Fermionic Mediators, Static Potentials, and Contact/Boundary-Condition Limits

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Abstract

The textbook derivation of a static potential from "field exchange" uses a bosonic mediator linearly sourced by a commuting classical density, yielding an effective action quadratic in the source and (in a static limit) a central Yukawa/Coulomb potential. This derivation does not transplant verbatim to fermionic fields: the linear source terms for fermions require Grassmann-valued sources, so there is no ordinary commuting classical source whose elimination produces a classical potential in the same way. This note isolates the precise obstruction and records the robust infrared replacement: when a microscopic description reduces to local operators at low resolution, the effective interaction is encoded by contact terms (delta kernels and their derivatives) or, equivalently, boundary-condition/self-adjoint-extension data, with renormalization-group running when the contact limit is singular.

This is a dependent note aligned with the broader refinement-compatibility program: contact terms are diagonal-support kernels, and their scale dependence is a compatibility condition rather than an afterthought.

1. Purpose and scope

This note answers a narrowly phrased question: what can it mean for a **fermionic** field to "generate a (central) potential" in the same sense that a massive bosonic field generates a Yukawa potential?

We keep the scope bounded: 1. state the bosonic sourcing $\Rightarrow$ potential mechanism (derivation-first, brief), 2. state the fermionic obstruction precisely (Grassmann sources), 3. give one explicit IR matching witness: **local operators $\Rightarrow$ contact/derivative-contact kernels**, 4. connect contact kernels to related point-interaction/RG witnesses.

We do **not** claim that fermions cannot affect forces; we only isolate which parts of the "classical source $\Rightarrow$ potential" story fail, and what the correct replacement statement is at low resolution.

2. What "a field generates a potential" means in the bosonic source story

The archetypal construction is a bosonic mediator φ linearly coupled to a commuting source $J(x)$:

$$S[\varphi; J] = \int d^D x \left(\frac{1}{2} \varphi\, K\, \varphi + J\, \varphi \right), \qquad K = (\Box + m^2) \text{ (example)}.$$

Integrating out φ (Gaussian elimination) yields an effective action quadratic in the source,

$$S_{\text{eff}}[J] = -\frac{1}{2} \int d^D x\, d^D y\; J(x)\, K^{-1}(x, y)\, J(y),$$

so the static, nonrelativistic limit of K^{-1} produces a central potential (Yukawa for $m \neq 0$, Coulomb-type when $m = 0$).

The key structural ingredient is that the source is an ordinary commuting function (a classical background density).

3. Fermionic fields: linear sources are Grassmann, so the classical-source story does not transplant

For a Dirac fermion Ψ, the generating functional with sources is written with **Grassmann-valued** sources $\eta, \bar{\eta}$:

$$Z[\bar{\eta}, \eta] = \int D\bar{\Psi}\, D\Psi\; \exp\left(i \int d^D x\, \bar{\Psi}\, (i\gamma^\mu \partial_\mu - m)\Psi + i \int d^D x\, (\bar{\eta}\, \Psi + \bar{\Psi}\, \eta) \right).$$

An explicit statement of this form, including that $\eta, \bar{\eta}$ are Grassmann-valued, is recorded in [Floerchinger2024QFT1Lecture21].

Two immediate consequences follow.

`Remark 3.1 (Obstruction statement).` The bosonic derivation "choose a commuting classical source J, integrate out the field, and read off a classical potential" does not directly apply to fermions, because the linear source terms that couple to Ψ require Grassmann sources rather than commuting c-number densities. Therefore, "fermion exchange generates a classical potential between commuting sources" is not a well-posed transplant of the bosonic story.

This does **not** mean fermions are irrelevant: fermions can and do affect effective interactions through loop effects, through bosonic composite modes (bilinears), and through low-energy EFT operators. The point is that the meaning of "generates a potential" must be stated through one of these controlled mechanisms.

3.1 The controlled alternative: fermion loops modify bosonic propagators

The standard example is vacuum polarization in quantum electrodynamics. A closed electron–positron loop inserted into the photon propagator gives a momentum-dependent correction to the effective electromagnetic coupling,

$$\alpha_{\text{eff}}(q^2) = \frac{\alpha}{1 - \Pi(q^2)}, \qquad \Pi(q^2) = -\frac{\alpha}{3\pi} \ln\left(\frac{q^2}{\mu^2}\right) + \cdots,$$

where $\Pi(q^2)$ is the vacuum polarization function (the photon self-energy from a fermion one-loop diagram). At low momentum transfer ($|q| \ll m_e$), the loop correction is analytic in q^2 and generates precisely the local operators $C_0 + C_2 q^2 + \cdots$ discussed in Section 4 below.

