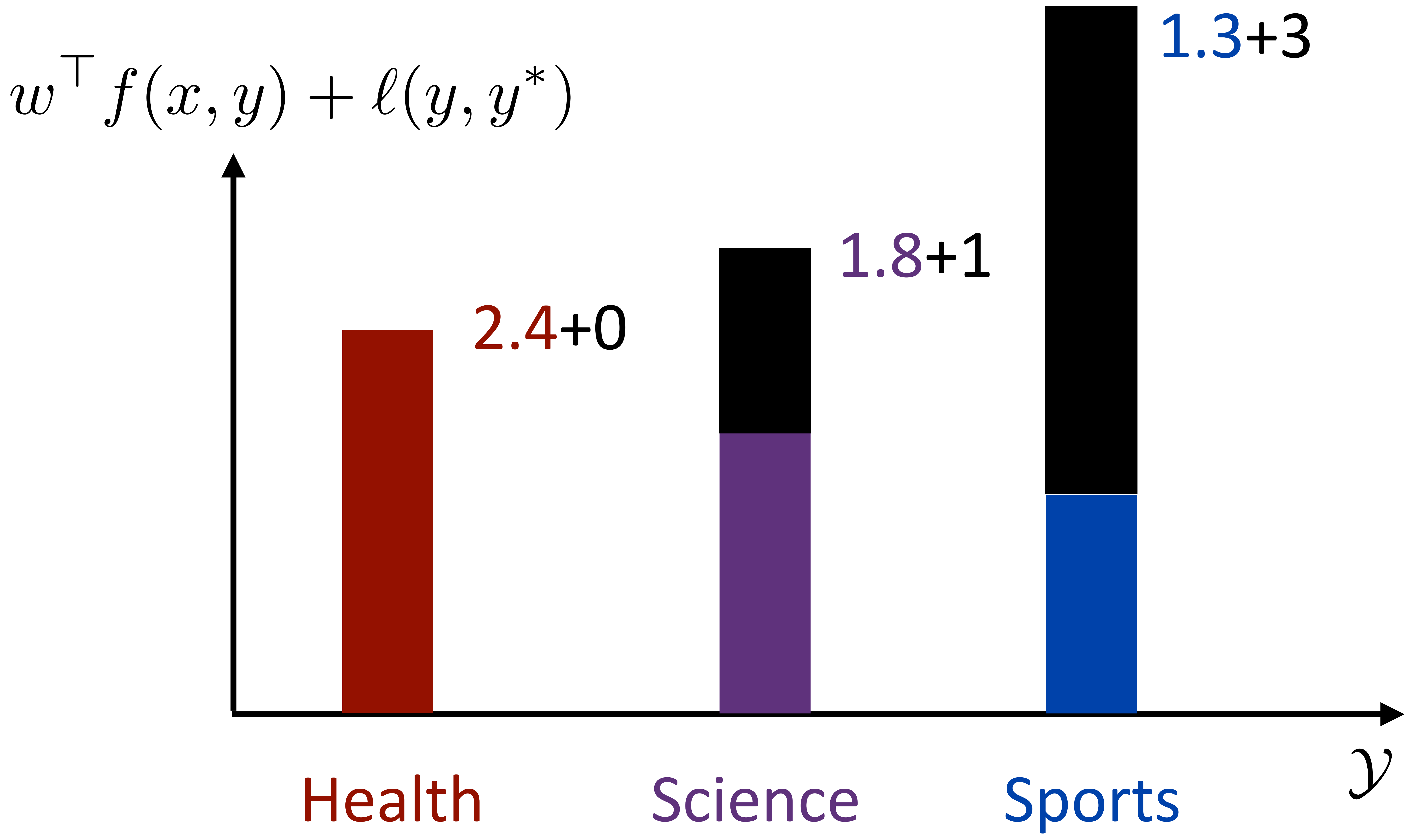

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