CSci 435: Formal Languages and Automata

Instructor: Dr. M. E. Kim Date: November 1st, 2018

Due: 5:00 PM, November 8th (Thr.), 2018. (No Extension) Name: **\_\_Aaron Johnson\_\_**

**Home Assignment 5: 80 points + 10 points (optional)**

**Abbreviation:**

CFL/CFG: (Nondeterministic) Context Free Language/Grammar.

DCFL/DCFG: Deterministic Context Free Language/Grammar.

RL/RG: Regular Language/Grammar.

Q1.[40] Prove if the following languages are CFL or not.

If L is a CFL, give its CFG. Otherwise, prove it by Pumping Lemma.

If any closure property of CFL is applicable, apply them to simplify it before its proof.

1. [10] L = {*wwRw* | *w* ∈ {*a, b*}\*}

**Let’s assume L is a CFL. Using pumping lemma with i as the parameter, consider the string s = 0i12i02i1i** ∈ **L. Because |s| ≥ i, we can split s into uvxyz, which satisfies the pumping lemma. We know that i ≥ |vxy|, so v and y can’t possibly have any zeros from 02i . If we take, for example, uv0xy0z = uxz, the string uxz should adhere to 0j1k02i1i, where k ≤ 2i and j < i. If uxz** ∈ **L, it should adhere to wwRw. However, uxz adheres to 0j1k02i1i where the length is ≥ 5i, so the first w should begin with j < i zeros with any number of ones after it. This means that wRw should only have at most 2j < 2i zeros, however uxz has 2k zeros, which shows L is not a CFL.**

1. [10] L = { *anwwRbn* | *n* ≥ 0, *w* ∈ {*a, b*}\*}

**This is a CFL. We get the language:**

**S → aSa | A**

**A → aAa | bAb |** 𝜆

1. [10] L = {*anbjanbj* | *n* ≥ 0, *j* ≥ 0}

**Let’s use pumping lemma with parameter m, pumping the string ambmambm. If c is in the first am and d is in the first bm, pumping c and d once will result in the string am+kbm+lambm, which is not consistent with L, so L is not a CFL.**

1. [10] L = {*anbjajbn* | *n* ≥ 0, *j* ≥ 0}

**This is a CFL. We get the language:**

**S → aSb | A**

**A → bAa |** 𝜆

1. [10, optional] L = {*an*| *n* is a prime number}

**Let’s assume L is a CFL. Using pumping lemma, letting m be the parameter. Let p be a prime number such that p is ≥ m. Pumping the string ap** ∈ **L, we get ap = cdfgh, where d = ak and g = al, k + l ≥ 1. From the pumping lemma, we get cd1+pfg1+ph** ∈ **L, so ap+kp+lp** ∈ **L. We can factor out p, so ap(1+k+l)** ∈ **L, which can’t possibly occur because p(1+k+l) is not a prime number. So, by this contradiction, L is not a CFL.**

Q2. [40] Prove the following properties clearly.

1. [10] The family of CFLs is closed under reversal.

**Let G be a CFG, let L be a CFL, and let G generate L. If we take G’ by reversing its productions, we can see that a string xR may only be generated by G’ if and only if the string x can be generated by G. Because of this, and since G’ generates LR, LR is a CFL.**

1. [10] The family of DCFL is closed under regular difference:

i.e. for a CFL L1 and a RL L2, L1 − L2 ∈ CFL.

**Since CFLs are closed under regular difference simply by L1 – L2, and since regular languages are closed under compliments by L2’, and since Theorem 8.5 in the text states CFLs are closed under regular intersection, L = L1 – L2 = L1 ∩ L2’. Similarly, for DCFLs, since they are a subset of CFLs, this logic can be applied as well, which shows that DCFLs are closed under regular difference.**

1. [10] The family of DCFL is not closed under union and intersection.

**Let L = {xnynzn| n≥0}, M = {xiyjzk|i,j,k≥0}, N = {xiyijj|i,j≥0}, O = {xiyjzj|i,j≥0}.**

**L is not a CFL. M, N, and O are DCFLs, and M is a RL. So, M’, N’ and O’ are all DCLFs.**

**So, L’ = M’** ⋃ **N’** ⋃ **O’. Since CFLs are closed under union and DCFLs are closed under complement, L’ should be a CFL. However, L’ is not a DCFL, so it can’t be closed under union or intersection.**

1. [10] The family of CFLs is not closed under complement. Give an example for it.

**Let L1 and L2 be CFLs and assume that the complements of all CFLs are also CFLs. We know CFLs are closed under union, and we are assuming that they are also closed under complement. This must mean they are closed under intersection. However, we know the intersection of two CFLs are not necessarily closed unless it is regular intersection, which proves the previous assumption incorrect, showing that CFLs are not closed under complement.**

**For example, let L1 = {xnynzn|n≥0}, which is not CFL. So, L1’ is a CFL.**