CSci 435: Formal Languages and Automata

Instructor: Dr. M. E. Kim Date: November 1st, 2018

Due: 5:00 PM, November 8th (Thr.), 2018. (No Extension) Name: **\_\_Aaron Johnson\_\_**

**Home Assignment 5: 80 points + 10 points (optional)**

**Abbreviation:**

CFL/CFG: (Nondeterministic) Context Free Language/Grammar.

DCFL/DCFG: Deterministic Context Free Language/Grammar.

RL/RG: Regular Language/Grammar.

Q1.[40] Prove if the following languages are CFL or not.

If L is a CFL, give its CFG. Otherwise, prove it by Pumping Lemma.

If any closure property of CFL is applicable, apply them to simplify it before its proof.

1. [10] L = {*wwRw* | *w* ∈ {*a, b*}\*}
2. [10] L = { *anwwRbn* | *n* ≥ 0, *w* ∈ {*a, b*}\*}

**This is a CFL. We get the language:**

**S → aSa | A**

**A → aAa | bAb |** 𝜆

1. [10] L = {*anbjanbj* | *n* ≥ 0, *j* ≥ 0}

**Let’s use pumping lemma with parameter m, pumping the string ambmambm. If c is in the first am and d is in the first bm, pumping c and d once will result in the string am+kbm+lambm, which is not consistent with L, so L is not a CFL.**

1. [10] L = {*anbjajbn* | *n* ≥ 0, *j* ≥ 0}

**This is a CFL. We get the language:**

**S → aSb | A**

**A → bAa |** 𝜆

1. [10, optional] L = {*an*| *n* is a prime number}

**Let’s assume L is a CFL. Using pumping lemma, letting m be the parameter. Let p be a prime number such that p is ≥ m. Pumping the string ap** ∈ **L, we get ap = cdfgh, where d = ak and g = al, k + l ≥ 1. From the pumping lemma, we get cd1+pfg1+ph** ∈ **L, so ap+kp+lp** ∈ **L. We can factor out p, so ap(1+k+l)** ∈ **L, which can’t possibly occur because p(1+k+l) is not a prime number. So, by this contradiction, L is not a CFL.**

Q2. [40] Prove the following properties clearly.

1. [10] The family of CFLs is closed under reversal.
2. [10] The family of DCFL is closed under regular difference:

i.e. for a CFL L1 and a RL L2, L1 − L2 ∈ CFL.

1. [10] The family of DCFL is not closed under union and intersection.
2. [10] The family of CFLs is not closed under complement. Give an example for it.