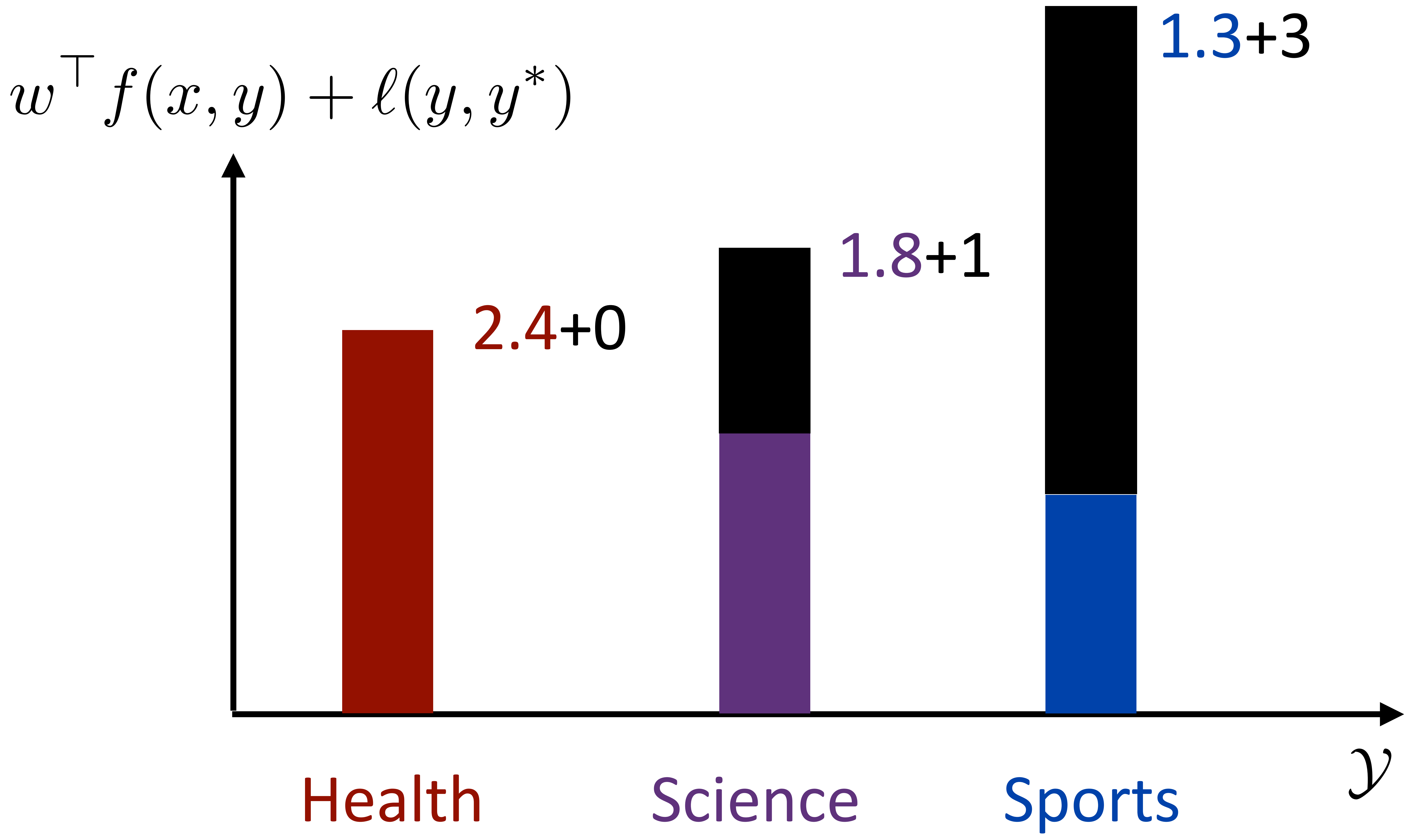

- Does gold beat every label + loss? No!

