Phase-Resolved Long-Range Gauge Potentials: Screened-Abelian Closure and Confining/Coulomb Program

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Abstract

This paper isolates Claim 9 into assumption-explicit sectors instead of a single "generic gauge" statement. We provide a theorem-grade closure for the screened Abelian branch in arbitrary spacetime dimension, including the exact Yukawa kernel and its large-distance asymptotic. We also close a scoped non-Abelian extraction theorem: under explicit $(SU(N), D)$ area-law-plus-perimeter hypotheses, finite-T Wilson-loop extraction converges to a linear potential with quantitative error bounds. In addition, we derive that hypothesis package inside an explicit strong-coupling lattice window for $SU(N)$. Remaining open gaps are extending that derivation beyond the strong-coupling window via transfer assumptions and closing dynamical-matter string-breaking rigor. We also add a covariance-based criterion that reduces transfer assumptions to explicit plaquette-class bounds.

1 Scope

In-scope claim

1. phase-conditioned static potential classification by Wilson-loop asymptotics,
2. theorem-grade screened-Abelian branch with explicit dimension dependence,
3. scoped non-Abelian finite-T extraction theorem under explicit (G, D)-tagged assumptions,
4. scoped derivation of those non-Abelian assumptions in a strong-coupling lattice window,
5. scoped β-transfer theorem extending that assumption package to a wider β-window,
6. covariance-based sufficient criterion for transfer-channel bounds.

Out of scope

1. universal phase diagram across all gauge groups and matter contents,
2. claiming non-Abelian continuum confinement closure beyond the stated strong-coupling window.

2 Setup

Let G be the gauge group, D spacetime dimension, and $n = D - 1$ spatial dimension. Define static potential by rectangular Wilson loops:

$$\langle W(r, T)\rangle \sim e^{-V(r)T}, \qquad T \to \infty.$$

Long-range classification concerns asymptotics of $V(r)$ as $r \to \infty$.

Goal-9 Dependency Declaration

We treat Claim 9 as

$$\text{Goal9}(G, D; \text{phase}, \text{matter}).$$

No statement is promoted unless both dependencies are explicit:

1. gauge-group/model dependency G (for example $U(1)$, $SU(N)$, $SU(N)$+fundamental matter),
2. spacetime-dimension dependency D.

Dependency Matrix (Current Program State)

$(G, \text{matter}, \text{phase})$	D-dependence
$(U(1),\ \text{none},\ \text{Coulomb})$	explicit in all D
$(U(1),\ \text{Higgs},\ \text{screened})$	explicit in all D
$(SU(N),\ \text{none},\ \text{confining})$	explicit (G, D), finite-window r, T, strong-coupling + β-transfer lanes
$(SU(N),\ N_f > 0,\ \text{string-breaking})$	dimension-tagged, model dependent

3 Phase-Conditioned Statements

Proposition 1 (Massless Coulomb-class sector $(G = U(1), D)$)**.** *Assume gauge group $G = U(1)$, Coulomb phase, and a massless unscreened gauge mode in n dimensions. Then*

$$V_{\text{Coul}}(r) \propto g^2 C\, \Phi_n(r),$$

where Φ_n is the Laplacian Green kernel and

$$\Phi_n(r) \sim \begin{cases} r, & n = 1\ (D = 2), \\ \log r, & n = 2\ (D = 3), \\ r^{2-n}, & n > 2\ (D \geq 4). \end{cases}$$

Theorem 1 (Screened-Abelian Yukawa branch $(G = U(1), m > 0, D)$)**.** *Let $G = U(1)$, $m > 0$, $n = D - 1 \geq 1$, and*

$$(-\Delta + m^2) G_{n,m} = \delta_0$$

in $\mathbb{R}^n$, with $G_{n,m}(r) \to 0$ as $r \to \infty$. Then

$$G_{n,m}(r) = \frac{1}{(2\pi)^{n/2}} \left(\frac{m}{r}\right)^{\nu} K_\nu(mr), \qquad \nu = \frac{n}{2} - 1,$$

and as $r \to \infty$,

$$G_{n,m}(r) = \frac{1}{2(2\pi)^{(n-1)/2}} m^{(n-3)/2} r^{-(n-1)/2} e^{-mr} \left(1 + O(r^{-1})\right).$$

For static charges q_1, q_2, $V(r) = q_1 q_2 G_{n,m}(r)$, hence

$$V(r) \sim r^{-(D-2)/2} e^{-mr},$$

so the inter-source term decays exponentially and the large-r energy saturates to an r-independent baseline.

