Lab 2

ECE 380 W21

Group 8

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Declaration of Authorship

We acknowledge and promise that:

a) We are the sole authors of this lab report and associated simulation files/code.
b) This work represents our original work.
c) We have not shared detailed analysis or detailed design results, computer code, or Simulink diagrams with any other student.
d) We have not obtained or looked at lab reports from any other current or former student of ECE/SE 380, and we have not let any other student access any part of our lab work.
e) We have completely and unambiguously acknowledged and referenced all persons and aids used to help us with our work.

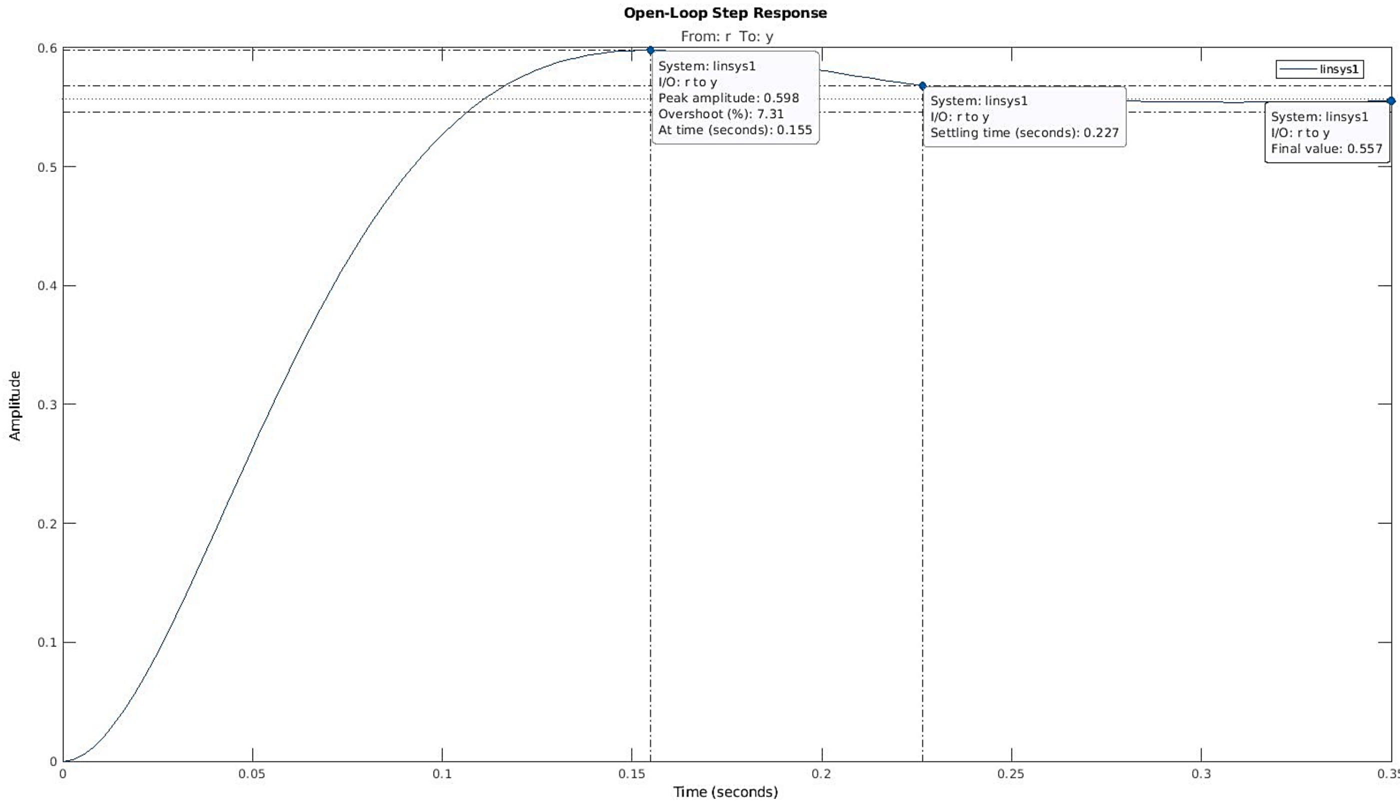
<u>Student1 Name and Signature:</u>

Arjun Bawa

<u>Student2 Name and Signature:</u>

Andrew Tran

2.1

