(11) How does closing the loop in Procedure 1.5 affect settling time and time-const.?

Open-loop time const.: $\tau_o = \dfrac{1}{aT}$

Closed-loop time const.: $\tau_c = \dfrac{1}{T(a+bk_p)}$

$$\boxed{\Rightarrow \frac{\tau_c}{\tau_o} = \frac{a}{a+bk_p} \Rightarrow \begin{array}{ll} \tau_c < \tau_o & \text{if } k_p > 0 \\ \tau_c > \tau_o & \text{if } k_p < 0 \end{array}}$$

$$Y(s) = G(s)R(s) = G(s)U(s) = \frac{\tau b T k_p}{s(\tau s+1)},$$

$$\tau = T^{-1}(a+bk_p)^{-1}$$

$$\Rightarrow = \frac{\tau b k_p T}{s} - \frac{\tau b k_p T}{s+\tau^{-1}}$$

$$\Rightarrow y(t) = \tau b k_p T\left(1(t) - e^{-\frac{t}{\tau}}\right), \; t>0$$

$$y(t) = \frac{bk_p}{a+bk_p}(0.98) \Rightarrow \frac{0.98}{a+bk_p} = \tau T\left(1-e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \frac{0.98}{\tau T(a+bk_p)} = 1 - e^{-t/\tau}$$

$$\Rightarrow 0.02 = e^{-\frac{t}{\tau}} \Rightarrow t = -\tau \ln(0.02)$$

$$\Rightarrow \boxed{t = \frac{-\ln(0.02)}{T(a+bk_p)}}$$

Open-loop settling time: $t_o = -\dfrac{\ln(0.02)}{aT}$

Closed-loop settling time: $t_c = \dfrac{-\ln(0.02)}{T(a+bk_p)}$

$$\Rightarrow \quad \frac{t_c}{t_o} = \frac{a}{a+bk_p} = \frac{1}{1+\frac{b}{a}k_p}$$

$$\Rightarrow \quad t_c < t_o \quad \text{if} \quad k_p > 0$$

$$t_c > t_o \quad \text{if} \quad k_p < 0$$