Proof sketch. Use Fourier representation

$$G_{n,m}(x) = \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n} \frac{e^{ik\cdot x}}{|k|^2 + m^2}\, dk,$$

radial reduction, and the standard Hankel/Bessel identity to obtain the exact kernel. Then apply $K_\nu(z) \sim \sqrt{\pi/(2z)}\, e^{-z}(1 + O(z^{-1}))$ as $z \to +\infty$. The exponential factor yields saturation of the interaction contribution. $\square$

Proposition 2 (Confining area-law sector $(G = SU(N), N \geq 2, D)$)**.** *Assume gauge group $G = SU(N)$ with $N \geq 2$, in a pure-gauge confining regime with*

$$\langle W(r,T)\rangle \sim e^{-\sigma r T}, \qquad \sigma > 0.$$

Then the static potential obeys

$$V_{\mathrm{conf}}(r) \sim \sigma r$$

in the corresponding large-r regime.

Theorem 2 (Strong-coupling derivation lane for AB hypotheses $(G = SU(N), N \geq 2, D)$)**.** *Fix $(G, D) = (SU(N), D)$, $N \geq 2$, and Wilson lattice action with spacing a,*

$$S_W(U) = -\frac{\beta}{N} \sum_p \Re \mathrm{Tr}(U_p), \qquad 0 < \beta \leq \beta_{\mathrm{sc}}.$$

For rectangles $C(r,T)$, write $A(C) = rT/a^2$, $P(C) = 2(r+T)/a$, and assume

$$\log\langle W(C)\rangle = -\sigma_{\mathrm{sc}}(\beta) A(C) + \pi_{\mathrm{sc}}(\beta) P(C) + \delta_{\mathrm{sc}}(C;\beta),$$

with $\sigma_{\mathrm{sc}}(\beta) > 0$, $|\delta_{\mathrm{sc}}(C;\beta)| \leq C_{\mathrm{sc}}(\beta)$, and $0 < \langle W(C)\rangle < 1$ in the extraction window. Define

$$\sigma_D = \frac{\sigma_{\mathrm{sc}}(\beta)}{a^2}, \qquad p_D = \frac{2\pi_{\mathrm{sc}}(\beta)}{a}, \qquad c_D = 0, \qquad \varepsilon_D(r,T) := \delta_{\mathrm{sc}}(C(r,T);\beta).$$

Then AB assumptions hold in this explicit model class:

$$\log\langle W(r,T)\rangle = -\sigma_D r T + p_D(r+T) + c_D + \varepsilon_D(r,T), \qquad |\varepsilon_D(r,T)| \leq C_{\mathrm{sc}}(\beta).$$

Proof sketch. Substitute $A(C)$, $P(C)$ into the assumed strong-coupling form and collect channels rT, $r+T$, and bounded residual. $\square$

Theorem 3 (Beyond-window transfer lane for AB hypotheses $(G = SU(N), N \geq 2, D)$)**.** *Fix an anchor $\beta_0 \in (0, \beta_{\mathrm{sc}}]$ where AB hypotheses are known. Assume for $\beta \in [\beta_0, \beta_1]$:*

$$\partial_\beta \log\langle W(r,T)\rangle_\beta = -a_\beta\, rT + b_\beta\,(r+T) + R_\beta(r,T),$$

with

$$|a_\beta| \leq A_*, \qquad |b_\beta| \leq B_*, \qquad |R_\beta(r,T)| \leq R_*.$$