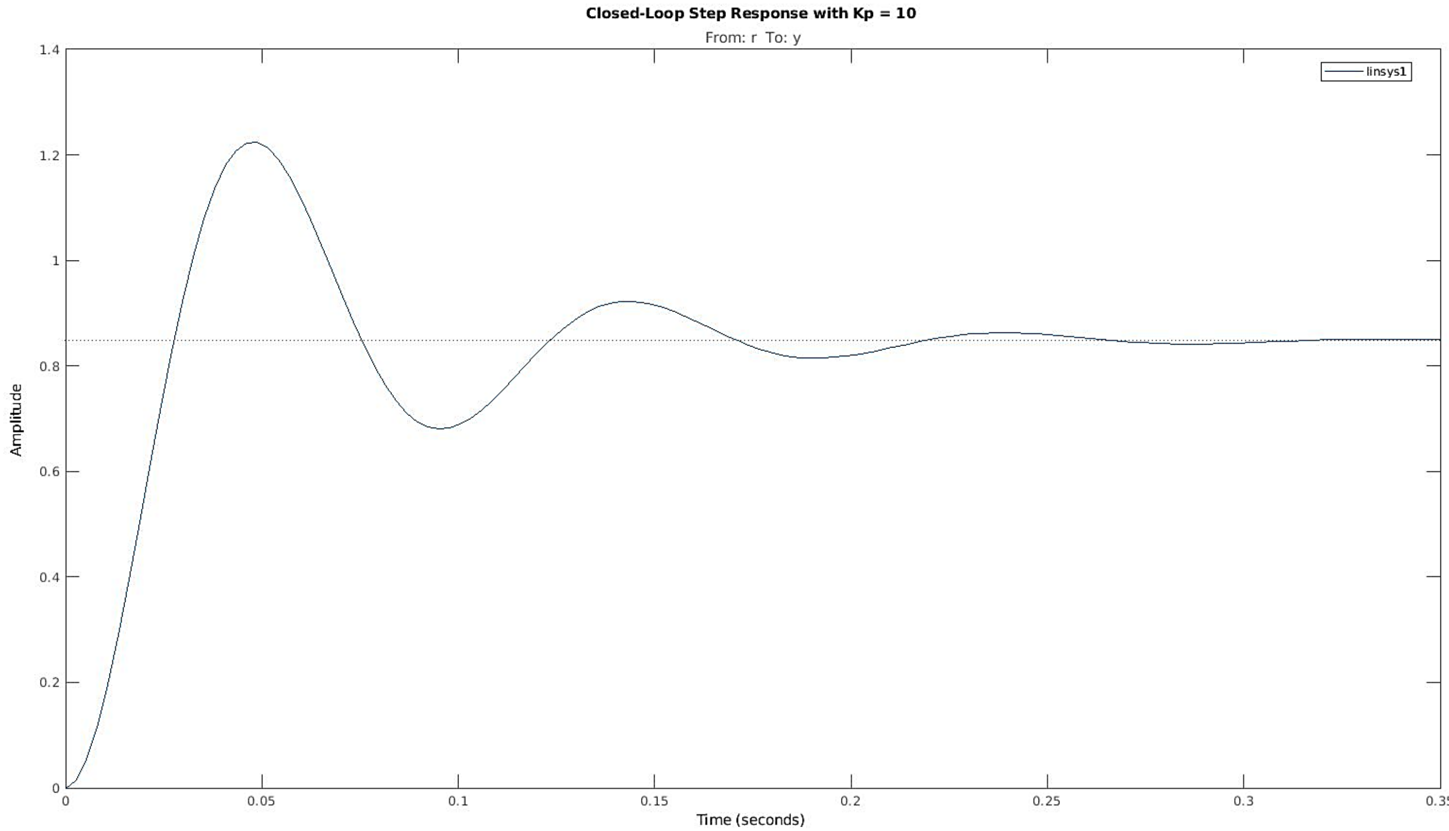


$$y_{ss} \approx 0.557$$

$$T_{peak} \approx 0.155\, s \quad , \quad y_{max} \approx 0.598$$

$$T_{2\%} \approx 0.227\, s$$

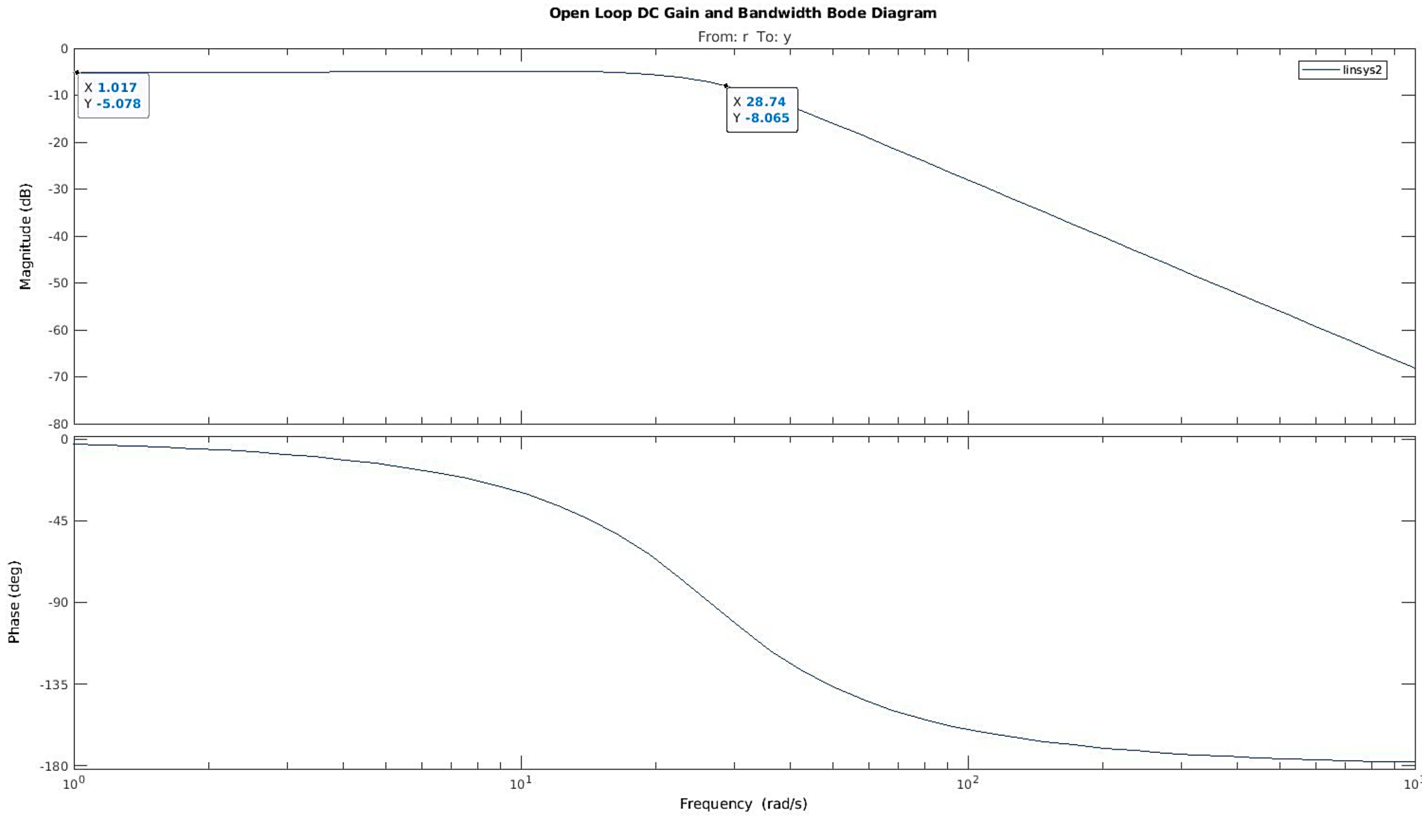
2.2



2.3

K_p *Gain*	y_{ss} **Steady-State Value**	y_{max} **Peak Value**	T_{peak} **Time-to-Peak**	$\%O.S.$ **Overshoot**
1	0.358	0.413	0.111 seconds	15.3 %
5	0.736	0.982	0.0652 seconds	33.5 %
10	0.848	1.22	0.0489 seconds	44.4 %

