Final Closure Synthesis: Project Status, Inventories, and Future-Work Gates

2026-02-13

Abstract

This document is the definitive closure synthesis for the `physbook` repository. It supersedes the earlier wind-down snapshot (2026-02-09) and records: (i) the project's mission and three-level program, (ii) the frozen claim maturity scores for all 17 claims, (iii) complete inventories of Lean formalizations, numerical simulations, theorem notes, and manuscripts, (iv) the three explicitly deferred future-work gates, and (v) reproducibility metadata. No new theorems are asserted.

Contents

1 Project Mission

The repository investigates how quantum formalism appears when variational mechanics is made distributionally consistent, through AI-assisted formal and numerical verification. The foundational axis has three cores:

1. **Variational-distribution core**: the chain $\delta(f') \to$ action extremals $\to$ path/field functional measure with oscillatory-amplitude interpretation.
2. **Geometry-of-force bridge**: action reduction yields force/orbit/geodesic structure across SR/GR/gauge settings.
3. **Scale-control core**: refinement/RG/tangent-groupoid control (τ-type flow, continuum/de-regularization existence, Schwinger–Dyson closure).

2 Three-Level Program (Goals 1A–1C)

Goal 1A (Statics) Prove consistency for static problems and package as a paper on probability amplitudes as geometric $\frac{1}{2}$-densities.
Status: **Closed.** Manuscript: `claim1-paper1-static-amplitude-qm-equivalence`.

Goal 1B (Dynamics) Prove consistency for time-line mechanics and package as a path-integral equivalence paper with historical section.
Status: **Closed.** Manuscript: `claim1-paper2-dynamics-path-integral-equivalence-history`.

Goal 1C (Fields) Prove consistency for field theory on spacetime via a dimension-indexed ladder $d{=}2 \to d{=}3 \to d{=}4$.
Status: **Partially closed.** The $d{=}2$ ultralocal interacting closure is complete; the $d{=}3$ branch is advanced through AN-42 (stochastic sub-Gaussian error-budget control); the $d{=}4$ interacting closure remains open (see §8). Manuscript: `claim1-paper3-field-theory-general-dim`

3 Frozen Claim Maturity Scores

Scores are frozen as of 2026-02-13. The deterministic adjudication rule (theorem-grade closure, scope widening, or assumption elimination) produced the final values below.

3.1 Top 10 Claims

Claim	Score	Label	Summary
1	9.80	heuristic	Scoped theorem package through AN-42: stochastic sub-Gaussian false-stop / error-budget control with three-way decomposition $\varepsilon = \varepsilon_{\text{reg}} + \varepsilon_{\text{tail}} + \varepsilon_{\text{stop}}$ in $d{=}3$ branch. Full global interacting closure remains open.
2	9.10	proved	SR center-access trichotomy with explicit global turning-set / capture map in $n{=}2$ lane.
3	9.00	proved	Relativistic Coulomb phase portrait: 5-regime taxonomy with root/rotation/global-time consistency bridge.

Claim	Score	Label	Summary
4	9.20	proved	n=3 Duffing reduction with robustness windows ($\|\delta\| < 0.35$, $\delta_{\text{crit}} \approx -0.697$) and time asymptotics.
5	9.20	proved	D-dimensional GR matching including D=3 log-potential branch and source-normalization crosswalk.
6	9.60	proved	Schwarzschild fixed-energy interval plus null impact-threshold analogue ($b_* = 3\sqrt{3}$).
7	9.65	proved	ISCO threshold with geometric-to-SI unit crosswalk.
8	8.15	heuristic	Static/rotating split with effective-potential channel verification (D=6: 93.3% unstable, 6.7% narrow stable window). Full all-spin closure open.
9	8.55	heuristic	Screened-Abelian theorem closure plus first-principles lattice plaquette-counting transfer control. Non-Abelian beyond-window transfer and dynamical-matter string-breaking open.
10	9.65	proved	Circular-orbit benchmark inequalities with deterministic regression checks.
Mean (Claims 1–10)	**9.19**		

3.2 Extended Claims (11–17)

Claim	Score	Label	Summary
11	5.5	heuristic	Planck area quantization forces 3+1 dimensions via inverse-square law. Algebraically verified; propagator-origin step assumed.
12	5.0	heuristic	Uncertainty principle forces $1/r^2$ force from virtual exchange. Correct scaling; $\Delta t = r/c$ ad hoc for massive case.
13	8.0	proved	Action additivity + composition uniquely forces $\exp(iS/\hbar)$ kernel, within stated axioms.
14	7.0	heuristic	Amplitudes as half-densities on tangent-groupoid algebras. Static case proved; infinite-dimensional extension scoped.
15	4.5	speculative	Democritean refinement argument $\Rightarrow$ quantum mechanics necessity. Mathematically motivated; philosophical rather than theorem-grade.
16	3.5	speculative	Two-body Planck area quantization and standard model mass scales. Algebraically explicit; physically speculative.
17	6.0	heuristic	Schwinger–Dyson identity as RG invariant via Ehrenfest correspondence. Correct in finite-dimensional models.
Mean (Claims 1–17)	**7.69**		

4 Lean Formalization Inventory

Twenty machine-checked modules in `Claim1lean/`, building on `mathlib` (15,798 compiled files). Toolchain: Lean 4 via `elan`; CI via `.github/workflows/lean.yml`.

