Engineering Drawing

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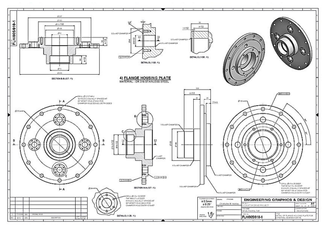


Figure 1: Rotation Matrices

1 Introduction

Engineering Drawing deals with projections of 3-Dimensional objects onto a 2-Dimensional plane by the virtue of many views or,combining the many given views of an object(The front view,top view,side view) in the 2-Dimensional plane and then coordinating and combining them to form a 3-Dimensional view of an object with the help of aligning the different views and then attaching them to one another in order to form a more precise 3-D model.

2 Problem

The problem that we have been given is to analyze the various views in which the object can be viewed as a 2-Dimensional image,or namely the different 2-Dimensional views of an object and then figure out how many of them can we use to accurately combine and give us the whole 3-Dimensional use of the object.(i.e. the minimum amount of 2-Dimensional views necessary to form the 3-Dimensional view of the object).Also if given the 3-Dimensional description of the object, how do we project it in the different necessary views in the 2 dimensional which in turn can be made to form the 3-Dimensional again without the need of another 2-Dimensional description.

3 Conversion Of 3-Dimensional objects to 2-Dimensional Descriptions

Given the 3-Dimensional description of an object,it can be converted to the many 2-Dimensional descriptions by projecting it to the various planes available,perpendicular to which,we will be able to view the specific 2-Dimensional description of the object in that plane.Here,we will be dealing with specifically 3 views of an object namely the Top View(perpendicular to the X-Y Plane),the Front View(Perpendicular to the X-Z Plane),and the Side View(Perpendicular to the Y-Z Plane).

4 Top View of a 3-Dimensional Object

The Top View of a 3-D object is the view which corresponds to being perpendicular to the X-Y Plane(or which is seen from the Z-Axis),from which,the top of the object can be seen. Let us solve this problem in a matrix form.Let the coordinates of any point of the object in the 3-Dimensional view be X(i),Y(i),Z(i).Let us also assume that the corresponding 2-Dimensional views of the objects are x(i) and y(i),(as the view lies in the X-Y plane).

by Matrix Multiplication,we know that multiplying a 2x3 matrix by a 3x1 matrix,we will get a 2x1 matrix which will be the desired value.

let the 3x1 matrix be

$$M = \begin{bmatrix} X(i) \\ Y(i) \\ Z(i) \end{bmatrix}$$

we have to convert it to the 2x1 matrix which is

$$m = \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}$$

So multiplying it by a matrix which has x-y coordinates and is of the form 2x3 we get that

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X(i) \\ Y(i) \\ Z(i) \end{bmatrix} = \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}$$

Hence in this way,we can get a projection of the 3-Dimensional view into the 2-Dimensional top view

5 Front View of a 3-Dimensional Object

The Front View of a 3-D object is the view which corresponds to being perpendicular to the X-Z Plane(or which is seen from the Y-Axis),from which,the front of the object can be seen. Let us solve this problem in a matrix form.Let the coordinates of any point of the object in the 3-Dimensional view be X(i),Y(i),Z(i).Let us also assume that the corresponding 2-Dimensional views of the objects are x(i) and z(i),(as the view lies in the X-Z plane).

by Matrix Multiplication,we know that multiplying a 2x3 matrix by a 3x1 matrix,we will get a 2x1 matrix which will be the desired value.

let the 3x1 matrix be

$$M = \begin{bmatrix} X(i) \\ Y(i) \\ Z(i) \end{bmatrix}$$

we have to convert it to the 2x1 matrix which is

$$m = \begin{bmatrix} x(i) \\ z(i) \end{bmatrix}$$

So multiplying it by a matrix which has x-y coordinates and is of the form 2x3 we get that

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X(i) \\ Y(i) \\ Z(i) \end{bmatrix} = \begin{bmatrix} x(i) \\ z(i) \end{bmatrix}$$

Hence in this way,we can get a projection of the 3-Dimensional view into the 2-Dimensional front view.

6 Side View of a 3-Dimensional Object

The Side View of a 3-D object is the view which corresponds to being perpendicular to the Y-Z Plane(or which is seen from the X-Axis),from which,the side of the object can be seen. Let us solve this problem in a matrix form.Let the coordinates of any point of the object in the 3-Dimensional view be X(i),Y(i),Z(i).Let us also assume that the corresponding 2-Dimensional views of the objects are x(i) and z(i),(as the view lies in the Y-Z plane).

by Matrix Multiplication,we know that multiplying a 2x3 matrix by a 3x1 matrix,we will get a 2x1 matrix which will be the desired value.

let the 3x1 matrix be

$$M = \begin{bmatrix} X(i) \\ Y(i) \\ Z(i) \end{bmatrix}$$

we have to convert it to the 2x1 matrix which is

$$m = \begin{bmatrix} y(i) \\ z(i) \end{bmatrix}$$

So multiplying it by a matrix which has x-y coordinates and is of the form 2x3 we get that

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X(i) \\ Y(i) \\ Z(i) \end{bmatrix} = \begin{bmatrix} y(i) \\ z(i) \end{bmatrix}$$

Hence in this way,we can get a projection of the 3-Dimensional view into the 2-Dimensional side view.