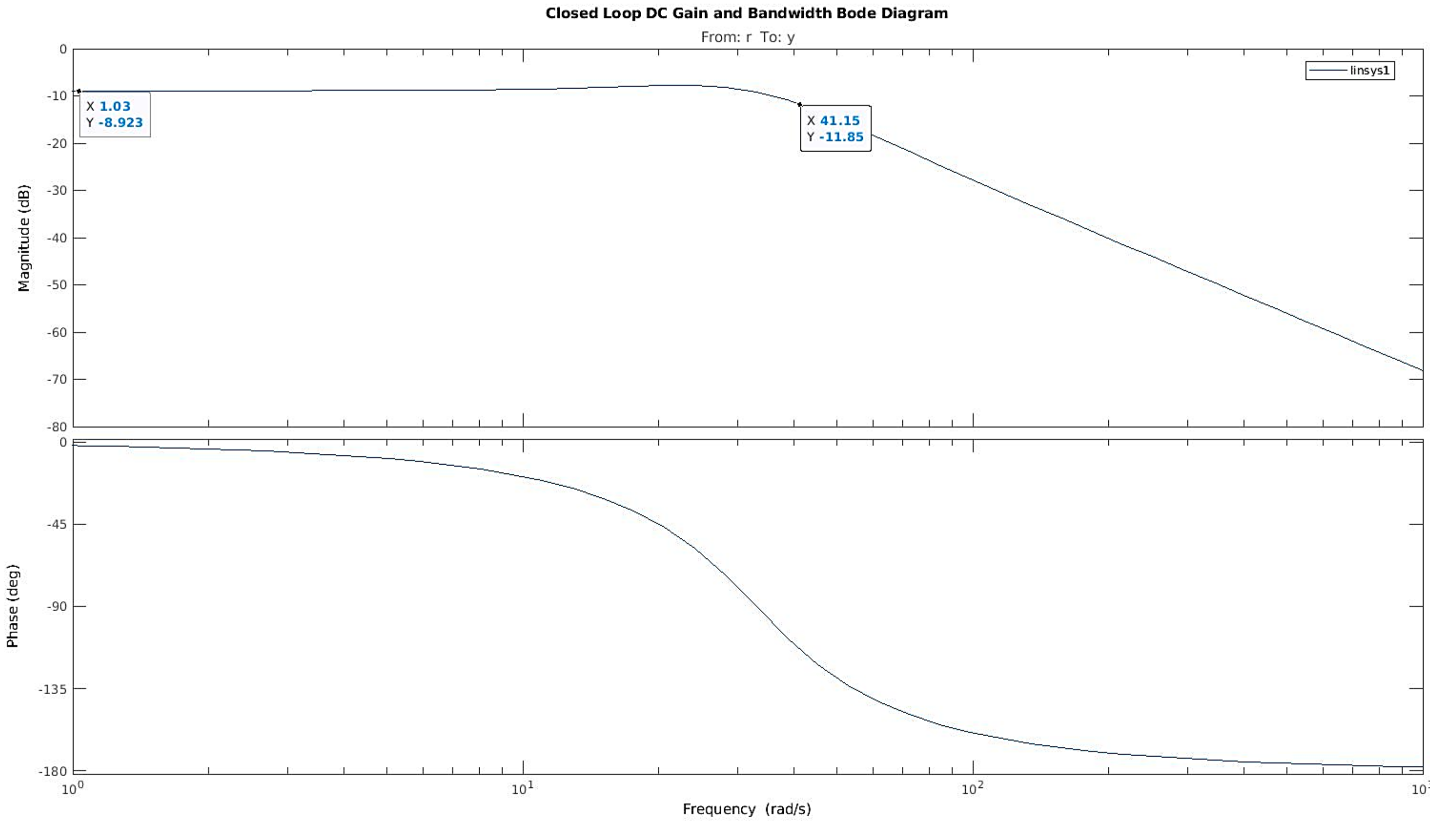
2.4



$$DC\ Gain \approx -5.078\ dB$$