The structural point: fermions affect forces, but the path from "fermion field" to "effective interaction" runs through a quantum loop (not through a tree-level Gaussian elimination of a classical source), and the low-energy residue takes the form of local/contact operators.

Example 3.2 (Uehling potential: the coordinate-space face of vacuum polarization). The momentum-dependent coupling above translates, via Fourier transform, into a coordinate-space correction to the Coulomb potential — the Uehling potential $V_{\text{Uehl}}(r) = -(Z_1 Z_2 \alpha/r) \cdot (2\alpha/3\pi) \int_1^\infty du \ (1 + 1/(2u^2))\sqrt{1 - 1/u^2} \, u^{-1} e^{-2m_e r u}$. At short distances ($r \ll 1/m_e$) the integral yields a logarithmic correction $\propto \ln(1/(m_e r))$, reflecting the running coupling and matching the analytic $C_0 + C_2 q^2 + \cdots$ expansion of Section 4; at long distances ($r \gg 1/m_e$) it is exponentially suppressed $\propto e^{-2m_e r}$, confirming that the fermion decouples below its mass threshold. The dominant observable consequence is the vacuum-polarization contribution to the hydrogen Lamb shift (≈ 27 MHz of the total ≈ 1058 MHz $2S$–$2P$ splitting), which probes the modified potential at nuclear distances where the S-wave wavefunction satisfies $|\psi(0)|^2 \neq 0$.

Remark 3.3 (Anomalous magnetic moment: the vertex correction). The vacuum-polarization modification of the photon propagator (Section 3.1) is one of three one-loop QED corrections. The vertex correction — a virtual photon emitted and reabsorbed by the electron line — modifies the electron's magnetic moment to $a_e = (g-2)/2 = \alpha/(2\pi) \approx 0.00116$ (Schwinger, 1948). This is a dimension-5 effective operator $\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$ in the EFT language of Section 4 — the magnetic analog of the electric contact expansion. Higher-order corrections involve both vacuum polarization insertions (the running coupling of Section 3.1) and light-by-light fermion loops, and the QED prediction agrees with the measured electron $g-2$ to better than one part in 10^{12} — the most precisely tested prediction in physics. The muon anomalous magnetic moment, being more sensitive to heavy virtual particles by a factor $\sim (m_\mu/m_e)^2$, probes the contact operators of heavier sectors: new physics at scale Λ contributes $\delta a_\mu \sim m_\mu^2/\Lambda^2$, the standard EFT suppression pattern.

`Remark 3.4 (Schwinger pair production and the Euler–Heisenberg`
`Lagrangian: the non-perturbative complement).` The perturbative vacuum-polarization story of Section 3.1 has a non-perturbative counterpart. In a constant electric field $\mathcal{E}$, the QED vacuum is unstable against spontaneous electron–positron pair creation at a rate per unit volume $\Gamma/V \propto m^2(\mathcal{E}/\mathcal{E}_{\mathrm{cr}})^2 \exp(-\pi\mathcal{E}_{\mathrm{cr}}/\mathcal{E})$, where $\mathcal{E}_{\mathrm{cr}} = m^2c^3/(e\hbar) \approx 1.3 \times 10^{18}$ V/m is the Schwinger critical field (Schwinger, 1951). The essential singularity $\exp(-\pi\mathcal{E}_{\mathrm{cr}}/\mathcal{E})$ makes this invisible to all orders of perturbation theory: the rate vanishes faster than any power of α as $\mathcal{E} \to 0$. The real part of the same one-loop effective action gives the Euler–Heisenberg Lagrangian $\mathcal{L}_{\mathrm{EH}} = \mathcal{L}_{\mathrm{Maxwell}} + (2\alpha^2/45m^4)[(\mathcal{E}^2 - \mathcal{B}^2)^2 + 7(\mathcal{E} \cdot \mathcal{B})^2] + \cdots$ (Euler and Heisenberg, 1936) — dimension-8 contact operators in the electromagnetic field, the QFT counterpart of the contact expansion in Section 4. Fermions thus affect forces both perturbatively (running coupling, Uehling potential) and non-perturbatively (pair creation, photon–photon scattering), with the contact-operator tower organizing both regimes.

