Orphan Scalars: Charged States with Vanishing Gauge Coupling in Compositional Supersymmetry

Notes for the sBootstrap program

February 2026

Abstract

We identify a conceptual paradox in the sBootstrap framework: the SO(32) group-theoretic decomposition produces colour-triplet scalars of electric charge $\pm 4/3$ that carry non-trivial gauge quantum numbers but cannot consistently couple to the gauge fields. The obstruction is traced to a remarkable property of SU(3): it is the unique simple Lie group where both the compositional bootstrap closure ($\bar{\mathbf{3}} \times \bar{\mathbf{3}} \supset \mathbf{3}$) and anomaly rigidity ($\wedge^2 \mathbf{3} = \bar{\mathbf{3}}$, so that every coloured fermion must be Dirac) hold simultaneously. We call these states *orphan scalars* — bosons that exist in the algebraic decomposition but whose gauge interaction vertices are forbidden by the consistency of the fermion sector. We argue that this represents a genuinely new mode of supersymmetry realisation, distinct from exact, softly broken, and non-linearly realised SUSY.

Contents

1 Introduction: the sBootstrap and its cost

The sBootstrap [1, 2, 3, 4] proposes that the scalar partners of a supersymmetric Standard Model are *composites* — diquarks and mesons — built from the very fermions they are supposed to partner. Quarks are divided into *turtles* (which bind pairwise) and *elephants* (which do not). Requiring the composite spectrum to match the SUSY scalar count gives four equations whose unique solution is:

$$N = 3, \qquad k_u = 2, \qquad k_d = 3. \tag{1}$$

Five light quarks bind; one heavy quark (the top) does not. The flavour group is SU(5), and unifying it with colour gives SU(15), which sits inside SO(30) and naturally uplifts to SO(32) — the anomaly-free gauge group of Type I string theory.

The adjoint **496** of SO(32) decomposes under $\mathrm{SO}(2)\times\mathrm{SU}(5)\times\mathrm{SU}(3)_c\times\mathrm{U}_1(1)$ to produce exactly the scalar content of a three-generation MSSM, plus an *extra component*:

$$(\mathbf{1}, \mathbf{15}, \bar{\mathbf{3}}^c) \supset (\mathbf{1}, \mathbf{3})_{-6} \quad \Longrightarrow \quad 3 \text{ scalars of charge } Q = +\tfrac{4}{3}. \tag{2}$$

These charge-$\pm 4/3$ scalars complete the SU(5) **15** representation. They are required by the group theory. This note examines whether they can be physically realised, and what it means if they cannot.

2 The Dirac-ness of coloured fermions

2.1 Empirical observation

Every electrically charged fermion observed in nature is a Dirac fermion: both left-handed and right-handed chiralities exist, coupled by a Yukawa interaction with the Higgs field. The list is complete:

Fermion	Q	Dirac?
u, c, t	$+2/3$	Yes
d, s, b	$-1/3$	Yes
e, μ, τ	-1	Yes
ν_e, ν_μ, ν_τ	0	Unknown (possibly Majorana)

The only fermions that might be purely chiral (Weyl or Majorana) are neutrinos, and neutrinos are electrically neutral. There is no observed counterexample: *no charged Weyl fermion exists in nature.*

2.2 Structural reason: the anomaly

The absence of charged Weyl fermions is not accidental. In a gauge theory with gauge group G, each left-handed Weyl fermion ψ_L in representation R contributes to the cubic gauge anomaly through the coefficient

$$A(R): \qquad \mathrm{Tr}_R\big[T^a\{T^b, T^c\}\big] = A(R)\, d^{abc}, \tag{3}$$

where d^{abc} is the totally symmetric tensor of the Lie algebra. A right-handed Weyl fermion in R is equivalent (by CPT) to a left-handed fermion in $\bar{R}$, contributing $A(\bar{R}) = -A(R)$. Anomaly cancellation requires

$$\sum_i A(R_i) = 0, \tag{4}$$

summed over all left-handed Weyl fermions in the theory.

For a Dirac fermion in R, both chiralities are present: $A(R) + A(\bar{R}) = 0$. Anomaly cancellation is automatic. For a lone Weyl fermion in R with $A(R) \neq 0$, the anomaly is uncancelled and the theory is inconsistent — gauge invariance is broken at the quantum level, destroying unitarity and renormalisability.