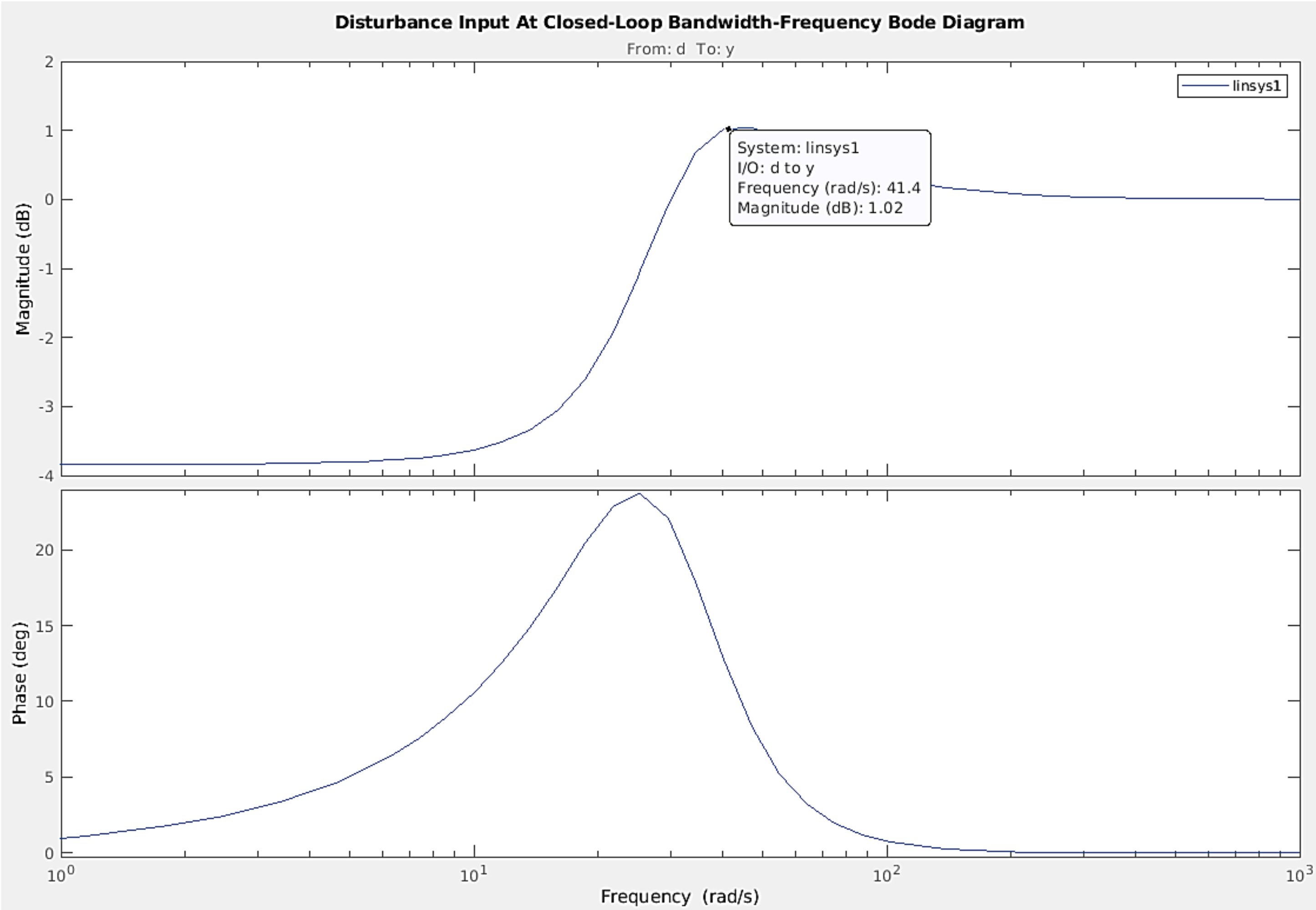
$$Bandwidth\ Frequency\ ,\ \ \omega_B \approx 28.74\,\frac{rad}{s}$$

