

(5) Find formula for τ in terms of a, b, T

$$P(s) = \frac{k}{\tau s + 1}, \quad k \text{ is DC Gain}$$

$$P(s) = \frac{bT}{s + aT} = \frac{\frac{bT}{aT}}{\frac{1}{aT}s + 1}$$

$$= \frac{b/a}{\frac{1}{aT}s + 1} \Rightarrow K = \frac{b}{a}, \quad \tau = \frac{1}{aT}$$

(6) Find formula for 2% settling time of $P(s)$ in terms of a, b, T

$$Y(s) = P(s)U(s) = \frac{bT}{s(s+aT)} = \frac{b}{as} - \frac{b}{a(s+aT)}$$

$$\Rightarrow y(t) = \frac{b}{a} 1(t) - \frac{b}{a} e^{-aTt}$$

$$y(t) = 0.98 \frac{b}{a} \Rightarrow 0.98 = 1(t) - e^{-aTt}$$

$$\Rightarrow 0.98 = 1 - e^{-aTt}, \quad t > 0$$

$$\Rightarrow \ln(0.02) = -aTt \Rightarrow t = -\frac{\ln(0.02)}{aT}$$

$$\doteq \frac{3.91}{aT}$$