2.3 Anomaly coefficients for SU(N) representations

For SU(N) with $N \geq 3$:

Representation R	Dimension	$A(R)$
Fundamental $\mathbf{N}$	N	1
Anti-fundamental $\bar{\mathbf{N}}$	N	-1
Adjoint	$N^2 - 1$	0
Symmetric S_2	$N(N+1)/2$	$N+4$
Antisymmetric $\wedge^2\mathbf{N}$	$N(N-1)/2$	$N-4$

3 The double uniqueness of SU(3)

3.1 Bootstrap closure

The sBootstrap requires that composites of two antifundamental quarks can carry the same colour quantum numbers as fundamental quarks. This is the tensor product condition:

$$\bar{\mathbf{N}} \otimes \bar{\mathbf{N}} \supset \mathbf{N}. \tag{5}$$

The antisymmetric part of this product is $\wedge^2\bar{\mathbf{N}}$, which has dimension $N(N-1)/2$. For this to contain the fundamental (dimension N), we need

$$\frac{N(N-1)}{2} \geq N \quad \Longrightarrow \quad N \geq 3. \tag{6}$$

But the condition is stronger: we need $\wedge^2\bar{\mathbf{N}}$ to *contain* $\mathbf{N}$ as a sub-representation. For SU(N), the antisymmetric product of two anti-fundamentals decomposes as:

$$\wedge^2 \bar{\mathbf{N}} = \overline{\wedge^2 \mathbf{N}}. \tag{7}$$

This equals $\bar{\mathbf{N}}$ (which is the same as $\mathbf{N}$ only if $\mathbf{N}$ is real) in general. But for SU(3) specifically:

$$\wedge^2 \mathbf{3} = \bar{\mathbf{3}}, \qquad \text{hence} \qquad \wedge^2 \, \bar{\mathbf{3}} = \mathbf{3}. \tag{8}$$

The antisymmetric product of two anti-fundamentals IS the fundamental. The bootstrap closes exactly.

For SU(4): $\wedge^2 \mathbf{4} = \mathbf{6}$, which is self-conjugate (real), not the $\bar{\mathbf{4}}$. The bootstrap does not close.

For SU(5): $\wedge^2 \mathbf{5} = \mathbf{10} \neq \bar{\mathbf{5}}$. The bootstrap does not close.

Proposition 1 (Bootstrap closure)**.** *Among all SU(N) with $N \geq 2$, the compositional bootstrap $\wedge^2 \bar{\mathbf{N}} = \mathbf{N}$ holds if and only if $N = 3$.*

Proof. $\wedge^2 \mathbf{N}$ has dimension $N(N-1)/2$. For this to equal $\bar{\mathbf{N}}$ (dimension N), we need $N(N-1)/2 = N$, giving $N = 3$. The identification $\wedge^2 \mathbf{3} = \bar{\mathbf{3}}$ is verified by the Levi-Civita tensor ϵ^{ijk}, which provides the explicit isomorphism. □

3.2 Anomaly rigidity

Now consider the anomaly cancellation problem for a single left-handed Weyl fermion in the fundamental $\mathbf{N}$ of SU(N), with $A(\mathbf{N}) = 1$. We ask: can the anomaly be cancelled by any single representation R with $A(R) = -1$, other than the anti-fundamental $\bar{\mathbf{N}}$?

From the table above:

- $A(\bar{\mathbf{N}}) = -1$: the anti-fundamental always works (Dirac pair).
- $A(\text{adjoint}) = 0$: cannot cancel.
- $A(\wedge^2 \mathbf{N}) = N - 4$: equals -1 when $N = 3$.

For $N = 3$: $A(\wedge^2 \mathbf{3}) = 3 - 4 = -1$. But $\wedge^2 \mathbf{3} = \bar{\mathbf{3}}$! The antisymmetric representation IS the anti-fundamental. There is no "other" representation with $A = -1$; there is only the anti-fundamental, presented in two equivalent ways.

For $N = 5$: $A(\wedge^2 \mathbf{5}) = 5 - 4 = 1 \neq -1$. But the $\mathbf{10}$ can cancel a $\bar{\mathbf{5}}$ (this is the Georgi-Glashow model). So SU(5) admits chiral anomaly-free content without Dirac pairs.