2.5



$$Bandwidth\ Frequency\ ,\ \ \omega_B \approx 41.15\,\frac{rad}{s}$$

2.6



Disturbance Input At Closed-Loop Bandwidth-Frequency Bode Diagram
From: d To: y
linsys1
System: linsys1
I/O: d to y
Frequency (rad/s): 41.4
Magnitude (dB): 1.02
Magnitude (dB)
Phase (deg)
Frequency (rad/s)

2.7

1

1) Determine $\hat{K}$, ω_n, ζ from procedure 2.1.4

$$\lim_{t\to\infty} y(t) = y_{ss}(t) = \boxed{\hat{K} \approx 0.557}$$

$$\%\,O.S. = 100\exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \approx 7.31\,\%$$

$$\Rightarrow \frac{-\zeta^2\pi^2}{1-\zeta^2} = 2\ln\left(\frac{\%\,O.S.}{100}\right)$$

$$\Rightarrow -\zeta^2\pi^2 = 2\ln(\|\%\,O.S.\|) - 2\zeta^2\ln(\|\%\,O.S.\|)$$

$$\Rightarrow -\zeta^2\left(\pi^2 - 2\ln(\|\%\,O.S.\|)\right) = 2\ln(\|\%\,O.S.\|)$$

$$\Rightarrow \boxed{\zeta} = \sqrt{\frac{-2\ln(\|\%\,O.S.\|)}{\pi^2 - 2\ln(\|\%\,O.S.\|)}} \boxed{\approx 0.589}$$

$$T_{2\%} \approx \frac{4}{\zeta\omega_n} \approx 0.227\ s$$

$$\Rightarrow \boxed{\omega_n} \approx \frac{4}{\zeta T_{2\%}} \boxed{\approx 29.9\ \frac{rad}{s}}$$

2) Estimate ω_n using time-to-peak. Which is a better estimator for ω_n, using time-to-peak or 2% settling time?

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \Rightarrow \omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}} \approx 21.577$$

Would rather use time-to-peak to estimate ω_n since the relationship between 2% settling time and ω_n for an underdamped, standard 2^{nd} order system is approximate whereas the relationship between time-to-peak and ω_n isn't.

3) Compute closed-loop transfer func. for Fig. 2.1.
Compute damping ratio, DC Gain and natural freq. in terms of plant's $\hat{k}$, ω_n and ζ and gain k_p

$$E(s) = R(s) - Y(s)$$
$$Y(s) = D(s) + P(s)E(s)k_p$$
$$\Rightarrow Y(s) = D(s) + k_p(R(s) - Y(s))P(s)$$
$$\Rightarrow Y(s) = D(s) + k_p P(s) R(s) - k_p P(s) Y(s), \quad D(s) = 0$$
$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{k_p P(s)}{1 + k_p P(s)}$$
$$= \frac{k_p \hat{k} \omega_n^2 (s^2 + 2\zeta\omega_n s + \omega_n^2)^{-1}}{1 + k_p \hat{k} \omega_n^2 (s^2 + 2\zeta\omega_n s + \omega_n^2)^{-1}}$$
$$= \frac{k_p \hat{k} \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2 + k_p \hat{k} \omega_n^2}$$
$$= \frac{k_p \hat{k} \omega_n^2 \cancel{(s^2 + 2\zeta\omega_n s + \omega_n^2)}}{\cancel{(s^2 + 2\zeta\omega_n s + \omega_n^2)} 1 + \cancel{(s^2 + 2\zeta\omega_n s + \omega_n^2)}(k_p \hat{k} \omega_n^2)}$$
$$= \frac{k_p \hat{k} \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2 + k_p \hat{k} \omega_n^2} = \frac{\hat{k}' \omega_n^2}{s^2 + 2\zeta' \omega_n' s + \omega_n'^2}$$

