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Unit 2 Seminar

Title: Sets, Set Theory, Truth Tables and Logic

1. Read Partee et al (1993) Chapter 1 and then attempt exercises 1 and 4, located at the end of the chapter.

Exercises

1. Given the following sets:

$A = \{a, b, c, 2, 3, 4\}$ $\quad E = \{a, b, \{c\}\}$

$B = \{a, b\}$ $\quad F = \emptyset$

$C = \{c, 2\}$ $\quad G = \{\{a, b\}, \{c, 2\}\}$

$D = \{b, c\}$

classify each of the following statements as true or false

(a)	$c \in A$	**(g)**	$D \subset A$	**(m)**	$B \subseteq G$
(b)	$c \in F$	**(h)**	$A \subseteq C$	**(n)**	$\{B\} \subseteq G$
(c)	$c \in E$	**(i)**	$D \subseteq E$	**(o)**	$D \subseteq G$
(d)	$\{c\} \in E$	**(j)**	$F \subseteq A$	**(p)**	$\{D\} \subseteq G$
(e)	$\{c\} \in C$	**(k)**	$E \subseteq F$	**(q)**	$G \subseteq A$
(f)	$B \subseteq A$	**(l)**	$B \in G$	**(r)**	$\{\{c\}\} \subseteq E$

Answer:

A	T
B	F
C	F
D	T
E	F
F	T
G	T
H	F

I	F
J	
K	
L	F
M	T
N	T
O	T
P	T
Q	F
R	T

4. Consider the following sets:

$$
\begin{array}{lcllcl}
S1 & = & \{\{\emptyset\},\{A\},A\} & S6 & = & \emptyset \\
S2 & = & A & S7 & = & \{\emptyset\} \\
S3 & = & \{A\} & S8 & = & \{\{\emptyset\}\} \\
S4 & = & \{\{A\}\} & S9 & = & \{\emptyset,\{\emptyset\}\} \\
S5 & = & \{\{A\},A\} & & &
\end{array}
$$

Answer the following questions. Remember that the members of a set are the items separated by commas, if there is more than one, between the outermost braces only; a subset is formed by enclosing within braces zero or more of the members of a given set, separated by commas.

(a) Of the sets $S1$ - $S9$ which are members of $S1$?

(b) which are subsets of $S1$?

(c) which are members of $S9$?

(d) which are subsets of $S9$?

(e) which are members of $S4$?

(f) which are subsets of $S4$?

Answer:

A. S2, S3, S4, S6, S7, S8.

B. S3, S4, S5, S7, S8.

C. S6, S7, S8.

D. S7, S8.

E S2, S3.

F. S3.

2. Read the wiki at Sharma et al (2022) and then attempt the exercises below:

A. For each clause (a) - (f) below, create truth tables for each to answer the question of when each statement is false.

1. **~ P**
2. **P ∧ Q**
3. **P ∨ Q**
4. **P → Q**
5. **P ←→ Q**
6. **P → (~ Q)**

Answer:

1. ~P (Not P)

~P	P
0	1
1	0

2. P ∧ Q (P and Q)

P	Q	P ∧ Q
0	0	0
0	1	0

P	Q	P ∧ Q
1	0	0
1	1	1

3. P ∨ Q (P or Q)

P	Q	P ∨ Q
0	0	0
0	1	1
1	0	1
1	1	1

4. P → Q (P implies Q)

P	Q	P → Q
0	0	1
0	1	1
1	0	0
1	1	1

5. P ←→ Q (P if and only if Q)

P	Q	P ←→ Q
0	0	1
0	1	0
1	0	0
1	1	1

6. P → (~Q) (P implies not Q)

P	Q	~Q	P → (~Q)

0	0	1	1
0	1	0	1
1	0	1	0
1	1	0	0

B. Consider the statement (~ Q) -> (~ P).

1. **When is it false?**
2. **Now consider P → Q. When is it false?**
3. **Do you believe these two compound statements mean the same thing?**
4. **Construct the truth table for the statement (~ Q) −> (~ P). Then revisit your answer to (c).**

Answer:

1. For the statement (~Q) -> (~P), it is false only when ~Q (not Q) is true and ~P (not P) is false. In other words, when Q is false and P is true.

2. For the statement P -> Q, it is false when P is true and Q is false.

3. These two compound statements do not mean the same thing. The first statement (~Q) -> (~P) states that if Q is false, then P must also be false. The second statement P -> Q states that if P is true, then Q must also be true.

4. Here's the truth table for the statement (~Q) -> (~P):

Q	P	~Q	~P	(~Q) -> (~P)
0	0	1	1	1
0	1	1	0	0
1	0	0	1	1
1	1	0	0	1

C. Construct the truth table for P XOR Q.

Answer:

P	Q	P XOR Q
0	0	0
0	1	1
1	0	1
1	1	0

D. Construct truth tables for the following statements.

1. **~ (P ∧ Q)**
2. **P ∨ (Q ∧ R)**
3. **P ∨ (Q ∨ R)**
4. **(P ∨Q) ∨ R (Compare to the previous statement.)**
5. **(P → Q) ∧ (Q → P)**

Answer:

1. ~ (P ∧ Q):

P	Q	P ∧ Q	~ (P ∧ Q)
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

2. P ∨ (Q ∧ R):

P	Q	R	Q ∧ R	P ∨ (Q ∧ R)
0	0	0	0	0
0	0	1	0	0

0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

3. P ∨ (Q ∨ R):

P	**Q**	**R**	**Q ∨ R**	**P ∨ (Q ∨ R)**
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

4. (P ∨ Q) ∨ R:

P	**Q**	**R**	**P ∨ Q**	**(P ∨ Q) ∨ R**
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1

1	1	1	1	1

5. $(P \rightarrow Q) \land (Q \rightarrow P)$:

P	**Q**	**$P \rightarrow Q$**	**$Q \rightarrow P$**	**$(P \rightarrow Q) \land (Q \rightarrow P)$**
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1