4. IR replacement: local operators $\Rightarrow$ contact kernels / boundary-condition data

At low resolution, integrating out heavy degrees of freedom typically produces local operators. In a two-body, nonrelativistic sector, this appears as an amplitude expansion analytic in momentum transfer q:

$$\mathcal{A}(q) = C_0 + C_2q^2 + O(q^4).$$

The coordinate-space interaction associated to such an analytic expansion is distributional and diagonal-supported. The invariant core is a Fourier-transform identity:

$$\int \frac{d^dq}{(2\pi)^d} \, e^{iq\cdot r} = \delta^{(d)}(r), \qquad \int \frac{d^dq}{(2\pi)^d} \, q^2 \, e^{iq\cdot r} = -\nabla^2\delta^{(d)}(r).$$

`Derivation 4.1 (Contact expansion gives \(\delta\) and derivative`
`contacts).` Interpreting the low-energy interaction kernel as the inverse Fourier transform of $\mathcal{A}(q)$ (Born-level language, up to overall convention-dependent factors), we obtain

$$V(r) \; \propto \; \int \frac{d^dq}{(2\pi)^d} \, e^{iq\cdot r} \, \mathcal{A}(q) \; \propto \; C_0 \, \delta^{(d)}(r) \; - \; C_2 \, \nabla^2\delta^{(d)}(r) \; + \cdots.$$

Thus locality at low energy naturally becomes **contact data**: delta kernels and their derivatives supported at a point (or on the diagonal, in kernel language).

In singular cases (notably δ interactions in $d \geq 2$ in various channels), this contact data is not simply a fixed number: it is defined by a renormalization condition and

can generate RG-invariant scales and bound states [Jackiw1991DeltaPotentials] [ManuelTarrach1994PertRenQM].

Remark 4.2 (Connection to the effective range expansion). In scattering theory the s-wave amplitude is parametrized by the effective range expansion (ERE) $k \cot \delta_0(k) = -1/a + r_0 k^2/2 + O(k^4)$, where a is the scattering length and r_0 the effective range. The contact expansion of Derivation 4.1 is the momentum-space counterpart: at Born level, C_0 determines a, C_2 determines r_0, and each higher C_{2n} maps to a shape parameter. Examples 5.1 and 5.2 below are the leading-order case $C_2 = 0$ (zero effective range, $r_0 = 0$), for which the full amplitude $f_0(k) = -a/(1 + ika)$ depends on a single parameter — the scattering length.

Remark 4.3 (Huang-Yang pseudopotential: regularization built into the operator). The Huang-Yang pseudopotential $V(r) = (4\pi\hbar^2 a/m)\,\delta^{(3)}(r)\,(\partial/\partial r)(r\,\cdot)$ builds the renormalization condition of Example 5.2 directly into the operator definition. The differential operator $(\partial/\partial r)(r\,\cdot)$ extracts the regular part of the wavefunction at the origin — for a function with the s-wave asymptotics $\psi(r) \sim A(1/r - 1/a) + (\text{regular})$, it yields the finite value $-A/a$ rather than the divergent A/r — and thereby automatically implements the scattering-length boundary condition without explicit cutoff manipulation. This is a "smart" contact kernel: the prescription for handling the $r = 0$ singularity is not an external regularization step but is part of the definition of the interaction, with different values of a selecting different self-adjoint extensions.

Remark 4.4 (Weinberg power counting: the systematic hierarchy for contact EFTs). The contact expansion of Derivation 4.1 is the leading term in a systematic effective field theory organized by Weinberg's power counting (1990, 1991). Each diagram is assigned a chiral order based on the number of loops L, the number of derivatives d_i at each vertex, and the number of nucleon lines n_i — with higher-order contributions suppressed by powers of Q/Λ, the ratio of typical momentum to the breakdown scale. At leading order (LO), only the non-derivative contact C_0 contributes; at next-to-leading order (NLO), the two-derivative contact $C_2 q^2$ and one-loop diagrams with C_0 insertions enter together; at NNLO, $C_4 q^4$, mixed one-loop diagrams, and two-loop diagrams appear. Crucially, loops generate non-analytic momentum dependence — terms like $q^2 \ln(q^2/\mu^2)$ in three dimensions — that cannot be absorbed into the polynomial contact expansion. These infrared logarithms distinguish the full quantum EFT from a naive polynomial fit and provide the physical content (e.g., the running coupling of Examples 5.1–5.2) that transcends tree-level contact operators. When the leading contact interaction is strong (large scattering length), an alternative counting due to Kaplan, Savage, and Wise (1998) resums C_0 to all orders — precisely the T-matrix resummation of Examples 5.1–5.2 — treating higher-order contacts and pion exchanges as perturbative corrections.