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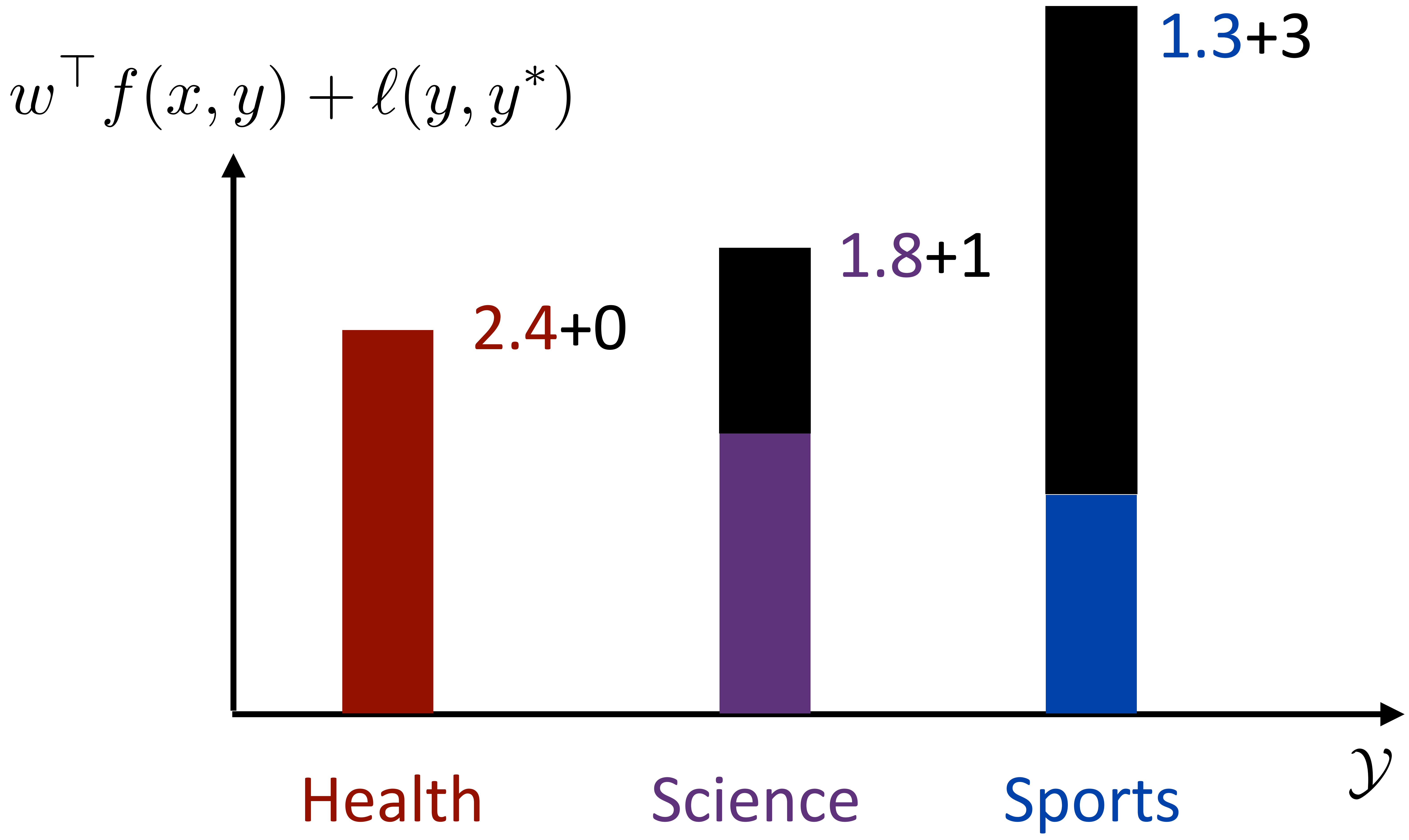

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- Most violated constraint is Sports; what is ξ_j ?

