Model Specifications

May 2022

1. Parameters

   $p =$ No. of Sectors

   $n =$ No. of Factors

   $f_{i,t}$ = return of the i-th factor at time t

   $r_{j,t}$ = return of the j-th sector at time t

   $T =$ No. of observations/time points (around 890)

   $k =$ No. of Exogenous Variable

2. Factor Model

   $$\vec{r}_{j,t} = \alpha_{j,t} + F\vec{\beta}_{j,t} + \vec{\varepsilon}_{j,t}$$

   for $j = 1, \ldots, p$, $t = 0, \ldots, T - 260$, where $\vec{r}_{j,t}$ is the return of sector j in the rolling window (260 weeks), F is the matrix containing factor return in the rolling window (size $260 \times n$)

   <u>Output:</u>

   $$\alpha_t = [\alpha_{1,t}, \cdots, \alpha_{p,t}]$$

   $$\beta_t = \left[\vec{\beta}_{1,t}, \cdots, \vec{\beta}_{p,t}\right]$$

   $$\varepsilon_t = [\vec{\varepsilon}_{1,t}, \cdots, \vec{\varepsilon}_{p,t}]$$

   $$\Sigma_{\varepsilon_t} = \text{covariance of residuals}$$

3. Vector Autoregression Model

   $$\begin{bmatrix} f_{1,t} \\ \vdots \\ f_{N,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_N \end{bmatrix} + \underbrace{\begin{bmatrix} a_{1,1} & \cdots & a_{1,N} \\ \vdots & & \vdots \\ a_{N,1} & \cdots & a_{N,N} \end{bmatrix}}_{A} \begin{bmatrix} f_{1,t-1} \\ \vdots \\ f_{N,t-1} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ \vdots \\ e_{N,t} \end{bmatrix}$$

   rolling window: 260 weeks

   <u>Output:</u>

   $$\text{vec}(\Sigma_t) = (I - A \otimes A)^{-1}\text{vec}(\Sigma_e)$$

$$\hat{f}_{t+4} = \text{predicted factor return 4 periods ahead}$$

$$MSE_t(f_{t+4}) = \sum_{i=0}^{3} A^i \Sigma_e A^{i'}$$

4. Vector Auto-regression Model with exogenous variable

$$\begin{bmatrix} f_{1,t} \\ \vdots \\ f_{N,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ \vdots \\ c_N \end{bmatrix} + \underbrace{\begin{bmatrix} a_{1,1} & \cdots & a_{1,N} \\ \vdots & & \vdots \\ a_{N,1} & \cdots & a_{N,N} \end{bmatrix}}_{A} \begin{bmatrix} f_{1,t-1} \\ \vdots \\ f_{N,t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} b_{1,1} & \cdots & b_{1,k} \\ \vdots & & \vdots \\ b_{k,1} & \cdots & b_{k,k} \end{bmatrix}}_{B} \begin{bmatrix} x_{1,t-1} \\ \vdots \\ x_{k,t-1} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ \vdots \\ e_{N,t} \end{bmatrix}$$

rolling window: 260 weeks

Output:

$$x_{t+i} = \text{average return of the past one year for } i = 1, 2, 3$$

$$\text{vec}(\hat{\Sigma}_t) = (I - A \otimes A)^{-1}(B \otimes B\text{vec}(\Sigma_x) + \text{vec}(\hat{\Sigma}_e))$$

$$\hat{f}_{t+4} = \text{predicted factor return 4 periods ahead}$$

$$MSE_t(f_{t+4}) = \sum_{i=0}^{3} A^i \Sigma_e A^{i'} + \sum_{i=0}^{2} A^i B \text{Var}_t(x_{t+3-i}) B' A^{i'}$$

5. Sector Expected Return(E) and Covariance(V)

$$E_t = \hat{f}_{t+4} \beta_t$$

$$V_t = \beta_t' \hat{\Sigma}_{t+1} \beta_t + \text{diag}\{\Sigma_{\varepsilon_t}\}$$

for $t = 1, \ldots, T - 260$

6. Portfolio Optimization

$$\text{maximize } U = \vec{w}_t' E_t - \frac{\gamma}{2} \vec{w}_t' V_t \vec{w}_t \text{ w.r.t } \vec{w}_t$$

subject to $\sum_{j=1}^{p} w_{j,t} = 1$, $w_{j,t} \geq 0$ for $j = 1, \ldots, p$