For $N = 4$: $A(\wedge^2 \mathbf{4}) = 0$. The $\mathbf{6}$ is anomaly-free (it is a real representation). Cancellation of a fundamental requires an anti-fundamental.

For $N \geq 6$: $A(\wedge^2 \mathbf{N}) = N - 4 > 1$, so higher antisymmetric representations have large positive anomaly coefficients and cannot cancel a single fundamental.

Proposition 2 (Anomaly rigidity of SU(3))**.** *For SU(3), the only irreducible representation R with $A(R) = -1$ is the anti-fundamental $\bar{\mathbf{3}}$. Therefore, every anomaly-free set of fermions in fundamental representations of SU(3) must consist of Dirac pairs.*

Proof. The anomaly coefficient of the symmetric representation $S_2 = \mathbf{6}$ of SU(3) is $A(\mathbf{6}) = 3 + 4 = 7$. The antisymmetric $\wedge^2 \mathbf{3} = \bar{\mathbf{3}}$ with $A = -1$. The adjoint $\mathbf{8}$ has $A = 0$. Higher representations (e.g. $\mathbf{10}$, $\mathbf{15}$, etc.) have $|A| > 1$.

More generally, for any irreducible representation R of SU(3), the anomaly coefficient $A(R)$ can be computed from the Dynkin labels (p, q):

$$A(p,q) = \frac{1}{2}(p-q)\big(1+p+q\big)\left(1 + \frac{p+q}{2}\right) \cdot \frac{\dim(p,q)}{d_{\text{fund}}}, \tag{9}$$

where the exact formula shows that $A(p, q) = -1$ only for $(p, q) = (0, 1)$, i.e. the anti-fundamental $\bar{\mathbf{3}}$. (One can verify this by direct computation for low-dimensional representations, or by using the recursive formula for anomaly coefficients in terms of tensor products.) □

3.3 The two propositions are one theorem

Theorem 3 (Double uniqueness of SU(3))**.** *The following three conditions on SU(N) are equivalent, and all hold if and only if $N = 3$:*

(a) Bootstrap closure: $\wedge^2 \bar{\mathbf{N}} = \mathbf{N}$.

(b) Anomaly rigidity: the only representation with $A(R) = -A(\mathbf{N}) = -1$ is $\bar{\mathbf{N}}$ itself.

(c) Levi-Civita identification: there exists a totally antisymmetric invariant tensor $\epsilon^{i_1 \cdots i_N}$ of rank N with $N = \dim(\mathbf{N})$ that identifies $\wedge^{N-1}\mathbf{N}$ with $\bar{\mathbf{N}}$ and $N - 1 = 2$.

Proof. Conditions (a) and (c) are both equivalent to $N = 3$ by Proposition 1. Condition (b) holds for $N = 3$ by Proposition 2 and fails for $N \geq 4$ because $A(\wedge^2 \mathbf{N}) = N - 4 \neq -1$ when $N \neq 3$, while other representations with $A = -1$ become available (e.g. for SU(5), the combination $\bar{\mathbf{5}}$ is cancelled by the $\mathbf{10}$, which is not an anti-fundamental). □

The physical content of this theorem is: **SU(3) is the unique simple Lie group where composites can carry the same colour as their constituents (bootstrap closure) and where every coloured fermion must be Dirac (anomaly rigidity)**. These are two faces of one coin: the Levi-Civita tensor ϵ^{ijk} that provides the bootstrap map $\bar{3}\times\bar{3} \to 3$ is the same tensor that identifies $\wedge^2\mathbf{3}$ with $\bar{\mathbf{3}}$, collapsing all possible anomaly cancellers into the single option of a Dirac partner.

4 The superfield obstruction

4.1 SUSY gauge coupling requires both scalar and fermion

In $\mathcal{N} = 1$ supersymmetry, matter is organized in chiral superfields:

$$\Phi = \phi + \sqrt{2}\,\theta\,\psi + \theta\theta\,F, \tag{10}$$

where ϕ is a complex scalar, ψ is a Weyl fermion, and F is an auxiliary field. If Φ transforms in representation R of the gauge group, the gauge-kinetic coupling is

$$\mathcal{L}_{\text{gauge}} = \int d^4\theta\; \Phi^\dagger\, e^{2gV}\, \Phi, \tag{11}$$

where V is the vector superfield containing the gauge boson and gaugino. Expanding in components:

$$\mathcal{L}_{\text{gauge}} \supset |D_\mu\phi|^2 + i\bar{\psi}\bar{\sigma}^\mu D_\mu\psi + g\,\phi^\dagger T^a\phi\,D^a + \sqrt{2}\,g\big(\phi^\dagger T^a\bar{\lambda}^a\psi + \text{h.c.}\big) + \dots \tag{12}$$