5. Boundary-condition reading (point interactions)

Point-supported interactions can be encoded as self-adjoint extension / boundary-condition data rather than as ordinary functions $V(r)$. This is the natural operator-theoretic counterpart of "diagonal-support kernels." For standard references and pedagogical framing, see [BonneauFarautValent2001SAE] and the delta-potential discussion in [Jackiw1991DeltaPotentials].

This viewpoint matches the controlled-refinement perspective: when a continuum description is defined as a refinement limit, UV data can survive in the limit precisely as boundary-condition parameters (contact terms), with RG flow expressing compatibility across resolutions.

Example 5.1 (Contact coupling generates a scale: 2D delta potential). In two spatial dimensions, a contact interaction $V(r) = g_0\,\delta^{(2)}(r)$ with bare coupling g_0 and UV cutoff Λ leads, after a standard loop integral, to a T-matrix with the structure

$$T(k)^{-1} = \frac{1}{g_0} + \frac{m}{\pi\hbar^2}\ln\left(\frac{\Lambda}{k}\right),$$

which diverges as $\Lambda \to \infty$ unless g_0 is tuned. Define a renormalized coupling at reference scale μ by absorbing the $\ln\Lambda$ divergence; cutoff independence then gives the beta function $\beta(g_R) = \mu\,dg_R/d\mu = (m/\pi\hbar^2)\,g_R^2$. This is a quadratic beta function of the same form as the toy logarithmic model in the cornerstone (Section 8.3), with solution producing a dynamically generated scale $\mu_* = \mu\,e^{\pi\hbar^2/(mg_R)}$. For attractive coupling ($g_R < 0$) this scale is below the reference scale and sets the bound-state energy: $E = -\hbar^2\mu_*^2/(2m)$.

The structural lesson: the "contact" limit of the effective interaction is not a number (coupling constant) but a flow — a scale-dependent parameter whose RG trajectory is part of the definition. This is "uncuttable" in the sense of the companion note: the continuum theory requires the refinement rule (cutoff removal + beta function) and not merely a single-cutoff value.

Example 5.2 (3D contact interaction: scattering length). In three spatial dimensions, the same contact interaction $V(r) = g_0\,\delta^{(3)}(r)$ with UV cutoff Λ produces a linearly divergent loop integral (compared to the logarithmic divergence in $d = 2$). After resummation, the s-wave scattering amplitude takes the standard effective-range form with zero effective range:

$$f_0(k) = \frac{-a}{1 + ika},$$

where the scattering length a is defined by absorbing the Λ-dependent bare coupling into a single physical parameter via a renormalization condition of the form $1/g_0 \propto \Lambda + $ (finite part depending on a). When $a > 0$, a pole at $k = i/a$ gives a bound state with energy $E = -\hbar^2/(2ma^2)$ [AlbeverioGesztesyHoeghKrohn-Holden2005]. The comparison with Example 5.1 highlights how the divergence

character changes with dimension — logarithmic ($d = 2$) versus linear ($d = 3$) — while the structural lesson is identical: the "coupling constant" of a contact interaction is not a bare number but a renormalization-group datum, defined only through a refinement rule (cutoff removal + physical matching condition).

Remark 5.3 (Unitarity limit: universality at the RG fixed point). When the scattering length diverges ($|a| \to \infty$), the contact coupling sits at a non-trivial RG fixed point and the theory becomes scale-invariant: no microscopic length survives beyond the interparticle spacing. Thermodynamic ratios become universal — for a spin-$\frac{1}{2}$ Fermi gas the ground-state energy is $E = \xi E_{\mathrm{FG}}$ with Bertsch parameter $\xi \approx 0.37$, independent of the short-range physics that produced the large scattering length. This fixed point controls the BEC–BCS crossover in cold atomic gases, where a magnetic Feshbach resonance tunes a through $\pm\infty$, providing a laboratory realization of the structural lesson in Examples 5.1–5.2: the contact coupling is not a number but a flow, and the fixed point of that flow generates universality.

