From Newton to the Path Integral: A Foundational Lecture Note

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Status: Working manuscript (foundational synthesis)

0. Aim

This note gives a single technical storyline for the corpus:

1. Newton's finite-step geometric conservation law,
2. action-based reduction and discriminant orbit structure,
3. distributional/oscillatory emergence of amplitude weighting,
4. controlled refinement ((h), (_), Schwinger-Dyson),
5. field-theoretic extension.

The target is a first-month QFT companion narrative with explicit mathematical dependencies.

Quick Definitions

1. `c-parameter`: [c=(-i/h).]
2. `c-invariant`: unchanged under parameter changes that keep (c) fixed (equivalently, constant on a (_)-orbit).
3. `de-regularization`: the one-sided limit (^+) from damped oscillatory kernels.
4. `terminology guardrail`: use probability/transition amplitude as default physics language; reserve geometric (1/2)-density for explicit kernel-bundle statements.

1. Newton's Seed: Finite-Step Invariant Before the Limit

For central-force polygonal evolution (equal time steps (t)), equal swept triangles are exact at the discrete level.
Angular momentum conservation is encoded geometrically: [A_n= |r_nr_n|, =.]
Central impulse gives (r_np_n=0), so (A_n) is step-invariant.

The continuum law is then a controlled refinement: [t, A_n, .] The important logic is: exact finite-step structure first, limit second.

2. Action Reduction as Common Mechanics

For a Lagrangian action (S=L(q,q),d):

1. time-translation symmetry gives conserved (E),
2. rotational symmetry gives conserved (L),
3. cyclic elimination reduces to a 1D radial problem [p_r^2=R(r;E,L,).]

Orbit classes are read from turning-point geometry:

1. allowed region: (R(r)),
2. bounded non-circular branch: two turning points,
3. branch boundaries: double root [R(r_)=0,R'(r_)=0.]

This is the same mechanism in:

1. SR Coulomb ((^2)-classified regimes),
2. Schwarzschild fixed-energy interval and separatrix/ISCO,
3. probe dynamics in gauge-defined static (V(r)).

References in workspace:

- `research/workspace/notes/theorems/2026-02-08-claim3-coulomb-phase-classification.md`
- `research/workspace/notes/theorems/2026-02-08-claim6-schwarzschild-fixed-energy-interval.md`
- `research/workspace/reports/2026-02-09-claim9-gauge-phase-long-range-paper.tex`
- `research/workspace/notes/theorems/2026-02-09-foundational-action-reduction-unification.md`

3. Static Variational Problem as Distribution

For smooth (f), classical extrema are encoded by [(f'(x)).] Under nondegenerate critical points (x_i): [(f'(x)) = _i .]

Fourier representation [(u)=e^{izu},dz] introduces oscillatory exponentials already at static level.

The oscillatory amplitude expression [A_(O)=$^{\{-1/2\}e}$\{f(x)\}O(x),dx] has [|A_(O)|^2 (f'),|O|^2] in the stationary-phase/nondegenerate setting.
This is the amplitude-to-density pattern and motivates probability-amplitude language; in geometric kernel calculus the same object is represented as a geometric (1/2)-density.

References in workspace:

- `research/workspace/notes/theorems/2026-02-08-claim1-discrete-variational-delta-lemmas.md`
- `research/workspace/notes/theorems/2026-02-08-claim1-manifold-halfdensity-convolution.md`
- `research/workspace/notes/theorems/2026-02-08-claim1-groupoid-halfdensity-theorem-pack.md`

4. From Static to Time Histories

Replacing (f) by action (S[]), classical paths solve [=0.] The formal object [!()] is regularized via oscillatory weighting of histories, leading to path-integral form.

Consistency of refinement/composition introduces a surviving scale parameter (h) (identified physically with () in standard QM/QFT usage), and scale-flow covariance: [*: (,,h)(,/,h),] with dressed-state invariance under (*) in the scoped framework.

References in workspace:

- `research/workspace/notes/theorems/2026-02-09-claim1-scale-flow-covariance.md`
- `research/workspace/notes/theorems/2026-02-09-claim1-fd-schwinger-dyson-identity.md`

5. Field-Theoretic Lift and Eq.(11)-Type Structure

In field form, Euler-Lagrange structure lifts to Schwinger-Dyson identities by functional integration by parts.
In the scoped finite-dimensional model family, this is theoremized as: [_i!(e^{-cS})=0 ;; _i = c,_i S.] This is the rigorous version of the Eq.(11)-type closure thread in the corpus.

Field-level existence is dimension-dependent and should be tracked explicitly:

1. (d=2): strongest constructive continuum control in many classes (first closure target).
2. (d=3): substantial control for superrenormalizable branches (second closure target).
3. (d=4): physically central but hardest/open in key interacting cases (frontier with explicit hypotheses).
4. (d>4): typically EFT/nonrenormalizable branch for generic local interactions.

So the Claim 1 field program should escalate (d=2d=3d=4), rather than claim one-shot dimension-independent closure.

6. Current Claim 1 Closure Boundary

The scoped bridge now includes:

1. exact projective cylinder consistency,
2. (^+) de-regularization in several interacting classes,
3. large-(N) non-factorized quadratic and quartic tails,
4. finite-(g) non-perturbative multi-mode quartic control (Euclidean and oscillatory regularized),
5. (^+) closure for that multi-mode quartic sector.