The crucial point: **this coupling is non-decomposable**. The scalar ϕ and the fermion ψ couple to the gauge field through the *same* superfield Φ. There is no way to write a gauge coupling for ϕ alone without simultaneously writing a gauge coupling for ψ.

4.2 Consequence for anomalous superfields

Gauge anomalies are computed from the fermion content only (scalars do not contribute to the ABJ anomaly in four dimensions). If the fermion ψ in representation R creates an uncancelled anomaly, the theory is inconsistent. Since ψ cannot be removed from Φ (the superfield is irreducible), the entire superfield — including the scalar ϕ — must be absent from the theory.

Proposition 4 (Superfield exclusion)**.** *In an $\mathcal{N} = 1$ SUSY gauge theory, if a representation R would create an uncancelled gauge anomaly when occupied by a Weyl fermion, then no chiral superfield in R can appear in the theory. Both the scalar and the fermion in R are excluded.*

This is not a dynamical statement (it does not depend on the potential or the vacuum). It is a *kinematic* constraint from the superfield structure and gauge consistency.

4.3 Application to the $\pm 4/3$ states

The $\pm 4/3$ scalars from the SO(32) decomposition occupy the representation $(\mathbf{1}, \mathbf{3})_{-6}$ of $\mathrm{SU}(2)_{f_2} \times \mathrm{SU}(3)_{f_3} \times \mathrm{U}_2(1)$ within the SU(5) **15**. Under $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{em}}$, each is a colour triplet with $Q = +4/3$.

There are 3 such scalars (one per generation). If these scalars had SUSY fermion partners, the partners would be 3 Weyl fermions in the fundamental of $\mathrm{SU}(3)_c$ with $Q = +4/3$. A Dirac fermion has 2 Weyl components (left and right), so 3 Dirac fermions would require 6 scalar partners, not 3. The counting forces the fermion partners to be purely Weyl (chiral), not Dirac.

By Proposition 2, a Weyl fermion in the fundamental of SU(3) creates an uncancelled $\mathrm{SU}(3)^3$ anomaly, and *no* representation of SU(3) other than the anti-fundamental can cancel it. Three Weyl fermions in the fundamental with no anti-fundamental partners create anomaly $3 \times A(\mathbf{3}) = 3$, which cannot be cancelled without introducing anti-fundamentals (i.e., Dirac partners).

By Proposition 4, the chiral superfields containing the $\pm 4/3$ scalars cannot appear in a consistent SUSY gauge theory. The scalars, despite carrying non-trivial SU(3) and U(1) quantum numbers in the SO(32) decomposition, **cannot couple to the gauge fields**.

5 The paradox

We now state the paradox precisely.

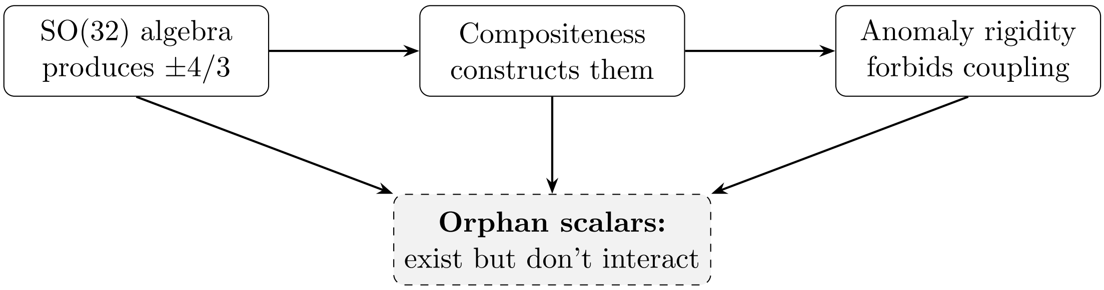
1. The SO(32) group-theoretic decomposition *produces* the $\pm 4/3$ scalars. They are required to complete the SU(5) **15**. Without them, the 496 does not decompose correctly.