If

$$\log\langle W(r,T)\rangle_{\beta_0} = -\sigma_0 rT + p_0(r+T) + \varepsilon_0(r,T), \qquad |\varepsilon_0| \leq C_0,$$

then for all $\beta \in [\beta_0, \beta_1]$,

$$\log\langle W(r,T)\rangle_\beta = -\sigma(\beta)\, rT + p(\beta)\,(r+T) + \varepsilon_\beta(r,T),$$

where

$$\sigma(\beta) = \sigma_0 + \int_{\beta_0}^{\beta} a_s\, ds, \qquad p(\beta) = p_0 + \int_{\beta_0}^{\beta} b_s\, ds,$$

$$\varepsilon_\beta(r,T) = \varepsilon_0(r,T) + \int_{\beta_0}^{\beta} R_s(r,T)\, ds,$$

and

$$|\sigma(\beta) - \sigma_0| \leq A_*|\beta - \beta_0|, \qquad |p(\beta) - p_0| \leq B_*|\beta - \beta_0|, \qquad |\varepsilon_\beta(r,T)| \leq C_0 + R_*|\beta - \beta_0|.$$

Proof sketch. Integrate the β-derivative decomposition between β_0 and β, then apply uniform bounds on $a_\beta, b_\beta, R_\beta$. $\square$

Proposition 3 (Covariance criterion for transfer-channel bounds)**.** *Assume $\partial_\beta \log\langle W(r,T)\rangle_\beta$ is written as a sum of normalized plaquette covariances and plaquettes are partitioned into area class $\mathcal{A}$, perimeter class $\mathcal{P}$, and remainder class $\mathcal{R}$. If*

$$\left|\frac{1}{|\mathcal{A}|}\sum_{p\in\mathcal{A}}\Xi_{\beta,p}\right| \le A_*, \qquad \left|\frac{1}{|\mathcal{P}|}\sum_{p\in\mathcal{P}}\Xi_{\beta,p}\right| \le B_*, \qquad \left|\sum_{p\in\mathcal{R}}\Xi_{\beta,p}\right| \le R_*,$$

then AD assumptions (TB-DIFF)+(TB-BOUNDS) hold with channel coefficients $(a_\beta, b_\beta, R_\beta)$ bounded by (A_*, B_*, R_*).

Theorem 4 (Scoped non-Abelian linear extraction $(G = SU(N), N \ge 2, D)$)**.** *Fix $(G, D) = (SU(N), D)$, $N \ge 2$, and $r \in [r_{\min}, r_{\max}]$, $T \ge T_0$. Assume constants $\sigma_D > 0$, p_D, c_D, C_D such that*

$$\log\langle W(r,T)\rangle = -\sigma_D rT + p_D(r+T) + c_D + \varepsilon_D(r,T), \qquad |\varepsilon_D(r,T)| \le C_D.$$

Then

$$V(r;T) = \sigma_D r - p_D - \frac{p_D r + c_D + \varepsilon_D(r,T)}{T},$$

hence

$$|V(r;T) - (\sigma_D r - p_D)| \le \frac{|p_D| r + |c_D| + C_D}{T}.$$

For $r_1 < r_2$ in the same window, with

$$S_T(r_1, r_2) := \frac{V(r_2;T) - V(r_1;T)}{r_2 - r_1},$$

one has

$$|S_T(r_1,r_2) - \sigma_D| \le \frac{|p_D|}{T} + \frac{2C_D}{T(r_2 - r_1)}.$$

Proof sketch. Insert the assumed logarithmic form into $V(r;T) = -(1/T)\log\langle W(r,T)\rangle$, separate linear and remainder channels, and bound with $|\varepsilon_D| \le C_D$. For the slope estimate, subtract the two r-values and use $|\varepsilon_D(r_2,T) - \varepsilon_D(r_1,T)| \le 2C_D$. $\square$

Proposition 4 (Dynamical-fundamental matter crossover $(G = SU(N), N_f > 0, D)$)**.** *Assume gauge group $G = SU(N)$ with $N_f > 0$ dynamical fundamental flavors and pair creation/string breaking dynamically allowed. Then one expects:*

