

From Newton to the Path Integral: A Foundational Lecture Note

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Status: Working manuscript (foundational synthesis)

0. Aim

This note gives a single technical storyline for the corpus:

1. Newton's finite-step geometric conservation law,
2. action-based reduction and discriminant orbit structure,
3. distributional/oscillatory emergence of amplitude weighting,
4. controlled refinement ((h) , $(_)$, Schwinger-Dyson),
5. field-theoretic extension.

The target is a first-month QFT companion narrative with explicit mathematical dependencies.

Quick Definitions

1. c -parameter: $[c = (-i/h).]$
2. c -invariant: unchanged under parameter changes that keep (c) fixed (equivalently, constant on a $(_)$ -orbit).
3. de-regularization: the one-sided limit $(^+)$ from damped oscillatory kernels.

1. Newton's Seed: Finite-Step Invariant Before the Limit

For central-force polygonal evolution (equal time steps (t)), equal swept triangles are exact at the discrete level.

Angular momentum conservation is encoded geometrically: $[A_n = |r_{nr_n}|, =.]$

Central impulse gives $(r_{np_n}=0)$, so (A_n) is step-invariant.

The continuum law is then a controlled refinement: $[t, A_n, .]$ The important logic is: exact finite-step structure first, limit second.

2. Action Reduction as Common Mechanics

For a Lagrangian action ($S=L(q,\dot{q},t)$):

1. time-translation symmetry gives conserved (E),
2. rotational symmetry gives conserved (L),
3. cyclic elimination reduces to a 1D radial problem [$p_r^2=R(r;E,L)$.]

Orbit classes are read from turning-point geometry:

1. allowed region: $R(r)$,
2. bounded non-circular branch: two turning points,
3. branch boundaries: double root [$R(r_*)=0, R'(r_*)=0$.]

This is the same mechanism in:

1. SR Coulomb (ℓ^2 -classified regimes),
2. Schwarzschild fixed-energy interval and separatrix/ISCO,
3. probe dynamics in gauge-defined static $V(r)$.

References in workspace:

- `research/workspace/notes/theorems/2026-02-08-claim3-coulomb-phase-classification.md`
- `research/workspace/notes/theorems/2026-02-08-claim6-schwarzschild-fixed-energy-interval.md`
- `research/workspace/notes/theorems/2026-02-09-foundational-action-reduction-unification.md`

3. Static Variational Problem as Distribution

For smooth f , classical extrema are encoded by [$f'(x)$.] Under nondegenerate critical points (x_i) : [$f'(x) = 0$.]

Fourier representation [$u=e^{i\phi}, d\phi$] introduces oscillatory exponentials already at static level.

The “halved expression” [$A(O)=\int \{f(x)\} O(x), dx$] has [$|A(O)|^2(f), |O|^2$] in the stationary-phase/nondegenerate setting.

This is the amplitude-to-density pattern and motivates half-density language.

References in workspace:

- `research/workspace/notes/theorems/2026-02-08-claim1-discrete-variational-delta-lemmas.md`
- `research/workspace/notes/theorems/2026-02-08-claim1-manifold-halfdensity-convolution.md`
- `research/workspace/notes/theorems/2026-02-08-claim1-groupoid-halfdensity-theorem-pack.md`

4. From Static to Time Histories

Replacing (f) by action (S[]), classical paths solve $[\neq 0.]$ The formal object $[!()]$ is regularized via oscillatory weighting of histories, leading to path-integral form.

Consistency of refinement/composition introduces a surviving scale parameter (h) (identified physically with \hbar in standard QM/QFT usage), and scale-flow covariance: $[: (.,h)(./,h),]$ with dressed-state invariance under $()$ in the scoped framework.

References in workspace:

- research/workspace/notes/theorems/2026-02-09-claim1-scale-flow-covariance.md
- research/workspace/notes/theorems/2026-02-09-claim1-fd-schwinger-dyson-identity.md

5. Field-Theoretic Lift and Eq.(11)-Type Structure

In field form, Euler-Lagrange structure lifts to Schwinger-Dyson identities by functional integration by parts.

In the scoped finite-dimensional model family, this is theoremized as: $[_i!(e^{\{-cS\}})=0 \;; _i = c, _i S.]$ This is the rigorous version of the Eq.(11)-type closure thread in the corpus.