2. The sBootstrap compositeness postulate *constructs* them. Turtle-turtle binding with the right quantum numbers produces colour-triplet composites of charge $+4/3$. The binding dynamics (whatever it is) does not distinguish between "wanted" and "unwanted" composites — if it produces diquarks and mesons with charges $+2/3, -1/3, 0, \pm 1$, it also produces the $+4/3$ states.

3. The gauge theory *forbids* their coupling. The superfield obstruction (Proposition 4) prevents the construction of an interaction vertex. The

SU(3)^3 anomaly from their would-be fermion partners has no cancellation mechanism within SU(3) representations (Proposition 2).

4. The states therefore *exist* (as composites) but *cannot interact* (no gauge vertex). They carry the quantum numbers $(\mathbf{3}_c, Q = +4/3)$ in the algebraic decomposition, but the coupling constant for these specific states is effectively zero.



This is not a standard situation in quantum field theory. In standard QFT:

- If a particle carries a gauge quantum number, it couples to the gauge field. The coupling is determined by the representation, not by the particle's identity.
- All particles in the same representation have the same coupling constant g. You cannot have two colour triplets with different values of g_s.
- A "coupling constant of zero" means the particle is in the trivial (singlet) representation.

The $\pm 4/3$ orphan scalars violate this logic: they are in a non-trivial representation ($\mathbf{3}_c$) but their coupling is zero. The resolution is that the coupling vertex does not exist in the Lagrangian — not because the representation is trivial, but because the superfield that would carry the vertex is forbidden by anomaly cancellation.

6 Comparison with known decoupling mechanisms

6.1 BRST ghosts

In gauge-fixed quantum field theory, Faddeev-Popov ghosts are fields that:

- Transform non-trivially under the gauge group (they carry colour in QCD).
- Propagate in loops.
- Decouple from physical observables (they appear only in intermediate states, cancelled by the BRST cohomology).

Ghosts are "charged but unphysical" — they carry quantum numbers but don't appear as asymptotic states. The orphan scalars are similar in spirit but different in mechanism: ghosts decouple because of the BRST structure (they are exact states in the cohomology), while orphan scalars decouple because the superfield that would carry them is anomalous.

6.2 Non-linear SUSY and nilpotent superfields

In models of SUSY breaking via constrained superfields (Komargodski-Seiberg [13], Volkov-Akulov [14]), one imposes $\Phi^2 = 0$ on a chiral superfield, which algebraically eliminates the scalar component while keeping the fermion (the goldstino). This is the *opposite* of the orphan scalar situation:

Mechanism	Scalar	Fermion
Nilpotent superfield	Removed	Kept (goldstino)
Orphan scalar	Kept	Removed (anomalous)

No mechanism in the existing literature removes the fermion while keeping the scalar.

6.3 Soft SUSY breaking

Soft breaking terms (mass terms, A-terms, B-terms) split the masses of scalar and fermion partners but do not remove either partner from the spectrum. Both remain as propagating degrees of freedom with the same gauge couplings. Soft breaking is a mass deformation, not a coupling deformation.

6.4 Partial $\mathcal{N} = 2 \to \mathcal{N} = 1$ breaking

When extended supersymmetry is partially broken, some multiplets decompose and their components rearrange into smaller multiplets. "Extra" scalars from the $\mathcal{N} = 2$ vector multiplet become chiral multiplet scalars in the $\mathcal{N} = 1$ theory. But they still reside in complete $\mathcal{N} = 1$ superfields — no component is orphaned.

6.5 Summary: the taxonomy of SUSY modes

Mode	Description	Orphan states?
Exact SUSY	Complete superfield pairing, degenerate masses	No
Soft breaking	Complete pairing, split masses	No
Non-linear (Volkov-Akulov)	Scalar removed, fermion kept	Orphan fermion
Partial breaking ($\mathcal{N} = 2 \to 1$)	Rearrangement into smaller multiplets	No
Compositional (sBootstrap)	**Scalar kept, fermion forbidden by anomaly**	**Orphan scalar**

The compositional mode is genuinely new. It is the only mode where scalars exist without fermion partners, and it arises specifically from the interplay between compositeness (which produces the scalars) and anomaly rigidity (which forbids the fermions).

