1) Derive transfer func. from $r(t)$ to $x(t)$

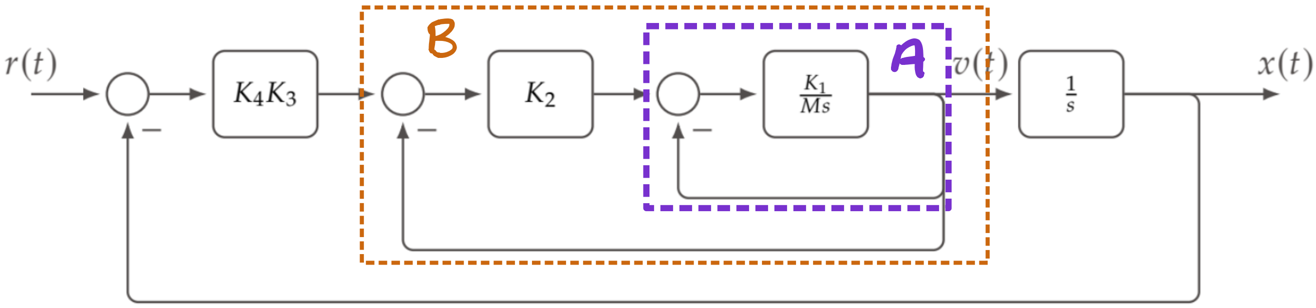

Figure 3.1: Entire closed-loop architecture for Lab 3.

$$A = \frac{k_1}{k_1 + Ms}$$

$$B = \frac{k_2 A}{1 + k_2 A} = \frac{k_1 k_2}{k_1 + Ms + k_2 k_1}$$

$$\Rightarrow \frac{X(s)}{R(s)} = \frac{\frac{1}{s} k_4 k_3 B}{1 + \frac{1}{s} k_4 k_3 B}$$

$$= \frac{\frac{1}{s} k_4 k_3 \frac{k_1 k_2}{k_1 + Ms + k_2 k_1}}{1 + \frac{1}{s} k_4 k_3 \frac{k_1 k_2}{k_1 + Ms + k_2 k_1}}$$

$$= \frac{k_1 k_2 k_3 k_4}{(k_1 + k_1 k_2 + Ms)s + k_1 k_2 k_3 k_4}$$

$$= \frac{k_1 k_2 k_3 k_4}{Ms^2 + (k_1 + k_1 k_2)s + k_1 k_2 k_3 k_4}$$

$$= \boxed{\frac{1}{M} \cdot \frac{k_1 k_2 k_3 k_4}{s^2 + \left(\frac{k_1 + k_1 k_2}{M}\right) s + \frac{k_1 k_2 k_3 k_4}{M}}}$$

2) Write T.F. in standard 2^{nd} order form. Find DC Gain $\hat{K}$, ω_n and ζ

$$\frac{X(s)}{R(s)} = \frac{\frac{1}{M}K_1K_2K_3K_4}{s^2 + \left(\frac{K_1 + K_1K_2}{M}\right)s + \frac{K_1K_2K_3K_4}{M}}$$

let $H(s) = \frac{X(s)}{R(s)} \Rightarrow \boxed{\hat{K} = |H(0)| = 1}$

$$\omega_n^2 = \frac{K_1K_2K_3K_4}{M} \Rightarrow \boxed{\omega_n = \sqrt{\frac{K_1K_2K_3K_4}{M}}}$$

$$\zeta = \frac{K_1 + K_1K_2}{2M\omega_n} = \frac{\sqrt{M}(K_1 + K_1K_2)}{2M\sqrt{K_1K_2K_3K_4}}$$

$$= \boxed{\frac{K_1 + K_1K_2}{2\sqrt{MK_1K_2K_3K_4}}}$$

3) Identify the two gains that affect $T_{2\%}$

$$T_{2\%} \approx \frac{4}{\zeta\omega_n} = \frac{4}{\frac{K_1 + K_1K_2}{2\sqrt{MK_1K_2K_3K_4}} \cdot \sqrt{\frac{K_1K_2K_3K_4}{M}}}$$

$$= \frac{4 \cdot 2 \cdot \sqrt{M}\sqrt{MK_4K_3K_2K_1}}{(K_1 + K_1K_2)\sqrt{K_4K_3K_2K_1}} = \boxed{\frac{8M}{K_1 + K_1K_2}}$$

$\Rightarrow K_1$ and K_2 affect $T_{2\%}$

4) Determine a gain that can be changed to reduce %OS w/out affecting $T_{2\%}$

$$\ln(O.S.) = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \Rightarrow \ln\left(\frac{1}{O.S.}\right) = \frac{\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow \ln\left(\frac{1}{O.S.}\right) = \frac{\dfrac{(k_1+k_1k_2)\pi}{2\sqrt{Mk_1k_2k_3k_4}}}{\sqrt{1-\dfrac{(k_1+k_1k_2)^2}{4Mk_1k_2k_3k_4}}}$$

$$= \frac{(k_1+k_1k_2)\pi}{\left(4Mk_1k_2k_3k_4-(k_1+k_1k_2)^2\right)^{1/2}}$$

$$\Rightarrow O.S. = \exp\left[\frac{-(k_1+k_1k_2)\pi}{\left(4Mk_1k_2k_3k_4-(k_1+k_1k_2)^2\right)^{1/2}}\right]$$

$a := k_1+k_1k_2 \qquad b := 4Mk_1k_2$

$$\Rightarrow O.S. = \exp\left(\frac{-a\pi}{\sqrt{bk_3k_4-a^2}}\right)$$

$\propto$:= proportional , $\propto^{-1}$:= inversely proportional

$$\Rightarrow O.S. \propto^{-1} \frac{a\pi}{\sqrt{bk_3k_4-a^2}} \propto^{-1} bk_3k_4-a^2$$

$$\Rightarrow k_3 k_4 \gg \frac{(k_1 + k_1 k_2)^2}{4 M k_1 k_2} : \text{O.S.} \longrightarrow 1$$

Increasing the product of k_3 and k_4 increases O.S. $\Rightarrow$ decreasing $k_3 k_4$ decreases O.S.