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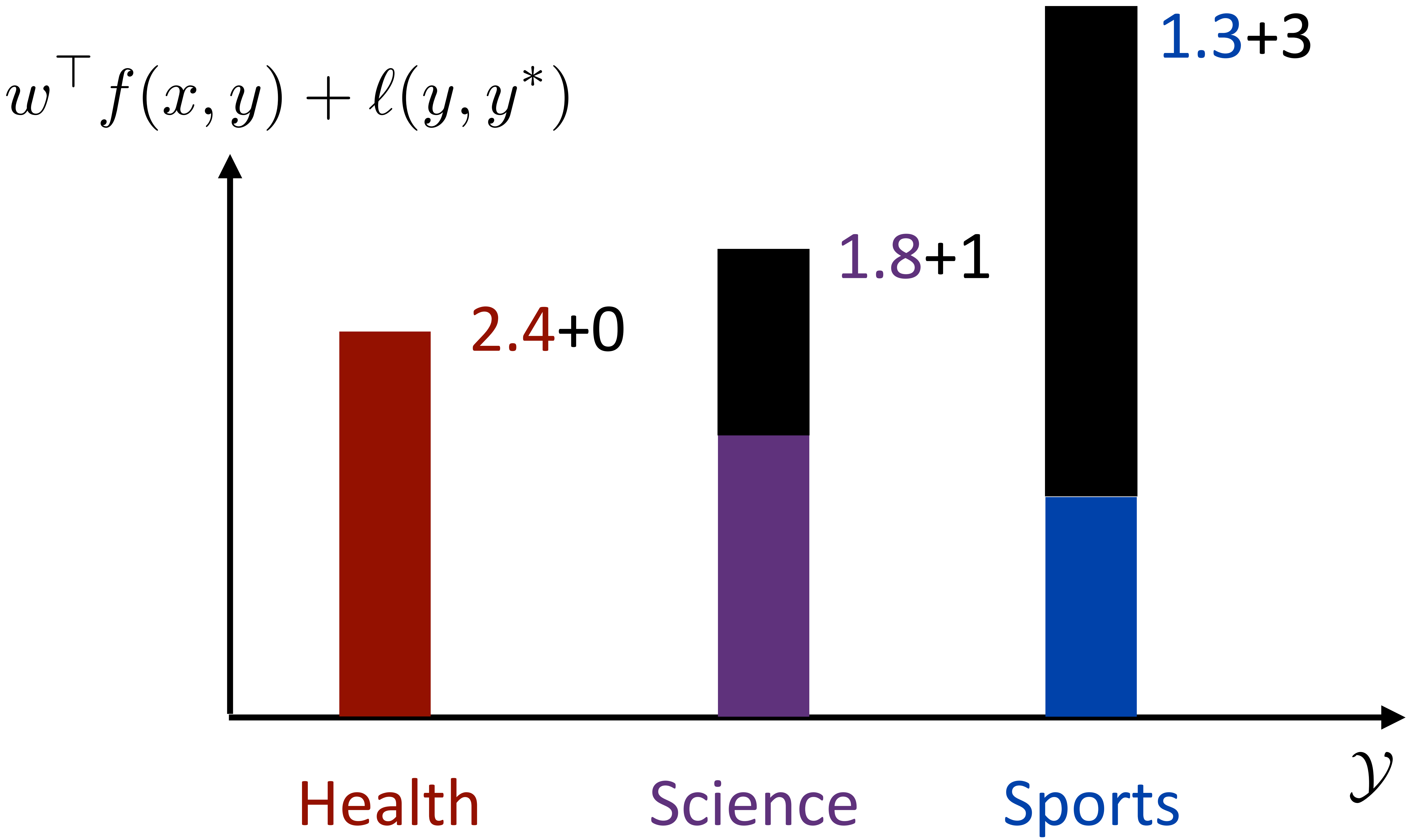