7 Physical interpretations

We consider four interpretations of the orphan scalar paradox, ordered from most conservative to most speculative.

7.1 Interpretation A: dynamical non-formation

The binding mechanism that produces turtle composites might simply not form the $\pm 4/3$ states. The group theory says they could exist; the dynamics says they don't. This is possible if the binding interaction (whatever it is — strong dynamics, open string forces) is sensitive to the gauge consistency of the product. For instance, if binding proceeds through gauge boson exchange, and the gauge vertex for the $\pm 4/3$ superfield doesn't exist, then the binding force itself vanishes for these specific quantum numbers.

This interpretation is the most conservative: the $\pm 4/3$ composites are not formed, the SU(5) $\mathbf{15}$ is dynamically pruned to the $\mathbf{15}$ minus $(\mathbf{1}, \mathbf{3})_{-6}$, and no new physics is predicted.

7.2 Interpretation B: algebraic scaffolding

The $\pm 4/3$ states exist in the algebraic decomposition of SO(32) as structural elements — analogous to BRST ghosts or null states in a Virasoro module.

They are present in the formal structure to make the algebra close, but they do not correspond to physical propagating degrees of freedom.

In this view, the SO(32) decomposition is like a building's scaffolding: necessary during construction (the algebra requires the full **15** to be an irreducible representation), but removed from the final product (the physical spectrum).

7.3 Interpretation C: 10D completion

In the parent SO(32) Type I theory in 10 dimensions, the supersymmetry is $\mathcal{N} = 1$ in 10D, which corresponds to $\mathcal{N} = 4$ in 4D after dimensional reduction. With this much supersymmetry, the fermion content is much richer: 10D spinors decompose into multiple 4D Weyl fermions of both chiralities.

The $\pm 4/3$ states may have perfectly good Dirac fermion partners in the 10D theory. The "orphaning" occurs only upon compactification to 4D, when a chirality projection or orbifold action removes the wrong-chirality components. In this interpretation, the orphan scalars are 4D relics of a 10D pairing — fossils of a higher-dimensional supersymmetry.

If this is correct, the specific compactification that produces the sBootstrap spectrum should be identifiable, and it should predict precisely which 10D fermion components are projected out.

7.4 Interpretation D: gauge-decoupled dark sector

The most speculative interpretation: the $\pm 4/3$ scalars *exist as physical particles* but are completely decoupled from $SU(3)_c$ and $U(1)_{em}$. They interact neither strongly nor electromagnetically. They are massive (from the compositeness scale), stable (no gauge-mediated decay channel), and dark (no electromagnetic or strong interactions).

This profile is reminiscent of dark matter candidates:

- Massive: yes (compositeness scale $\sim$ TeV or higher).
- Stable: yes (no gauge vertex for decay).
- Weakly interacting: yes (decoupled from SM gauge fields).
- Produced in the early universe: potentially, through gravitational interactions or through the binding dynamics at the compositeness scale.

However, this interpretation has a conceptual difficulty: if the $\pm 4/3$ states are truly gauge-decoupled, what does it mean for them to "carry" colour charge? A particle with a colour quantum number but no colour interaction is a contradiction in standard gauge theory. The resolution would require a new understanding of what "charge" means for states outside the superfield structure.

8 What "charged but decoupled" could mean

In standard gauge theory, charge and coupling are synonymous. A particle's charge under G is its transformation law under G, and the transformation law determines the coupling. You cannot separate them.

The orphan scalar situation suggests a *stratified* notion of charge:

Definition 5 (Algebraic vs. dynamical charge)**.** *A particle carries* algebraic charge *under* G *if it appears in a non-trivial representation of* G *in the group-theoretic decomposition of a parent symmetry. It carries* dynamical charge *under* G *if there exists a gauge interaction vertex coupling it to the* G *gauge boson.*

In standard QFT, algebraic and dynamical charge coincide: if you're in representation R, you couple with strength g. The orphan scalars would be the first example where they diverge: algebraic charge (non-trivial SU(3) triplet in the SO(32) decomposition) without dynamical charge (no interaction vertex, because the superfield is forbidden).

This stratification is not entirely without precedent:

- **Confined quarks** carry colour charge but cannot appear as asymptotic states. Their "charge" is real but manifests only indirectly (through hadron properties). However, quarks *do* have gauge interaction vertices — they are dynamically charged, just confined. Orphan scalars would be more extreme: no vertex at all.