7 Rotation of Views

As we saw above,a view of an object and it's coordinates can be represented by the virtue of a matrix.However we can also rotate a projection by the use of another matrix which can be termed as the rotation matrix.The rotation matrix is differently defined to rotate any view about an axis clockwise or counter clockwise.The Rotations matrix is defined as follows for rotating about the following 3 axes.

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 2: Rotation Matrices

The Rotation matrix then which is a 3x3 Matrix,can be used to rotate any given 2-Dimensional View in the following manner:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} R \begin{bmatrix} X(i) \\ Y(i) \\ Z(i) \end{bmatrix} = \begin{bmatrix} x(i) \\ y(i) \end{bmatrix}$$

for the top view, where R defines the rotation matrix,which can be used to rotate the projection in any given direction,depending on the value of R.

similarly for the front view,it will be something like

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} R \begin{bmatrix} X(i) \\ Y(i) \\ Z(i) \end{bmatrix} = \begin{bmatrix} x(i) \\ z(i) \end{bmatrix}$$

where R defines the amount,the axis, and the direction of rotation of the view.

and for the side view,it will be something like

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} R \begin{bmatrix} X(i) \\ Y(i) \\ Z(i) \end{bmatrix} = \begin{bmatrix} y(i) \\ z(i) \end{bmatrix}$$

where R will define the angle by which the view rotates,about which axis,and the direction(clockwise or counter-clockwise) of the rotation.

So in such a way,we can rotate all the 2-Dimensional Descriptions of a 3-Dimensional object,with the help of the Rotation Matrix.

8 Conversion of 2-Dimensional Descriptions to a 3-Dimensional object

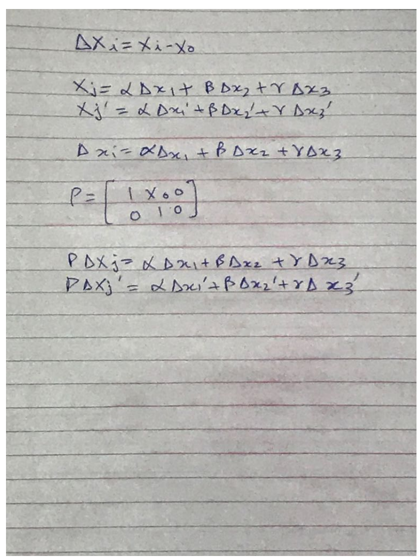
We have previously defined 3 2-Dimensional views of an object namely Top view,Front view,and the side view.These are the 3 views which are necessary and sufficient to compute the 3-Dimensional description of an object accurately and uniquely.Missing any of these views,and computing the 3-Dimensional Description of the object will result in a non-unique result and will result in multiple objects,which will satisfy the two views. When we have to convert 3 views from different angles to a 3D view, we can do that by making the correspondence between the points of each of the views and then assigning them to a single point of 3 dimensions which corresponds to the 3D view.

Let the point (a,b) in the 2-Dimensional X-Y plane correspond to a line in the 3-Dimensional plane.

Let the point (c,d) in the 2-Dimensional X-Z plane correspond to a line in the 3-Dimensional plane.

Let the point (e,f) in the 2-Dimensional Y-Z plane correspond to a line in the 3-Dimensional plane.

The intersection of these lines will correspond to the points which we will get in the 3-Dimensional view and will be the part of the 3-Dimensional Description of the object.



$$\Delta X_i = X_i - X_0$$

$$X_j = \alpha \Delta x_1 + \beta \Delta x_2 + \gamma \Delta x_3$$
$$X_j' = \alpha \Delta x_1' + \beta \Delta x_2' + \gamma \Delta x_3'$$

$$\Delta x_i = \alpha \Delta x_1 + \beta \Delta x_2 + \gamma \Delta x_3$$

$$P = \begin{bmatrix} 1 & X & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P \Delta X_j = \alpha \Delta x_1 + \beta \Delta x_2 + \gamma \Delta x_3$$
$$P \Delta X_j' = \alpha \Delta x_1' + \beta \Delta x_2' + \gamma \Delta x_3'$$

Figure 3: 2D-3D

Rotation of a 3-Dimensional matrix will be similar and will be done using the rotation Matrix R.

9 Conclusion

So in conclusion,by combining 2-Dimensional Descriptions to make a 3-Dimensional Object or converting a 3-Dimensional description to 2-Dimensional views, we can study an object in greater detail and have much more easy in making or producing it.Also by the use of matrices in depicting the views of the object, we can easily convert or rotate them from one description to the other without much difficulty.