Field-level existence is dimension-dependent and should be tracked explicitly:

1. (d=2): strongest constructive continuum control in many classes (first closure target).
2. (d=3): substantial control for superrenormalizable branches (second closure target).
3. (d=4): physically central but hardest/open in key interacting cases (frontier with explicit hypotheses).
4. (d>4): typically EFT/nonrenormalizable branch for generic local interactions.

So the Claim 1 field program should escalate (d=2d=3d=4), rather than claim one-shot dimension-independent closure.

6. Current Claim 1 Closure Boundary

The scoped bridge now includes:

1. exact projective cylinder consistency,
2. $(^+)$ de-regularization in several interacting classes,
3. large-(N) non-factorized quadratic and quartic tails,
4. finite-(g) non-perturbative multi-mode quartic control (Euclidean and oscillatory regularized),
5. $(^+)$ closure for that multi-mode quartic sector.

Primary artifacts:

- research/workspace/reports/2026-02-09-claim1-scoped-complete-proof.tex
- research/workspace/notes/theorems/2026-02-09-claim1-multimode-quartic-dereg-eta0.md

Remaining frontier:

1. full continuum/global interacting equivalence beyond scoped classes,
2. uniform renormalization/channel control in truly field-theoretic limits.

Execution notes for this frontier:

- research/workspace/notes/theorems/2026-02-09-claim1-three-level-program.md
- research/workspace/notes/theorems/2026-02-09-claim1-field-dimension-existence-roadmap.md
- research/workspace/notes/theorems/2026-02-09-claim1-d2-field-cylinder-candidate.md

7. Dependency Graph (Explicit)

Newton finite-step area invariance

- > action additivity + symmetry charges (E, L)
- > 1D radial reduction $p_r^2 = R(r)$
- > double-root boundaries (circular/separatrix/threshold)
- > SR Coulomb regimes (Claim 3)
- > Schwarzschild intervals + ISCO (Claims 6,7)
- > gauge-phase static probe branches (Claim 9 map)

Static variational distribution $\delta(f')$

- > Fourier/oscillatory representation
- > halved amplitude $|A|^2$ structure
- > half-density/groupoid formulation
- > path-time slicing + RG-style control (τ flow)
- > Schwinger-Dyson lifted equations
- > scoped Claim 1 theorem chain
- > large-N non-factorized interacting tails
- > finite-g nonperturbative + $\eta \rightarrow 0+$

closures

8. Groupoid/()/Schwinger-Dyson Unified Sheet

The dependency requested on the foundational queue is now formalized in:

- research/workspace/notes/theorems/2026-02-09-claim1-groupoid-tau-sd-dependency-sheet.md

Core fixed-parameter identity: $[c=(-i/h)]$ is preserved by $[_:(.,h)(./,h),]$ and Schwinger-Dyson Eq.(11)-type identities are invariant because they depend on the kernel only through (c).

This closes the conceptual link: groupoid scaling intuition () dressed flow covariance () SD closures.

9. Minimal “What Is Forced” Statement

In this program, path-integral-type oscillatory weighting is not introduced as optional aesthetics; it is the structurally stable way to combine:

1. localization on variational extrema (distributional viewpoint),
2. multiplicative composition under slicing (action additivity),
3. controlled refinement with a surviving scale parameter.

That is the precise sense in which quantum weighting appears as the consistent correction/completion of naive classical refinement.

10. Reproducibility Index

Core diagnostic scripts:

1. `python3.12 research/workspace/simulations/foundation_action_reduction_unification_check.py`
2. `python3.12 research/workspace/simulations/claim1_multimode_quartic_dereg_eta0_check.py`
3. `python3.12 research/workspace/simulations/claim1_multimode_quartic_nonperturbative_oscillatory_check.py`
4. `python3.12 research/workspace/simulations/claim6_schwarzschild_interval_scan.py`
5. `python3.12 research/workspace/simulations/claim3_coulomb_classification_scan.py`
6. `python3.12 research/workspace/simulations/claim1_groupoid_tau_sd_dependency_check.py`

11. Next Formal Target

Integrate the new dependency sheet directly into the scoped Claim 1 theorem manuscript so the same (c)-invariance logic is explicit in the main paper chain.