Module	Content
`Basic.lean`	Utility definitions
`CInvariant.lean`	c-invariance under τ reparameterization
`SmallKappaLipschitz.lean`	$O(\kappa)$ increment inequality on $[0, \kappa]$
`CovarianceDerivative.lean`	Quotient-derivative identity for $\omega = N/Z$
`FiniteCovarianceBound.lean`	Finite-support centered-product covariance
`RatioStateDerivativeBound.lean`	Abstract $\lvert\partial\omega\rvert$ bound from AN-7/AN-8
`RatioStateIncrementBound.lean`	Interval increment bound from derivatives
`FiniteExponentialFamilyDeriv.lean`	Finite-sum exponential-family derivatives
`FiniteExponentialRepresentation.lean`	Centered representation bridge
`FiniteExponentialDerivativeBound.lean`	Model-internal derivative bounds
`FiniteExponentialIncrementBound.lean`	Model-internal $C\kappa$ interval bounds
`FiniteExponentialRegularity.lean`	Automatic regularity (Zsum > 0, differentiability)
`WeightedLocalGraphDecay.lean`	Weighted-local truncation, graph-decay, denominator-rate bookkeeping
`SemigroupGenerator.lean`	Semigroup $\to$ generator lemma
`SchurComplementDeterminant.lean`	Schur-complement determinant prefactor template
`SchurComplementElimination.lean`	Block-LDU elimination identity
`GaussianSemigroupNormalization.lean`	1D Gaussian convolution semigroup prefactor anchor
`GaussianSemigroupNormalizationNd.lean`	Dimension-indexed d-power prefactor formulas
`GaussianSemigroupScalingRigidity.lean`	Additive semigroup law fixes prefactor
`VanVleckPrefactorBridge.lean`	$\lvert\det\rvert^{-1/2}$ product identities

5 Manuscript Inventory

All manuscripts reside in `research/workspace/reports/`.

File stem	Content
`claim1-paper1-static-amplitude-qm-equivalence`	Goal 1A: static variational consistency, probability amplitudes as $\frac{1}{2}$-densities
`claim1-paper2-dynamics-path-integral-equivalence-history`	Goal 1B: dynamics, path-integral chain, historical section
`claim1-paper3-field-theory-general-dimension-roadmap`	Goal 1C: dimension-indexed field program with G1/G2/G3 gates
`claim1-scoped-complete-proof`	Consolidated Claim 1 scoped theorem chain (static through interacting field)
`claim1-variational-delta-note`	First statement of Goal 1 as three-level ladder
`claim9-gauge-phase-long-range-paper`	Standalone Claim 9: gauge potentials with screened-Abelian closure
`conv-research-program`	Overview: what was claimed, what was proved
`newton-action-path-integral-lecture`	Foundational narrative: Newton $\to$ action $\to$ path integral
`foundations-refinement-compatibility`	Foundations submission: refinement compatibility principle

File stem	Content
winddown-status-snapshot	Earlier wind-down synthesis (now superseded by this document)
synthesis-proof-dependency	Dependency graph of scoped proof chain

6 Numerical Simulation Inventory

106 Python scripts reside in research/workspace/simulations/. All referenced diagnostics report all_checks_pass=True. Scripts cover:

- Claim 1 (AN-1 through AN-42): 55+ scripts for continuum, c-invariance, field-level, envelope, schedule, and stochastic checks across d=2, d=3, and d=4 branches.
- Claims 2–7, 10: orbital mechanics, phase portrait, Duffing, GR matching, Schwarzschild, ISCO, and benchmark regression checks.
- Claim 8 (AB–AF): D=6 multi-spin discriminant, regime partition, margin robustness, outer-branch horizon, effective-potential channel.
- Claim 9 (AG–AL): non-Abelian transfer positivity, adaptive window, budget tightening, segmented cover, Lipschitz budget, first-principles transfer derivative control.
- Claims 11–17: dimensional analysis, uncertainty scaling, action-additivity kernel, half-density Born rule, refinement, Planck area, SD/RG invariant.
- Themes 1–2: action-angle EBK cross-check, fermionic mediation contact cross-check.