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- ξ_j = 4.3 - 2.4 = 1.9

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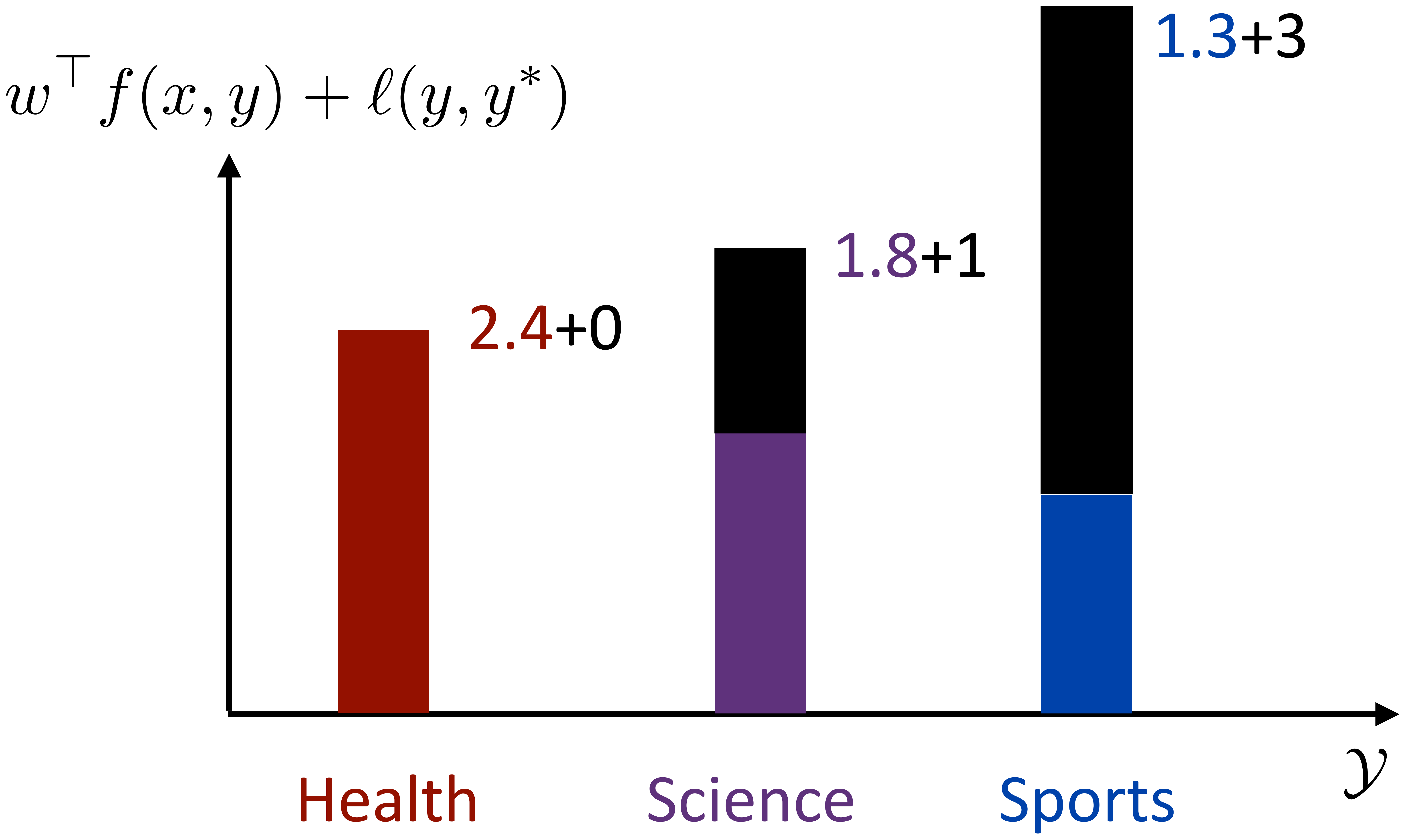

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- Most violated constraint is Sports; what is ξ_j ?
- ξ_j = 4.3 - 2.4 = 1.9
- Perceptron would make no update here

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Minimize $\lambda\|w\|_2^2 + \sum_{j=1}^{m} \xi_j$

s.t. $\forall j \quad \xi_j \geq 0$

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- One slack variable per example, so it's set to be whatever the *most violated constraint* is for that example

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s.t. $\forall j \quad \xi_j \geq 0$

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- One slack variable per example, so it's set to be whatever the *most violated constraint* is for that example

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- Plug in the gold *y* and you get 0, so slack is always nonnegative!

