

(9) Find transfer func. from input to output for closed-loop system.

$$U(s) = K_p (R(s) - Y(s))$$

$$R(s) = U(s) - Y(s)$$

$$Y(s) = P(s)U(s)$$

$$\Rightarrow Y(s) = P(s)K_p(R(s) - Y(s))$$

$$\Rightarrow Y(s) = P(s)K_p R(s) - P(s)K_p Y(s)$$

$$\Rightarrow Y(s) = \frac{P(s)K_p R(s)}{1 + P(s)K_p}$$

$$\text{let } G(s) = \frac{K_p P(s)}{1 + K_p P(s)} \Rightarrow Y(s) = G(s)R(s)$$

$$P(s) = \frac{bT}{s+aT} \Rightarrow G(s) = \frac{\frac{bT}{s+aT} \cdot K_p}{1 + \frac{bT}{s+aT} \cdot K_p}$$

$$\Rightarrow G(s) = \frac{bTK_p}{s+aT+bTK_p}, \text{ let } \tau = \frac{1}{T(a+bK_p)}$$

$$\Rightarrow G(s) = \frac{\tau bTK_p}{\tau s + 1}, \left(\text{or } G(s) = \frac{\frac{bK_p}{a+bK_p}}{\frac{s}{T(a+bK_p)} + 1} \right)$$

where $\tau = T^{-1}(a+bK_p)^{-1}$ is time const. and $K = \tau bTK_p$ is DC Gain