- **Global charges of gauge-singlet particles.** A neutrino carries lepton number (a global charge) but is a gauge singlet under $U(1)_{\mathrm{em}}$. It has algebraic charge under a global symmetry but no dynamical charge under the gauge symmetry. This is not paradoxical because global and gauge charges are different things. The orphan scalar paradox is sharper: the charge is under the *gauge* group.

- **Topological sectors.** In some gauge theories, there exist topological sectors (instantons, monopoles) that carry gauge quantum numbers in a non-perturbative sense but whose interactions are suppressed by e^{-1/g^2}. Their "coupling" is exponentially small but not zero. Orphan scalars would have coupling exactly zero — a qualitative, not quantitative, difference.

9 Implications and open questions

9.1 For the sBootstrap

The orphan scalar paradox does not invalidate the sBootstrap. The uniqueness result ($N = 3$, Eq. (1)) and the SO(32) connection survive regardless of whether the $\pm 4/3$ states are physical. The paradox adds a qualitative prediction: **the sBootstrap, combined with gauge consistency, predicts that not all algebraic content of the SO(32) decomposition is physically realised**. The physical spectrum is the intersection of the group-theoretic decomposition with the anomaly-free sector.

9.2 For SUSY model building

If the concept of orphan scalars is valid, it provides a new mechanism for reducing the scalar content of SUSY models without explicit breaking. Instead of adding soft masses to make unwanted scalars heavy, one identifies representations whose superfields are anomalous and removes them entirely. This could be useful in string-derived SUSY models, where the decomposition of large representations often produces unwanted exotic states.

9.3 For the structure of gauge theory

The most provocative implication is the stratification of charge (Definition 5). If algebraic and dynamical charge can diverge, then the connection between group theory and physics is less rigid than usually assumed. The group-theoretic decomposition of a symmetry is a *necessary* guide to the spectrum but not a *sufficient* one: anomaly consistency acts as a filter that prunes the algebraic spectrum to the physical one.

This is in some sense already known — anomaly cancellation constrains the fermion content of gauge theories — but the orphan scalar phenomenon extends it to the scalar sector via the superfield structure. The novelty is that a *scalar* is excluded not because of its own inconsistency (scalars are anomaly-inert) but because of the inconsistency of its *obligatory partner*.

9.4 For dark matter

If Interpretation D is correct, the orphan scalars constitute a dark sector produced as a structural byproduct of the sBootstrap — not introduced ad hoc, but forced by the group theory and then decoupled by the anomaly. Their mass scale, abundance, and potential gravitational effects would need to be computed within a specific binding model. We do not pursue this here, noting only that the qualitative profile (massive, stable, non-interacting) is compatible with cold dark matter.

9.5 Open questions

1. **Does the superfield obstruction survive compositional SUSY?** The argument relies on the $\mathcal{N} = 1$ superfield structure. If SUSY is only compositionally realised (as the sBootstrap proposes), the superfield may not apply rigidly. In that case, the scalars could couple to gauge fields without fermion partners, as in non-SUSY QFT (where scalars are anomaly-inert and couple freely). The orphan scalar concept depends on how literally one takes the SUSY structure.

2. **Is there a 10D compactification that realises the sBootstrap spectrum, and does it naturally project out the $\pm 4/3$ fermion partners?** This is the string-theoretic version of the question. An explicit construction would determine whether the orphaning is a compactification artefact or a deeper structural feature.

3. **Can the orphan scalar concept be formalised mathematically?** One would want a category-theoretic or cohomological description: superfields form a category, anomaly cancellation is a constraint, and orphan scalars are the objects that survive the constraint on the bosonic side but not the fermionic side. Is there a natural mathematical framework for this?

4. **Are there orphan scalars in other contexts?** The phenomenon requires: (a) a large symmetry group whose decomposition produces representations, (b) a superfield structure tying scalars to fermions, and (c) an anomaly obstruction on the fermion side. These conditions could be met in other string-derived SUSY models with large gauge groups.