Computing the Subgradient

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- Perceptron-like, but we update away from *loss-augmented* prediction

Putting it Together

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- SVM: max over ys to compute gradient. LR: need to sum over ys

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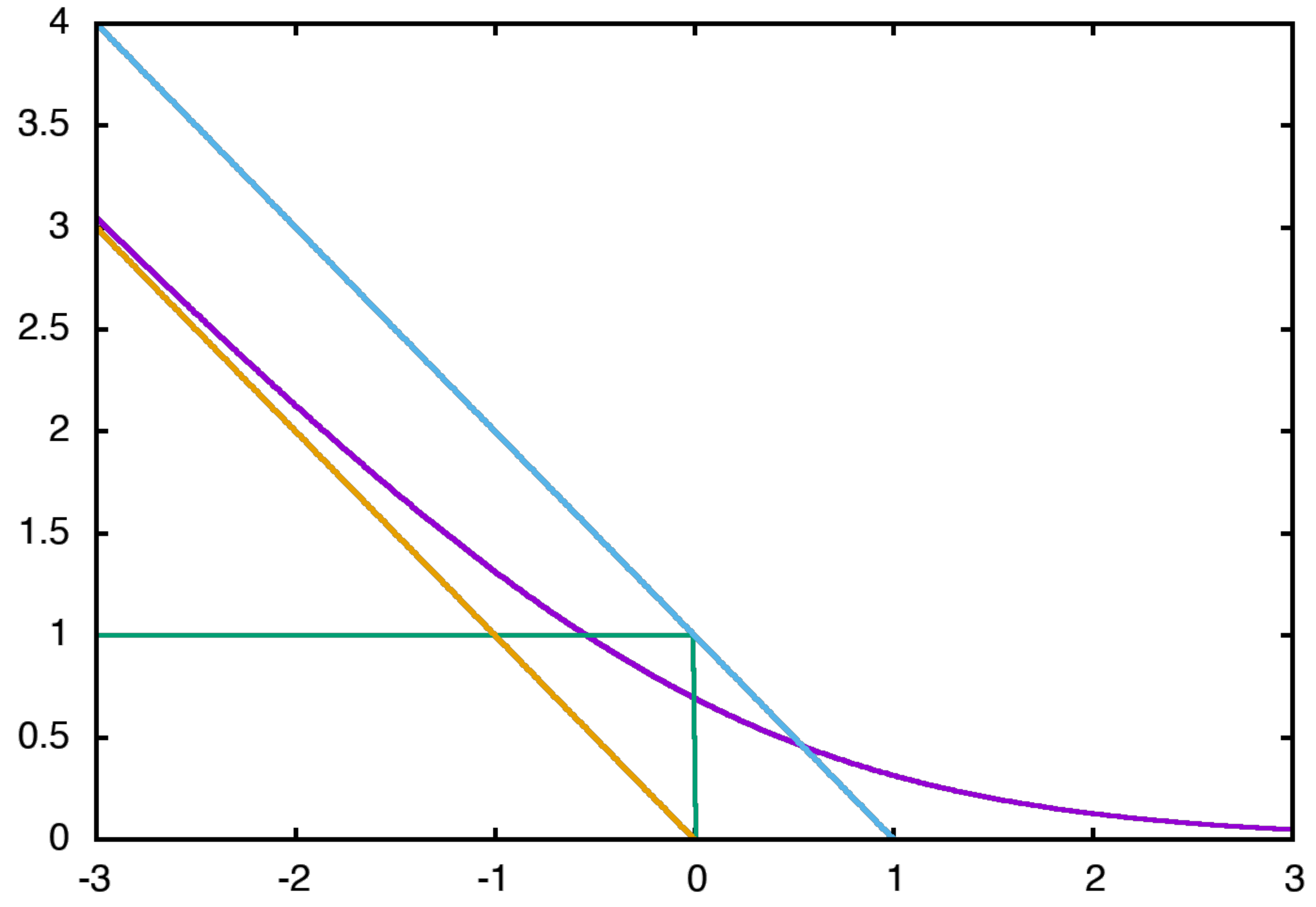