$\hat{k}$, ω_n and ζ from part 1 & 2

$$\Rightarrow \omega_n'^2 = \omega_n^2 (1 + k_p \hat{k})$$
$$\Rightarrow \boxed{\text{natural freq. } \omega_n' = \omega_n \sqrt{1 + k_p \hat{k}}}$$
$$\Rightarrow \cancel{2}\zeta\omega_n \cancel{s} = \cancel{2}\zeta'\omega_n' \cancel{s}$$
$$\Rightarrow \boxed{\text{damping ratio } \zeta' = \frac{\zeta}{\sqrt{1 + k_p \hat{k}}}}$$
$$\Rightarrow \boxed{\text{DC Gain } \hat{k}' = \frac{k_p \hat{k}}{1 + k_p \hat{k}}}$$

The closed-loop damping ratio ζ_c, DC gain $\hat{K}_c$ and natural frequency ω_{n_c} are given by

$$\omega_{n_c} = \omega_n\sqrt{1 + K_p\hat{K}} \qquad \zeta_c = \frac{\zeta}{\sqrt{1 + K_p\hat{K}}} \qquad \hat{K}_c = \frac{K_p\hat{K}}{1 + K_p\hat{K}}$$

Where ω_n , ζ , $\hat{K}$ are open-loop natural frequency, damping ratio and DC gain, respectively.

4

Increasing K_p increases the steady-state value y_{ss}, decreases the time-to-peak T_{peak} and increases the percent overshoot $\%O.S.$ of the closed-loop step response.

From the derived equation for $\hat{K}_c$ above, increasing K_p increases $\hat{K}_c$ (i.e. increases y_{ss}), making it approach 1 in the limit.

From the derived equation for ζ_c above, increasing K_p decreases $\zeta_c \propto \frac{1}{\sqrt{K_p}}$.

From the formulas in parts 1 and 2, this leads to an increase of $\%O.S.$ because the exponent $\frac{-\zeta\pi}{\sqrt{1+\zeta^2}} \to 0$ as ζ_c decreases. Similarly, $T_{peak} \to \frac{\pi}{\omega_{n_c}}$ as ζ_c decreases because the denominator $\sqrt{1-\zeta_c^2} \to 1$ as ζ_c decreases. We see that ω_{n_c} increases as K_p increases, meaning T_{peak} decreases as K_p increases.

From the approximation in part 1, $T_{2\%} \approx \frac{4}{\zeta_c\omega_{n_c}}$ for the closed-loop 2% settling time. Using the derived equations above, with some substitution we see that $\omega_{n_c} = \frac{\omega_n\zeta}{\zeta_c}$. That is, $\omega_{n_c} \propto \frac{1}{\zeta_c}$.

This means that theoretically any change in ω_{n_c} or ζ_c that a change in K_p causes don't affect the approximation of 2% settling time. Using some substitution, the closed loop 2% settling time simplifies to $T_{2\%} \approx \frac{4}{\omega_n\zeta}$ which isn't reliant on any of the closed-loop parameters and thus unaffected by a change in K_p.

5

Proportional error feedback control cannot always reliably correct error in the presence of disturbance (which physical systems suffer from). This is because proportional error feedback alone doesn't consider the possibility that disturbance offsets the error calculated during feedback.

6

Going from an open-loop system to a closed-loop system increased the bandwidth frequency ω_B from $28.74\frac{rad}{s}$ to $41.15\frac{rad}{s}$.

