

# “Uncuttable” as Controlled Refinement

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## Abstract

This note fixes a meaning of “uncuttable” aligned with the refinement-compatibility thesis of the cornerstone manuscript. Here **uncuttable** does not mean “indivisible.” It means: the quantity of interest is not determined by any *finite* dissection alone; it is a **limit object** whose definition requires a refinement rule and a comparison structure across refinements.

The point is structural and mathematical: once a theory is built from composable local pieces, the continuum theory is the stable target of a refinement limit, and extra control data may be required for that limit to exist or be unique.

## 1. Definition

Call a quantity  $Q$  **uncuttable** (in this note’s sense) if: 1. there exists a family of finite approximants  $Q_N$  produced by a finite dissection/refinement scheme of depth  $N$ , but 2. the value of interest is not any finite  $Q_N$ ; it is a controlled limit  $Q = \lim_{N \rightarrow \infty} Q_N$ , and 3. specifying the *rule of refinement* and the *comparison across refinements* is part of the definition of  $Q$ .

This is the ordinary situation in analysis: finite partitions approximate, but the object is defined by a limiting procedure together with hypotheses that ensure convergence/uniqueness.

## 2. Toy model: an integral is already a refinement limit

Let  $f : [a, b] \rightarrow \mathbb{R}$ . A prototypical refinement family is a partition  $P_N = \{a = t_0 < \dots < t_N = b\}$  with mesh  $\|P_N\| := \max_k (t_{k+1} - t_k) \rightarrow 0$ . Define the Riemann-sum approximants

$$Q_N := \sum_{k=0}^{N-1} f(\xi_k) (t_{k+1} - t_k), \quad \xi_k \in [t_k, t_{k+1}].$$

In good cases,  $Q_N \rightarrow \int_a^b f(t) dt$  as  $\|P_N\| \rightarrow 0$ , and the limit is independent of the tags  $\xi_k$ . But this is not a tautology: the limit can fail to exist or can depend on the refinement rule unless hypotheses are stated.

In the present program, this is the basic moral: finite cuts approximate, but the value is defined by **controlled refinement**.

### 3. Dynamics: action, stationarity, and the need for control data

The cornerstone manuscript uses the same template in mechanics. Given a partition of time, the discrete action

$$S_N[q] = \sum_k \mathcal{L}\left(q_k, \frac{q_{k+1} - q_k}{\Delta t_k}, t_k\right) \Delta t_k$$

is a finite refinement approximant. The continuum action  $S[q] = \int \mathcal{L} dt$  is a refinement limit.

Two “uncuttable” features appear immediately when one pushes beyond smooth classical paths: 1. **Singular probes and corners:** stationarity must be interpreted in weak/distributional form; point-supported variations require mollification. 2. **Non-uniqueness of refinement schemes:** different discretization conventions (even if classically equivalent) can produce distinct refined objects unless an equivalence or control map is specified.

These are exactly the obstructions discussed in the cornerstone manuscript: the point is not indivisible atoms, but limit control.

### 4. Outlook: refinement compatibility as “the extra structure”

In the companion papers, the “extra structure” used to control refinement limits is made explicit: - half-densities make kernel composition coordinate-free without hidden measure choices, - control maps  $\tau$  encode how parameters must flow under refinement to maintain stability, - renormalization is the compatibility rule when naive refinement limits diverge.

This note is therefore a small conceptual bridge: it isolates an early, analysis-level instance of the same meta-problem that reappears in quantization and in QFT.

## References

1. [BatesWeinstein1997] Sean Bates and Alan Weinstein, *Lectures on the Geometry of Quantization*, Berkeley Mathematics Lecture Notes, vol. 8,

- AMS, 1997. OA: <https://math.berkeley.edu/~alanw/GofQ.pdf>. (Half-density formalism for coordinate-free kernel composition.)
2. [FeynmanHibbs1965] Richard P. Feynman and Albert R. Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill, 1965. (Path integral as a refinement limit of time-sliced amplitudes.)