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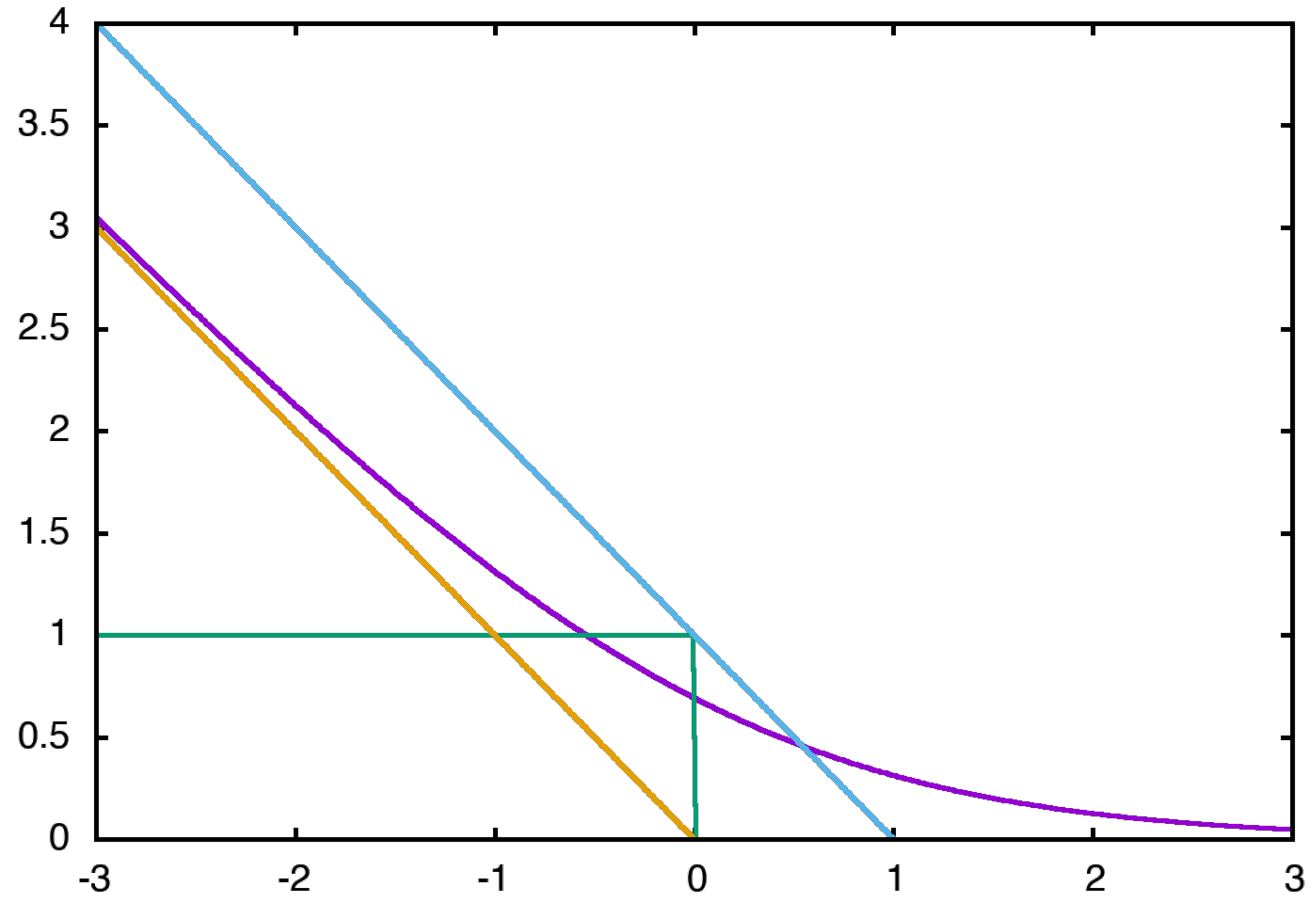