7

The closed-loop system rejects disturbance signals well for frequencies well below the closed-loop bandwidth frequency. When disturbance rejection is high, the effect of the disturbance signal on the output is low. This means the magnitude of the disturbance-response transfer function is very small. When disturbance input frequency is small, the plot shows a very small dB value, which indicates a small magnitude of transfer function for small frequencies.

For frequencies approaching the closed-loop bandwidth frequency, disturbance rejection gets worse. The decibel value of the disturbance-response peaks at $\approx 1\ dB$ near the bandwidth frequency before dropping to 0 dB for frequencies higher than the bandwidth. This means that for high frequencies (higher than bandwidth), the effect of the disturbance signal on the output is very significant; the output is almost solely affected by the disturbance signal at high frequencies.

8) Derive closed-loop transfer func. from d to y.

What happens when k_p changes?

$$Y(s) = D(s) + k_p P(s) R(s) - k_p P(s) Y(s), \quad R(s) = 0$$

$$\Rightarrow Y(s) = D(s) - k_p P(s) Y(s)$$

$$\Rightarrow \frac{Y(s)}{D(s)} = \frac{1}{1 + k_p P(s)} = \frac{1}{1 + \dfrac{k_p \hat{k}_c \omega_{n_c}^2}{s^2 + 2\zeta_c \omega_{n_c} s + \omega_{n_c}^2}}$$

$$= \frac{s^2 + 2\zeta_c \omega_{n_c} s + \omega_{n_c}^2}{s^2 + 2\zeta_c \omega_{n_c} s + \omega_{n_c}^2 (1 + k_p \hat{k}_c)}$$

$$= (s^2 + 2\zeta_c \omega_{n_c} s + \omega_{n_c}^2) \cdot \boxed{\frac{1}{\omega_{n_c}^2 (1 + k_p \hat{k}_c)} \cdot \frac{\omega_{n_c}^2 (1 + k_p \hat{k}_c)}{s^2 + 2\zeta_c \omega_{n_c} s + \omega_{n_c}^2 (1 + k_p \hat{k}_c)}}$$

Only latter half of transfer func. depends on k_p

$$\Rightarrow \dot{\omega}_{n_c}^2 = \omega_{n_c}^4 (1 + k_p \hat{k}_c)^2 \Rightarrow \boxed{\dot{\omega}_{n_c} = \omega_{n_c}^2 (1 + k_p \hat{k}_c)}$$

$$\cancel{2} \zeta_c \omega_{n_c}^3 (1 + k_p \hat{k}_c) = \cancel{2} \dot{\zeta}_c \dot{\omega}_{n_c}$$

$$\Rightarrow \boxed{\dot{\zeta}_c} = \frac{\zeta_c \omega_{n_c}^{\cancel{3}} \cancel{(1 + k_p \hat{k}_c)}}{\cancel{\omega_{n_c}^2} \cancel{(1 + k_p \hat{k}_c)}} = \boxed{\zeta_c \omega_{n_c}}$$

$$\dot{\hat{K}}_c \dot{\omega}_{n_c}^2 = \omega_{n_c}^2 (1 + K_p \hat{K}_c) \Rightarrow \dot{\hat{K}}_c = \frac{\cancel{\omega_{n_c}^2 (1 + K_p \hat{K}_c)}}{\omega_{n_c}^{\cancel{2}4} (1 + K_p \hat{K}_c)^{\cancel{2}}}$$

$$\Rightarrow \boxed{\dot{\hat{K}}_c = \frac{1}{\omega_{n_c}^2 (1 + K_p \hat{K}_c)}}$$

$\Rightarrow$ as K_p increases, natural frequency increases

$\Rightarrow$ damping ratio is unaffected by K_p

$\Rightarrow$ DC Gain decreases as K_p increases.

DC Gain $= \omega_{n_c}^{-2}$ when $K_p = 0$

Increasing $K_p \Rightarrow$ decrease in magnitude of transfer function $\Rightarrow$ better disturbance rejection (since $20 \log |G(s)| \to -\infty$ as $|G(s)| \to 0$)

When K_p increases, disturbance rejection properties of system improve.