7 Theorem Notes Inventory

128 theorem-style notes reside in research/workspace/notes/theorems/, each with explicit assumptions, proof, and validation contract. The notes index is maintained in research/workspace/notes/REA

Audit and tracking files reside in research/workspace/notes/audits/:

- 2026-02-08-top10-claim-audit.md (144 work items, 140 done)
- 2026-02-11-extended-claims-audit.md (Claims 11–17)
- 2026-02-10-weekly-score-portfolio.md (Tasks A–F, all done)
- 2026-02-10-weekly-score-retro.md
- 2026-02-10-research-pool-themes.md (2 themes, deferred)

8 Deferred Future-Work Gates

The following open proof gates are explicitly deferred at project closure. Each represents a substantial research program beyond scoped closure.

8.1 Gate F1: Claim 1 — Full d=4 Interacting Closure

The d=3 branch is advanced through AN-42 with stochastic error-budget control. Full d=4 interacting closure requires:

1. G2: continuum existence (ultraviolet renormalization beyond superrenormalizable models),
2. G3: reconstruction (recovering Wightman/OS axioms from the regulated family).

This is a problem of comparable difficulty to constructive QFT in four dimensions.

8.2 Gate F2: Claim 8 — $D \geq 6$ All-Spin Rotating Closure

The static Tangherlini sector and several rotating subclasses are closed (including the D=6 doubly-spinning effective-potential channel). The remaining open region is a narrow D=6 stable window (6.7% of scan grid) and all $D \geq 7$ multi-spin sectors. Full closure requires extending the discriminant/margin analysis to arbitrary spin configurations in higher dimensions.

8.3 Gate F3: Claim 9 — Non-Abelian Beyond-Window Transfer and String Breaking

Screened-Abelian closure is theorem-grade. Non-Abelian transfer control has been tightened through AL (first-principles lattice plaquette-counting bounds), but:

1. beyond-window transfer: weak-coupling cluster-expansion control is needed,
2. dynamical-matter string-breaking crossover: requires explicit (G, D, N_f)-tagged analysis beyond program-level framing.

9 Items at Closure: In-Progress Lanes

Two work items from the audit remain marked `[in progress]` at closure:

1. **Item 96 (Physarticle scan backlog)**: remaining targets include half-density scalarization unitarity, full Gaussian kernel $t^{-d/2}$ semigroup normalization in d dimensions (kernel-level), point-splitting δ' lemma, and Planck-area hypothesis cleanup. *Deferred.*
2. **Item 105 (Newton-limit paradox support)**: next target is the kernel-level semigroup/normalization constraint behind Gaussian/Van-Vleck prefactors and its link to amplitude/half-density necessity. *Deferred.*

Two bibliography anchors (Item 97: `Connes1994`, `Landsman1998`) remain `PENDING`. *Deferred.*

10 Research Pool Themes

Two non-core themes were queued but are **deferred at closure**:

1. **Theme 1**: action–angle "undeterminacy" for central potentials. Initial deliverable (Coulomb EBK cross-check) completed; full formalization deferred.
2. **Theme 2**: fermionic field mediation of central potentials. Initial deliverable (contact-interaction cross-check) completed; full formalization deferred.

11 Repository Consistency Checks

- **Git state**: clean (only `.DS_Store` uncommitted). No open PRs or issues.
- **Lean CI**: last run passed; 15,798/15,798 files compiled.
- **Simulation diagnostics**: all 106 scripts report `all_checks_pass=True` as documented in audit trail.
- **Audit trail**: top-10 audit has 144 work items (140 done, 2 in-progress now deferred, 1 queued now deferred, 1 score-update done). Weekly portfolio tasks A–F all completed.
- **Score consistency**: scores in `top10-claim-audit.md`, `weekly-score-portfolio.md`, and this document are mutually consistent at $\bar{S} = 9.19$.

12 Validation Contract

Assumptions

1. This is a status synthesis; no new theorem is claimed.
2. All closure/open labels are inherited from tracked artifacts.

Units and Dimensions

No new dimensional claim is introduced.

Symmetry/Conservation

No new symmetry or conservation claim is asserted.

Independent Cross-Check Path

1. Compare score tables across audit, portfolio, and this document.
2. Verify Lean module list against `ls Claim1lean/`.
3. Verify simulation count against `ls simulations/ | wc -l`.

Confidence

High confidence for inventory extraction and score consistency. Medium confidence for completeness of the deferred-gate descriptions, which summarize open research problems.

13 Reproducibility Metadata

- Date anchor: 2026-02-13 (US).
- Repository: `/Users/arivero/physbook`, branch `main`.
- Lean toolchain: Lean 4 via `elan`, mathlib dependency.
- Python: 3.12 with numpy, scipy, sympy, matplotlib.
- TEX: TeX Live 2025 (`pdflatex`).
- This document supersedes `2026-02-09-winddown-status-snapshot.tex`.