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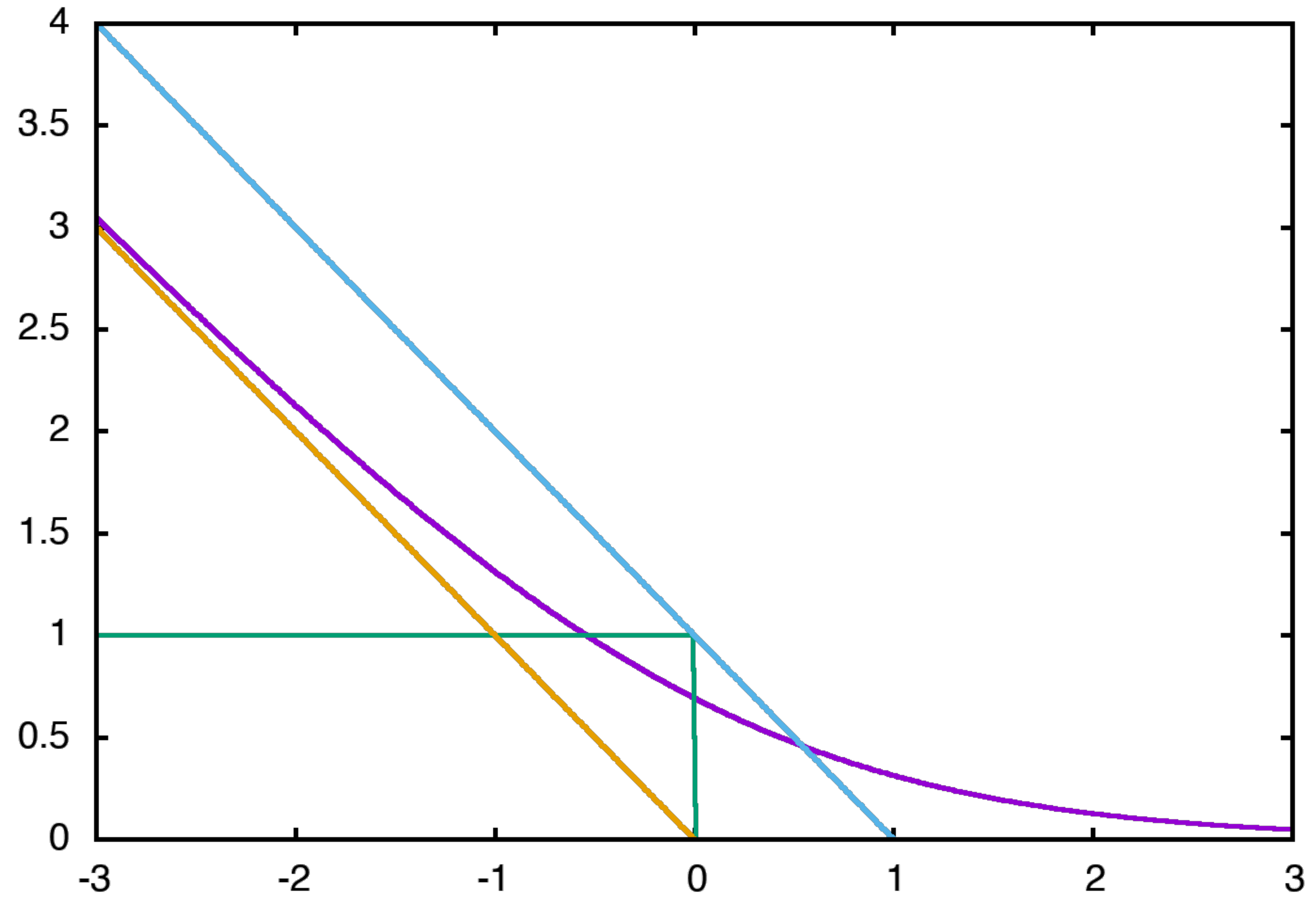

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  - Training: gradient descent?

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- Quasi-Newton methods: L-BFGS, etc. approximate inverse Hessian

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- Other techniques for optimizing deep models — more later!

Duchi et al. (2011)

Summary

- You’ve now seen everything you need to implement multi-class classification models

- Next time: HMMs (POS tagging)

- In 2 lectures: CRFs (NER)