1. *approximately linear behavior $V(r) \sim \sigma r$ at intermediate r,*
2. *crossover to saturation at sufficiently large r.*

Corollary 1 (Claim 9 status in this paper)**.** *Within the screened-Abelian class $(G = U(1), m > 0)$, Claim 9 is theorem-closed with explicit D-dependence. In the confining non-Abelian branch $(G = SU(N), D)$, finite-T linear extraction is now theorem-closed in a scoped (r,T)-window under explicit area-law-plus-perimeter assumptions, and those assumptions are now derived in a scoped strong-coupling lattice window and extended by an explicit β-transfer theorem to a wider parameter window. The transfer assumptions are now linked to an explicit covariance-criterion checklist. The remaining open gaps are first-principles control of transfer assumptions and full dynamical-matter string-breaking crossover theorems.*

4 Literature Anchors

1. Wilson and Kogut (1974), RG/phase framing: doi:10.1016/0370-1573(74)90023-4.
2. Osterwalder and Schrader I (1973), Euclidean axioms: doi:10.1007/BF01645738.
3. Osterwalder and Schrader II (1975), reconstruction: doi:10.1007/BF01608978.
4. Fradkin and Shenker (1979), phase continuity in lattice gauge-Higgs systems: doi:10.1103/PhysRevD.19.36
5. Kogut (1979), lattice gauge theory review (strong-coupling background): doi:10.1103/RevModPhys.51.659
6. Seiler (1982), Gauge Theories as a Problem of Constructive Quantum Field Theory: doi:10.1007/978-3-540-39023-7.

5 Validation Contract

Assumptions

1. each statement is conditioned on explicit (G, D), phase, and matter tags,
2. screened-Abelian closure uses $m > 0$ and linearized static kernel framework,
3. scoped non-Abelian theorem uses explicit finite-window area-law/perimeter hypotheses.

Units and dimensions check

1. mr is dimensionless in Yukawa factors,
2. dimensional prefactors in $G_{n,m}$ match the $n = D - 1$ Green-kernel scaling.

Independent cross-check paths

1. analytic Green-kernel and asymptotic derivation (this paper),
2. executable checks:
   - `python3.12 research/workspace/simulations/claim9_phase_longrange_table.py`,
   - `python3.12 research/workspace/simulations/claim9_model_class_table.py`,
   - `python3.12 research/workspace/simulations/claim9_abelian_screened_asymptotic_check.p`
   - `python3.12 research/workspace/simulations/claim9_nonabelian_arealaw_linear_check.py`,
   - `python3.12 research/workspace/simulations/claim9_nonabelian_strong_coupling_window_c`
   - `python3.12 research/workspace/simulations/claim9_nonabelian_beyond_window_transfer_c`
   - `python3.12 research/workspace/simulations/claim9_nonabelian_derivative_covariance_ch`

Confidence statement

Screened-Abelian long-range behavior is theorem-grade in this scoped class. The confining non-Abelian extraction lane is now theorem-grade under explicit (G, D)-tagged finite-window assumptions, and those assumptions are derived in a scoped strong-coupling lattice window with an explicit β-transfer lane and covariance-based sufficient criterion. First-principles control beyond this scoped transfer model and closing string-breaking with dynamical matter remain open.

6 Reproducibility Metadata

- date anchor: 2026-02-09 (US),
- build toolchain tested here: `/Library/TeX/texbin/pdflatex` (TeX Live 2025),
- safe build script: `~/.codex/skills/pdflatex-safe-build/scripts/build_pdflatex_safe.sh`.

7 Conclusion

Claim 9 should be read as phase-resolved, not universal. The screened-Abelian branch is fully closed at theorem level in this manuscript. The non-Abelian confining branch is now closed at scoped extraction-theorem level under explicit (G, D)-tagged area-law/perimeter assumptions, with a scoped derivation lane grounded in an explicit strong-coupling lattice model and extended by an explicit β-transfer theorem and covariance criterion. First-principles control of the transfer assumptions and full string-breaking crossover remain explicit (G, D)-indexed next targets.