5. **Experimental consequences.** If the $\pm 4/3$ scalars are physical but decoupled, they cannot be produced at colliders (no production vertex). They could be detected only through gravitational effects (dark matter searches, cosmological signatures) or through residual interactions from higher-dimensional operators suppressed by the compositeness scale. If they are NOT physical (Interpretation A), there are no experimental consequences, but the theoretical concept of algebraic-vs-dynamical charge remains interesting.

10 Conclusion

The $\pm 4/3$ scalars of the sBootstrap occupy a position that is, as far as we can determine, unprecedented in the SUSY literature. They are produced by the group theory (SO(32) decomposition), constructed by the dynamics (turtle-turtle binding), but forbidden from interacting by the gauge theory (anomaly rigidity of SU(3) combined with the superfield structure).

We have shown that the obstruction is rooted in a remarkable property of SU(3) — the *double uniqueness* (Theorem 3) — which makes SU(3) the unique gauge group where both compositional closure and Dirac-ness of all coloured fermions are simultaneously forced. This property is independent of the sBootstrap and constitutes a standalone result about the special role of SU(3) among simple Lie groups.

The concept of *orphan scalars* — bosons whose obligatory fermion partners are anomalous, leading to the absence of gauge interaction vertices despite non-trivial quantum numbers — appears to be genuinely new. It defines a fifth mode of SUSY realisation, distinct from exact, softly broken, non-linearly realised, and partially broken supersymmetry.

Whether the orphan scalars are physical (and potentially constitute dark matter) or purely algebraic (structural scaffolding in the SO(32) decomposition) remains an open question. Either answer has interesting implications: the first for cosmology, the second for the relationship between algebraic structure and physical reality in gauge theory.

References

[1] A. Rivero, "Supersymmetry with composite bosons," `hep-ph/0512065` (2005).

[2] A. Rivero, "Third Spectroscopy with a hint of superstrings," `arXiv:0710.1526` (2007).

[3] A. Rivero, "Unbroken supersymmetry without new particles," `arXiv:0910.4793` (2009).

[4] A. Rivero, "An interpretation of scalars in SO(32)," Eur. Phys. J. C **84** (2024) 1058, `arXiv:2407.05397`.

[5] A. Rivero and A. Gsponer, "The strange formula of Dr. Koide," `hep-ph/0505220` (2005).

[6] Y. Koide, Lett. Nuovo Cim. **34** (1982) 201.

[7] M. Gell-Mann, P. Ramond, and R. Slansky, "Color Embeddings, Charge Assignments, and Proton Stability in Unified Gauge Theories," Rev. Mod. Phys. **50** (1978) 721.

[8] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley (1995), Ch. 19.

[9] S. Weinberg, *The Quantum Theory of Fields*, Vol. 2, Cambridge University Press (1996), Ch. 22.

[10] R. A. Bertlmann, *Anomalies in Quantum Field Theory*, Oxford University Press (1996).

[11] C. Vafa and E. Witten, "Restrictions on symmetry breaking in vector-like gauge theories," Nucl. Phys. B **234** (1984) 173.

[12] E. Witten, "An SU(2) anomaly," Phys. Lett. B **117** (1982) 324.

[13] Z. Komargodski and N. Seiberg, "From linear SUSY to constrained superfields," JHEP **09** (2009) 066, `arXiv:0907.2441`.

[14] D. V. Volkov and V. P. Akulov, "Is the neutrino a Goldstone particle?" Phys. Lett. B **46** (1973) 109.

[15] H. Miyazawa, "Spinor Currents and Symmetries of Baryons and Mesons," Phys. Rev. **170** (1968) 1586.

[16] A. Armoni, M. Shifman, and G. Veneziano, "SUSY Relics in One-Flavor QCD from a New $1/N$ Expansion," Phys. Rev. Lett. **91** (2003) 191601, `hep-th/0307097`.

[17] K. Blum, A. Efrati, C. Frugiuele, and Y. Nir, "Exotic colored scalars at the LHC," JHEP **02** (2017) 104, `arXiv:1610.06582`.

[18] C. T. Hill, P. A. N. Machado, A. E. Thomsen, and J. Turner, "Scalar Democracy," Phys. Rev. D **100** (2019) 015015, `arXiv:1902.07214`.

[19] Y. Sumino, `arXiv:0812.2103` (2008).

[20] P. H. Frampton, "Unifiable chiral color with natural GIM mechanism," Phys. Rev. Lett. **58** (1987) 2168.