Remark 5.4 (Dimensional dependence: from UV-finite to cutoff-dependent extension data). The same contact operator $\delta^{(d)}(r)$ requires qualitatively different control data across dimensions. In $d = 1$, the delta potential defines a self-adjoint extension of the free Hamiltonian whose boundary-condition parameter (the coupling g) is UV-finite — no cutoff dependence, no RG flow, just a number. In $d = 2$ (Example 5.1), the extension parameter diverges logarithmically with the cutoff, producing a running coupling and dimensional transmutation. In $d = 3$ (Example 5.2), the divergence is linear. The critical dimension is $d = 2$, where the contact coupling is classically marginal; above it, the coupling is classically relevant and power-law subtractions are needed. In all dimensions the interaction requires self-adjoint extension theory (it is never a "plain operator perturbation"), but the extension datum transitions from UV-finite ($d = 1$) to cutoff-dependent ($d \geq 2$) — and it is this transition that makes renormalization group flow part of the definition.

Remark 5.5 (Efimov effect: 3-body limit cycle from 2-body contact). At the unitarity limit of Remark 5.3, the 3-body sector exhibits a qualitatively new RG phenomenon: the running 3-body coupling is log-periodic in the cutoff with period π/s_0 (where $s_0 \approx 1.006$ for identical bosons), corresponding to a geometric energy scaling factor $e^{\pi/s_0} \approx 22.7$. This RG limit cycle — as opposed to the 2-body fixed point — produces an infinite tower of 3-body bound states with geometrically spaced energies $E_n \propto e^{-2\pi n/s_0}$, a signature of discrete scale invariance first predicted by Efimov (1970) and observed in cesium cold atoms by Kraemer et al. (2006). Crucially, the absolute position of the tower requires an additional 3-body parameter beyond the scattering length a — a new piece of self-adjoint extension data not determined by 2-body physics, illustrating that the contact/extension hierarchy grows richer with particle number.

Remark 5.6 (Feshbach resonances: laboratory realization of the contact-coupling RG flow). The theoretical framework of Examples 5.1–5.2 has a direct experimental counterpart: a magnetic Feshbach resonance occurs

when a bound state in a closed hyperfine channel is Zeeman-tuned into near-degeneracy with the open scattering threshold, producing a scattering length $a(B) = a_{\text{bg}}(1 - \Delta B/(B - B_0))$ that diverges as $B \to B_0$ (Chin et al., 2010). Sweeping B through the resonance traces the full one-parameter family of self-adjoint extensions: from weakly attractive ($a < 0$, BCS pairing) through the scale-invariant unitarity fixed point of Remark 5.3 ($|a| = \infty$) to the BEC regime of tightly bound dimers ($a > 0$), thereby realizing the BEC–BCS crossover as a continuous traversal of the RG flow. The Efimov states of Remark 5.5 have been observed near such Feshbach resonances in cesium (Kraemer et al., 2006). Structurally, the magnetic field B is an external knob that tunes the renormalized contact coupling at a fixed energy scale — the experimentalist's version of the beta function controlling the coupling flow in Examples 5.1–5.2.

Remark 5.7 (Casimir effect: macroscopic force from boundary conditions alone). The self-adjoint-extension/boundary-condition paradigm of Examples 5.1–5.2 generates a measurable macroscopic force in the Casimir effect (Casimir, 1948). Two perfectly conducting parallel plates separated by distance d impose boundary conditions that restrict the electromagnetic vacuum modes to a discrete set between the plates. The regulated difference of zero-point energies — computed via zeta-function regularization, exponential cutoff, or dimensional methods, all yielding the same result — gives an attractive force per unit area $F/A = -\pi^2 \hbar c/(240\,d^4)$, experimentally verified to percent-level precision (Lamoreaux, 1997; Mohideen and Roy, 1998). No classical source $J(x)$ is needed: the force arises purely from how the boundary conditions modify the vacuum fluctuation spectrum. The regularization-scheme independence of the result echoes the cutoff independence of the renormalized observables in Examples 5.1–5.2: the physically meaningful Casimir energy is the refinement limit (regulator removed), not any single regulated approximant. This is the boundary-condition paradigm of Section 5 elevated from a mathematical framework to a laboratory-observable phenomenon.