Primary artifacts:

- `research/workspace/reports/2026-02-09-claim1-scoped-complete-proof.tex`
- `research/workspace/notes/theorems/2026-02-09-claim1-multimode-quartic-dereg-eta0.md`

Remaining frontier:

1. full continuum/global interacting equivalence beyond scoped classes,
2. uniform renormalization/channel control in truly field-theoretic limits.

Execution notes for this frontier:

- `research/workspace/notes/theorems/2026-02-09-claim1-three-level-program.md`
- `research/workspace/notes/theorems/2026-02-09-claim1-field-dimension-existence-roadmap.md`
- `research/workspace/notes/theorems/2026-02-09-claim1-d2-field-cylinder-candidate.md`

Claim maturity snapshot (audit, 2026-02-10 US)

The canonical score table lives in: `research/workspace/notes/audits/2026-02-08-top10-claim-audit.md` (Section "Claim Maturity Scores (0-10)").

Claim	Score	Closure boundary (date-anchored summary)
1	9.6	Scoped theorem-grade closure in a nontrivial oscillatory/projective class (statics/dynamics plus a dimension-gated (d=2d=3) field branch), with explicit remaining global interacting and reconstruction gaps.
2	9.0	Local asymptotic theorem closure is strong; global phase-space completion remains open.
3	8.9	SR Coulomb phase portrait and global/asymptotic time structure are

Claim	Score	Closure boundary (date-anchored summary)
		theorem-closed in the scoped model.
4	9.0	(n=3) Duffing reduction and global-time/ topology classification are theorem-grade in scoped model.
5	9.0	D-dimensional GR matching is closed in the conventions used.
6	9.5	Fixed-energy Schwarzschild bound-orbit interval and separatrix structure are fully explicit and cross-checked.
7	9.5	ISCO threshold statement is canonical and correctly framed with unit conventions.
8	7.8	Static baseline and rotating regime maps exist with explicit unresolved sectors (especially in multi-spin (D) lanes).
9	8.2	Screened-Abelian Yukawa branch is theorem-closed; non-Abelian confining branch is closed at scoped extraction-theorem level with strong-coupling derivation and a ()-transfer lane; first-principles transfer control and dynamical-matter string-breaking remain open.

Claim	Score	Closure boundary (date-anchored summary)
10	9.5	Benchmark inequalities and threshold regimes are explicit and validated.

7. Dependency Graph (Explicit)

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Newton finite-step area invariance
  -> action additivity + symmetry charges (E, L)
     -> 1D radial reduction p_r^2 = R(r)
        -> double-root boundaries (circular/separatrix/threshold)
           -> SR Coulomb regimes (Claim 3)
           -> Schwarzschild intervals + ISCO (Claims 6,7)
           -> gauge-phase static probe branches (Claim 9 map)

Static variational distribution δ(f')
  -> Fourier/oscillatory representation
     -> oscillatory amplitude |A|^2 structure
        -> geometric 1/2-density/groupoid formulation
           -> path-time slicing + RG-style control (tau flow)
              -> Schwinger-Dyson lifted equations
                 -> scoped Claim 1 theorem chain
                    -> large-N non-factorized interacting tails
                       -> finite-g nonperturbative + η->0+
closures
```

8. Groupoid/(_)/Schwinger-Dyson Unified Sheet

The dependency requested on the foundational queue is now formalized in:

- `research/workspace/notes/theorems/2026-02-09-claim1-groupoid-tau-sd-dependency-sheet.md`

Core fixed-parameter identity: [c=(-i/h)] is preserved by [_:(,,h)(,/,h),] and Schwinger-Dyson Eq.(11)-type identities are invariant because they depend on the kernel only through (c).

This closes the conceptual link: groupoid scaling intuition () dressed flow covariance () SD closures.

9. Minimal “What Is Forced” Statement

In this program, path-integral-type oscillatory weighting is not introduced as optional aesthetics; it is the structurally stable way to combine:

1. localization on variational extrema (distributional viewpoint),
2. multiplicative composition under slicing (action additivity),
3. controlled refinement with a surviving scale parameter.

That is the precise sense in which quantum weighting appears as the consistent correction/completion of naive classical refinement.

10. Reproducibility Index

Core diagnostic scripts:

1. `python3.12 research/workspace/simulations/foundation_action_reduction_unification_check.py`
2. `python3.12 research/workspace/simulations/claim1_multimode_quartic_dereg_eta0_check.py`
3. `python3.12 research/workspace/simulations/claim1_multimode_quartic_nonperturbative_oscillatory_check.py`
4. `python3.12 research/workspace/simulations/claim6_schwarzschild_interval_scan.py`
5. `python3.12 research/workspace/simulations/claim3_coulomb_classification_scan.py`
6. `python3.12 research/workspace/simulations/claim1_groupoid_tau_sd_dependency_check.py`

11. Next Formal Target

Finish the current foundations-facing closure loop by (i) wiring the concrete exhaustion/regularization envelopes into the AN-33L-C commuting-limit wrapper on the field side (so the Lean wrapper can be invoked without hidden hypotheses), and (ii) continuing the Newton-limit paradox support lane (kernel-level (t^{-d/2}) semigroup normalization and Van Vleck/Schur prefactor links).