6. Outlook: emergent bosonic composites

Remark 6.1 (Cooper pairing: fermion bilinears as emergent bosonic mediators). The Grassmann-source obstruction of Section 3 is bypassed when fermion bilinears condense into bosonic composite modes. The canonical example is BCS superconductivity (Bardeen, Cooper, and Schrieffer, 1957): any attractive interaction in the Cooper channel between fermions near a Fermi surface produces a pairing instability with a non-perturbative gap $\Delta \propto \omega_D \exp(-1/(N(0)|V|))$, where $N(0)$ is the density of states and $|V|$ the interaction strength. The resulting condensate — described by the bosonic order parameter $\langle \psi_\downarrow \psi_\uparrow \rangle$ — gives mass to the photon via the Anderson–Higgs mechanism, with the effective photon mass $m_A \propto \sqrt{n_s}\,e^2$ set by the superfluid density n_s (proportional to $|\Delta|^2$ at zero temperature). This generates the Meissner effect: magnetic fields decay exponentially inside the superconductor

with penetration depth $\lambda = 1/m_A$. In the contact-interaction limit, the BCS gap equation requires the same renormalization as Examples 5.1–5.2: the bare coupling diverges with the cutoff, and the physical gap is determined by the renormalized scattering length. The BEC–BCS crossover of Remark 5.6 then interpolates continuously between overlapping Cooper pairs (BCS) and tightly bound bosonic molecules (BEC), demonstrating that the "emergent boson" spans the full range from collective many-body mode to genuine two-body bound state.

Remark 6.2 (Neutrino-mediated forces: the direct fermionic potential). The paper's motivating question — "can a fermionic field generate a potential?" — has a direct quantitative answer. Feinberg and Sucher (1968) computed the long-range force between fermions mediated by virtual neutrino-antineutrino pair exchange (a box diagram with two neutrino propagators and two weak vertices). The result is $V(r) \propto G_F^2/r^5$ — a power-law potential steeper than the Coulomb $1/r$ or Yukawa e^{-mr}/r of bosonic exchange, with the $1/r^5$ scaling reflecting the two-fermion intermediate state. Crucially, this is not a tree-level exchange (which would require Grassmann sources, per Remark 3.1) but a loop effect: two fermion propagators carrying the force, exactly as Section 3.1's controlled alternative predicts. The power law (rather than exponential suppression) arises because the neutrino is (nearly) massless; for massive neutrinos, the potential crosses over to exponential decay at distances $r \gg 1/m_\nu$. At short distances $r \ll 1/M_W$, the two W-boson propagators collapse to the Fermi four-fermion vertex, and the force reduces to the contact expansion of Section 4 — the neutrino-mediated potential is the long-range tail of a contact interaction viewed at low resolution.

Remark 6.3 (Nuclear forces from pion exchange: the paradigmatic fermion-to-boson-to-potential chain). The Grassmann-source obstruction of Section 3 is bypassed most famously in the nuclear force. Quarks (fermions) bind into pions — pseudoscalar bosonic composites with the quantum numbers of $\bar{q}\gamma_5\boldsymbol{\tau}q$ (where $\boldsymbol{\tau}$ are isospin Pauli matrices) — which are the pseudo-Goldstone bosons of spontaneously broken chiral symmetry. Once formed, pions mediate the longest-range part of the nucleon-nucleon interaction through one-pion exchange (OPEP), giving a Yukawa potential $V(r) \propto e^{-m_\pi r}/r$ with range $\sim 1/m_\pi \approx 1.4$ fm, as predicted by Yukawa (1935). The OPEP includes a tensor component (proportional to the spin-dependent operator $S_{12} = 3(\boldsymbol{\sigma}_1 \cdot \hat{r})(\boldsymbol{\sigma}_2 \cdot \hat{r}) - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$) responsible for the deuteron quadrupole moment and D-state admixture. At distances shorter than the pion Compton wavelength, the pion-exchange potential is not resolved and collapses into the contact expansion of Section 4 — the same structural pattern as the neutrino-mediated force of Remark 6.2. Modern chiral effective field theory (Weinberg, 1990; Epelbaum, Hammer, and Meißner, 2009) organizes the nuclear force as a systematic hierarchy: pion exchanges provide the long-range, non-analytic-in-momentum contributions, while short-range physics is encoded by the contact operators $C_0, C_2, \dots$ of Derivation 4.1, with the expansion parameter Q/Λ_χ (where $\Lambda_\chi \sim 1$ GeV is the chiral symmetry breaking scale).

This is the complete realization of the paper's structural chain: fermions (quarks) $\to$ emergent bosons (pions) $\to$ Yukawa potential (long range) + contact operators (short range), with the EFT framework providing the systematic power counting that relates the two regimes.

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